Quantum computation of Quantum Electrodynamics beyond one spatial dimension

Bipasha Chakraborty
University of Southampton

In collaboration with Zong-Gang Mou

Chaos, Scars, Thermalization, CERN, 15th May 2026

Motivation

LGTs, particularly for QED, provides a powerful platform for studying non-equilibrium dynamics, including quantum many-body scars, thermalisation, and quantum chaos.

⟶ characterised by local gauge invariance, which constrains dynamics and allows for non-ergodic behaviour, where thermalisation is evaded.

Higher-dimensional gauge dynamics are classically hard

Hilbert space grows exponentially; exact diagonalisation fails rapidly; tensor-network methods become difficult in 2+1D and 3+1D

Need scalable real-time simulation frameworks; Quantum computation offers a natural route

Our goal is to develop scalable gauge-invariant real-time simulations of QED beyond one spatial dimension.

Recipe for digital QC of QFT's

- Handle infinite-dimensional Hilbert space in the Hamiltonian
- Discretize (and eventually digitize) fermion and gauge fields
- Use Gauss Law and boundary conditions (or other formulations) to remove (or truncate) gauge degrees of freedom

Recipe for digital QC of QFT's

- Handle infinite-dimensional Hilbert space in the Hamiltonian
- Discretize (and eventually digitize) fermion and gauge fields
- Use Gauss Law and boundary conditions (or other formulations) to remove (or truncate) gauge degrees of freedom

- Convert Fermion fields to Pauli matrices using Jordan Wigner (or similar) transformations
- Write down Hamiltonian in terms of Pauli matrices and thus quantum gates

Recipe for digital QC of QFT's

- Handle infinite-dimensional Hilbert space in the Hamiltonian
- Discretize (and eventually digitize) fermion and gauge fields
- Use Gauss Law and boundary conditions (or other formulations) to remove (or truncate) gauge degrees of freedom

- Convert Fermion fields to Pauli matrices using Jordan Wigner (or similar) transformations
- Write down Hamiltonian in terms of Pauli matrices and thus quantum gates

- Create eigen states of the Hamiltonian – adiabatic approximation, variational approaches
- Use optimized methods for time evolution – for example, Suzuki-Trotter decomposition

Recipe for digital QC of QFT's

- Handle infinite-dimensional Hilbert space in the Hamiltonian
- Discretize (and eventually digitize) fermion and gauge fields
- Use Gauss Law and boundary conditions (or other formulations) to remove (or truncate) gauge degrees of freedom

- Convert Fermion fields to Pauli matrices using Jordan Wigner (or similar) transformations
- Write down Hamiltonian in terms of Pauli matrices and thus quantum gates

- Create eigen states of the Hamiltonian – adiabatic approximation, variational approaches
- Use optimized methods for time evolution – for example, Suzuki-Trotter decomposition

- Take measurements and compute expectation values of Hermitian operators
- Estimate systematic errors and noise

Recipe for digital QC of QFT's

- Handle infinite-dimensional Hilbert space in the Hamiltonian
- Discretize (and eventually digitize) fermion and gauge fields
- Use Gauss Law and boundary conditions (or other formulations) to remove (or truncate) gauge degrees of freedom

- Convert Fermion fields to Pauli matrices using Jordan Wigner (or similar) transformations
- Write down Hamiltonian in terms of Pauli matrices and thus quantum gates

- Create eigen states of the Hamiltonian – adiabatic approximation, variational approaches
- Use optimized methods for time evolution – for example, Suzuki-Trotter decomposition

- Take measurements and compute expectation values of Hermitian operators
- Estimate systematic errors and noise

Digital quantum simulation of Schwinger Model with topological θ-term (1+1 d)

Continuum: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$

Consider Chiral rotation: $\psi \rightarrow e^{i\frac{\theta}{2}\gamma_5}\psi$ to obtain alternative Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi$$ [Fujikawa'79]

