Magnetic properties of hot hadronic medium

Rupam Samanta

in collaboration with Wojciech Broniowski

[based on PRC 112 (2025) 4,045202 and arXiv:2511.19255]

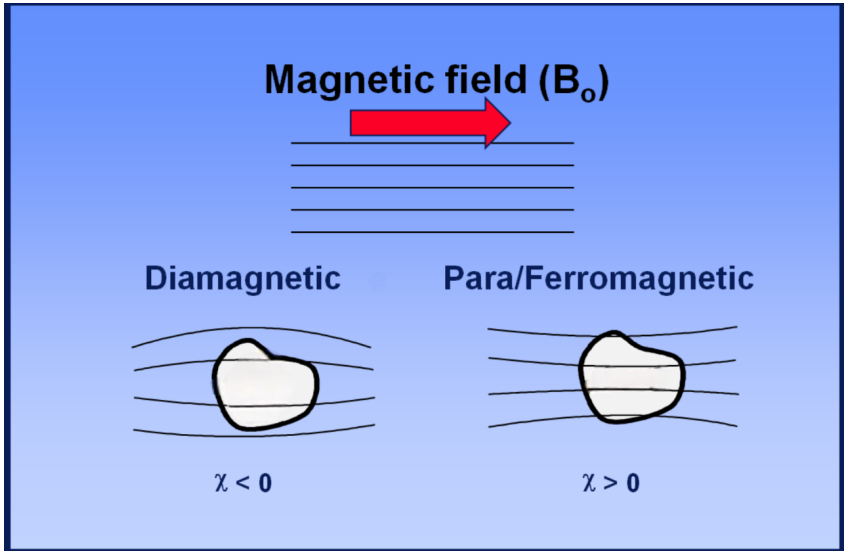
Institute of Nuclear Physics, PAS, Krakow

Magnetic Susceptibility

- **Magnetic response of a medium to the applied magnetic field**,

$$\chi_B = \left.\frac{\partial \mathcal{M}}{\partial B}\right|_{B=0}$$

$\mathcal{M} \rightarrow$ Magnetization of the system



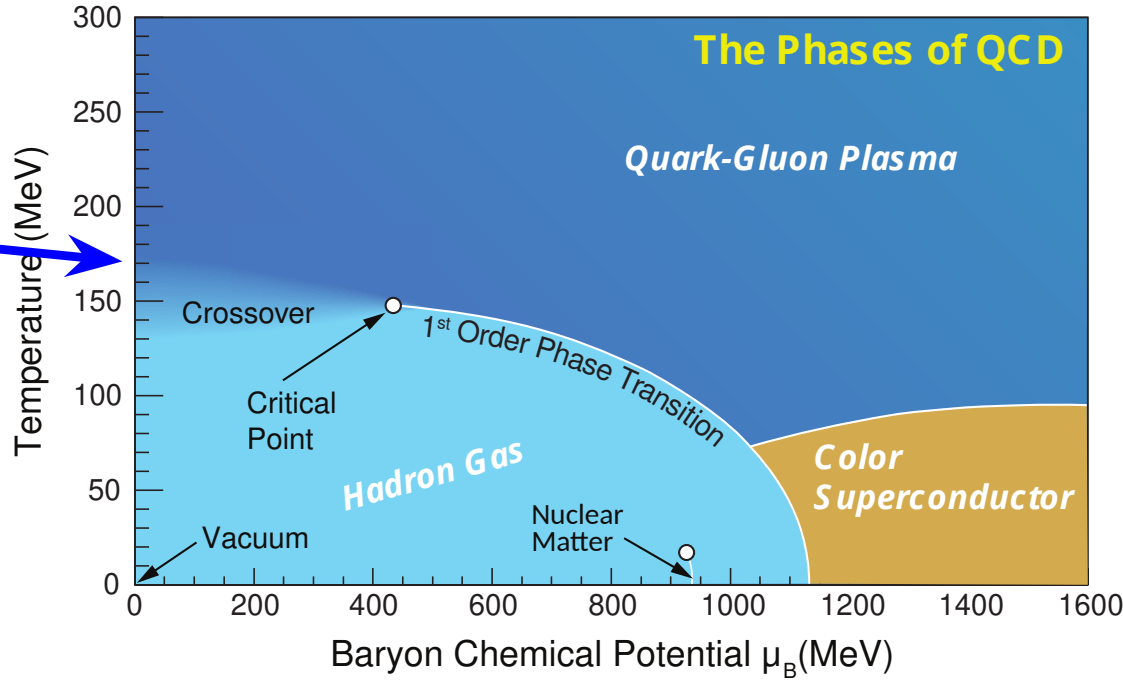
Thermodynamic system under magnetic field

- The grand potential: $\Omega = U - TS - B\mathcal{M} - \mu_{\rm ch} N = -P\,V$.
- Magnetization and magnetic susceptibility :

$$\mathcal{M} = \frac{\partial P}{\partial B} \quad \text{and} \quad \chi_B = \left.\frac{\partial^2 P}{\partial B^2}\right|_{B=0}$$

Our goal

- At LHC and RHIC highest energy, $\mu_{\mathcal{B}} \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV

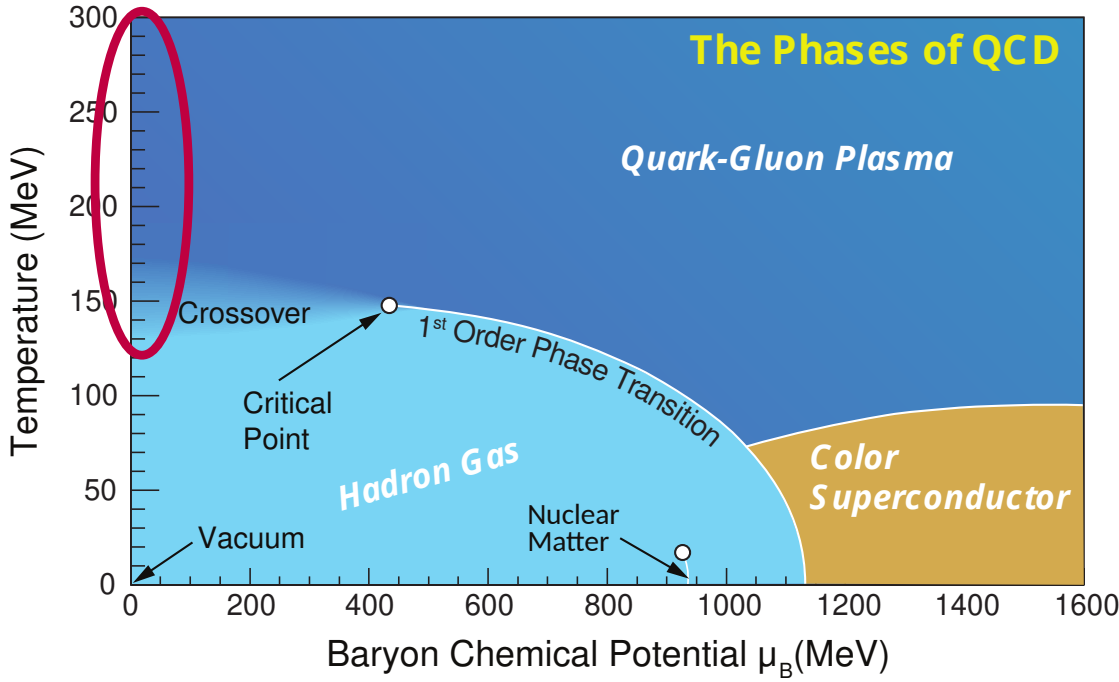


Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

Outline

Our goal

- At LHC and RHIC highest energy, $\mu_{\mathcal{B}} \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV
- **For $T \gtrsim 120$ MeV: lattice QCD (first principle description)**

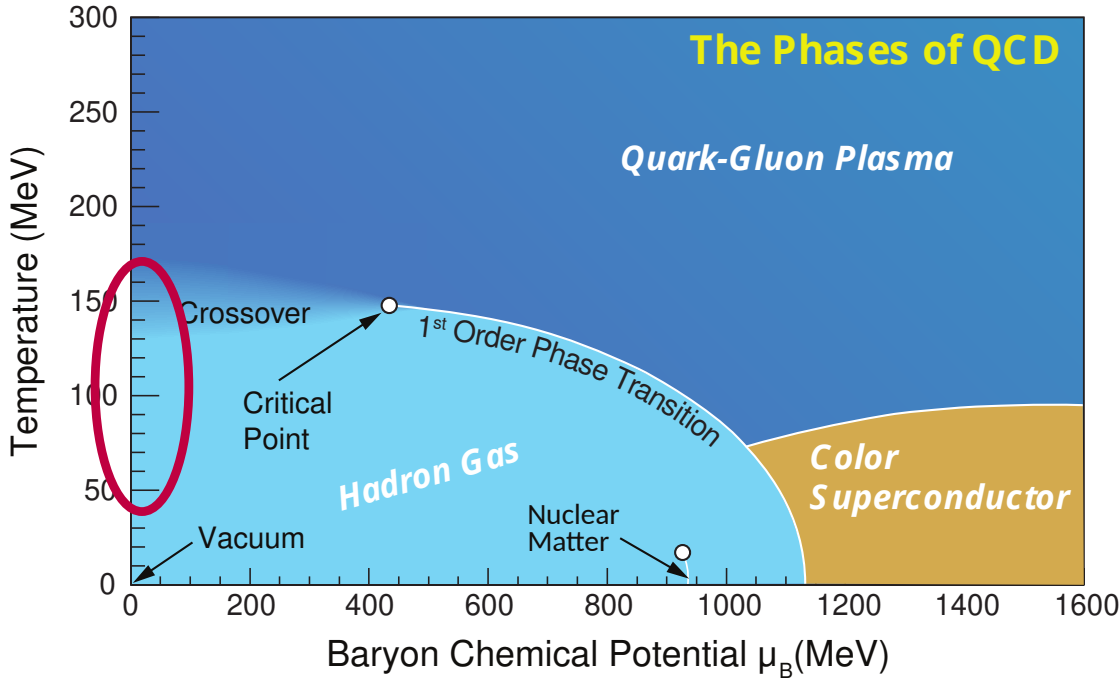


Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

Outline

Our goal

- At LHC and RHIC highest energy, $\mu_{\mathcal{B}} \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV
- **For $T \gtrsim 120$ MeV: lattice QCD (first principle description)**
- Below T_c: Hadron Resonance Gas (HRG) $\longrightarrow$ successfully describes numerous lattice data

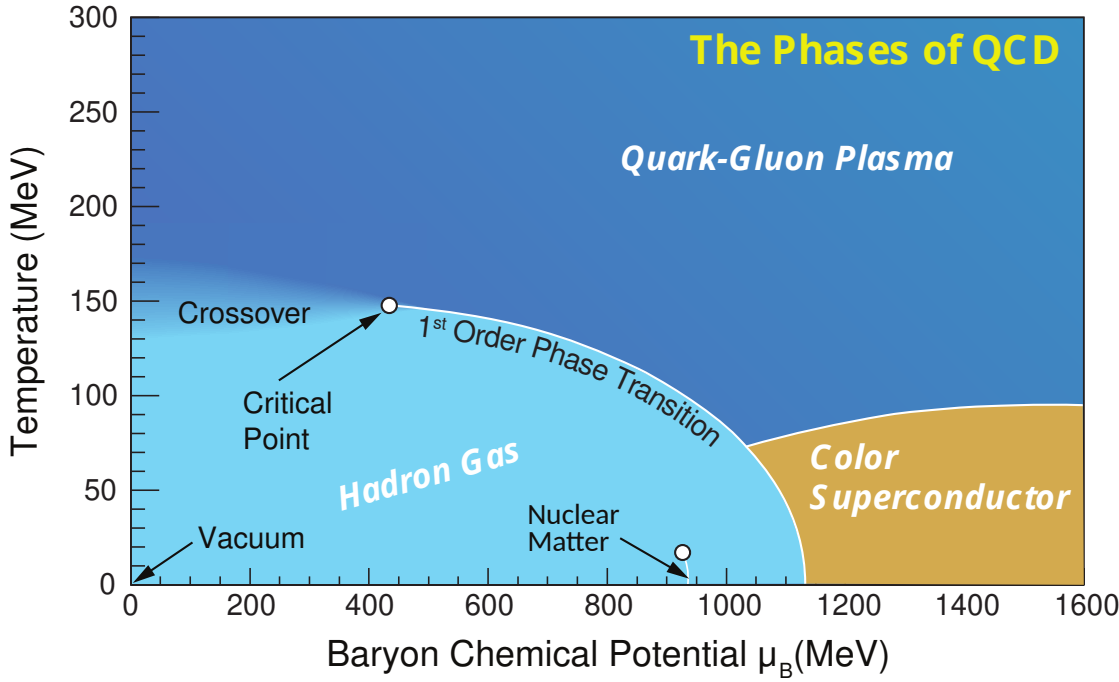


Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

Outline

Our goal

- At LHC and RHIC highest energy, $\mu_{\mathcal{B}} \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV
- **For $T \gtrsim 120$ MeV: lattice QCD (first principle description)**
- Below T_c: Hadron Resonance Gas (HRG) $\longrightarrow$ successfully describes numerous lattice data



