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Strongly interacting matter under imaginary rotation

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Abstract

At low energies, QCD becomes strongly coupled, requiring the use of effective field theories. Lattice QCD simulations at finite rotation are hindered by the sign problem, which can be circumvented by introducing an imaginary angular velocity. To facilitate direct comparison with such simulations, theoretical frameworks must be formulated in the context of imaginary rotation. This motivates our current work, in which we study interacting quark matter within the linear sigma model subjected to an imaginary angular velocity, $\Omega = i\Omega_I$. We analyze tasymhe resulting thermodynamic properties and plot the corresponding phase diagram. Fractalization, the property of thermal expectation values to have a Thomae-like dependence, established for free, massless fermions in [1], also appears for the interacting case, massive case. The contribution that survives at large distances away from the rotation axis or at high termperatures, referred to as fractal, displays ninionic statistics, as introduced in [2]. This generalized statistics emerges for matter rotating with an imaginary rational frequency $\nu = \beta\Omega_I/2\pi = p/q$ and interpolates between fermionic and bosonic behavior, controlled by the parameter $k = p + q$. Its presence modifies the structure of the phase diagram by inducing qualitatively distinct thermodynamic behavior.

Rotating quark matter in the $LSM_{i\Omega_I q}$ model

- The (comoving) Lagrangian of the $LSM_{i\Omega_I q}$ with rotating quarks

$$\mathcal{L}(\psi,\sigma,\vec{\pi}) = \mathcal{L}_M(\sigma,\vec{\pi}) + \mathcal{L}_q^{i\Omega_I}(\psi,\sigma,\vec{\pi})$$

- Mesonic contribution

$$\mathcal{L}_M(\sigma,\vec{\pi}) = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - U(\sigma,\vec{\pi}),$$
$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 - h\sigma$$

- Quark contribution with rotation along $z-$axis (laboratory frame)

$$\mathcal{L}_q^{i\Omega_I}(\psi,\sigma,\vec{\pi}) = \bar{\psi}\left[\gamma^0(i\partial_t + i\Omega_I J^z) + i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} - g\left(\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi}\right)\right]\psi$$

- First approximation, classical homogeneous fields:

$$\bar{\sigma}(x) = \bar{\sigma} = \text{const}, \quad \bar{\vec{\pi}}(x) = \bar{\vec{\pi}} = \text{const}$$

- Classical EOMs for the mean condensates in the mean field approximation

$$\lambda\left(\bar{\sigma}^2 + \bar{\vec{\pi}}^2 - v^2\right)\bar{\sigma} = h - \frac{g}{V}\int_{\mathcal{M}_3}\langle\bar{\psi}\psi\rangle_\beta^{i\Omega_I}(x),$$
$$\lambda\left(\bar{\sigma}^2 + \bar{\vec{\pi}}^2 - v^2\right)\bar{\vec{\pi}} = -i\frac{g}{V}\int_{\mathcal{M}_3}\underbrace{\langle\bar{\psi}\gamma^5\vec{\tau}\psi\rangle_\beta(x)}_{\equiv 0} \implies \bar{\vec{\pi}} = 0.$$

Rotating quark matter in the macrocanonical ensemble

- Thermodynamics: $\Phi = -T\ln\mathcal{Z}_E$ with the partition function

$$\mathcal{Z}_E = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\sigma\mathcal{D}\vec{\pi}e^{-S_E}$$