Now apply temporal gauge: $A_0 = 0$ and define conjugate momentum: $\Pi = \dot{A}^1$

$$\hat{H} = \int dx \left[-i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}e^{i\theta\gamma^5}\psi + \frac{1}{2}\Pi^2 \right]$$

B. Chakraborty et. al. Phys. Rev. D (2022)]

Discretize theory on a lattice

Staggered fermions on lattice sites:

$$\frac{\chi_n}{\sqrt{a}} \leftrightarrow \begin{cases} \psi_u(x) & n : \text{even} \\ \psi_d(x) & n : \text{odd} \end{cases}$$

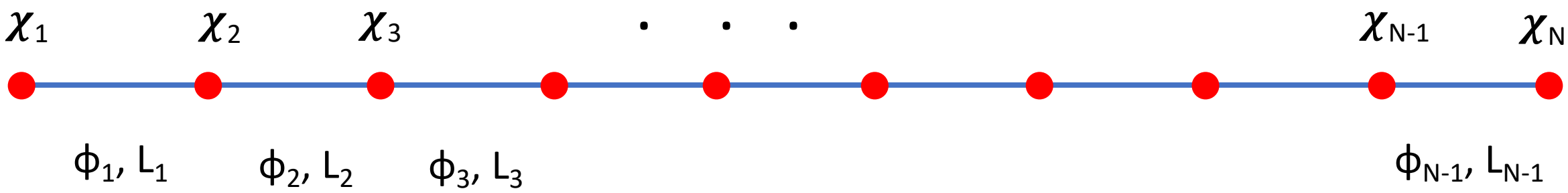
[Kogut-Susskind '75]

Gauge fields on links: (rescaled)

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$

Lattice (anti-) commutation relations:

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0, \ [\phi_n, L_m] = i\delta_{mn}$$



Lattice Hamiltonian:

$$H = -\mathrm{i}\sum_{n=1}^{N-1}\left(w-(-1)^n\frac{m}{2}\sin\theta\right)\left[\chi_n^\dagger e^{\mathrm{i}\phi_n}\chi_{n+1}-\mathrm{h.c.}\right]$$
$$+m\cos\theta\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n+J\sum_{n=1}^{N-1}L_n^2,$$

Where, $w = 1/(2a)$ and $J = g^2a/2$

Eliminate gauge degrees of freedom

To regularize the theory and remove infinite dimensional Hilbert space for gauge fields –

1. Lattice discretization
2. Use Gauss' Law and taken open boundary condition

- Gauss' Law for QFT: $0 = -\partial_1 \Pi - g\bar{\psi}\gamma^0\psi$

  On lattice it becomes - $L_n - L_{n-1} = \chi_n^\dagger \chi n - \frac{1-(-1)^n}{2}$

- Solve Gauss' Law by taking open B.C:

$$L_n = L_0 + \frac{1}{2}\sum_{\ell=1}^{n}\left(Z_\ell + (-1)^\ell\right) \qquad \text{where} \qquad L_0 = 0$$

- Now redefine staggered fermionic field : $\chi_n \to \prod_{\ell<n}\left[e^{-i\phi_\ell}\right]\chi_n$

