Lévy distributed sources from relativistic hydrodynamics

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Problem

- Measurement: freeze-out two-particle spatial distributions of collisions are Lévy-stable distributions ($\alpha \neq 2$).
- Hydrodynamic models predict a Gaussian ($\alpha = 2$).
- Possible explanation: hadron dynamics (Gaussian gets distorted to Lévy).
- $R - m_T$ hydro prediction is true.→**no distortion!**

Observable distributions

Source function definition:

$$S(x,p) = \mathcal{N} n(x) \exp\left(- \frac{p_\mu u^\mu}{T(x)} \right) \frac{p_\mu u^\mu}{u_0}.$$

Important distributions:

- $N_1(p_t) = \frac{\mathrm{d}N}{2\pi p_t \mathrm{d}p_t}$ (transverse momentum distribution)→calculated as cross-check,
- $D(r,K) = \int_{-\infty}^{\infty} S(\rho + r/2) S(\rho - r/2) \mathrm{d}^4\rho$ (spatial distribution),

where $N = \int S(x,p) \mathrm{d}^4x \mathrm{d}^3\mathbf{p}/E$.

Approach

- Original solution yields a spatial distribution described by a Gaussian (Lévy-stable distribution with $\alpha = 2$).
- Idea: Hydro-phase already creates a distribution with a power-law tail.

Modifications:

- $s = \left(\frac{r_x}{V_x \tau}\right)^2 + \left(\frac{r_y}{V_y \tau}\right)^2 + \left(\frac{r_z}{V_z \tau}\right)^2$,
- $\nu(s) = \left(\frac{s_0}{(s+\epsilon)}\right)^\beta$,
- Still a solution. ✓

Lévy-stable distribution:

$$\mathcal{L}(\alpha, R, \mathbf{r}) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3\mathbf{q} e^{i\mathbf{qr}} e^{-1/2|\mathbf{qR}|^\alpha}.$$

→used for fitting.

Future steps

- Check $m_T \propto 1/R^2$ relation, through fitting $\mathcal{L}(\alpha, R, \mathbf{r})$;
- Reduce numerical errors in $D(r,K)$;
- Calculate elliptic flow→solution can also incorporate anisotropy in s.

Online version

Relativistic hydrodynamics

The equations of relativistic hydrodynamics:

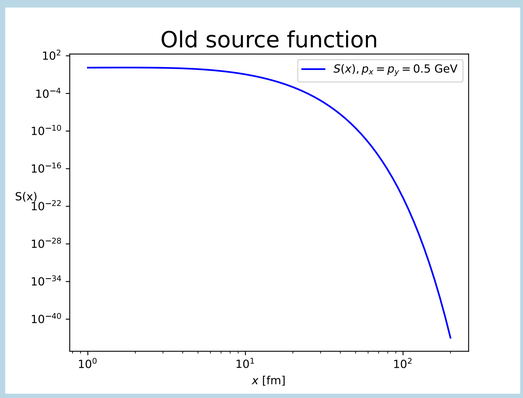
$$\partial_\mu T^{\mu\nu} = 0; \quad \partial_\mu (n u^\mu) = 0$$

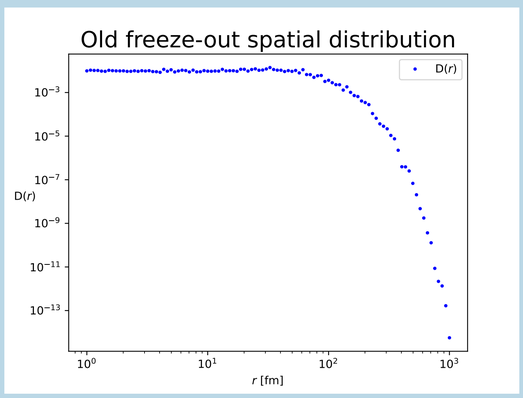
(for perfect fluid).
Analytic solution assuming ellipsoidal symmetry [Acta Phys. Hung. A 21, (2004) 73]:

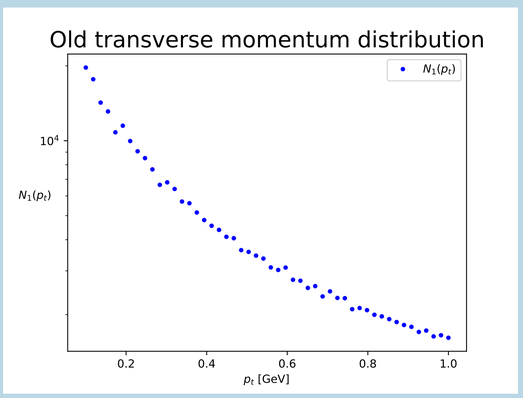
- $u^\mu = x^\mu/\tau$,
- $n(x) = n_0(\frac{\tau_0}{\tau})^3 \nu(s)$,
- $T(x) = T_0(\frac{\tau_0}{\tau})^{3/\kappa} \frac{1}{\nu(s)}$,
- $p(x) = p_0(\frac{\tau_0}{\tau})^{3(\kappa+1)/\kappa}$,
- scale factor: $s = \frac{r_x^2}{X(t)^2} + \frac{r_y^2}{Y(t)^2} + \frac{r_z^2}{Z(t)^2}$ $\to \nu(s) := \exp(-bs)$,
- constant freeze-out: $\tau \to \tau_0$,
- constant velocities: $\dot{X}_0, \dot{Y}_0, \dot{Z}_0$,
- known to reproduce spectra, flow, HBT [Eur.Phys.J.A 44 (2010) 473].

