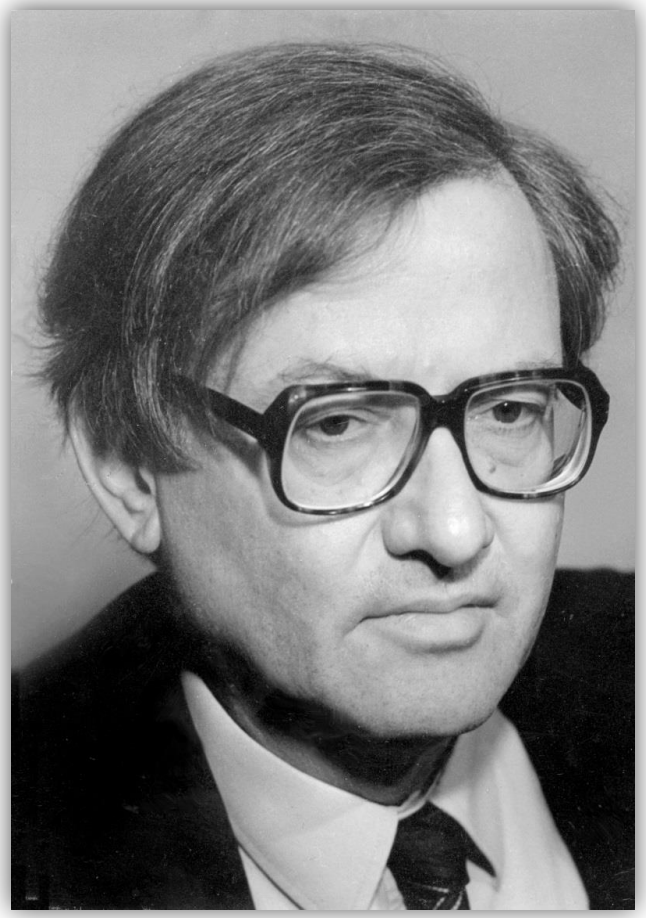


Gábor Kasza

25th Zimányi School

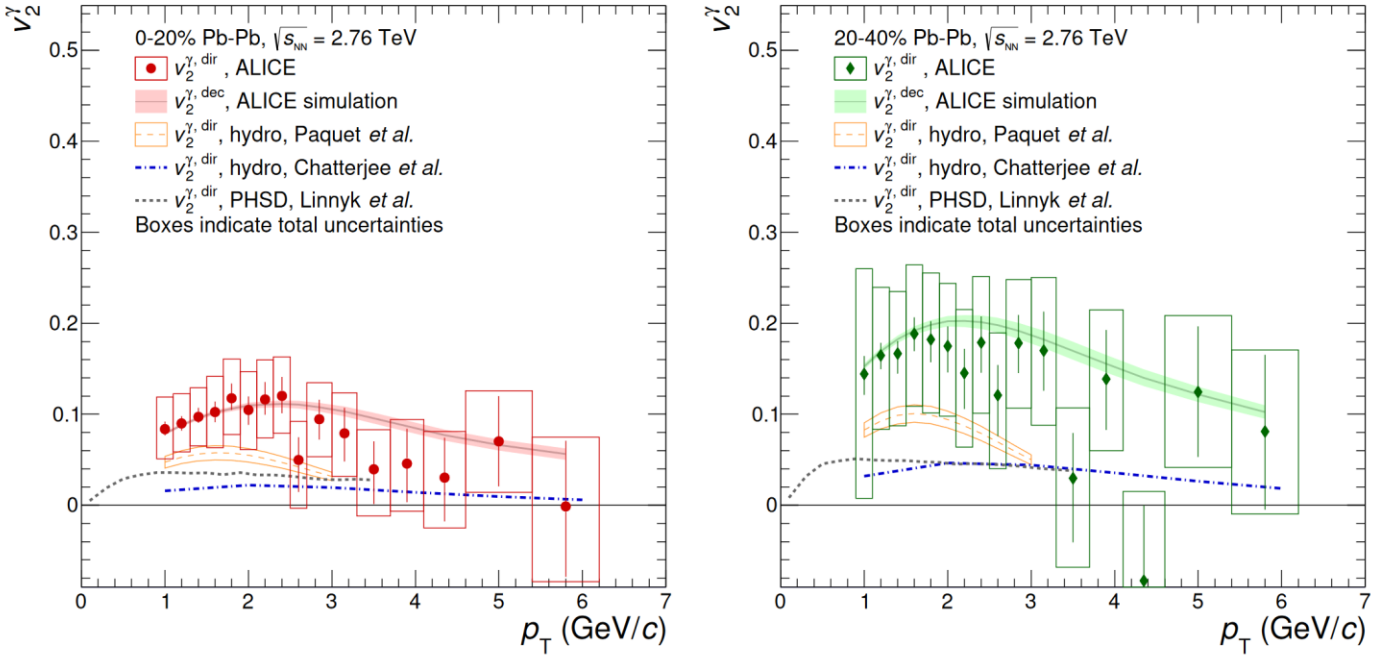
Budapest, 03/12/2025

Thermal Radiation from an Analytic Hydrodynamic Model in Heavy-Ion Collisions

with Hadronic and QGP Sources

Why direct photons?

- ***Direct photon puzzle***: the measured v_2 of direct photons is of the same order of magnitude as for hadrons.
- v_2 cannot be described simultaneously with direct photon spectra using the theoretical models known so far.