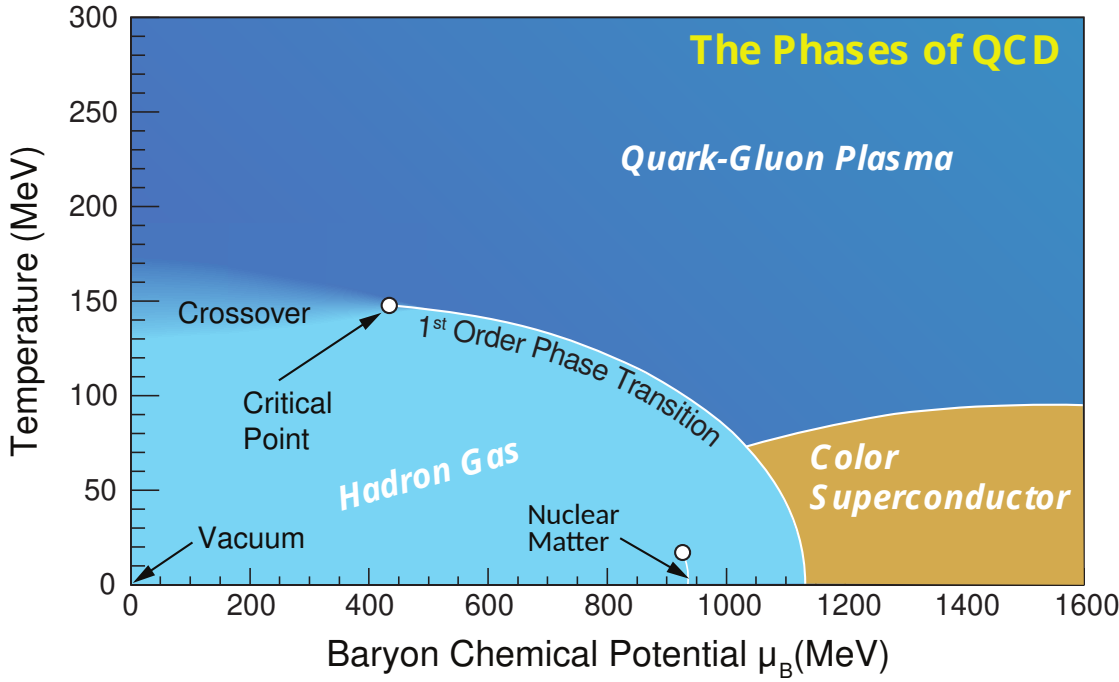
Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

Outline

- Can HRG simultaneously describe lattice data for conserved charge susceptibility (χ_{BB}, χ_{BS}) and magnetic susceptibility (χ_B) ?

Our goal

- At LHC and RHIC highest energy, $\mu_{\mathcal{B}} \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV
- **For $T \gtrsim 120$ MeV: lattice QCD (first principle description)**
- Below T_c: Hadron Resonance Gas (HRG) $\longrightarrow$ successfully describes numerous lattice data



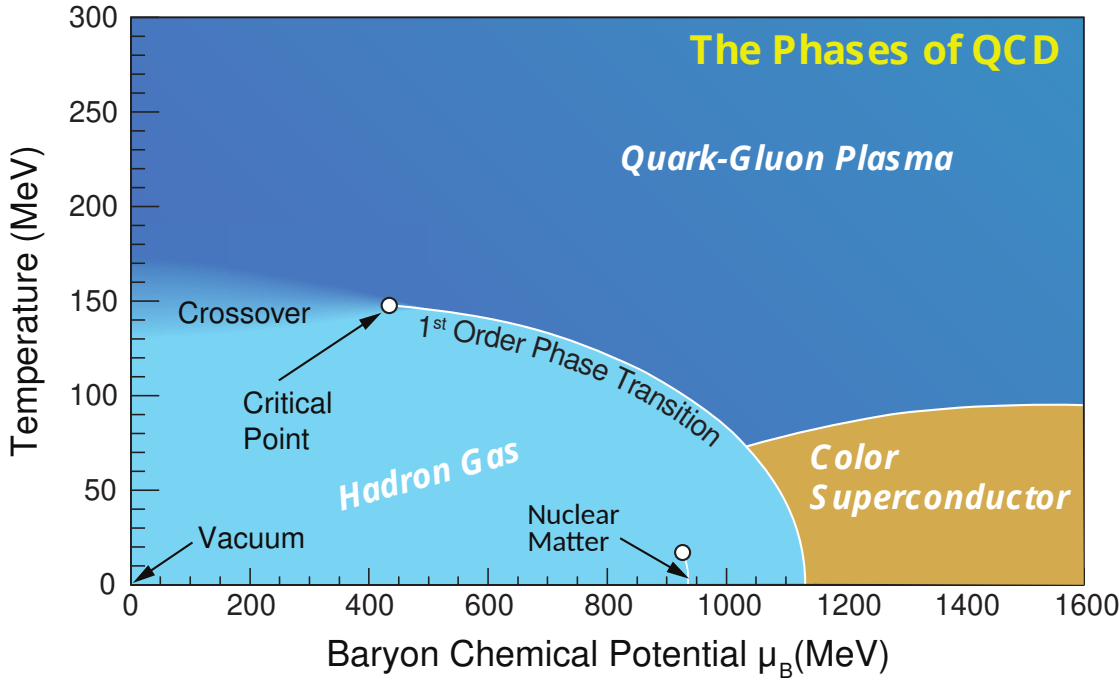
Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

Outline

- Can HRG simultaneously describe lattice data for conserved charge susceptibility (χ_{BB}, χ_{BS}) and magnetic susceptibility (χ_B) ?
- We propose a quark-meson framework to address the issue.

Our goal

- At LHC and RHIC highest energy, $\mu_{\mathcal{B}} \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV
- **For $T \gtrsim 120$ MeV: lattice QCD (first principle description)**
- Below T_c: Hadron Resonance Gas (HRG) $\longrightarrow$ successfully describes numerous lattice data



Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

Outline

- Can HRG simultaneously describe lattice data for conserved charge susceptibility (χ_{BB}, χ_{BS}) and magnetic susceptibility (χ_B) ?
- We propose a quark-meson framework to address the issue.
- Importance of vacuum contribution and anomalous magnetic moment.

Charged particle in a magnetic field

- Relativistic energy :

$$E = \left(M^2 + p_z^2 + \underbrace{B|Q|(2l+1)}_{\text{Landau diamagnetism}} \underbrace{-2QBs_z}_{\text{Pauli paramagnetism}} \right)^{1/2}$$

Charged particle in a magnetic field

- Relativistic energy :

$$E = \left(M^2 + p_z^2 + \underbrace{B|Q|(2l+1)}_{\text{Landau diamagnetism}} \underbrace{-2QBs_z}_{\text{Pauli paramagnetism}} \right)^{1/2}$$

- Thermal pressure:

$$P^{\rm th} = -\eta T \frac{B|Q|}{2\pi^2} \sum_{l=0}^{\infty} \sum_{s_z} \int_0^{\infty} dp_z \log[1 - \eta\, f]$$

where $f(E(B,p_z),\mu_{ch},T) = \frac{1}{\exp\left(\frac{E(B,p_z)-\mu_{ch}}{T}\right)+\eta}$, $\quad \mu_{ch} = \mu_{\mathcal{B}}\mathcal{B} + \mu_{\mathcal{S}}\mathcal{S} + \mu_{\mathcal{Q}}\mathcal{Q}$

η=+ 1 (fermion) or -1 (boson)

Charged particle in a magnetic field

- Relativistic energy :

$$E = \left(M^2 + p_z^2 + \underbrace{B|Q|(2l+1)}_{\text{Landau diamagnetism}} \underbrace{-2QBs_z}_{\text{Pauli paramagnetism}} \right)^{1/2}$$

- Thermal pressure:

$$P^{\rm th} = -\eta T \frac{B|Q|}{2\pi^2} \sum_{l=0}^{\infty} \sum_{s_z} \int_0^{\infty} dp_z \log[1 - \eta\, f]$$

where $f(E(B,p_z),\mu_{ch},T) = \frac{1}{\exp\left(\frac{E(B,p_z)-\mu_{ch}}{T}\right)+\eta}$, $\quad \mu_{ch} = \mu_{\mathcal{B}}\mathcal{B} + \mu_{\mathcal{S}}\mathcal{S} + \mu_{\mathcal{Q}}\mathcal{Q}$

η=+ 1 (fermion) or -1 (boson)

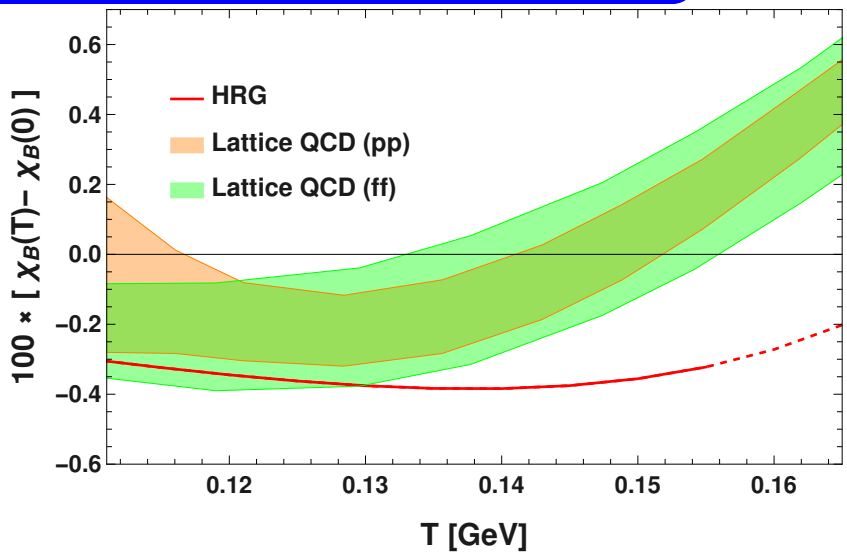
- Magnetic susceptibility:

$$\chi_B^{\rm th} = \frac{\partial^2 P^{\rm th}}{\partial B^2}|_{B=0} = \begin{cases} + & \text{if } s \neq 0 : \text{Paramagnetic} \quad \text{e.g } p, \rho, \Delta \\ - & \text{if } s = 0\text{: Diamagnetic} \quad \text{e.g. } \pi, K \end{cases}$$

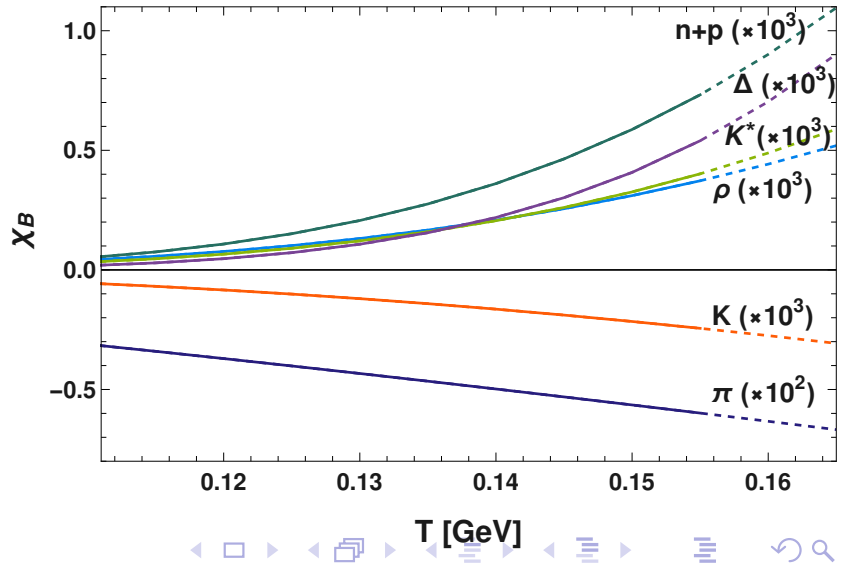
Magnetic Susceptibility (χ_B): HRG vs lattice QCD

- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$



Bali *et al.* (2020), JHEP; Brandt *et al.* (2024), JHEP

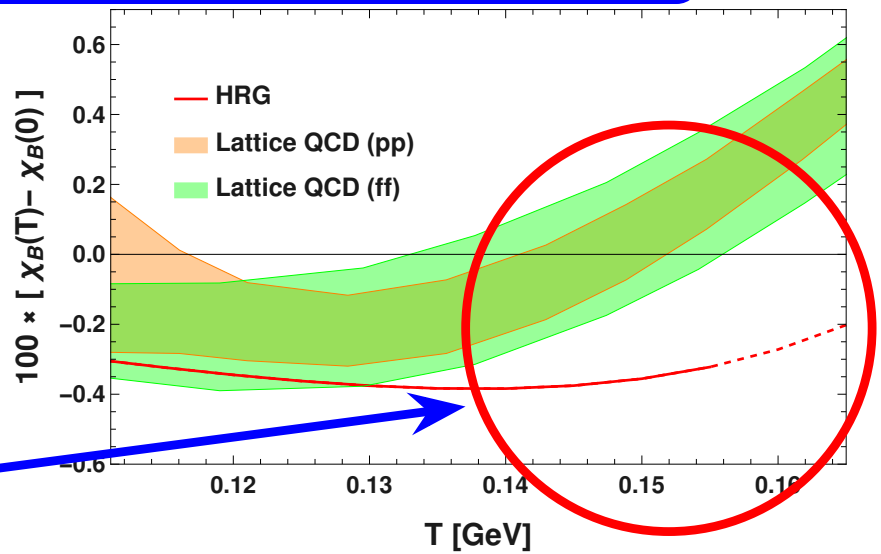


Magnetic Susceptibility (χ_B): HRG vs lattice QCD

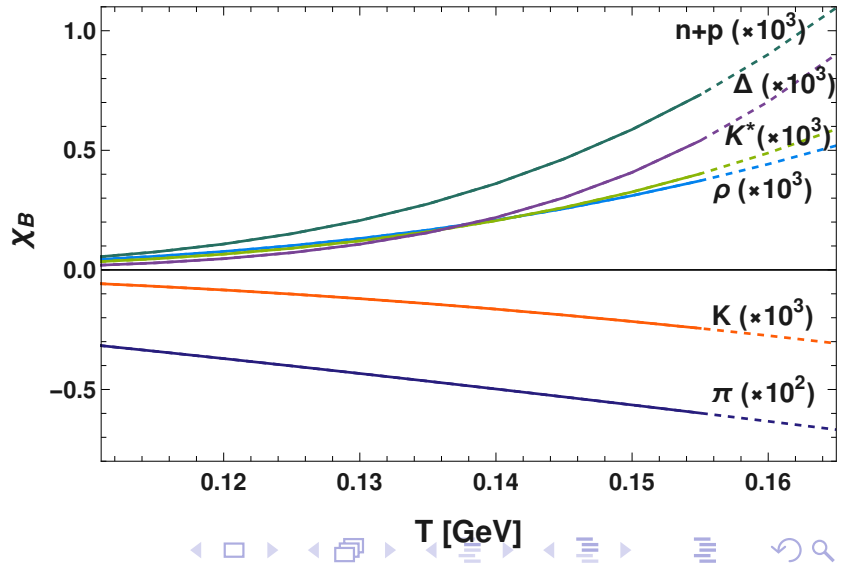
- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$

- HRG fails to describe the lattice χ_B (low-B feature !!)



Bali *et al.* (2020), JHEP; Brandt *et al.* (2024), JHEP

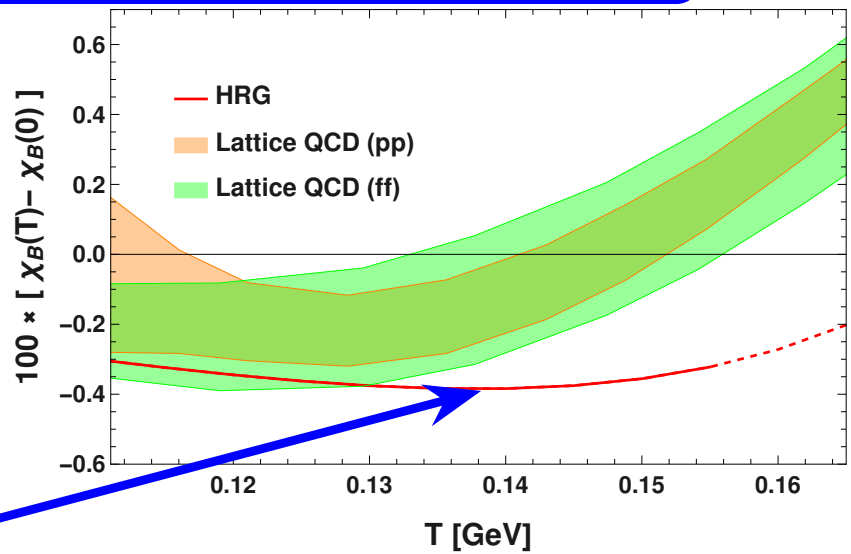


Magnetic Susceptibility (χ_B): HRG vs lattice QCD

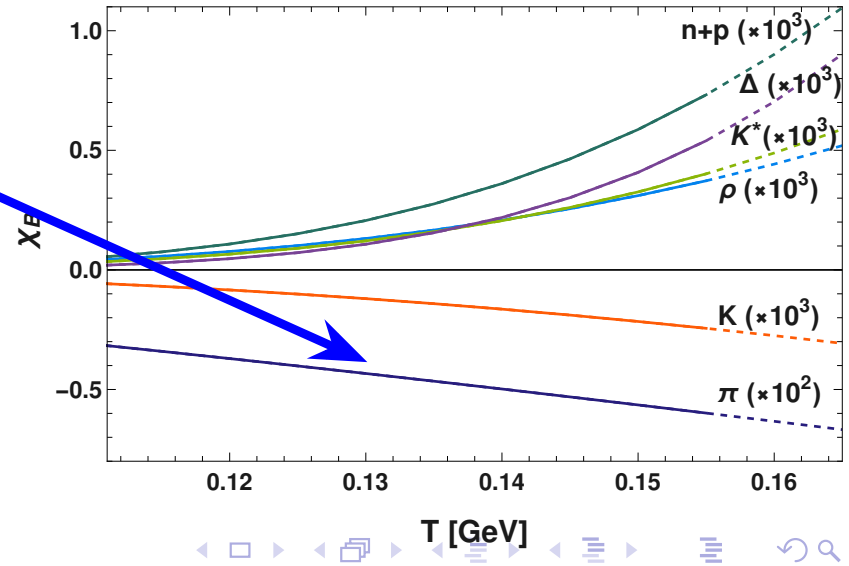
- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$

- HRG fails to describe the lattice χ_B (low-B feature !!)
- HRG is too diamagnetic $\longrightarrow$ (dominated by pion's contribution)



Bali *et al.* (2020), JHEP; Brandt *et al.* (2024), JHEP

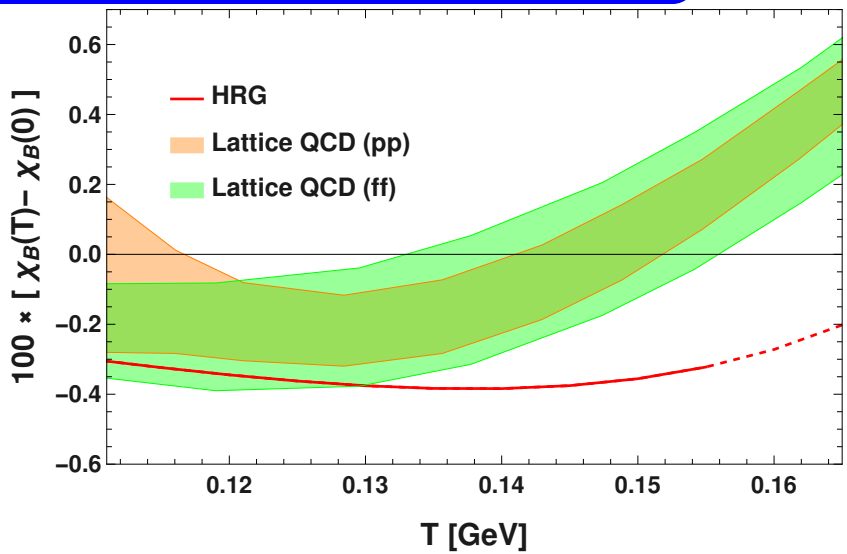


Magnetic Susceptibility (χ_B): HRG vs lattice QCD

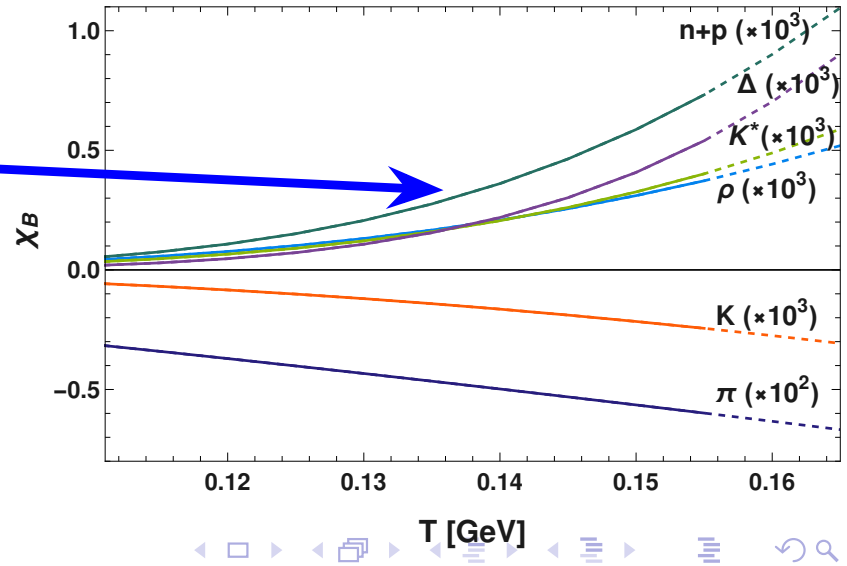
- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$

- HRG fails to describe the lattice χ_B (low-B feature !!)
- HRG is too diamagnetic $\longrightarrow$ (dominated by pion's contribution)
- States are thermally suppressed ($\sim e^{-M/T}$) $\longrightarrow$ paramagnetic contributions (heavy baryons) are not enough.



Bali *et al.* (2020), JHEP; Brandt *et al.* (2024), JHEP

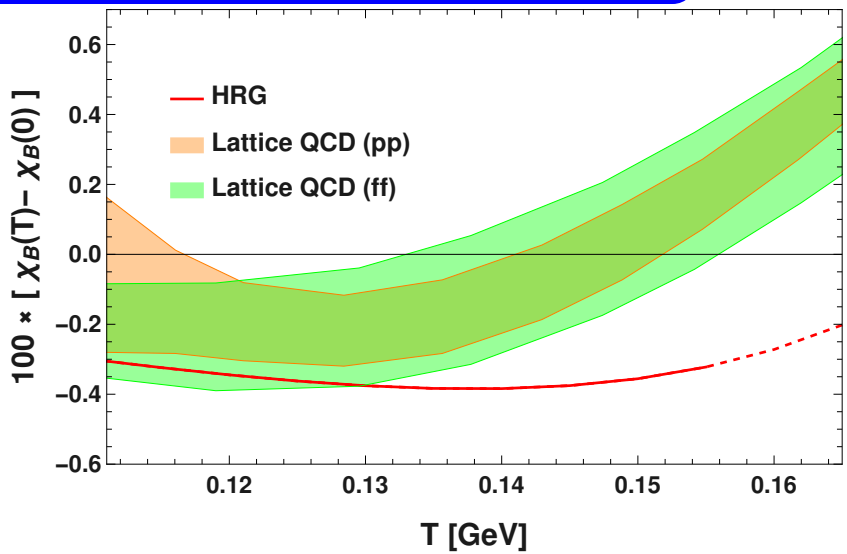


Magnetic Susceptibility (χ_B): HRG vs lattice QCD

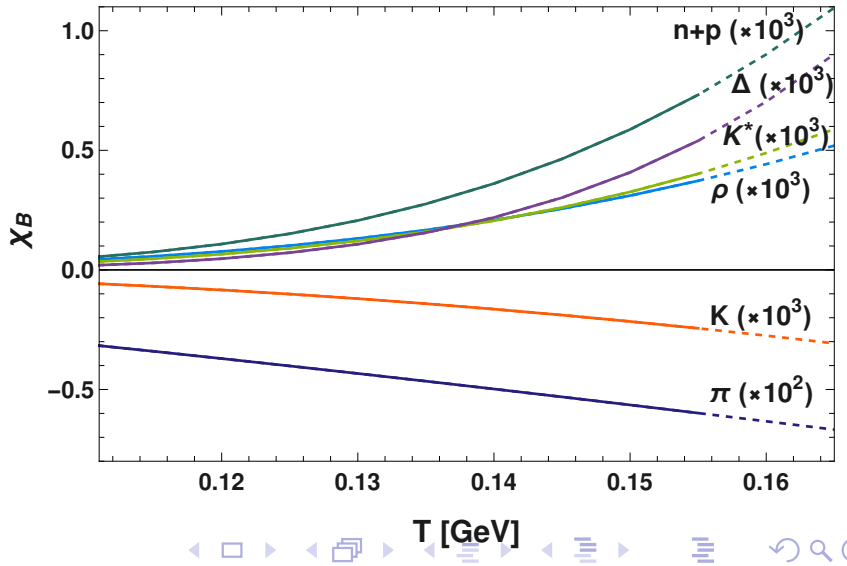
- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$

- HRG fails to describe the lattice χ_B (low-B feature !!)
- HRG is too diamagnetic $\longrightarrow$ (dominated by pion's contribution)
- States are thermally suppressed ($\sim e^{-M/T}$) $\longrightarrow$ paramagnetic contributions (heavy baryons) are not enough.
- **Genuine need for light paramagnetic source (e.g. quarks?)**