Numerical calculations

- Original solution:

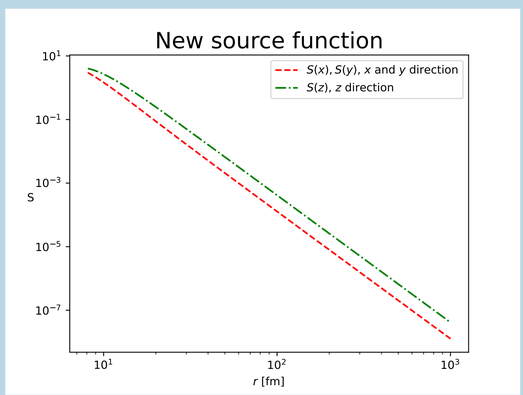


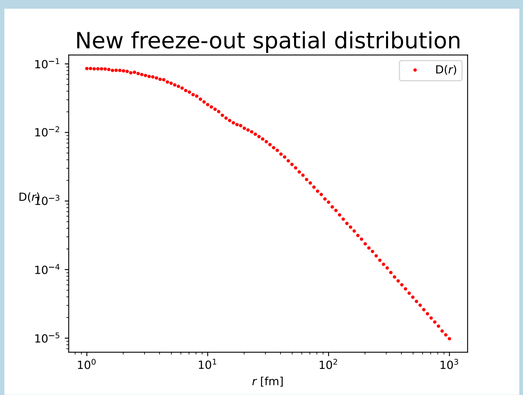


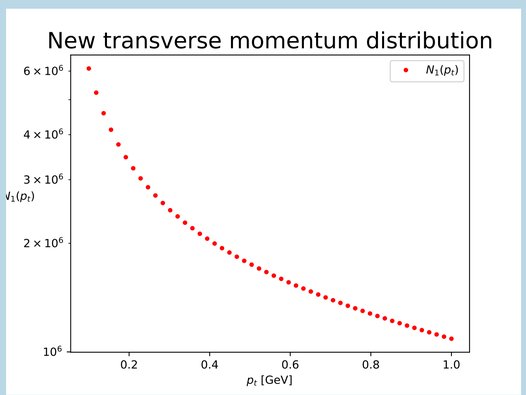


→**Gaussian** is clearly visible.

- New solution:



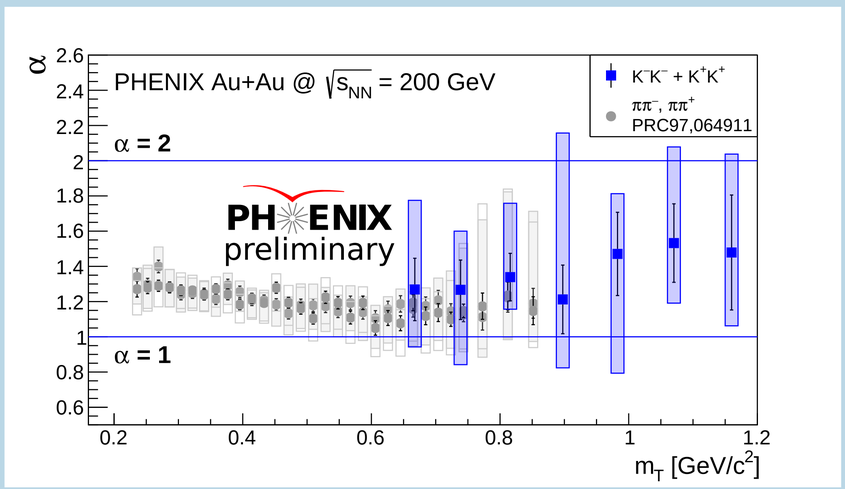


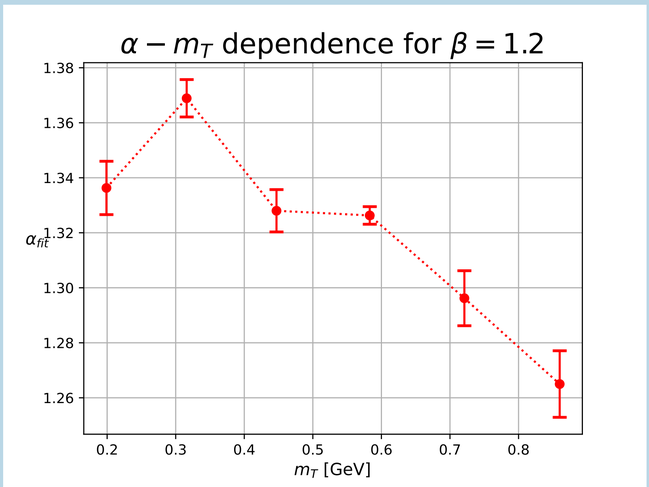


→**Power-law** in $D(r,K)$! ✓

Dependence on m_T

- Transverse mass: $m_T = \sqrt{m^2 + K_T^2}$,
- $\alpha - m_T$ from [arXiv:2307.09573] compared to the new solution:





- No strong dependence! ✓