Moving forward towards higher dimension

Multiple ways of handling gauge degrees of freedom

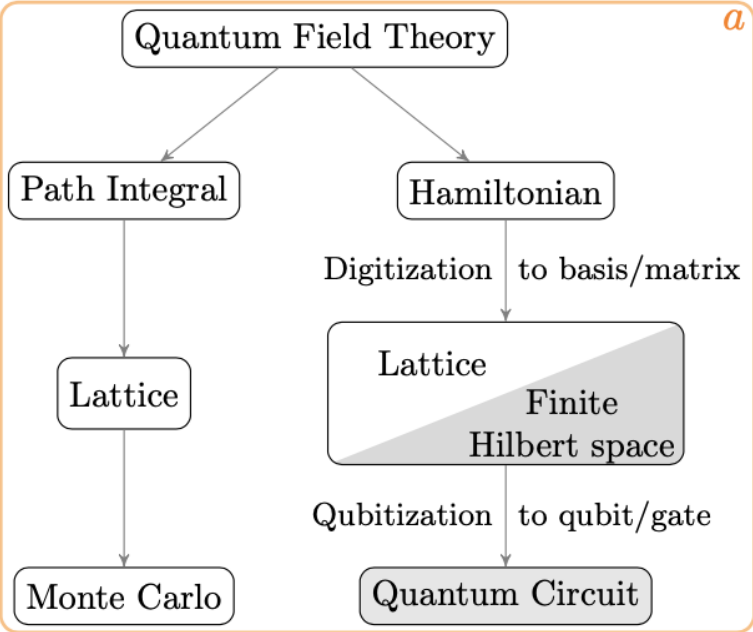
- Quantum link model or qubitisation formulations of gauge theories
- Irreducible representation or electric-field bases
- Prepotential and loop-string-hadron bases
- Group-element bases
- Dual or magnetic basis which diagonalises the magnetic Hamiltonian
- Light-cone quantisation of fields
- Matrix models using dimensional reduction

Ways of handling fermions

For near term devices, variational methods preferred compared to adiabatic approaches

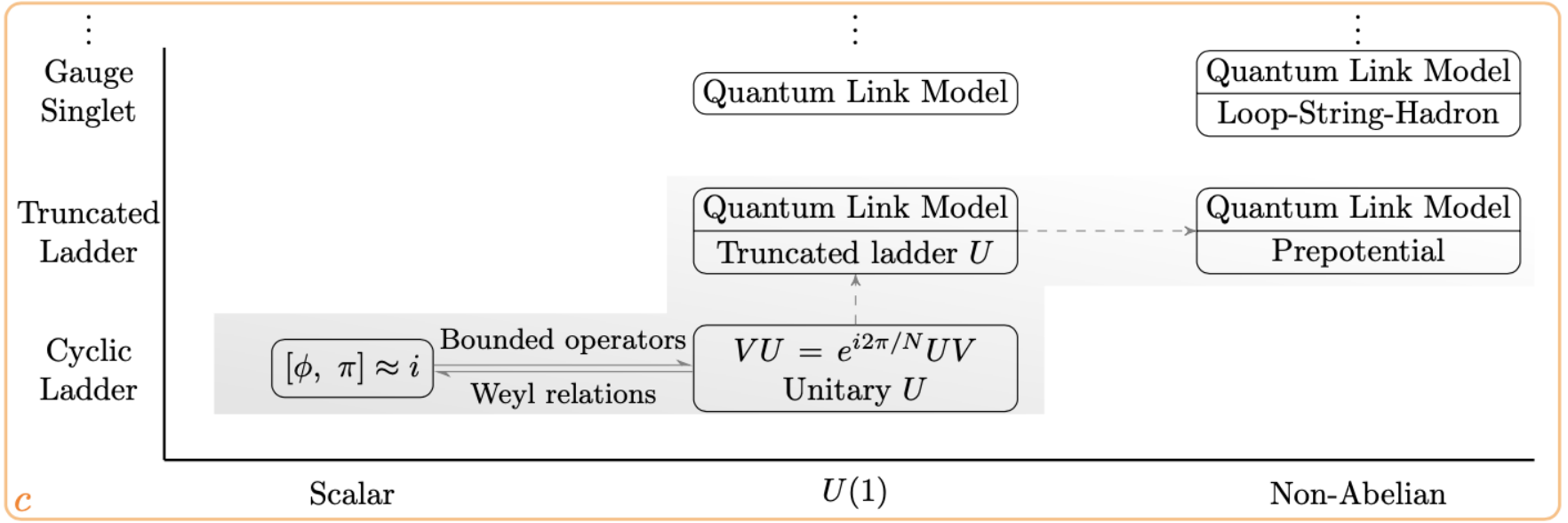
L. Funcke, PoS LATTICE2022, 228 (2023)
J. Osborne et. al. arXiv:2403.08858v1 (2024)
Z. Davoudi, arXiv:2507.15840 (2025)