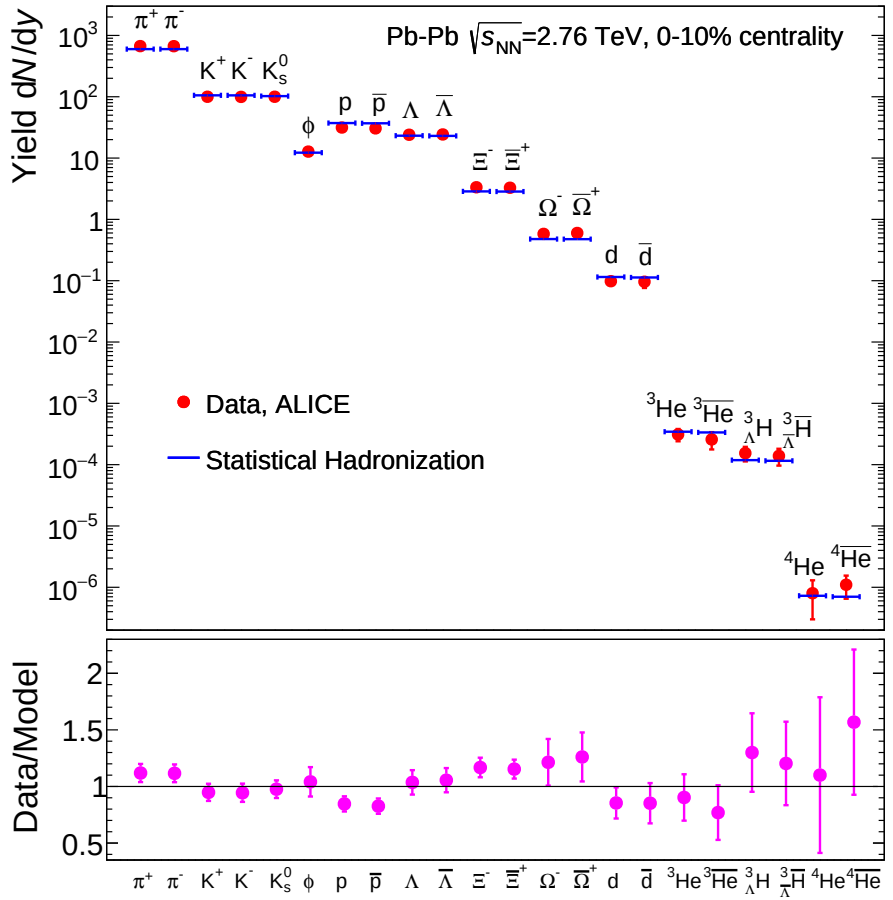


Bali *et al.* (2020), JHEP; Brandt *et al.* (2024), JHEP



Successes of HRG

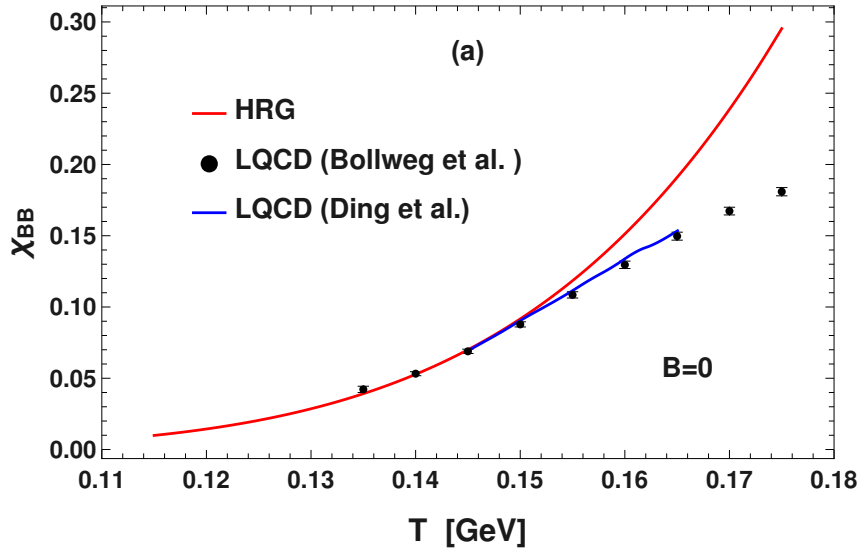
- Remarkably successful to reproduce particle multiplicity seen in heavy-ion collision



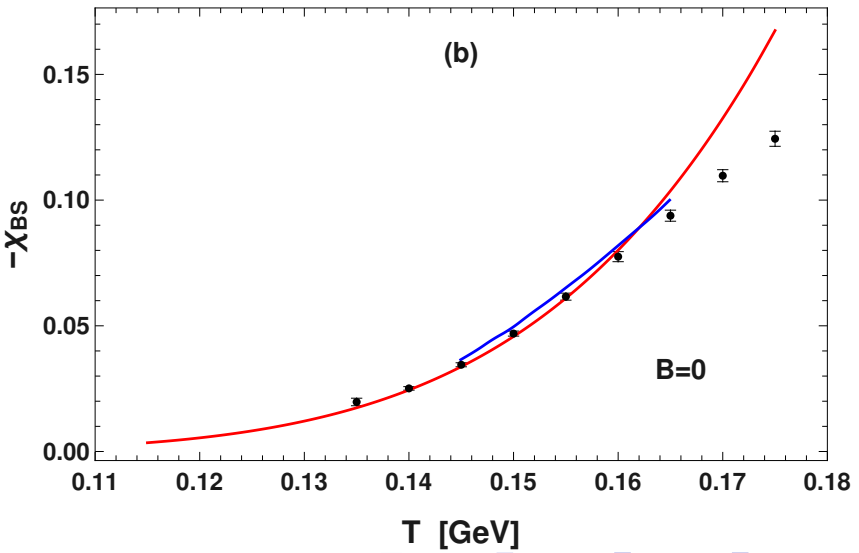
Andronic *et al.* (2018), Nature

Successes of HRG

- Remarkably successful to reproduce particle multiplicity seen in heavy-ion collision
- Successfully describes lattice data for χ_{QQ} at $B = 0$

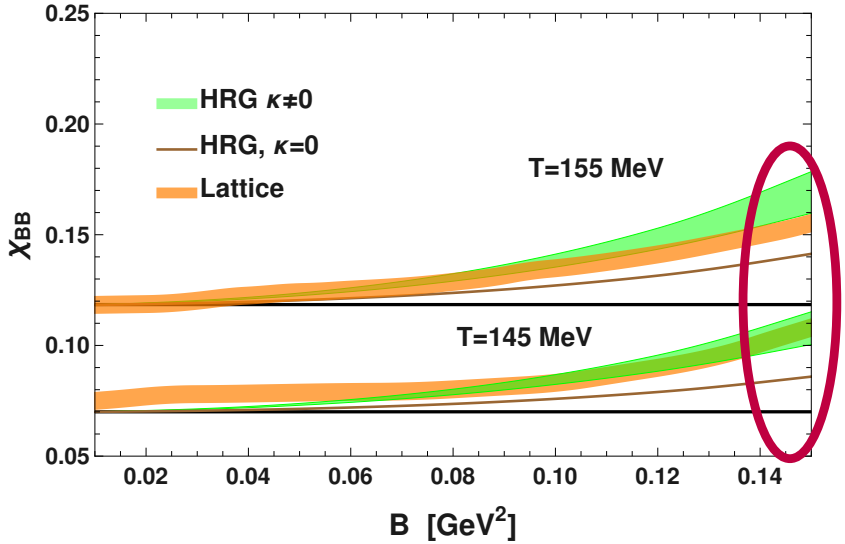


Bollweg *et al.* (HotQCD) (2021), PRD

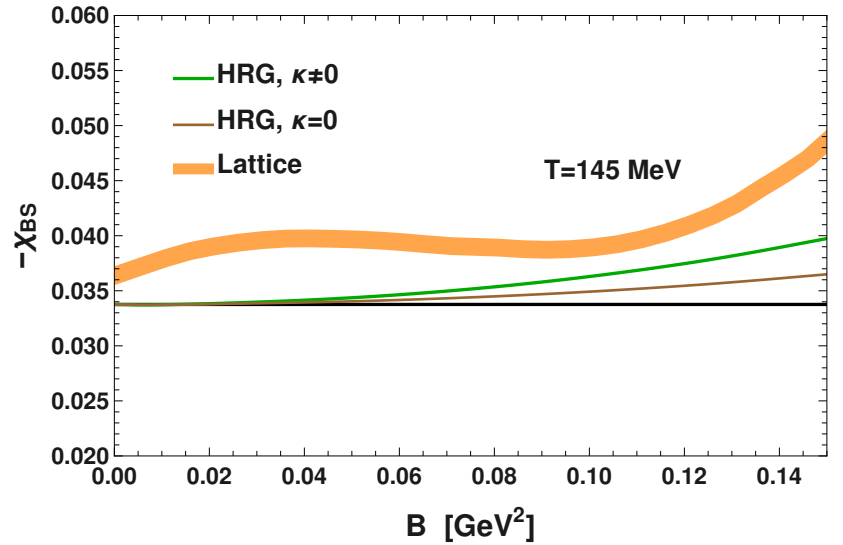


Successes of HRG

- Remarkably successful to reproduce particle multiplicity seen in heavy-ion collision
- Successfully describes lattice data for χ_{QQ} at $B = 0$
- Describes χ_{QQ} in presence of B (need hadron anomalous magnetic moment κ, RS *et al.* (2025), PRC)

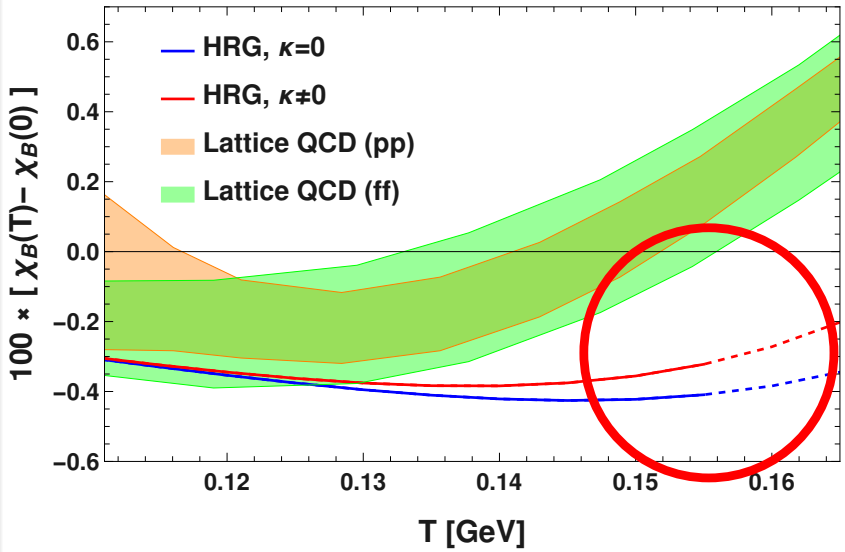


RS *et al.* (2025), PRC; Ding *et al.* (2024), PRL



Successes of HRG

- Remarkably successful to reproduce particle multiplicity seen in heavy-ion collision
- Successfully describes lattice data for χ_{QQ} at $B = 0$
- Describes χ_{QQ} in presence of B (need hadron anomalous magnetic moment κ, RS *et al.* (2025), PRC)
- Non-zero κ of hadrons are not sufficient for χ_B



We address this failure as a genuine problem

Importance of vacuum pressure

- The total pressure can be decomposed:

$$P = \underbrace{P^{\mathrm{vac}}(B;m)}_{\text{Vacuum part}} + \underbrace{P^{\mathrm{th}}(T,\mu,B;m)}_{\text{Thermal part}}$$

Importance of vacuum pressure

- The total pressure can be decomposed:

$$P = \underbrace{P^{\rm vac}(B;m)}_{\text{Vacuum part}} + \underbrace{P^{\rm th}(T,\mu,B;m)}_{\text{Thermal part}}$$

- The vacuum pressure:

$$P^{\rm vac}(B;m) = -\eta \frac{B|Q|}{4\pi^2} \sum_{l=0}^{\infty} \sum_{s_z} \int_0^{\infty} dp_z E(p_z,B) \quad \textbf{(Divergent !)}$$

Importance of vacuum pressure

- The total pressure can be decomposed:

$$P = \underbrace{P^{\rm vac}(B;m)}_{\text{Vacuum part}} + \underbrace{P^{\rm th}(T,\mu,B;m)}_{\text{Thermal part}}$$

- The vacuum pressure:

$$P^{\rm vac}(B;m) = -\eta\frac{B|Q|}{4\pi^2}\sum_{l=0}^{\infty}\sum_{s_z}\int_0^{\infty} dp_z E(p_z,B) \quad \textbf{(Divergent !)}$$