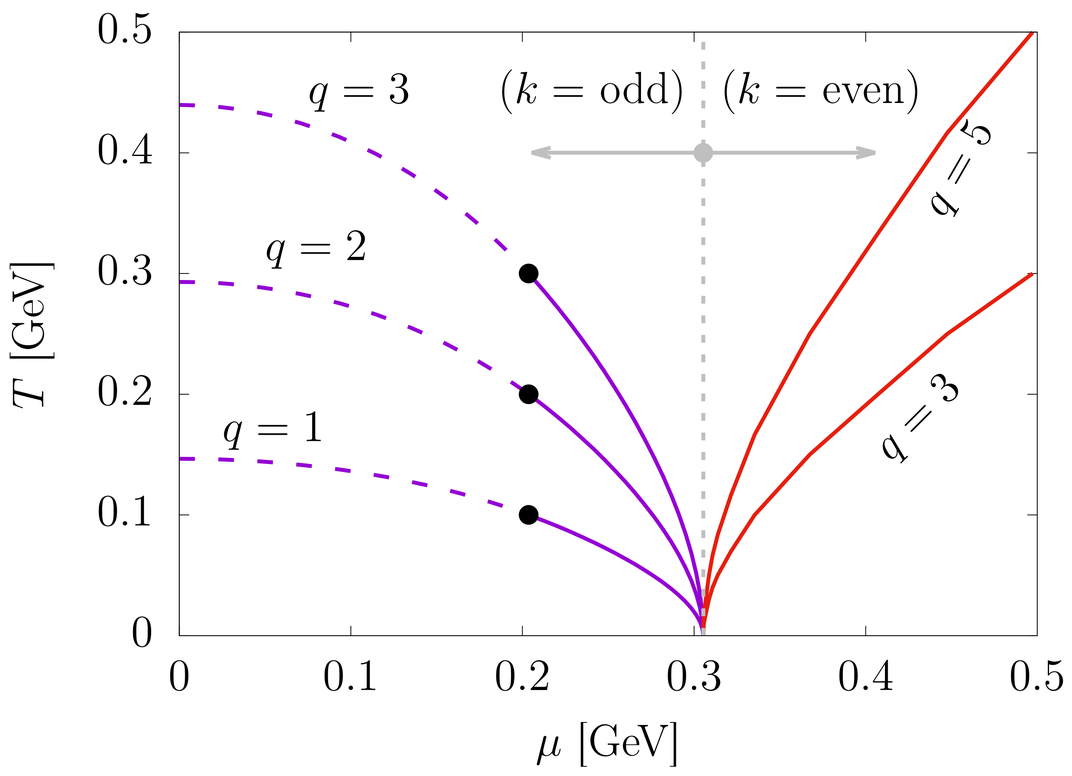
where $S_E = -iS_M = \int_0^\beta d\tau\int_{\mathcal{M}_3} d^3x\mathcal{L}_E$, with $\mathcal{L}_E(\tau,\vec{x}) = -\mathcal{L}(it,\vec{x})$.

- Density operator (laboratory frame) describind rigidly-rotating thermal states

$$\hat{\rho}^{i\Omega_I} = \exp\left\{-\beta\left(:\widehat{H}: - \mu:\widehat{Q}: - i\Omega_I:\widehat{J}^z:\right)\right\}.$$

- Thermal expectation values $\langle\widehat{A}\rangle^{i\Omega_I} = \mathcal{Z}_E^{-1}\,\mathrm{Tr}(\hat{\rho}^{i\Omega_I}\widehat{A})$, where $\mathcal{Z}_E = \mathrm{Tr}(\hat{\rho}^{i\Omega_I})$.

The phase diagram for homogeneous σ in $LSM_{i\Omega_I q}$



Imaginary thermal expectation values

- Thermal expectation values become for operator $\widehat{A}$

$$A_\beta^{i\Omega_I} \equiv \langle:\widehat{A}:\rangle_\beta^{i\Omega_I} = \sum_{j,\sigma}\mathcal{C}_\sigma(\mathcal{A})\frac{\mathcal{A}(U_j,U_j)}{e^{\beta\tilde{\mathcal{E}}_j^\sigma}+1}.$$

- Thermal factor becomes (expansion holds for any $\mathcal{E}_j^\sigma$)

$$\frac{1}{\exp(\beta\tilde{\mathcal{E}}_j^\sigma)+1} = \theta(\mathcal{E}_j^\sigma)\sum_{v=1}^\infty(-1)^{v+1}e^{-v\beta\mathcal{E}_j^\sigma}e^{iv\beta\Omega_I m_j} + \theta(-\mathcal{E}_j^\sigma)\sum_{v=0}^\infty(-1)^v e^{v\beta\mathcal{E}_j^\sigma}e^{-iv\beta\Omega_I m_j}.$$

- Thermal expectation values split as: $\boldsymbol{A_\beta^{i\Omega_I} \equiv \langle:\widehat{A}:\rangle_\beta^{i\Omega_I} = A_{v=0}^{i\Omega_I} + \Delta A^{i\Omega_I}}$.