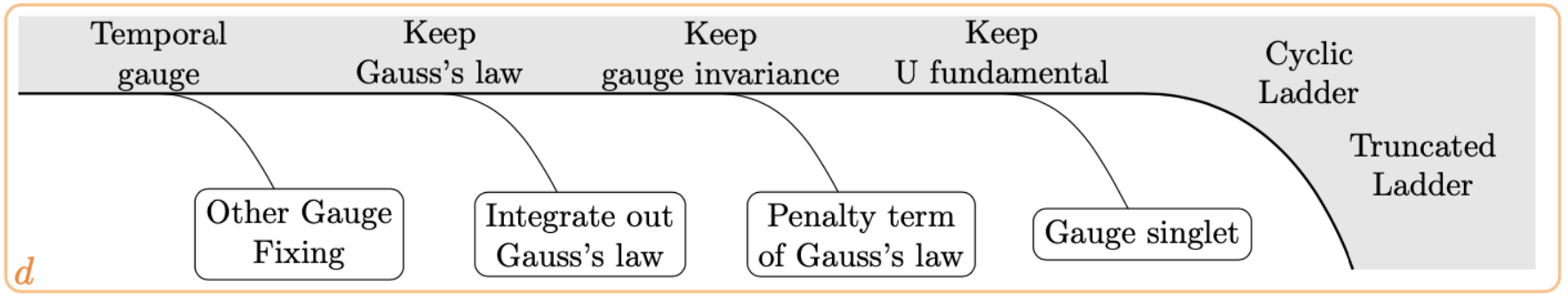
Candidates for quantum simulation of gauge theories



b

$dx \to 0$	Continuum limit
$L \to \infty$	Infinite volume
$dt \to 0$	Lie-Trotter-Suzuki limit
$N \to \infty$	Canonical Commutation Relation
	Z_N-gauge to compact QED
	Infinite energy range





Z. Mou, B. Chakraborty, arXiv: 2510.27668 (2025)
Zohreh Davoudi, arXiv:2507.15840 (2025).

QED Hamiltonian in the continuum

Hamiltonian in Temporal gauge:

$$H = \int \mathrm{d}^3x \left[\frac{1}{2}\boldsymbol{\Pi}^2 + \frac{1}{2}\boldsymbol{B}^2 + \bar{\psi}\gamma^i \left(\partial_i - igA_i\right)\psi + m\bar{\psi}\psi \right]$$

Where canonical commutation relation:

$$\left[A_i(x), \Pi^j(y)\right] = i\delta_i^j \delta^3(x-y), \; \left\{\psi(x), \psi^\dagger(y)\right\} = \delta^3(x-y)$$

Electric field: $\boldsymbol{E} = -\boldsymbol{\Pi}$ Magnetic field: $\boldsymbol{B} = \nabla \times \boldsymbol{A}$

Remaining constraint – Gauss's law: $\nabla \cdot \boldsymbol{\Pi} + g\psi^\dagger\psi = 0$

Lattice discretisation (naive)

$$H = \mathrm{d}^3x \sum_x \left[\frac{1}{2} \sum_i \Pi_i^2 + \frac{1}{4g^2} \sum_{i,j} \frac{2 - P_{ij} - P_{ji}}{(\mathrm{d}x^i \mathrm{d}x^j)^2} \right.$$

$$\left. + \sum_i \bar{\psi}(x) \gamma^i \frac{U_i(x)\psi(x+i) - U_i^\dagger(x-i)\psi(x-i)}{2\mathrm{d}x} + m\bar{\psi}\psi \right]$$