- χ_{QQ} depends only on the thermal pressure ($P^{\rm th}$)

$$\chi_{\mathcal{B}\mathcal{B}} = \frac{1}{T^2}\left.\frac{\partial^2 P^{\rm th}}{\partial\mu_{\mathcal{B}}^2}\right|_{\mu_{\mathcal{B}}=0} \quad \text{and} \quad \chi_{\mathcal{B}S} = \frac{1}{T^2}\left.\frac{\partial^2 P^{\rm th}}{\partial\mu_{\mathcal{B}}\partial\mu_S}\right|_{\mu_{\mathcal{B}},S=0}$$

Importance of vacuum pressure

- The total pressure can be decomposed:

$$P = \underbrace{P^{\rm vac}(B;m)}_{\text{Vacuum part}} + \underbrace{P^{\rm th}(T,\mu,B;m)}_{\text{Thermal part}}$$

- The vacuum pressure:

$$P^{\rm vac}(B;m) = -\eta\frac{B|Q|}{4\pi^2}\sum_{l=0}^{\infty}\sum_{s_z}\int_0^{\infty} dp_z E(p_z,B) \quad \textbf{(Divergent !)}$$

- χ_{QQ} depends only on the thermal pressure ($P^{\rm th}$)

$$\chi_{\mathcal{B}\mathcal{B}} = \frac{1}{T^2}\left.\frac{\partial^2 P^{\rm th}}{\partial\mu_{\mathcal{B}}^2}\right|_{\mu_{\mathcal{B}}=0} \quad \text{and} \quad \chi_{\mathcal{B}S} = \frac{1}{T^2}\left.\frac{\partial^2 P^{\rm th}}{\partial\mu_{\mathcal{B}}\partial\mu_S}\right|_{\mu_{\mathcal{B}},S=0}$$

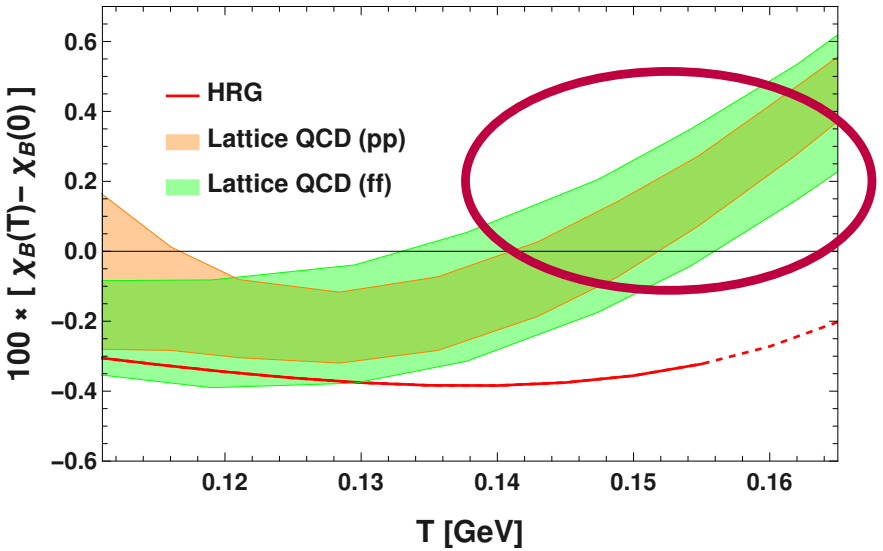
- Explicit B-dependence of $P^{\rm vac}$:

$$\chi_B^{\rm vac} \neq 0$$

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks

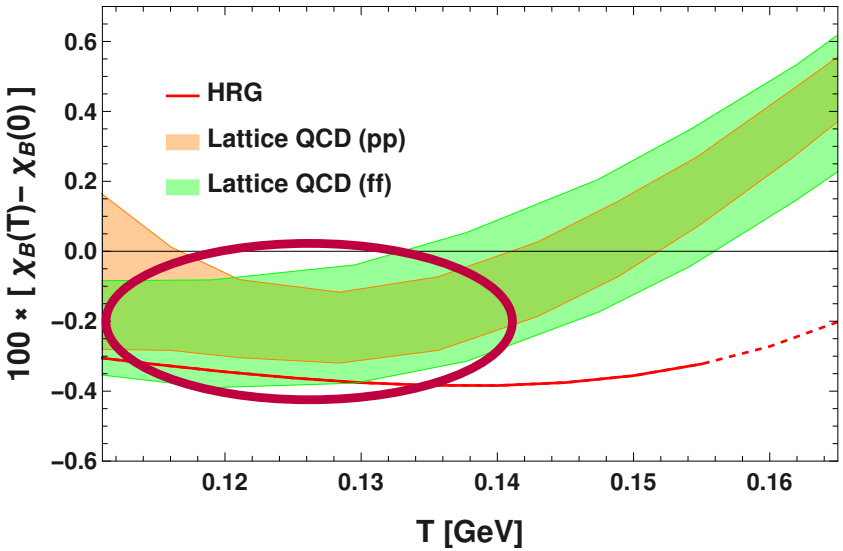


Temp-dependent quark masses

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons

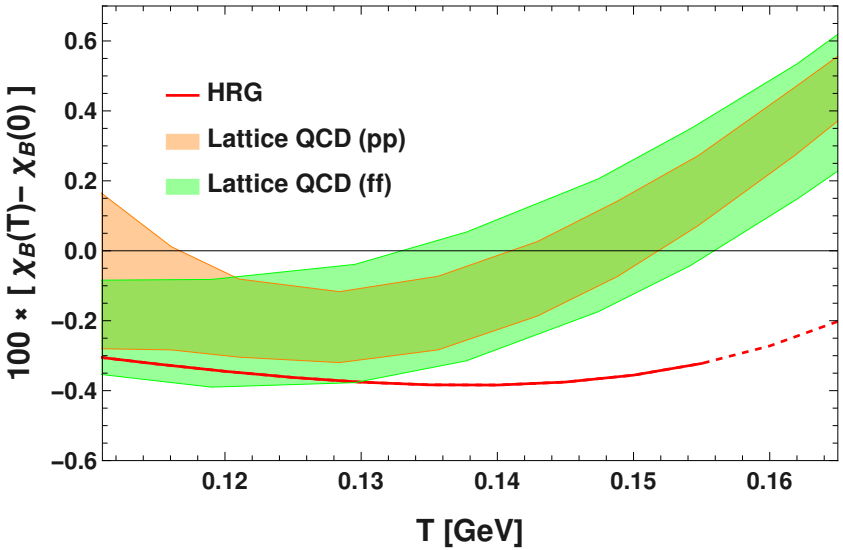


Temp-dependent quark masses

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons
- Alternative framework: **non-interacting quark-meson model with vacuum terms**

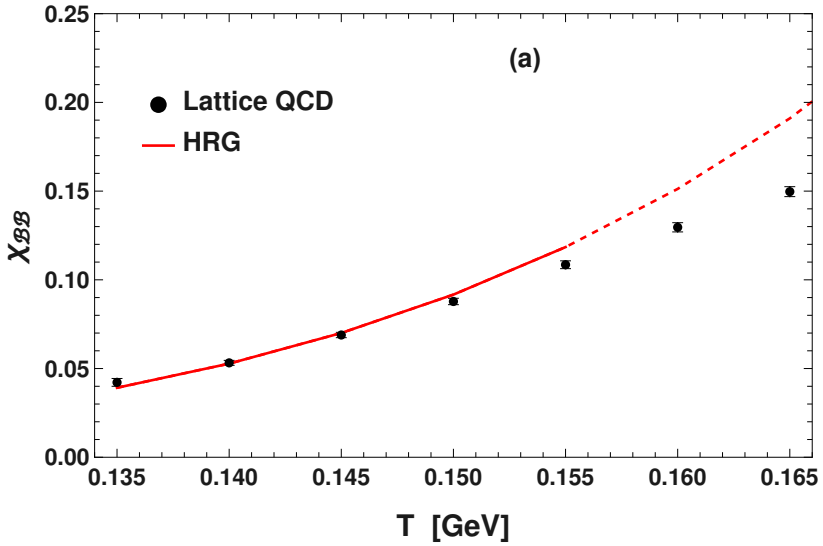


Temp-dependent quark masses

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons
- Alternative framework: **non-interacting quark-meson model with vacuum terms**
- Consistency requirement: simultaneous description of $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B

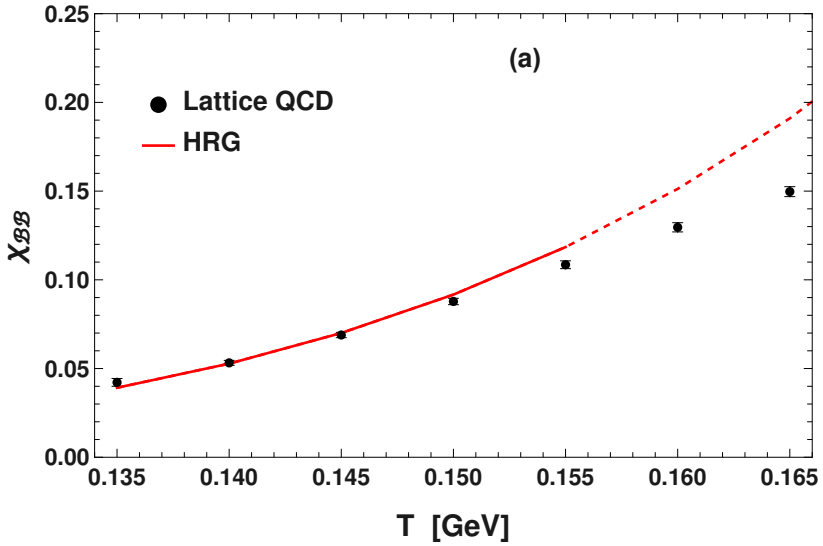


Temp-dependent quark masses

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons
- Alternative framework: **non-interacting quark-meson model with vacuum terms**
- Consistency requirement: simultaneous description of $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B
- We consider three quark flavors (u, d, s) with colors $(N_c = 3)$ and π, K

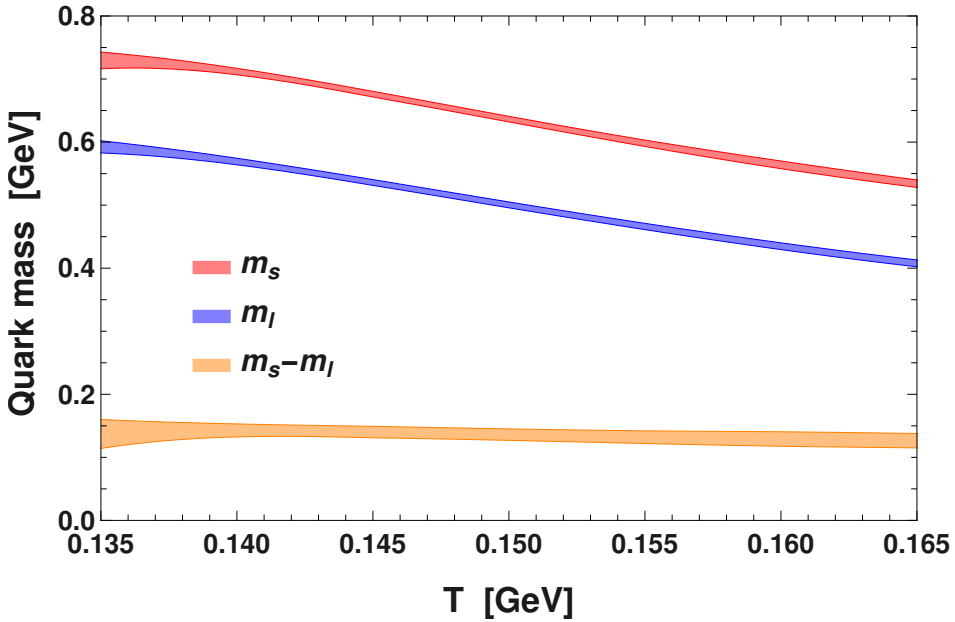


Temp-dependent quark masses

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons
- Alternative framework: **non-interacting quark-meson model with vacuum terms**
- Consistency requirement: simultaneous description of $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B
- We consider three quark flavors (u, d, s) with colors $(N_c = 3)$ and π, K