Fermion condensate fractalization

- In the massless case, the fermion condensate $FC_\beta^{i\Omega_I} = \langle:\bar{\Psi}\Psi:\rangle_\beta$ becomes

$$\frac{FC_{v=0}^{i\Omega_I}}{M} = \frac{\mu^2}{2\pi^2} \to \rho - \text{independent}$$

$$\frac{\Delta FC^{i\Omega_I}}{M} = \frac{2}{\pi^2}\sum_{v=1}^\infty\frac{1}{(v\beta)^2}(-1)^{v+1}\frac{c_v}{1+{\alpha_v}^2}\cos(v\beta\mu\alpha_v) \xrightarrow[\alpha_v\neq 0]{\rho\to\infty} 0,$$

where $c_v = \cos\left(\frac{v\beta\Omega_I}{2}\right) = \cos(\pi\nu v)$ and $\alpha_v = \frac{2\rho}{v\beta}\sin(\pi\nu v)$.

- Consider $\nu = \beta\Omega_I/2\pi = p/q$ rational frequencies (as irreducible fractions).
- Take $v = r + qQ$, denote $l = 2\pi\rho/\beta$ and $x_r = ls_r/\pi q$.
- Split $\sum_{v=1}^\infty \equiv \sum_{r+qQ=1}^\infty \equiv \sum_{Q=0}^\infty\sum_{r=1}^q$.
- If $\nu = p/q$ rational (as irreducible fractions) and $v = q+qQ$ ($r = q$), then $\alpha_v = 0$.
- The term $r = q$ (**fractal**) survives as $l\to\infty$ (or $\rho\to\infty$)

$$FC_\beta^{i\Omega_I} \xrightarrow{\rho\to\infty} \boldsymbol{FC_{\beta,\infty}^{i\Omega_I}} = FC_{v=0}^{i\Omega_I} + \boldsymbol{\Delta FC_{r=q}^{i\Omega_I}},$$

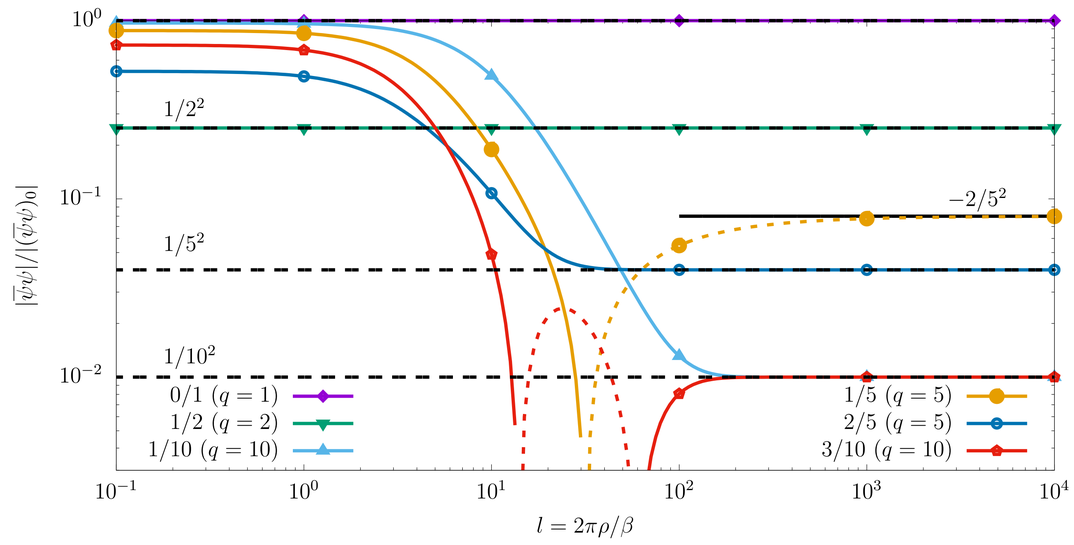
whereas the terms with $1\le r<q$ vanish when $l\to\infty$.

- The **asymptotic term** $FC_{\beta,\infty}^{i\Omega_I}$, that survives as $l\to\infty$, for massive quarks, is

$$FC_{\beta,\infty}^{i\Omega_I} = -\frac{M}{\pi^2}\sum_{\sigma=\pm1}\int_M^\infty dE\,p f_q^\sigma(E),$$

where $f_q^\sigma(E) = 1/(e^{\beta_q(E-\sigma|\mu|)} - (-1)^k)$ is the **ninionic distribution function** [2], corresponding to an effective **fractal temperature** $T_q = T/q$, or $\beta_q = q\beta$ [2].

- The chemical potential contribution $FC_{v=0}^{i\Omega_I}$ to the asymptotic term is independent of $\nu = \beta\Omega_I/2\pi$!



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[2] M. N. Chernodub, arXiv:2210.05651 [quant-ph].

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