where,

$$P_{ij}(x) = U_i(x) U_j(x+i) U_i^\dagger(x+j) U_j^\dagger(x)$$

$$U_i(x) \sim \exp\left(-igA_i(x)\mathrm{d}x\right)$$

$$\mathrm{d}x \to 0$$

Non-trivial commutation relations in discrete form:

$$[\Pi_i(x), U_j(y)] = -\frac{g}{\mathrm{d}^2x} \delta_{ij} \delta_{x,y} U_j(y),$$

$$\{\psi_\alpha(x), \psi_\beta^\dagger(y)\} = \frac{1}{\mathrm{d}^3x} \delta_{\alpha\beta} \delta_{x,y}.$$

Gauss's law on a single site: $G(x) \equiv \sum_i \frac{\Pi_i(x) - \Pi_i(x-i)}{\mathrm{d}x} + g\psi^\dagger(x)\psi(x) = 0$

Discretisation of the representation

After rescaling, commutator algebra: $[\Pi,\ U] = -U, \quad [\Pi,\ U^\dagger] = U^\dagger, \quad [U,\ U^\dagger] = 0.$

$$\Pi = \begin{pmatrix} \ddots & & & & \\ & -1 & & & \\ & & 0 & & \\ & & & 1 & \\ & & & & \ddots \end{pmatrix}, \quad U = \begin{pmatrix} \ddots & \ddots & & & \\ & 0 & 1 & & \\ & & 0 & 1 & \\ & & & 0 & \ddots \\ & & & & \ddots \end{pmatrix}$$

Ladder operators in Electric basis

Hard truncation violates Unitarity of the gauge fields:

$$[\Pi,\ U'] = -U', \quad [\Pi,\ U'^\dagger] = U'^\dagger, \quad [U',\ U'^\dagger] \approx 0,$$

$$\Pi = \begin{pmatrix} -2 & & & \\ & -1 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}, \quad U' = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Our formalism

Restoring the unitarity will violate the commutation relation between the electric field and the gauge-link field $[\Pi,\ U] \approx -U, \quad [\Pi,\ U^\dagger] \approx U^\dagger, \quad [U,\ U^\dagger] = 0$

e.g. $\Pi = \begin{pmatrix} -2 & & & \\ & -1 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

We rephrase the unbounded operators into bounded ones

$$[\Pi, U] \approx -U \ \Leftrightarrow e^{-i\alpha\Pi} U e^{i\alpha\Pi} = e^{-i\alpha} U$$

→ U(1) gauge theory is reduced to Z_N theory in the unitary matrix approach and will Be recovered in the large-N limit (*Phys. Rev. D 19 (1979) 3715.*)

For a matrix M of a large dimension, the precise qubit implementation of the rotation will rely on the eigen-decomposition

$$e^{-i\theta f(M,M^\dagger)} = \mathscr{U}^\dagger e^{-i\theta f(\Lambda,\Lambda^\dagger)} \mathscr{U}, \quad M = \mathscr{U}^\dagger \Lambda \mathscr{U}$$

So, the challenges are: (1) whether we can find the eigen-decomposition and (2) whether we can implement the unitary U exactly.

Unitary matrix and Fourier Transform

	Canonical		Gauge-Link		Weyl
Infinite	$[A,\Pi]=i$ Hermitian A Hermitian Π	$\Leftrightarrow$	$[\Pi,U]=-U$ unitary U Hermitian Π	$\Leftrightarrow$	$V^\alpha U^\beta = e^{i\alpha\beta}U^\beta V^\alpha$ unitary U unitary V
Finite			$[\Pi,U]=-U$ truncated U		
	$[A,\Pi]\approx i$	$\Leftrightarrow$	$[\Pi,U]\approx -U$	$\Leftrightarrow$	$V^k U^m = e^{i\frac{2km\pi}{N}} U^m V^k$

Note:
The unitary representation of the gauge-link field coincides with both the standard digitisation of the scalar field (*Science 336 (2012) 1130–1133*) and the finite representation of Weyl commutation relation (*Weyl, 1950; Found Phys 6 (1976) 583–587*).