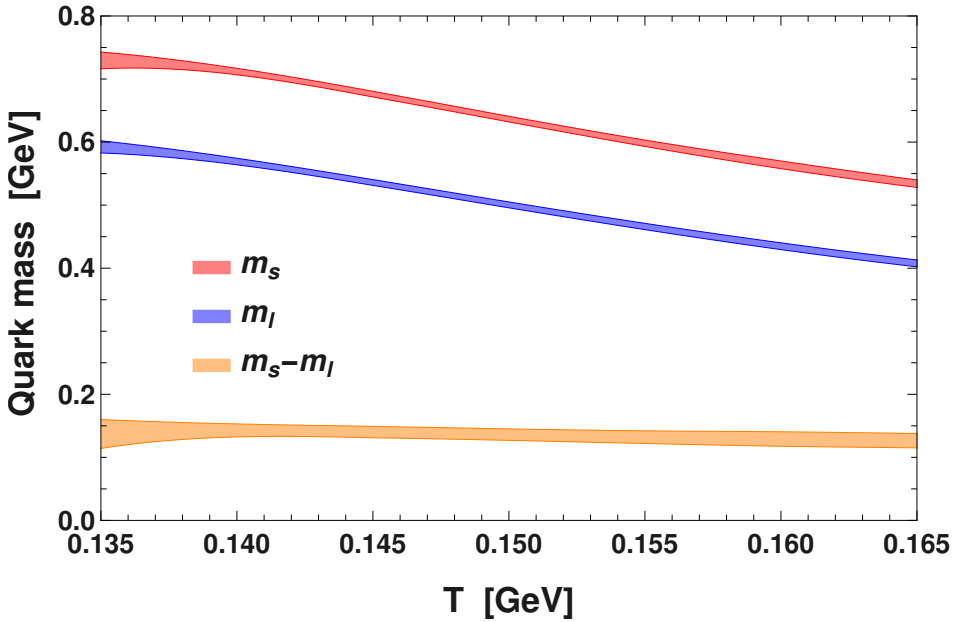
Temp-dependent quark masses

- We fit quark masses($m_u = m_d \equiv m_l$) to $\chi_{\mathcal{BB}}$ and $\chi_{\mathcal{BS}}$ lattice data at $B = 0$

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons
- Alternative framework: **non-interacting quark-meson model with vacuum terms**
- Consistency requirement: simultaneous description of $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B
- We consider three quark flavors (u, d, s) with colors $(N_c = 3)$ and π, K



Temp-dependent quark masses

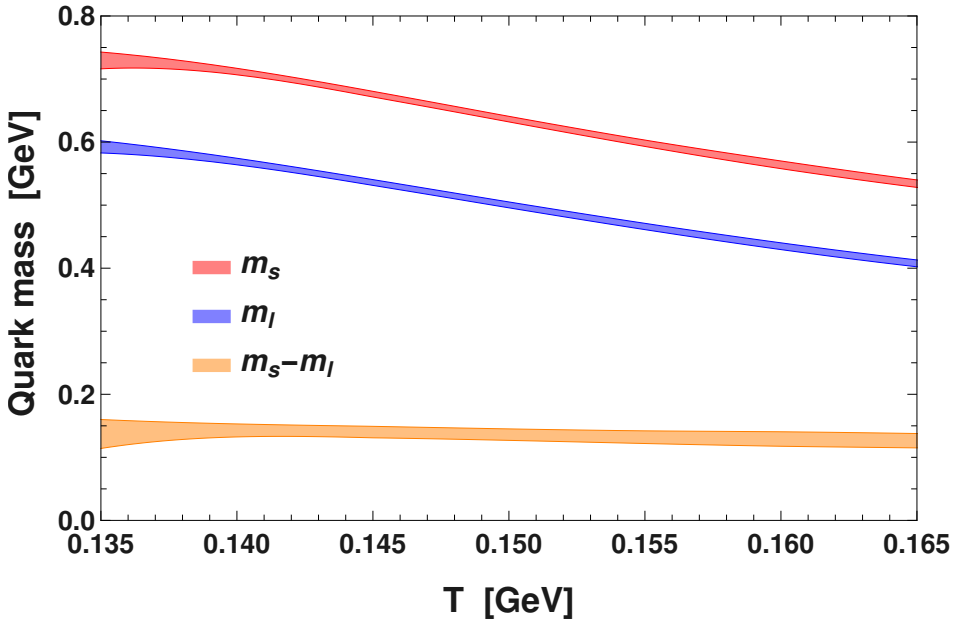
- We fit quark masses($m_u = m_d \equiv m_l$) to $\chi_{\mathcal{BB}}$ and $\chi_{\mathcal{BS}}$ lattice data at $B = 0$
- **Fitted quark masses are large** $\longrightarrow$ consistent with quasiparticle picture,

  Mykhaylova et al. (2019), PRD

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks
- We also need diamagnetism at low T : scalar mesons
- Alternative framework: **non-interacting quark-meson model with vacuum terms**
- Consistency requirement: simultaneous description of $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B
- We consider three quark flavors (u, d, s) with colors $(N_c = 3)$ and π, K



Temp-dependent quark masses

- We fit quark masses($m_u = m_d \equiv m_l$) to $\chi_{\mathcal{BB}}$ and $\chi_{\mathcal{BS}}$ lattice data at $B = 0$
- **Fitted quark masses are large** $\longrightarrow$ consistent with quasiparticle picture, Mykhaylova et al. (2019), PRD
- $m_s - m_l$ is roughly constant ~ 120 MeV

The vacuum contributions of quarks and mesons

Endrodi *et al.* (2013), JHEP; Kamikado *et al.* (2015), JHEP

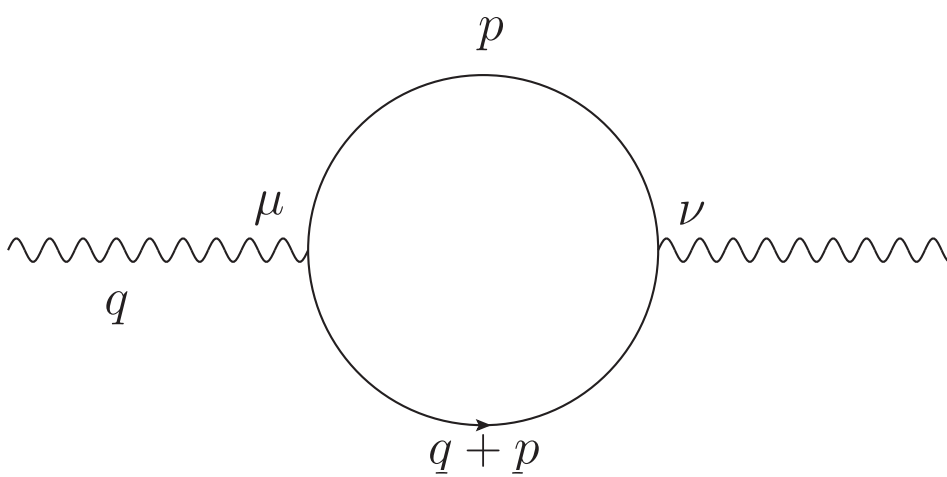
- Can be calculated from photon vacuum polarization involving a quark/pion loop ($\Pi^{\mu\nu}$).

$$\chi_B^{\rm vac} = \frac{1}{2} \lim_{\vec{q}\to 0} \lim_{q_0 \to 0} \frac{\partial^2 {\rm Re}[\Pi_s^{\rm vac}]}{\partial q^2}$$

where $\Pi_s^{\rm vac} = \frac{1}{2} \sum_{i=1}^{3} \Pi_{ii}^{\rm vac}$

Coupling with anomalous magnetic moment (κ)

$$\Gamma^\mu = \gamma^\mu + \frac{\kappa}{2m} \sigma^{\mu\nu} q^\nu$$



Photon vacuum polarization (quark loop)

Endrodi *et al.* (2022), JHEP

The vacuum contributions of quarks and mesons

Endrodi *et al.* (2013), JHEP; Kamikado *et al.* (2015), JHEP

- Can be calculated from photon vacuum polarization involving a quark/pion loop ($\Pi^{\mu\nu}$).

$$\chi_B^{\rm vac} = \frac{1}{2} \lim_{\vec{q}\to 0} \lim_{q_0 \to 0} \frac{\partial^2 {\rm Re}[\Pi_s^{\rm vac}]}{\partial q^2}$$

where $\Pi_s^{\rm vac} = \frac{1}{2} \sum_{i=1}^{3} \Pi_{ii}^{\rm vac}$

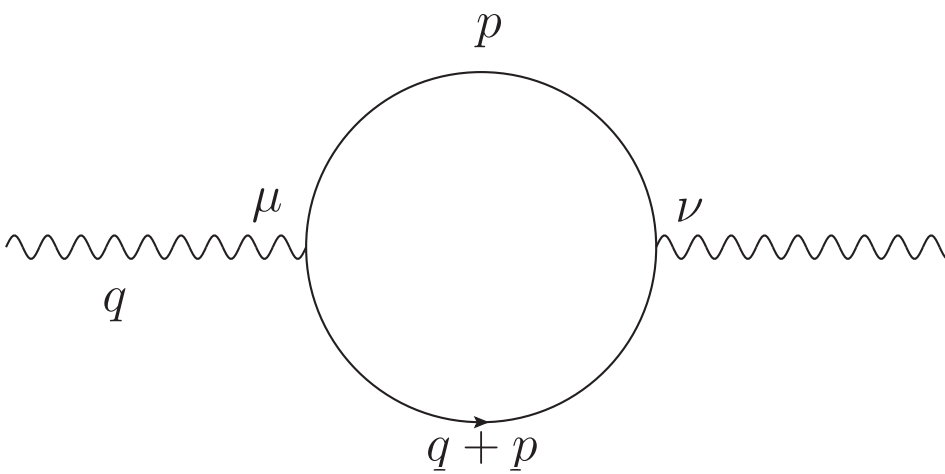
- After regularizing the divergence:

$$\chi_B^{\rm vac}(T) - \chi_B^{\rm vac}(0) = f(m(T), m(0), \kappa, \Lambda)$$

$\Lambda \sim 800$ MeV $\longrightarrow$ regularization scale

Coupling with anomalous magnetic moment (κ)

$$\Gamma^\mu = \gamma^\mu + \frac{\kappa}{2m} \sigma^{\mu\nu} q^\nu$$

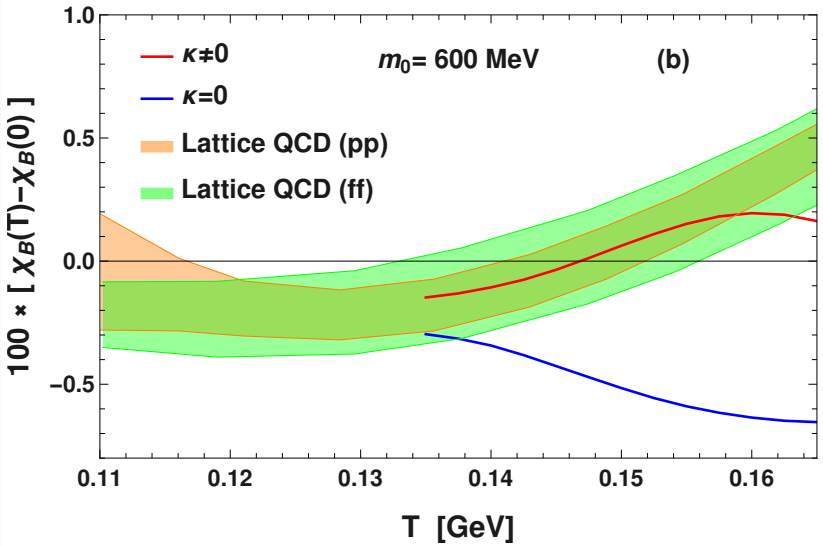


Photon vacuum polarization (quark loop)

Endrodi *et al.* (2022), JHEP

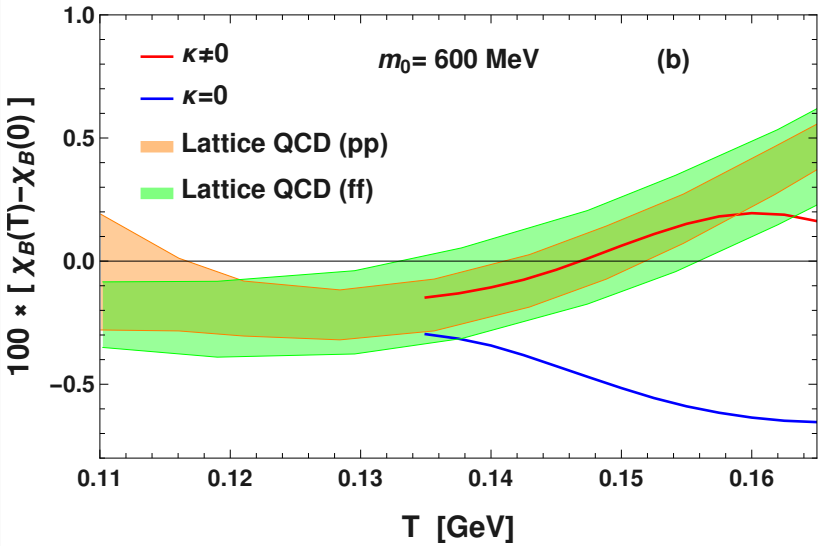
Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP