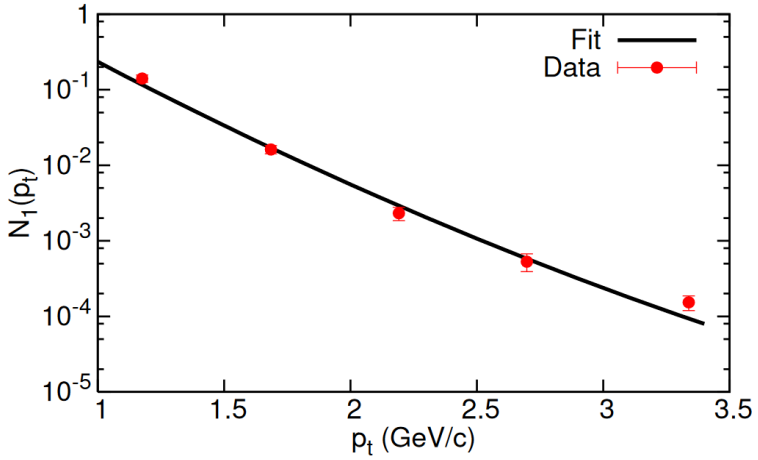
Phys.Lett.B 789 (2019) 308-322

Previous results from 1+3d analytic hydro

- Based on the Csörgő-Csernai-Hama-Kodama solution of relativistic hydro

Acta Phys.Hung.A 21 (2004) 73-84

- Boost-invariant velocity field, Gaussian temperature profile

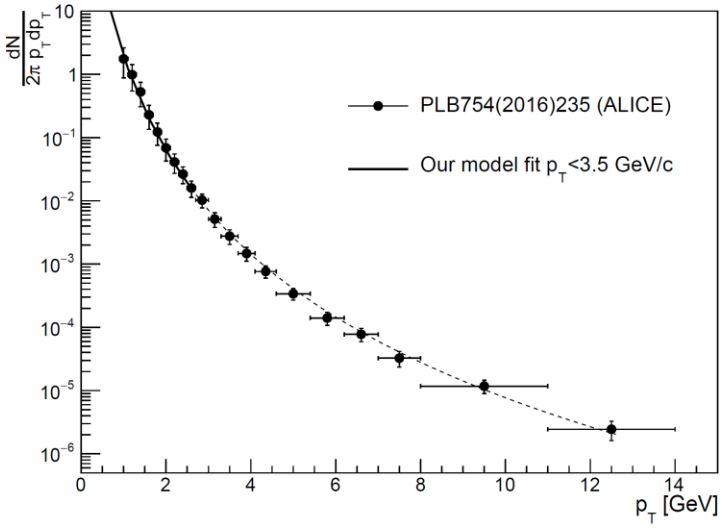


Fitted to PHENIX Au+Au @ 200 GeV

Completely analytic modell

$T_0 > 507 \pm 12$ MeV

M. Csanád and I. Májer: Central Eur.J.Phys. 10 (2012) 850-857



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S. Lökös and G. K.: Eur.Phys.J.A 61 (2025) 7, 157

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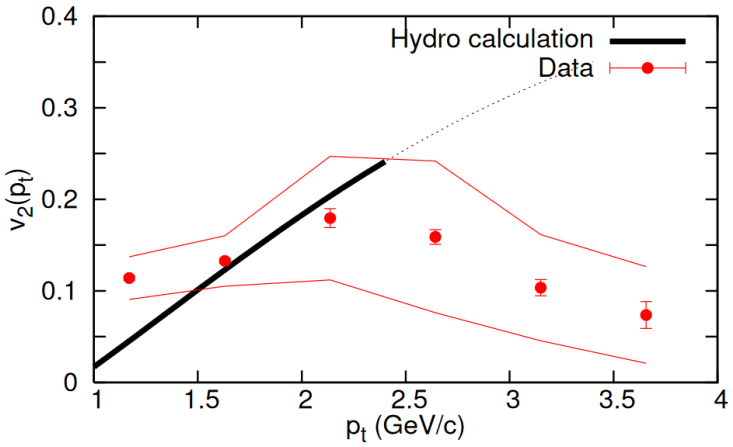

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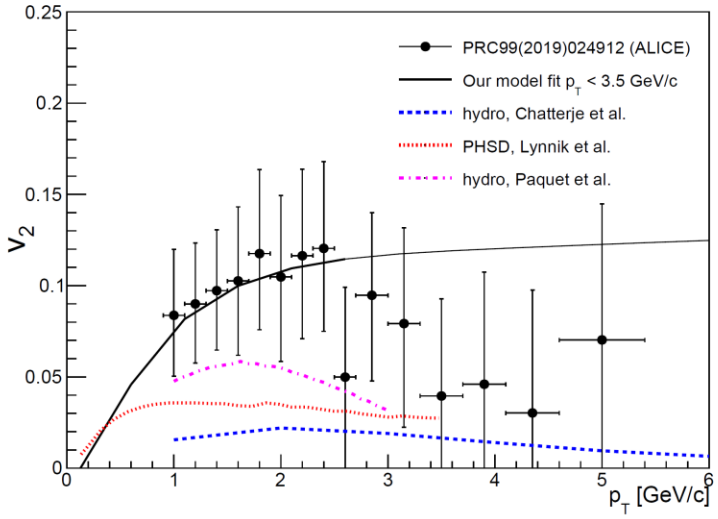

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In Today's presentation

- 1+1 dimensional *analytic* model (no transverse dynamics)
- v_2 cannot be investigated
- Focusing on thermal history
- Hadronic and QGP components of thermal radiation
- EoS constrained by lattice QCD calculations
- Locally accelerating velocity field

New 1+1d analytic solution(s)

- Generalization of an earlier perfect fluid solution:

T. Csörgő, G.K., M. Csanád, Z. Jiang: Universe 4 (2018) 6, 69

EoS: $\mu=0$, $\varepsilon=\kappa p$ → $\mu=0$, $\varepsilon=\kappa(T)p$

- Energy conservation:

$$\left[\kappa(T)+\frac{T}{1+\kappa(T)}\frac{\partial\kappa(T)}{\partial T}\right]u^\mu\partial_\mu T+T\partial_\mu u^\mu=0$$

