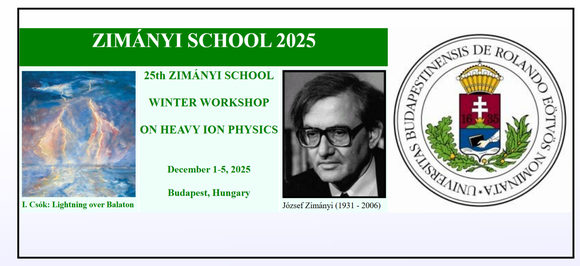
Pion emission source from UrQMD at STAR energies ranging from $\sqrt{s_{NN}}$ =3 to 27 GeV

Zimányi School Winter Workshop, Budapest, 1–5 December 2025

Mátyás Molnár, in collaboration with Máté Csanád and others
molnarmatyas@student.elte.hu



Correlation functions in femtoscopy

Correlation function for bosons with the invariant momentum distributions N_1, N_2:

$$C_2(p_1,p_2) = \frac{N_2(p_1,p_2)}{N_1(p_1)N_1(p_1)}$$

$$\Longrightarrow C_2(q,K) \approx 1 + \left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|^2,$$

where

- $N_2(p_1,p_2) = \int S(x_2,p_2)|\Psi_2(x_1,x_2)|^2 \mathrm{d}^4x_2 \mathrm{d}^4x_1$,
- $\tilde{S} = \int S(x,k) e^{iqx} \mathrm{d}^4x$,
- $K = \frac{k_1+k_2}{2}$ (or p_1, p_2) and $q = k_1 - k_2$.

The *event shape* (distance distribution): $D(r,K) = \int S(x_1,K)S(x_2,K)\mathrm{d}^4\rho = \int S(\rho + \frac{r}{2},K)S(\rho - \frac{r}{2},K)$ with avg. $\rho = \frac{x_1+x_2}{2}$ and rel. $r = x_1 - x_2$

The momentum correlation $C(q)$ can be related to source $S(r)$[a]:

$$C(q) \cong 1 + \left|\int S(r) e^{iqr} \mathrm{d}r\right|^2$$

Can be expressed with distance distribution $D(r)$:

$$C(q,K) \cong 1 + \int D(r,K) e^{-iqr} \mathrm{d}r$$

The Lévy-type source function

Lévy-stable distribution can be assumed and tested for the source function [2, 3].

$$S = S_{\text{core}} + S_{\text{halo}} \equiv \mathcal{L}(\mathbf{x}, \alpha, R)$$

where

- $\mathcal{L} = \frac{1}{(2\pi)^2} \int \mathrm{d}^3q \; e^{iqr} \; e^{-\frac{1}{2}|q\,R|^\alpha}$.
- Thus, the 2-particle correlation function $C_2(|k|) = 1 + \lambda\, e^{-\left(2\,R\,|k|\right)^\alpha}$ (note: $|k| = \frac{|q|}{2}$).
- Parameters: strength λ, scale R, shape α
- spatial distribution $D(\mathbf{r}) = \mathcal{L}\left(\mathbf{r}, \alpha, 2^{\frac{1}{\alpha}} R\right)$.
- In LCMS frame already K-independent: $D(r_{\text{LCMS}}) = \int \mathrm{d}\Omega_{\text{LCMS}}\, \mathrm{d}t \; D(\mathbf{r}_{\text{LCMS}}, t)$
  - $\mathbf{r}_{\text{LCMS}}$: pair-separation vector in the lab frame
- Central limit theorem, diffusion, thermodynamics: lead to Gaussian, corresponding to $\alpha = 2 \Rightarrow\Leftarrow$ Results in 1D: indicate non-Gaussian behaviour [b], Lévy distribution describes the data better [4].
- $\alpha < 2 \Longrightarrow \sim r^{-1-\alpha}$ power-law tail