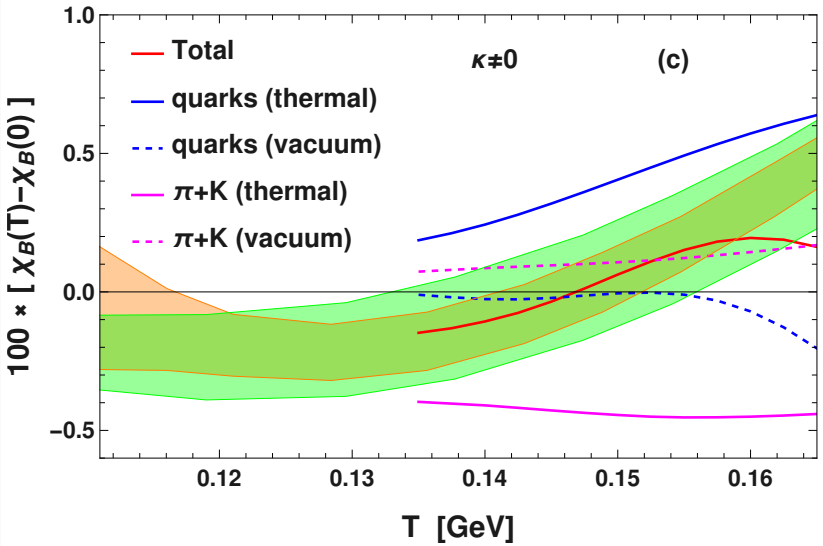
Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP
- Quark anomalous magnetic moment (κ) is essential



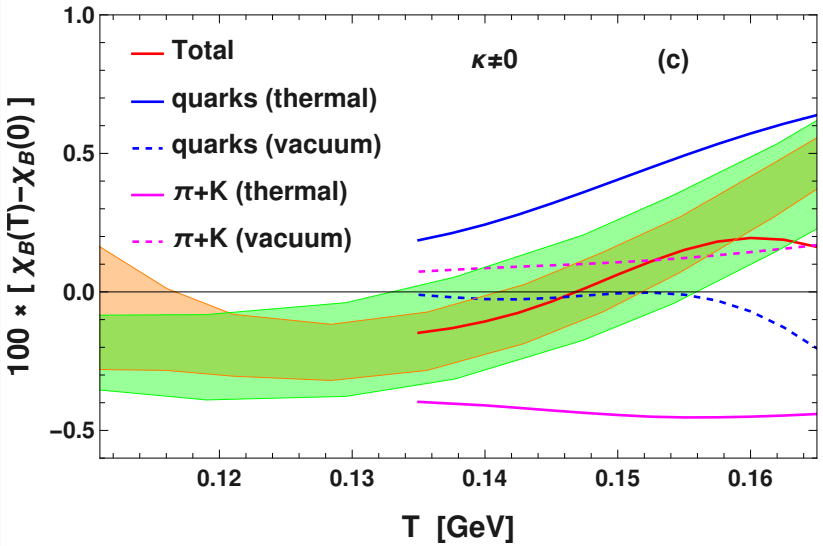
Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP
- Quark anomalous magnetic moment (κ) is essential
- $\chi_B^{\rm vac}$(mesons): paramagnetic (temperature dependence of mass flattens the thermal part)



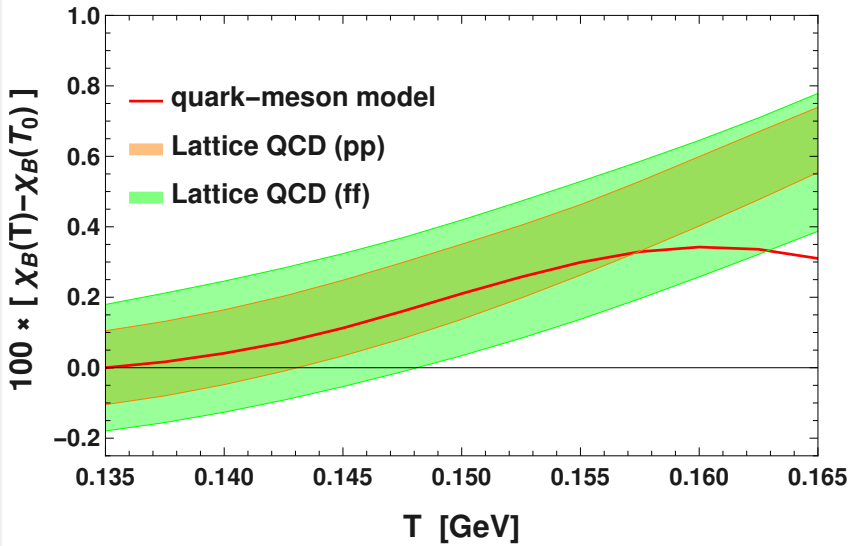
Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP
- Quark anomalous magnetic moment (κ) is essential
- $\chi_B^{\rm vac}$(mesons): paramagnetic (temperature dependence of mass flattens the thermal part)
- $\chi_B^{\rm vac}$(quarks): small, decreases sharply at large T.



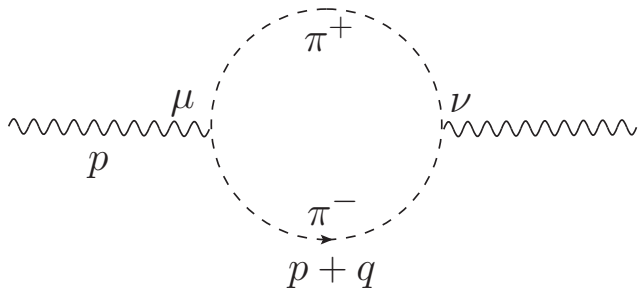
Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP
- Quark anomalous magnetic moment (κ) is essential
- $\chi_B^{\rm vac}$(mesons): paramagnetic (temperature dependence of mass flattens the thermal part)
- $\chi_B^{\rm vac}$(quarks): small, decreases sharply at large T.
- Accurate knowledge of $m_l(0)$ is missing $\longrightarrow$ **the model reproduces χ_B subtracted at $T_0 = 135$ MeV**

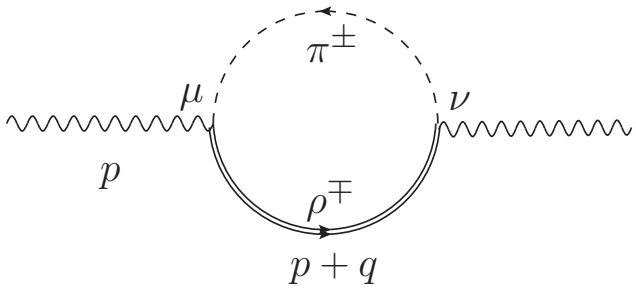


Contribution of π-vector meson loop

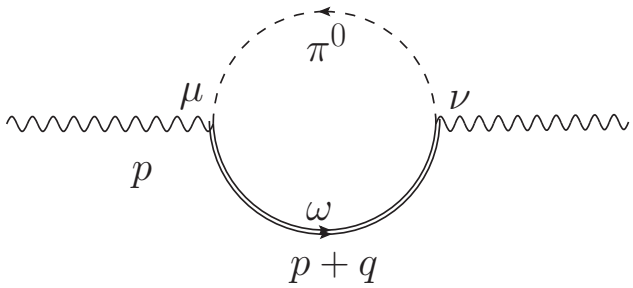
- In hadronic picture: pion-vector meson loop in $\Pi^{\mu\nu}$.



a. $\pi - \pi$ *loop*



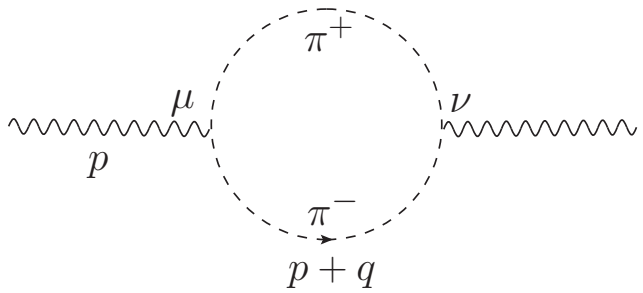
b. $\pi - \rho$ *loop*



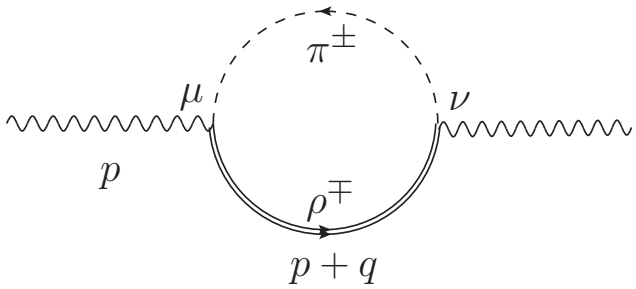
c. $\pi - \omega$ *loop*

Contribution of π-vector meson loop

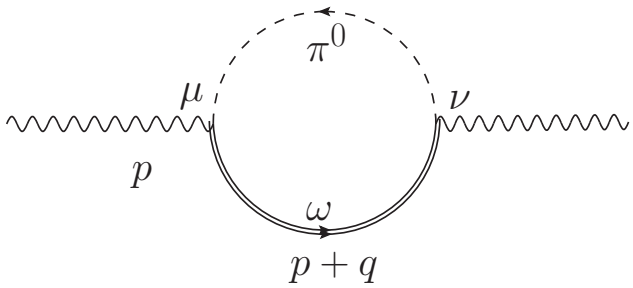
- In hadronic picture: pion-vector meson loop in $\Pi^{\mu\nu}$.
- $\pi - \rho$ loop contributes more than ρ meson : $\chi_B^{\pi-\rho} \gg \chi_B^{\rho-\rho}$



a. $\pi - \pi$ loop



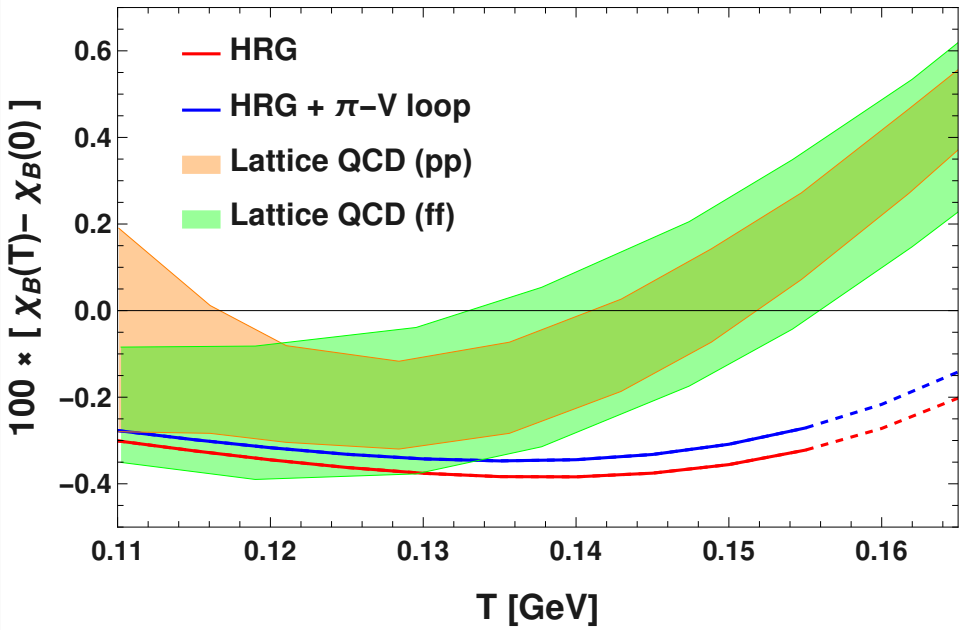
b. $\pi - \rho$ loop



c. $\pi - \omega$ loop

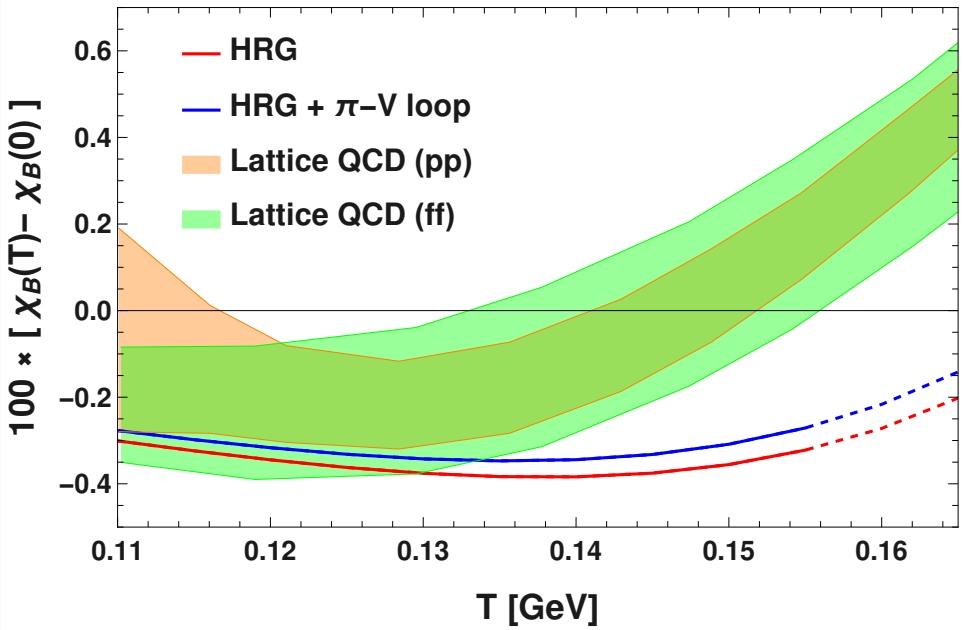
Contribution of π-vector meson loop

- In hadronic picture: pion-vector meson loop in $\Pi^{\mu\nu}$.
- $\pi - \rho$ loop contributes more than ρ meson : $\chi_B^{\pi-\rho} \gg \chi_B^{\rho-\rho}$
- **The effect is small and paramagnetic**



Contribution of π-vector meson loop

- In hadronic picture: pion-vector meson loop in $\Pi^{\mu\nu}$.
- $\pi - \rho$ loop contributes more than ρ meson : $\chi_B^{\pi-\rho} \gg \chi_B^{\rho-\rho}$
- **The effect is small and paramagnetic**
- In comparison to HRG , the contribution is at the level of 10-20 % $\longrightarrow$ comparable to two-loop χPT calculation.