Requiring temperature inhomogeneity: strong constraint for $\kappa(T)$:

$$\kappa(T)+\frac{T}{1+\kappa(T)}\frac{\partial\kappa(T)}{\partial T}=C=\text{const.}$$ → *leads to new solutions*

New 1+1d analytic solution(s)

- Low- and high-temperature solutions:

$$\kappa(T) + \frac{T}{1+\kappa(T)}\frac{\partial \kappa(T)}{\partial T} = C = \text{const.}$$

$$\kappa_{\mathrm{h}}(T) = \frac{C_{\mathrm{h}}\left(\frac{T}{T_{\mathrm{fo}}}\right)^{1+C_{\mathrm{h}}} - \frac{C_{\mathrm{h}}-\kappa_{\mathrm{fo}}}{\kappa_{\mathrm{fo}}+1}}{\left(\frac{T}{T_{\mathrm{fo}}}\right)^{1+C_{\mathrm{h}}} + \frac{C_{\mathrm{h}}-\kappa_{\mathrm{fo}}}{\kappa_{\mathrm{fo}}+1}}$$

$$\kappa_{\mathrm{q}}(T) = \frac{C_{\mathrm{q}}\left(\frac{T}{T_{\mathrm{tr}}}\right)^{1+C_{\mathrm{q}}} - \frac{C_{\mathrm{q}}-\kappa_{\mathrm{tr}}}{\kappa_{\mathrm{tr}}+1}}{\left(\frac{T}{T_{\mathrm{tr}}}\right)^{1+C_{\mathrm{q}}} + \frac{C_{\mathrm{q}}-\kappa_{\mathrm{tr}}}{\kappa_{\mathrm{tr}}+1}}$$

$T < T_{\mathrm{tr}}$

$$T_{\mathrm{h}}(\tau,\eta_z) = T_{\mathrm{tr},0}\left(\frac{\tau_{\mathrm{tr}}}{\tau}\right)^{\frac{\lambda}{C_{\mathrm{h}}}}\left[1+\frac{C_{\mathrm{h}}-1}{\lambda-1}\sinh^2\left(\Omega(\eta_z)-\eta_z\right)\right]^{-\frac{\lambda}{2C_{\mathrm{h}}}}$$

$T > T_{\mathrm{tr}}$

$$T_{\mathrm{q}}(\tau,\eta_z) = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{C_{\mathrm{q}}}}\left[1+\frac{C_{\mathrm{q}}-1}{\lambda-1}\sinh^2\left(\Omega(\eta_z)-\eta_z\right)\right]^{-\frac{\lambda}{2C_{\mathrm{q}}}}$$

Main goal

Analyic solutions and models provides reference points to complex numeric calculation

Improving analytic models, make them more general (accelerating velocity field, more realistic EoS)

Better reference points

Application of the new solutions

- Test whether the new hydrodynamic solution predicts a realistic EoS.
- Is the model capable of providing a good *qualitative* description of data?
- Fix several free parameters of κ using the lQCD EoS (e.g. T_{tr}, $\kappa(T_{tr})$)
- Remaining free parameters:
  - fitting the non-prompt direct photon spectrum (init. temp. T_0)
  - the pseudorapidity distribution of charged hadrons (acc. param. λ)

Source function and spectra

- Simple assumption for source function:

$$S\left(x^{\mu}, p^{\mu}\right) d^4x = \frac{g}{(2\pi\hbar)^3} \frac{H(\tau)}{\tau_{\mathrm{R}}} \exp\left(-\frac{p^{\mu}u_{\mu}}{T}\right) p_{\mu} d\Sigma^{\mu}$$

- Calculating p_{T} spectrum:

$$\left.\frac{d^2N}{2\pi p_{\mathrm{T}} dp_{\mathrm{T}} dy}\right|_{y=0} = \int_{-\infty}^{\infty} d\eta_z \int_{\tau_0}^{\infty} \tau A_{\mathrm{T}}(\tau) S\left(x^{\mu}, p^{\mu}\right) d\tau$$

$T > T_{\mathrm{tr}}$

$$\left.\frac{d^2N_{\mathrm{q}}}{2\pi p_{\mathrm{T}} dp_{\mathrm{T}} dy}\right|_{y=0} = N_{0,\mathrm{q}} \frac{2\alpha_{\mathrm{q}}}{3\pi^{3/2}} \left[\frac{1}{T_{\mathrm{tr}}^{\alpha_{\mathrm{q}}}} - \frac{1}{T_0^{\alpha_{\mathrm{q}}}}\right]^{-1} p_{\mathrm{T}}^{-\alpha_{\mathrm{q}}-2} \Gamma\left(\alpha_{\mathrm{q}} + \frac{5}{2}, \frac{p_{\mathrm{T}}}{T}\right)\Bigg|_{T=T_{\mathrm{tr}}}^{T=T_0}$$

$$\left.\frac{d^2N_{\mathrm{h}}}{2\pi p_{\mathrm{T}} dp_{\mathrm{T}} dy}\right|_{y=0} = N_{0,\mathrm{h}} \frac{2\alpha_{\mathrm{h}}}{3\pi^{3/2}} \left[\frac{1}{T_{\mathrm{f}}^{\alpha_{\mathrm{h}}}} - \frac{1}{T_{\mathrm{tr}}^{\alpha_{\mathrm{h}}}}\right]^{-1} p_{\mathrm{T}}^{-\alpha_{\mathrm{h}}-2} \Gamma\left(\alpha_{\mathrm{h}} + \frac{5}{2}, \frac{p_{\mathrm{T}}}{T}\right)\Bigg|_{T=T_{\mathrm{f}}}^{T=T_{\mathrm{tr}}}$$

