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Quantum corrections in quark-meson model under rotation

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Introduction

Effects of rotation on the chiral phase transition have been previously studied in Ref. [1] using two simplifications — that the fermion condensate follows the Tolman–Ehrenfest law and that the mass of condensate is homogeneous. However, only the leading term of the Tolman-Ehrenfest law, without explicit rotational dependence, was considered, while other contributions were neglected. In this work, we show that these contributions have significant implications for the behaviour of the condensate and the phase diagram.

Theory

We consider the fermion condensate, thermal expectation value of $\hat{\bar{\psi}}\hat{\psi}$ at temperature T, chemical potential μ, under the rotation Ω as [2]

$$\langle\hat{\bar{\psi}}\hat{\psi}\rangle = \frac{M}{2\pi^2}\sum_{\varsigma=\pm1}\sum_{m=-\infty}^{\infty}\int_M^\infty \frac{dE}{e^{\beta(\tilde{E}-\varsigma\mu)}+1}\int_0^p dk\, J_m^+\left(\rho\sqrt{p^2-k^2}\right), \tag{1}$$

where $\varsigma=\pm1$ for particles and anti-particles, $\tilde{E}=E-\hbar\Omega m$ is the corotating energy and $J_m^+(z)=J^2_{m-\frac{1}{2}}(z)+J^2_{m+\frac{1}{2}}(z)$. Analogously, fermions develop free energy as

$$\Phi_q^{\text{m.f.}} = \int dM\langle\hat{\bar{\psi}}\hat{\psi}\rangle = -\frac{T_0}{4\pi^2}\sum_{\varsigma=\pm1}\sum_{m=-\infty}^{\infty}\int_M^\infty dE\, E\, \ln\left(1+e^{-\beta(\tilde{E}-\varsigma\mu_0)}\right) \times\int_{-p}^{p} dk\, J_m^+(\rho\sqrt{p^2-k^2}). \tag{2}$$

In order to study chiral symmetry breaking under the rotation, we employ linear sigma model coupled to quark (LSMq).

$$\mathcal{L}_{\text{LSM}_\text{q}} = \mathcal{L}_M+\mathcal{L}_q, \qquad \text{with} \tag{3}$$

$$\mathcal{L}_M = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})-U(\sigma,\vec{\pi}), \tag{4}$$

$$\mathcal{L}_q = \bar{\psi}\left[\frac{i}{2}\overleftrightarrow{\not\partial}-g(\sigma+i\gamma^5\vec{\tau}\cdot\vec{\pi})\right]\psi, \tag{5}$$

where the mesonic potential responsible for symmetry breaking is

$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2-h\sigma \tag{6}$$

Considering mean-field approximation, where meson and pion fields take their corresponding thermal expectation values without quantum fluctuations, we construct mean-field free energy by

$$\Phi^{\text{m.f.}}[\sigma,\vec{\pi}] = -\frac{1}{\beta}\log\mathcal{Z}_E^{\text{m.f.}}, \quad \mathcal{Z}_E^{\text{m.f.}} = \int\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S_E}. \tag{7}$$

By extremizing free energy with respect to sigma and pion fields, we obtain equations of motion

$$\left[\Box+\lambda(\sigma^2+\vec{\pi}^2-v^2)\right]\sigma = h-g\langle\bar{\psi}\psi\rangle, \tag{8}$$

$$\left[\Box+\lambda(\sigma^2+\vec{\pi}^2-v^2)\right]\vec{\pi} = -g\langle i\bar{\psi}\gamma^5\vec{\tau}\psi\rangle, \tag{9}$$

Since the pion expectation value vanishes, we obtain unique equation governing behaviour of the condensate as

$$\left[-\frac{1}{\rho}\partial_\rho(\rho\partial_\rho)+\lambda(\sigma^2-v^2)\right]\sigma = h-g\langle\bar{\psi}\psi\rangle, \tag{10}$$