$D(\rho)$ analysis in UrQMD 3.4

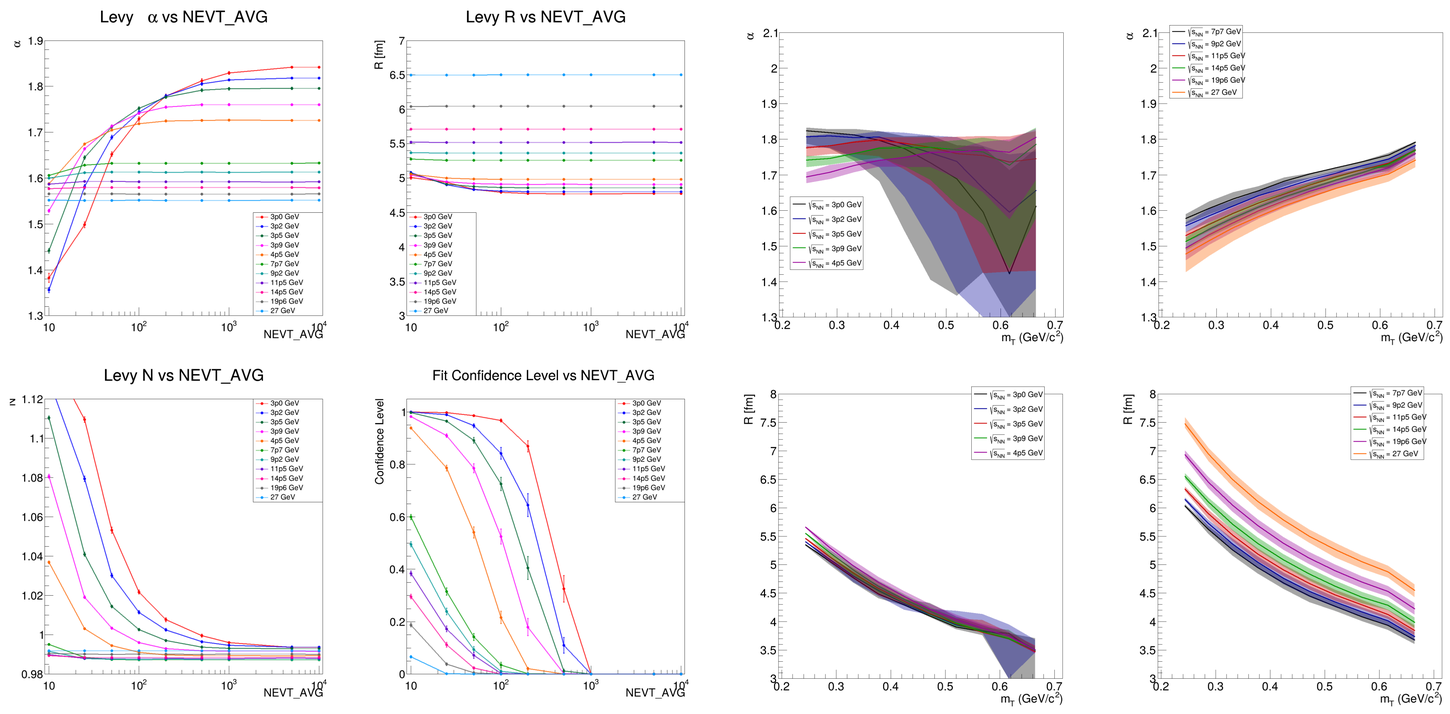
UrQMD is a Monte Carlo tool using a microscopic transport model with binary hadronic and string interactions to simulate high-energy hadronic collisions. I does not contain an explicit QGP or hydrodynamic evolution phase, though it is still excellent for studying hadronic transport and rescattering. [5]

Method

- 10000 Au+Au collision events simulated, 0–10% centrality [c]
- Pion pairs ($\pi^\pm$) separated into 10 $K_T = \frac{1}{2}\sqrt{K_x^2 + K_y^2}$ bins [d]
- 1-dimensional projection along Bertsch–Pratt coordinates, with default cuts as in the STAR experiment: $Q_{LCMS} < \sqrt{150\,\text{MeV} \cdot m_T}$, $0.15 < p_T\,[\text{GeV}] < 1.0$, $|\eta| < 1.0$
- Lévy-distributions fitted to $D(\rho)$

Event averaging

At lower energies, averaging is necessary before fitting due to the low number of pairs. However, averaging together too much events results in a less meaningful fit (confidence level tends to zero). With sensible averaging, the resulting Lévy parameters quickly saturate towards a constant value. The default event averaging chosen in this analysis is 500 events summed.