$T < T_{\mathrm{tr}}$

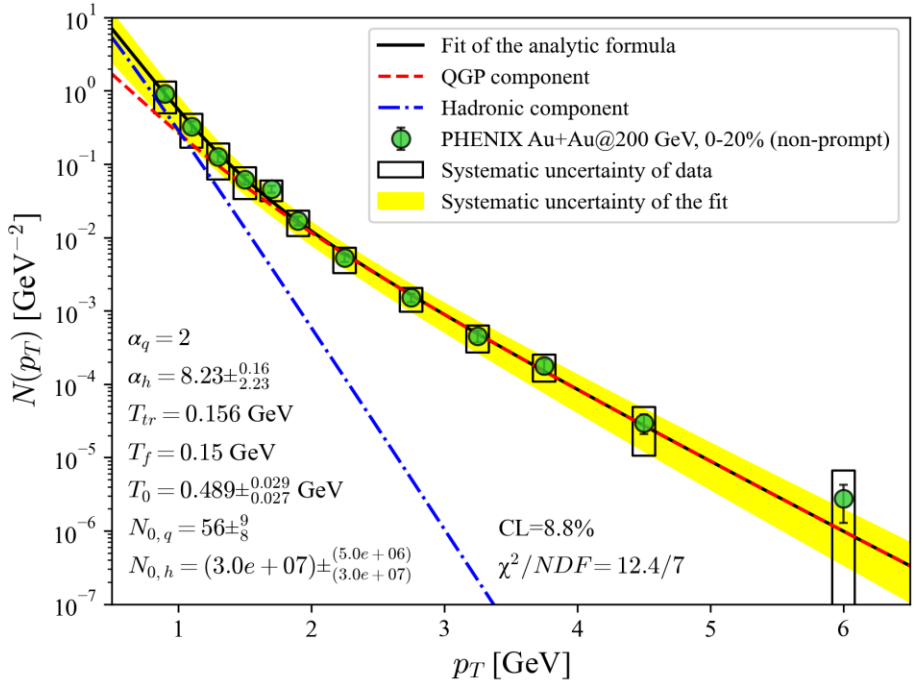
New notations: $\alpha_i = \frac{2C_i}{\lambda} - 3$ $\quad T_{\mathrm{tr}} = T_0 \left(\frac{\tau_0}{\tau_{\mathrm{tr}}}\right)^{\frac{\lambda}{C_{\mathrm{q}}}}$ $\quad T_{\mathrm{f}} = T_{\mathrm{tr},0} \left(\frac{\tau_{\mathrm{tr}}}{\tau_{\mathrm{f}}}\right)^{\frac{\lambda}{C_{\mathrm{h}}}}$ $\quad N_{0,i} = \left.\frac{dN_i}{dy}\right|_{y=0}$

Fits to PHENIX Au+Au@200 GeV

- Fit to the **non-prompt** data component across four centrality classes:

Data:
PHENIX: Phys.Rev.C 109 (2024) 4, 044912

0-20%



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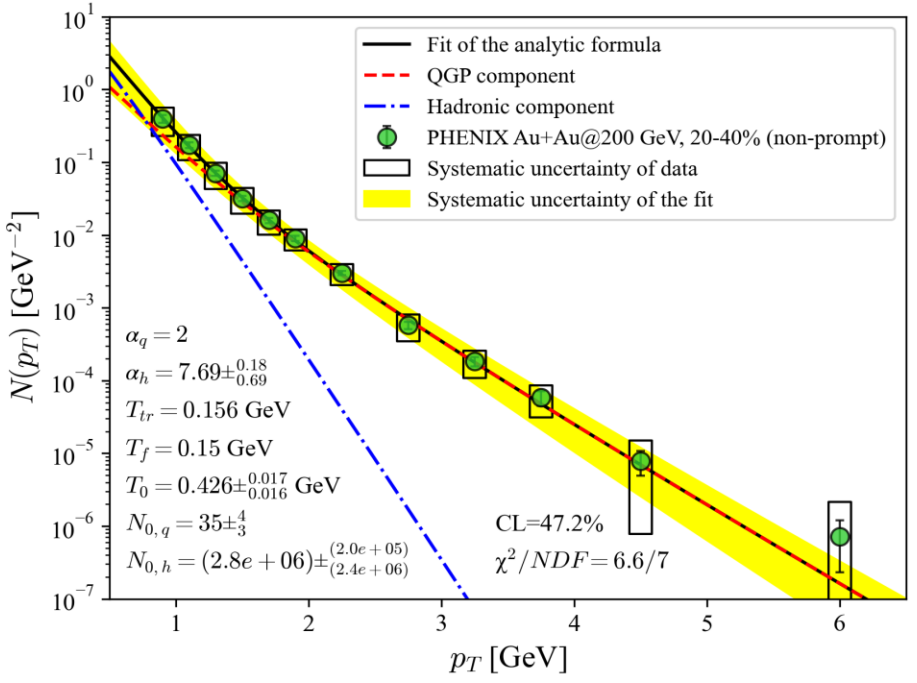

Good description of data!

Hadronic > QGP at p_T < 1 GeV, *supporting examples:*

S. Turbide, R. Rapp, C. Gale:
Phys.Rev.C 69 (2004) 014903

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Phys.Rev.C 77 (2008) 024909

Fits to PHENIX Au+Au@200 GeV

- Fit to the **non-prompt** data component across four centrality classes:

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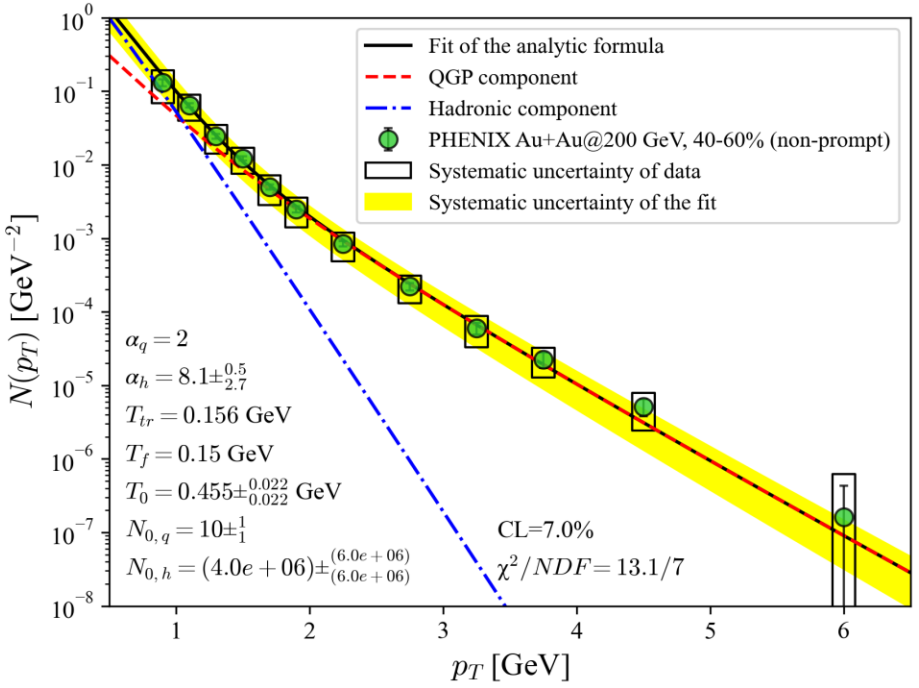
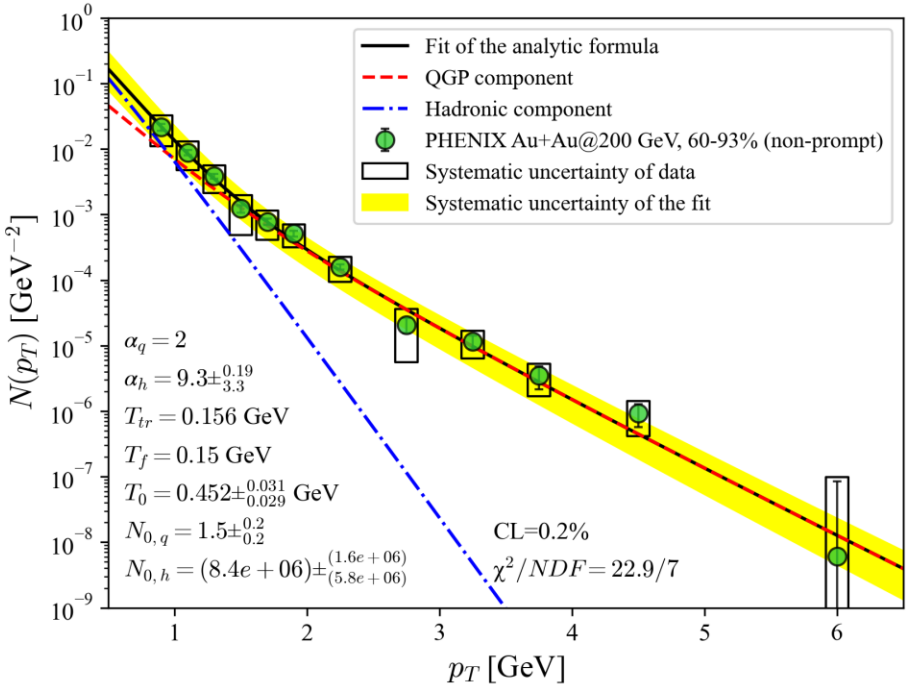
PHENIX: Phys.Rev.C 109 (2024) 4, 044912

40-60%



60-93%



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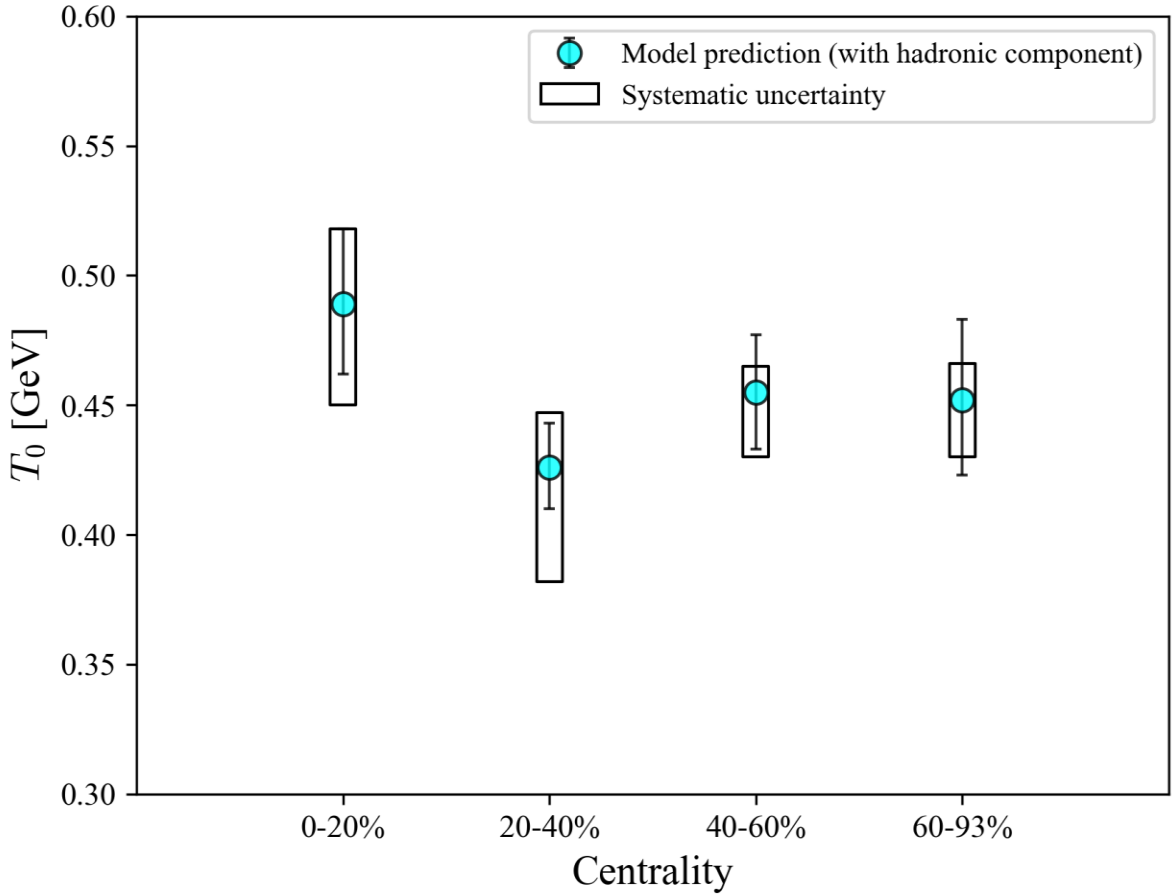
Fits to PHENIX Au+Au@200 GeV