Semi-classical expansion

Differentiation between classical and quantum contributions, introduced by rotation, can be achieved by employing expansion of corotating energy $\tilde{E}=E-m\hbar\Omega$ with respect to angular velocity

$$\langle\bar{\psi}\psi\rangle = \sum_{n=0}^{\infty}(\hbar\Omega)^{2n}\langle\bar{\psi}\psi\rangle^{(2n)} = \langle\bar{\psi}\psi\rangle^{\text{TE.}}+(\hbar\Omega)^2\langle\bar{\psi}\psi\rangle^{(2)}+\dots \tag{11}$$

By Taylor expanding the Fermi-Dirac distribution in Eq. [1] with respect to angular velocity, summing over $m^{2n}J_m^+$ terms, integrating over momentum k and performing the summation in respect to odd or even derivatives of FD distribution [2], it is possible to obtain the condensate expressed in terms of classical contribution and quantum correction. As a final step, we perform transformations from the co-rotating to inertial frame. Classical contribution is given as

$$\langle\hat{\bar{\psi}}\hat{\psi}\rangle^{\text{TE.}} = \frac{M}{\pi^2}\sum_{\varsigma=\pm1}\int_M^\infty dE\, p\, f_\rho(E) \tag{12}$$

and the first quantum correction can be obtained in the form

$$\langle\hat{\bar{\psi}}\hat{\psi}\rangle^{(2)} = \frac{M}{24\pi^2}\Gamma_\rho^4\sum_{\varsigma=\pm1}\int_M^\infty dE\frac{p^2(3-2(\rho\Omega)^2)+E^2(\rho\Omega)^2}{p}\frac{d^2}{dE^2}f_\rho(E), \tag{13}$$

where the distribution function depends on local parameters as

$$f_\rho(E) = \frac{1}{e^{\beta_\rho(E-\varsigma\mu_\rho)}+1}, \quad \beta_\rho=\Gamma_\rho^{-1}\beta, \quad \mu_\rho=\Gamma_\rho\mu, \quad \Gamma_\rho=(1-\rho^2\Omega^2)^{-1/2}. \tag{14}$$

Condensate on the axis and near light-cylinder

Considering the behaviour of the condensate on the axis, it is possible to obtain an exact solution. We see that the exact solution can be written in terms of classical condensates at vanishing rotation and modified chemical potentials as

$$\langle\bar{\psi}\psi\rangle_{\rho\to0} = \frac{M}{4\pi^2}\sum_{\varsigma=\pm1}\int_M^\infty dE\left[\frac{1}{e^{\beta_0(E-\frac{1}{2}\Omega-\mu)}+1}+\frac{1}{e^{\beta_0(E+\frac{1}{2}\Omega-\mu)}+1}\right]\int_{-p}^{p}dk = \frac{1}{2}\left[\langle\overline{\psi}\psi\rangle^{\text{TE.}}_{\rho=0}\left(\mu\to\mu+\tfrac{1}{2}\Omega\right)+\langle\overline{\psi}\psi\rangle^{\text{TE.}}_{\rho=0}\left(\mu\to\mu-\tfrac{1}{2}\Omega\right)\right]. \tag{15}$$

This corresponds exactly to the case of fermions at finite helical chemical potential [3]. Regarding the near light-cylinder ($\rho\Omega\to1$), we expect the chiral restoration and hence consider small mass expansion which yields [2]

$$\langle\hat{\bar{\psi}}\hat{\psi}\rangle_{M\to0} = M\left(\frac{T_\rho^2}{6}+\frac{\mu_\rho^2}{2\pi^2}+(\hbar\Omega)^2\Gamma_\rho^4\frac{3-(\rho\Omega)^2}{24\pi^2}\right)+O(M^2). \tag{16}$$

Local Density Approximation

In order to solve mass gap equation, slowly varying condensate with vanishing gradients can be considered as

$$\lambda(\sigma(\rho)^2-v^2)\sigma(\rho) = h-g\left[\langle\overline{\psi}\psi\rangle^{\text{TE.}}+\Omega^2\langle\overline{\psi}\psi\rangle^{(2)}\right]. \tag{17}$$

Solving previous equation yields condensate of behaviour with respect to normalized distance from the axis of rotation, as depicted in Fig. 1. In the case where three solutions appear, correspodning to the first-order phase transition, the physical one corresponds to the smallest free energy. The remaining branches are shown in grey.

