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West University of Timișoara and Tours University

Strongly interacting matter under imaginary rotation

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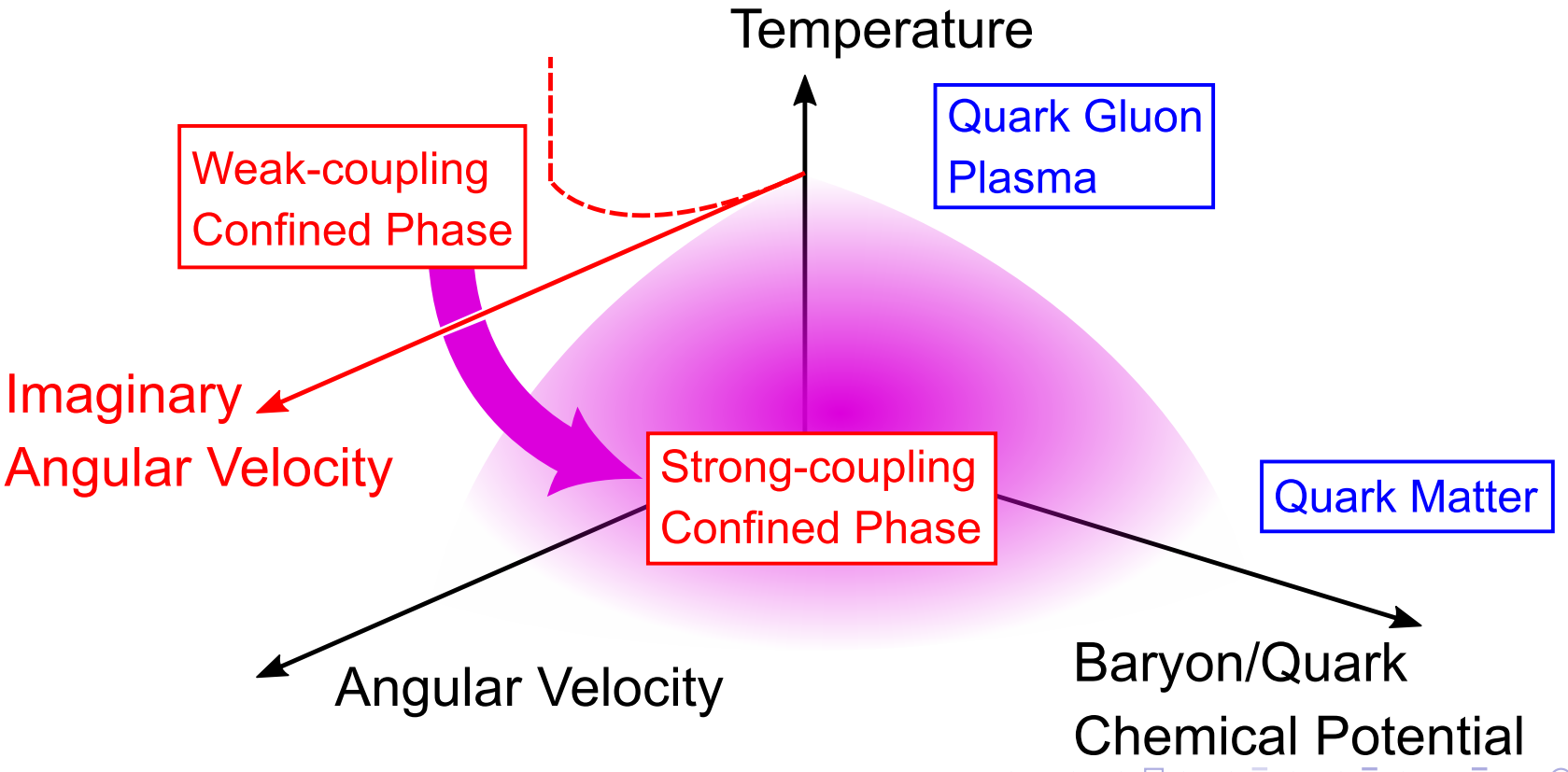
Motivation: study the phase diagram

- Lattice gauge simulations encounter sign problem when including rotation
- Because rotation term breaks hermiticity of Dirac operator $J^z\Omega \sim N\mu$
- **Idea** to go around the sign problem
  1. Go to imaginary rotation $\Omega \to i\Omega_I$
  2. Compare model results with lattice simulations
  3. Analytically continue back $\Omega_I \to -i\Omega$ to obtain the real system



$LSM_{i\Omega_I q}$ Lagrangian with rotating quarks

- The (comoving) Lagrangian of the LSM_q model **with rotating quarks**

$$\mathcal{L}(\psi, \sigma, \vec{\pi}) = \mathcal{L}_M(\sigma, \vec{\pi}) + \mathcal{L}_q^{i\Omega_I}(\psi, \sigma, \vec{\pi})$$

- Mesonic contribution

$$\mathcal{L}_M(\sigma, \vec{\pi}) = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi}),$$
$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - h\sigma$$

- Quark contribution with a new term for rotation along $z-$axis

$$\mathcal{L}_q^{i\Omega_I}(\psi, \sigma, \vec{\pi}) = \bar{\psi}\left[\gamma^0(i\partial_t + i\Omega_I J^z) + i\boldsymbol{\gamma} \cdot \boldsymbol{\nabla} - g\left(\sigma + i\gamma^5 \vec{\tau} \cdot \vec{\pi}\right)\right]\psi$$

- Model parameters λ, v, h, and g fixed by m_π, m_σ, f_π, m_q.

Macrocanonical ensemble with rotation

- Rotation frequency $i\Omega_I$ and chemical potential $\mu \implies$ laboratory frame
- Thermodynamics described by the grand canonical potential

$$\Phi = -T \ln \mathcal{Z}_E$$

with the partition function

$$\mathcal{Z}_E = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\sigma \mathcal{D}\vec{\pi} e^{-S_E}$$

where $S_E = -iS_M = \int_0^\beta d\tau \int_{\mathcal{M}_3} d^3x \mathcal{L}_E$, with $\mathcal{L}_E(\tau, \vec{x}) = -\mathcal{L}_M(it, \vec{x})$.

- Density operator describind rigidly-rotating thermal states

$$\hat{\rho}^{i\Omega_I} = \exp\left\{-\beta\left(:\widehat{H}: -\mu :\widehat{Q}: -i\Omega_I :\widehat{J^z}:\right)\right\},$$

- Partition function $\mathcal{Z}_E = \mathrm{Tr}(\hat{\rho}^{i\Omega_I})$.
- Thermal expectation values

$$\boxed{\langle \widehat{A} \rangle^{i\Omega_I} = \mathcal{Z}_E^{-1} \mathrm{Tr}(\hat{\rho}^{i\Omega_I} \widehat{A})}.$$

Next, in my poster...

- How fractalization affects the phase diagram within the $LSM_{i\Omega_l q}$ model;
- How ninionic statistics appears in the fermion condensate;
- How the phase diagram for the $LSM_{i\Omega_l q}$ model look like?