- Initial temperatures extracted:



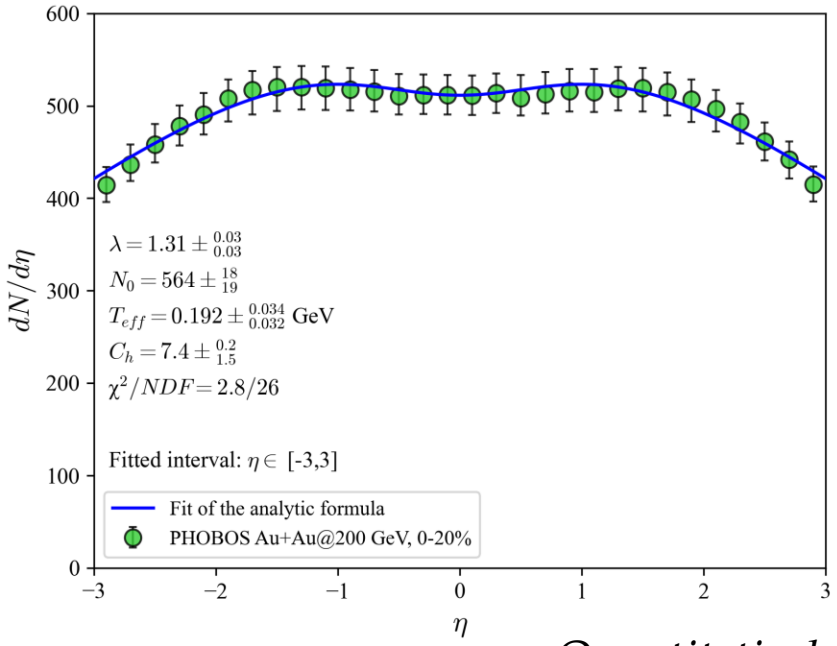
In more central collisions decreasing trend is obtained.

The model's validity for more peripheral collisions is questionable.

Test the model in hadronic channel

- Fit to the $dN/d\eta$ data of PHOBOS Au+Au@200 GeV *Data: PHOBOS: Phys.Rev.C* 83 (2011) 024913
- The fitted formula was already known: *G. K. and T. Csörgő: Int.J.Mod.Phys.A* 34 (2019) 26, 1950147
- Only fine-tuning was needed to account for the new hadronic solution

0-20%



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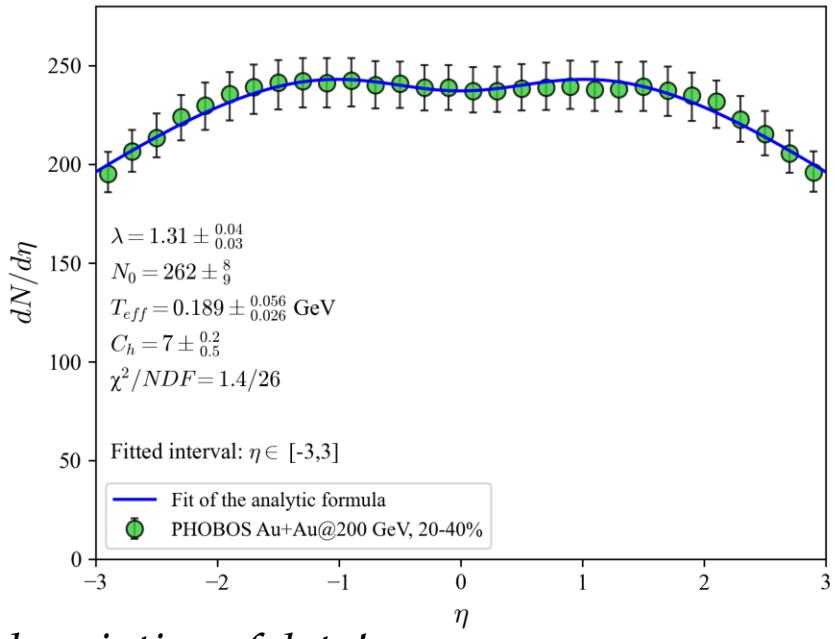


Quantitatively good description of data!