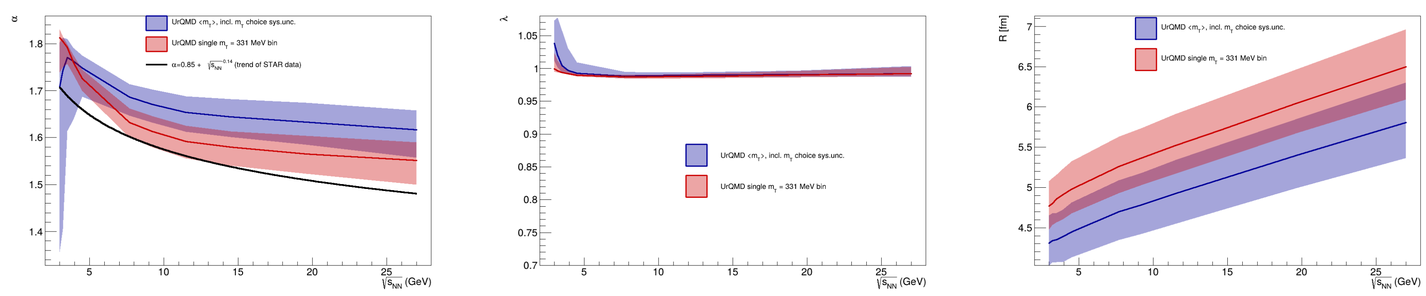
Systematic uncertainty checks

At this stage, three sources checked:

- Q_{LCMS} cut dependence: $\sqrt{A \cdot m_T}$ with $A = 150$ (strict 50, loose 25) [MeV/c]
- $\rho_{\text{max}}^{\text{fit}}$ upper limit of fit range 50–150 fm; default fit range: 1–100 fm
- Event averaging, 200–1000 events, default 500.

Collision energy dependence

Results obtained with averaging over the whole k_T range (175–625 MeV/c^2) and with a choice of a single k_T bin (with $m_T = 331$ MeV center).
Systematic uncertainty of m_T choice also obtained by a ±50 MeV variation.



Compared to analytical function fitted to STAR preliminary data, increasing discrepancy towards < 7.7 GeV – statistical analysis of the discrepancy to be done.

References

[1] Kincses, Dániel and Nagy, Márton and Csanád, Máté *Lévy walk of pions in heavy-ion collisions*, Communications Physics Vol. 8 no.1 (2025), p. 55

[2] T. Csörgő, S. Hegyi and W. A. Zajc *Bose-Einstein correlations for Levy stable source distributions*, Eur. Phys. J. C 36 (2004), 67-78

[3] T. Csörgő, S. Hegyi, T. Novak and W. A. Zajc *Bose-Einstein or HBT Correlation Signals of a Second Order QCD Phase Transition*, AIP Conf. Proc. 828 (2006) no.1, 525-532

[4] D. Kincses, M. Stefaniak and M. Csanád *Event-by-event investigation of the two-particle source function in heavy-ion collisions with EPOS*, Entropy 24 (2022) no.3, 308

[5] Petersen, Hannah et al. *Urqmd-2.3-changes and comparisons*, arXiv preprint arXiv:0805.0567 (2008)

[6] Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337

[7] Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532]

Notes

[a] Assuming neglected pair reconstruction, final state interactions, multi-particle correlations, coherence...

[b] Possible reasons: jet fragmentation, critical phenomena, Lévy walk [1, 6, 7]

[c] With preset 0–4.73 fm impact parameter

[d] $m_T = \sqrt{K_T^2 + m_{\pi^\pm}^2}$