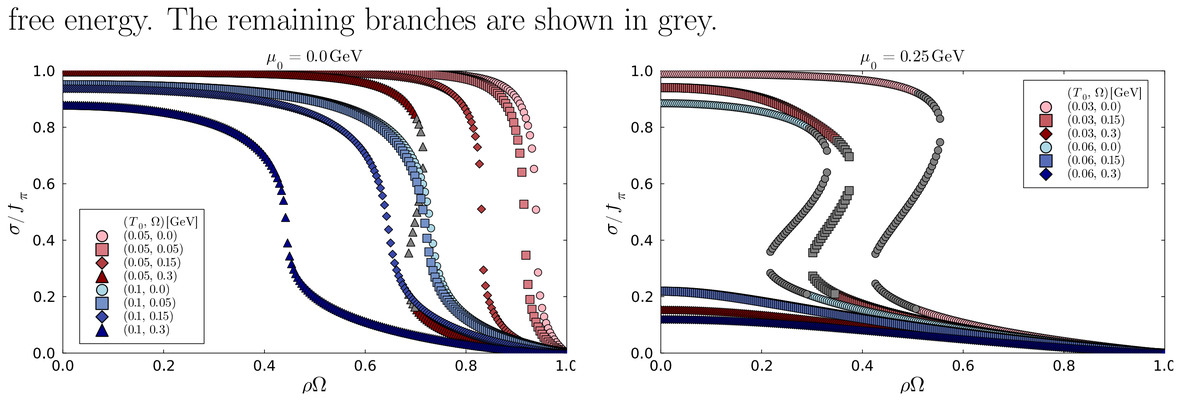

Fig. 1: σ/f_π with respect to $\rho\Omega$ for $\mu_0=0$ (left) and 0.25 GeV (right).

Since the condensate exhibits a monotonically decreasing behaviour, being chirally broken near the axis of rotation and chirally restored toward the light-cylinder, it is sufficient to consider only the values on axis when constructing the phase diagram. Fig. 2 shows the phase diagrams for the previously examined condensates at different angular velocities. Below each line lies the chirally broken phase, while the chirally restored phase is above it. Dashed lines correspond to second-order phase transitions, whereas solid lines indicate first-order transitions. The critical point is labeled by a star.

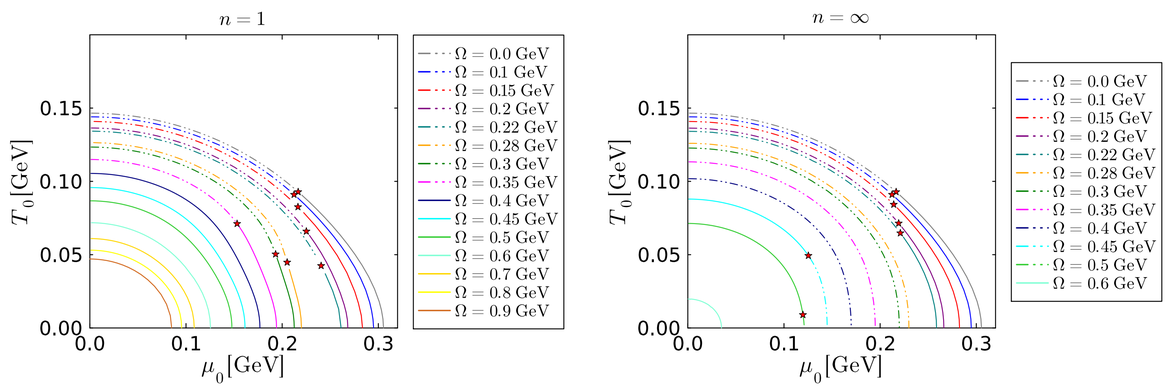

Fig. 2: Phase diagram for condensate including first quantum correction (left) and the exact one (right) – see also Ref. [3].

Main References

[1] S. Morales-Tejera, V. E. Ambruș and M. N. Chernodub, Phys. Rev. D **112** (2025) 054031.

[2] V.E.Ambruș, JHEP **08** (2020) 016.

[3] M.N.Chernodub, V.E.Ambruș, Phys. Rev. D **103** (2021) 094015.

Acknowledgements

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