EoS "predicted" by the model (0-20%)

- Fits to hadronic data
- Fits to photonic data
- lQCD simulations

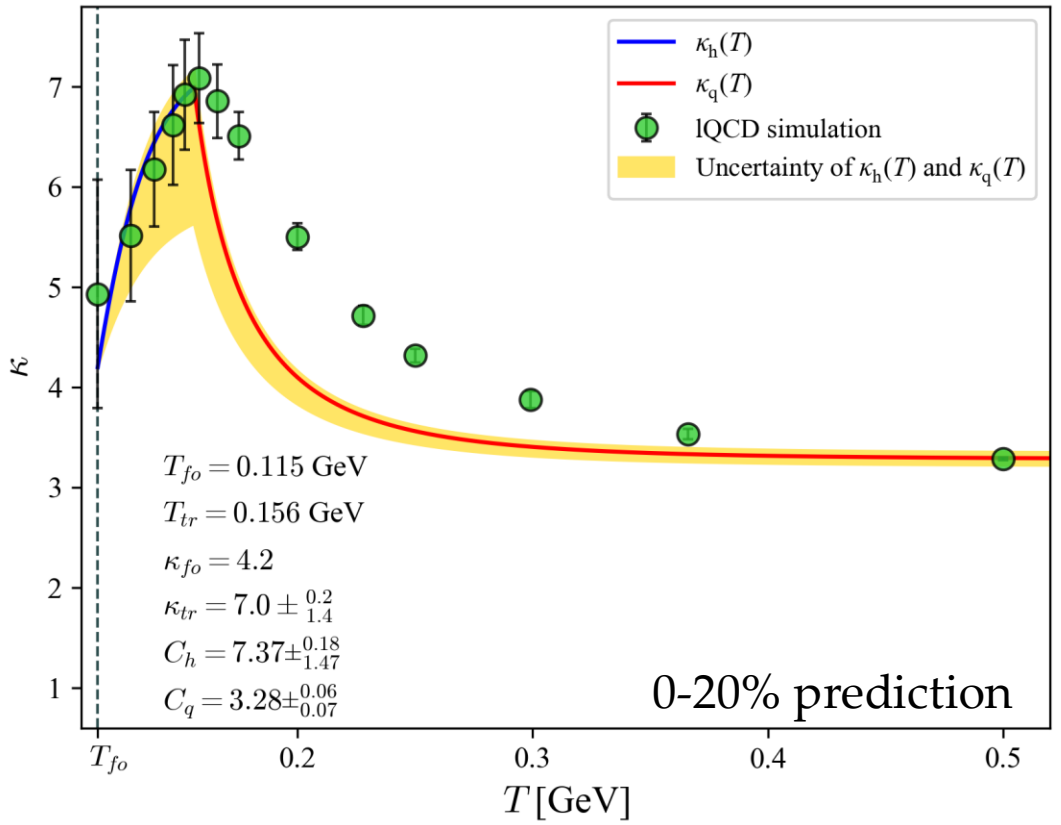
} constrain $\kappa(T) = \varepsilon/p$

Sz. Borsányi et. al.: JHEP 11 (2010) 077

- T_{fo}: varies widely in the literature, an intermediate value was chosen

PHENIX: Phys.Rev.C 109 (2024) 4, 044912; *STAR: Phys.Rev.C* 79 (2009) 034909

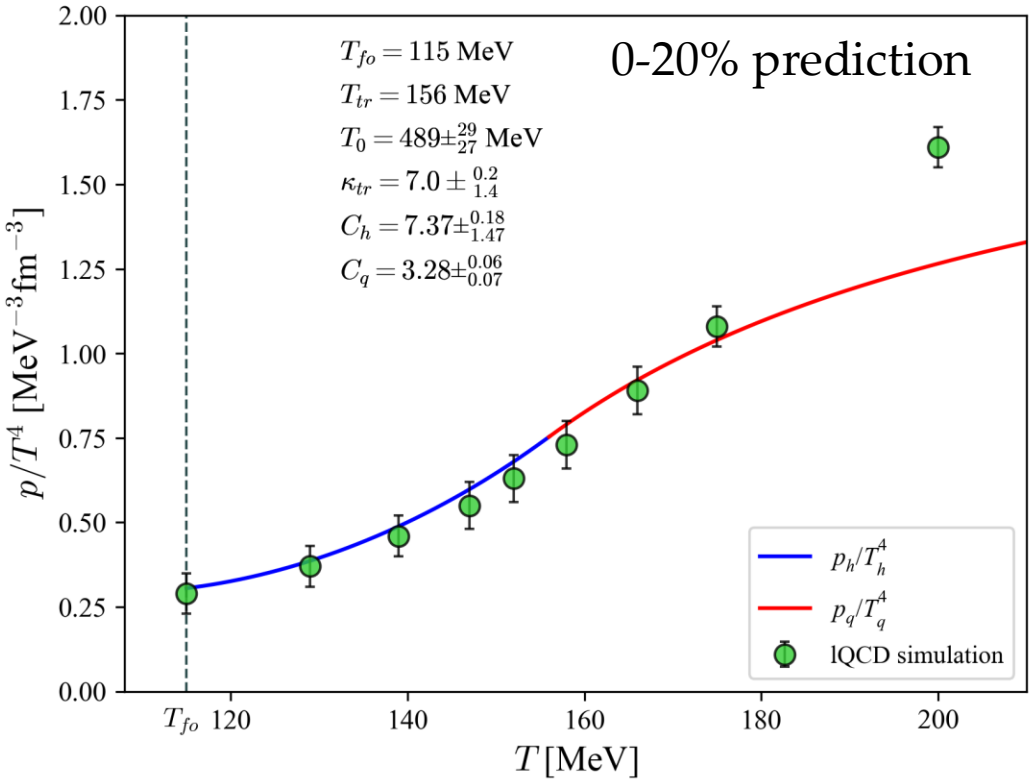
- *Low-temperature side:* good ***quantitative agreement*** achievable
- *High-temperature side:* only ***qualitative agreement*** is achived



EoS "predicted" by the model (0-20%)

$\kappa_q(T)$ compatible with lattice at high T → It does not guarantee same agreement for p or ε

- *Transition area:* good ***quantitative agreement*** achievable
- *For $T > 200$ MeV* ***the agreement breaks down***
- Better agreement achieved by not matching $\kappa_q(T)$ to high-T limit



Summary

- Perfect fluid models works well
  - → *Including viscosity would offer a valuable fine-tuning of the extracted parameters.*
- Two component model in analytic framework
  - → *Well describes the non-prompt photon spectrum across multiple centralities.*
  - → *Hadronic emission outshines the thermal QGP emission around $p_T < 1$ GeV.*
  - → *The model works well also in the hadronic channel.*
- Hydro-based EoS constrained by lQCD EoS
  - → *Further improvement in agreement is possible.*

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Thank you for your attention!