Conclusions

- HRG cannot describe simultaneously the lattice data for $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B (a low-B feature) $\longrightarrow$ genuine need for light paramagnetic state below T_c

Conclusions

- HRG cannot describe simultaneously the lattice data for $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B (a low-B feature) $\longrightarrow$ genuine need for light paramagnetic state below T_c

- Non-interacting quark-meson framework $\longrightarrow$ larger constituent quark-mass (quasi-particle picture)

Conclusions

- HRG cannot describe simultaneously the lattice data for $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B (a low-B feature) $\longrightarrow$ genuine need for light paramagnetic state below T_c
- Non-interacting quark-meson framework $\longrightarrow$ larger constituent quark-mass (quasi-particle picture)
- **The vacuum contribution directly affects** χ_B $\longrightarrow$ need better modeling

Conclusions

- HRG cannot describe simultaneously the lattice data for $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B (a low-B feature) $\longrightarrow$ genuine need for light paramagnetic state below T_c
- Non-interacting quark-meson framework $\longrightarrow$ larger constituent quark-mass (quasi-particle picture)
- **The vacuum contribution directly affects** χ_B $\longrightarrow$ need better modeling
- **The quark-meson model can successfully describe lattice data for** χ_B **while simultaneously reproducing** $\chi_{\mathcal{BB}}$ **and** $\chi_{\mathcal{BS}}$

Thank you !

Backup

Self-consistency requirement

- Charge-conjugation symmetry $\longrightarrow$ particle and anti-particle have same energy:

$$P^{\rm th}(T,\mu,B;\{m\}) = P^{\rm th}(T,-\mu,B;\{m\})$$

- The self-consistency condition:

$$\frac{\partial P}{\partial m} = 0$$

- thus obtained mass must be a symmetric function of μ:

$$\frac{\partial m}{\partial \mu}\,|_{\mu=0} = 0$$

Energy spectra of hadrons in uniform B with κ

- For **spin-1/2** states ($g = 2Q + 2\kappa$): Exact [Tsai and Yildiz , Phys. Rev. D (1971)]

$$E_{ch} = \sqrt{\left(\sqrt{M^2 + \mathcal{B}|Q|(2l+1) - 2Q\mathcal{B}s_z)} - \mu_M \mathcal{B} 2\kappa s_z\right)^2 + p_z^2}$$

$$E_{neu} = \sqrt{\left(\sqrt{M^2 + p^2 - p_z^2} - \mu_M \mathcal{B} 2\kappa s_z\right)^2 + p_z^2}$$

- For **spin 1 and 3/2**($g = 2Q + 2\kappa$) : Good approximation [Ferrar, Phys. Rev. D (1992); Belinfante, Phys. Rev. (1953); Paoli, J. Phys. G (2013)]

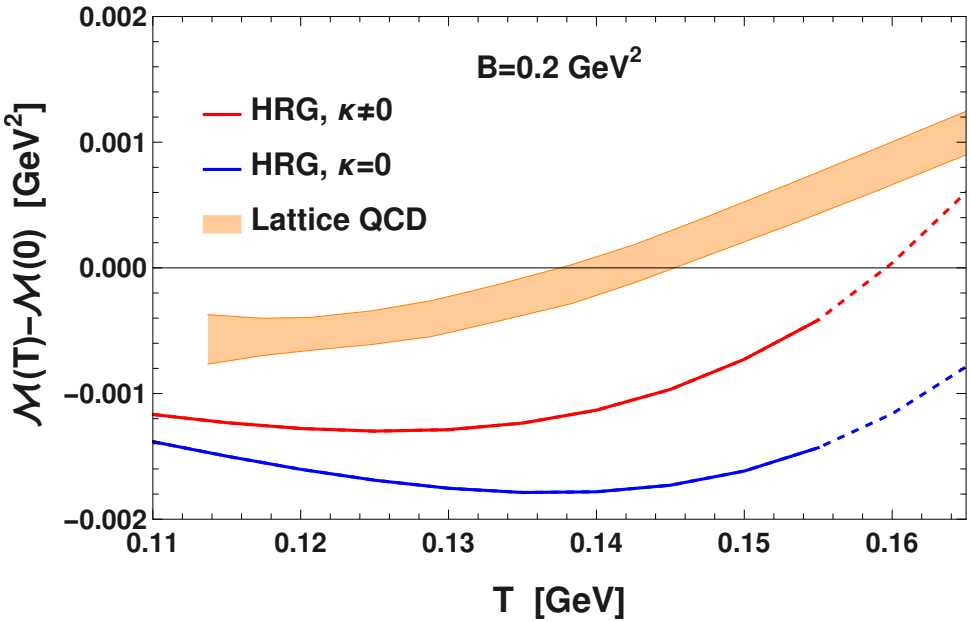
$$E_{ch} = \sqrt{M^2 + p_z^2 + \mathcal{B}|Q|(2l+1) - 2Q\mathcal{B}s_z} - \mu_M \mathcal{B} 2\kappa s_z \ ,$$

$$E_{neu} = \sqrt{M^2 + p^2} - \mu_M \mathcal{B} 2\kappa s_z$$

- For **spin** $>$ **3/2** ($g = 2Q + 2\kappa$): Approximation

$$E_{ch} = \sqrt{M^2 + p_z^2 + \mathcal{B}|Q|(2l+1)} - \mu_M \mathcal{B} g s_z \ , \ E_{neu} = \sqrt{M^2 + p^2} - \mu_M \mathcal{B} g s_z$$

Magnetization



Same problem !!

Pauli-Villars Regularization

- Vacuum contribution to quark:

$$\chi_B^{\rm vac} = R_0 + R_1\kappa + R_2\kappa^2$$

where , e.g. $R_0 = \frac{4}{3}Q^2\pi^2\left[(q^2+2m^2)B_0(q^2,m^2,m^2) - 2A_0(m^2)\right]$

- A_0 and B_0 are the one-loop Passarino-Veltman (PaVe) functions

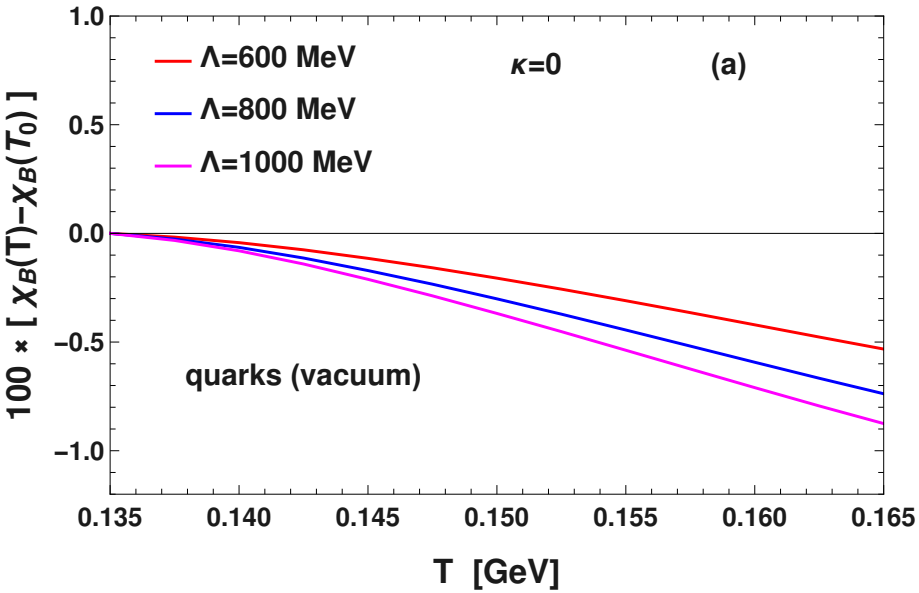
$$i\pi^2 A_0(m^2) = \int d^4p\, \frac{1}{p^2 - m^2 + i\epsilon}$$

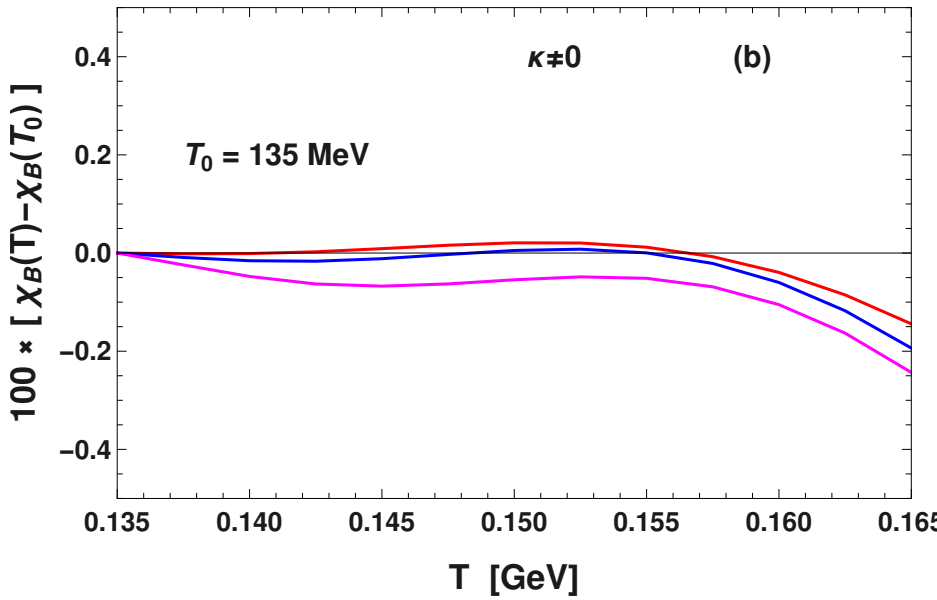
$$i\pi^2 B_0(q^2, m_1^2, m_2^2) = \int d^4p \left[\frac{1}{(p+q)^2 - m_1^2 + i\epsilon}\frac{1}{p^2 - m_2^2 + i\epsilon}\right]$$

- Replace $F(\{m^2\})$ with

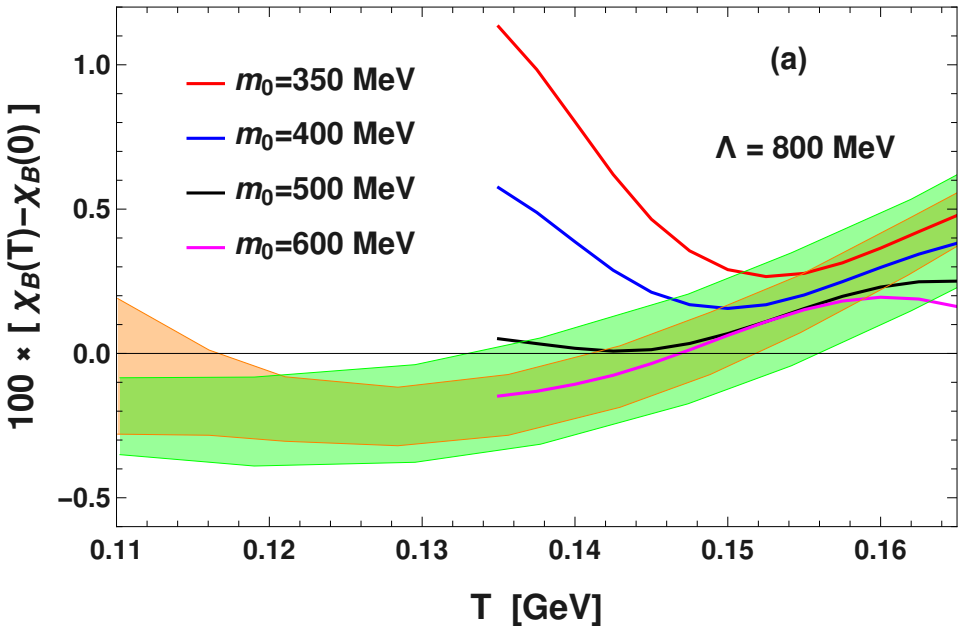
$$F^\Lambda(\{m^2\}) = F(\{m^2\}) - F(\{m^2+\Lambda^2\}) + \Lambda^2\frac{dF(\{M^2+\Lambda^2\})}{d\Lambda^2}$$

Effect of Λ





Fixing $m_q(T=0)$



We try several values of $m_l(0)$ ($m_s(0) = m_l(0) + 120$ MeV) and fix it to $m_l(0) = 600$ MeV $\longrightarrow$ best agreement with the data