Backup slides

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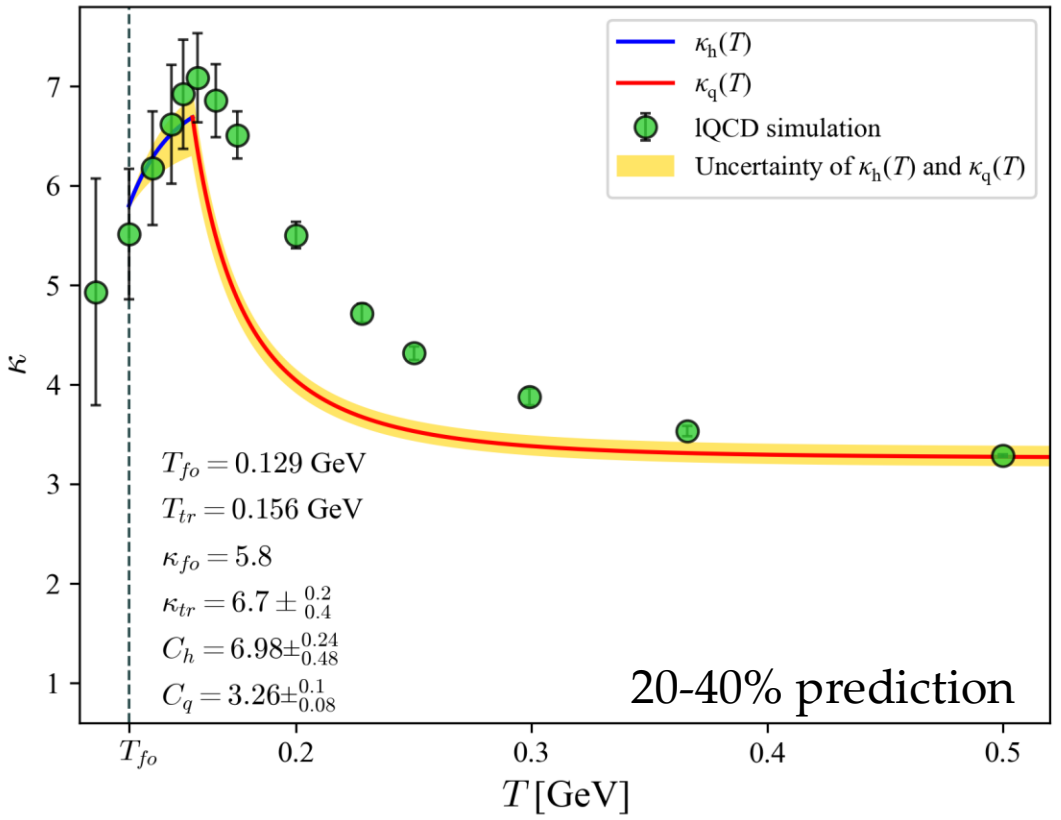

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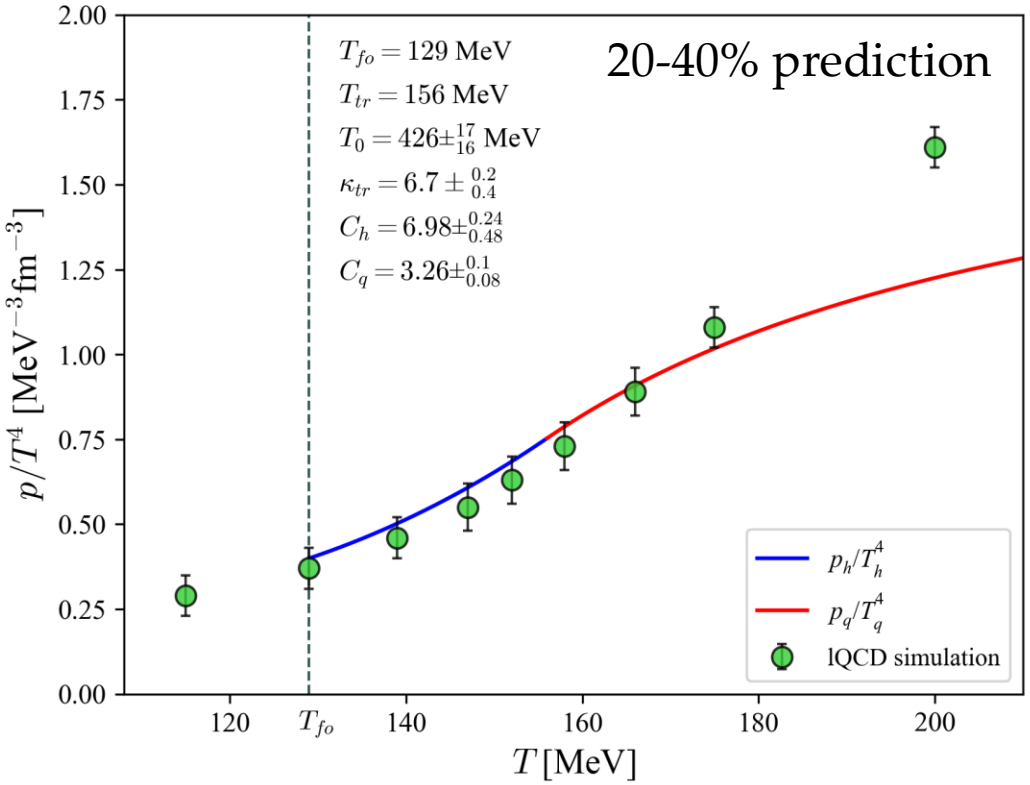
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