A finite dimensional representation exists via Sylvester's generalization of Pauli matrices with the shift matrix and the clock matrix, we can choose:

$$U = -F^\dagger \exp\left(-i\frac{2\pi}{N}\Pi\right) F, \quad V = \exp\left(-i\frac{2\pi}{N}\Pi\right)$$

$$V^k U^m = U^m V^k e^{i2mk\pi/N}.$$

For example, for N = 4, we get

$$\Pi = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$F = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix}, \quad \omega = e^{i2\pi/4}.$$

The advantages of our formulation:

- Weyl commutation relation is so that the setup can well be constructed up to the $N \to \infty$ limit under which we will not only recover the canonical commutation relation but also restore the U(1) gauge.

- The Fourier transform matrix F allows us to switch the eigen-basis between the electric and gauge potential fields ⇒ crucial when we construct the quantum circuit.

Z. Mou, B. Chakraborty, arXiv: 2510.27668 (2025)

Truncated matrix

Hard truncated matrix U doesn't directly admit the eigen-decomposition; in the end, it is the exponent f(M,M†) that we wish to decompose.

This includes the gauge-fermion interaction which will be in a form

$$\begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix} = \begin{pmatrix} \mathscr{V} & 0 \\ 0 & \mathscr{W} \end{pmatrix} \mathbf{H} \begin{pmatrix} S & \\ & -S \end{pmatrix} \mathbf{H}^\dagger \begin{pmatrix} \mathscr{V}^\dagger & 0 \\ 0 & \mathscr{W}^\dagger \end{pmatrix}$$

The plaquette term: we split a ladder matrix into two nilpotent matrices where $N^2 = 0$

$$\begin{pmatrix} & N+N^\dagger \\ N+N^\dagger & \end{pmatrix} = \begin{pmatrix} & N \\ N^\dagger & \end{pmatrix} \begin{pmatrix} & N^\dagger \\ N & \end{pmatrix}$$

Therefore, it can be shown that we can introduce an ancilla to perform the plaquette interaction.

QED in qubit

Qubitisation of gauge fields:

Via the Fourier transform, we can simply switch between the electric basis and the potential basis

$$\Pi = -\frac{1}{2}\sum_{j=0}^{n-1} 2^j Z_j - \frac{1}{2}, \text{ with } Z_j \equiv \underset{(n-1)\text{-th}}{I} \otimes \cdots \overset{j\text{-th from right}}{Z} \cdots \otimes \underset{0\text{-th}}{I}$$

$$U = F^\dagger \exp\left(-i\frac{2\pi}{N}\left(\Pi + \frac{N}{2}\right)\right) F.$$

- In the potential basis, the U- relevant terms is in a diagonal shape, and the relevant Trotterization consists of only rotations of Z's, so can be implemented precisely.

- This precise implementation is crucial, as the Gauss's law is satisfied only when each elementary Trotter term is done exactly.

Thus, we reply on two basic operations – Quantum Fourier Transform, and multi-Z rotations and "stator" method (*Phys. Rev. A 65 no. 3, (2002); Phys. Rev. A 95 no. 2, (2017) ; Phys. Rev. Lett. 118 no. 7, (2017) 070501*) can reduce the number of CX gates with the use of ancillary qubits.

QED in qubit

Qubitisation of fermionic fields:

A typical gauge-fermion interaction term can be split into three two-field-operations through

$$\exp\left(-i\frac{\theta}{2}\left[i\phi_P^\dagger U_{PQ}\phi_Q - i\phi_Q^\dagger U_{PQ}^\dagger \phi_P\right]\right)$$
$$= R_P \exp\left(-i\frac{\theta}{2}\left[i\phi_P^\dagger \phi_Q - i\phi_Q^\dagger \phi_P\right]\right) R_P^\dagger,$$

with the unitary transformation

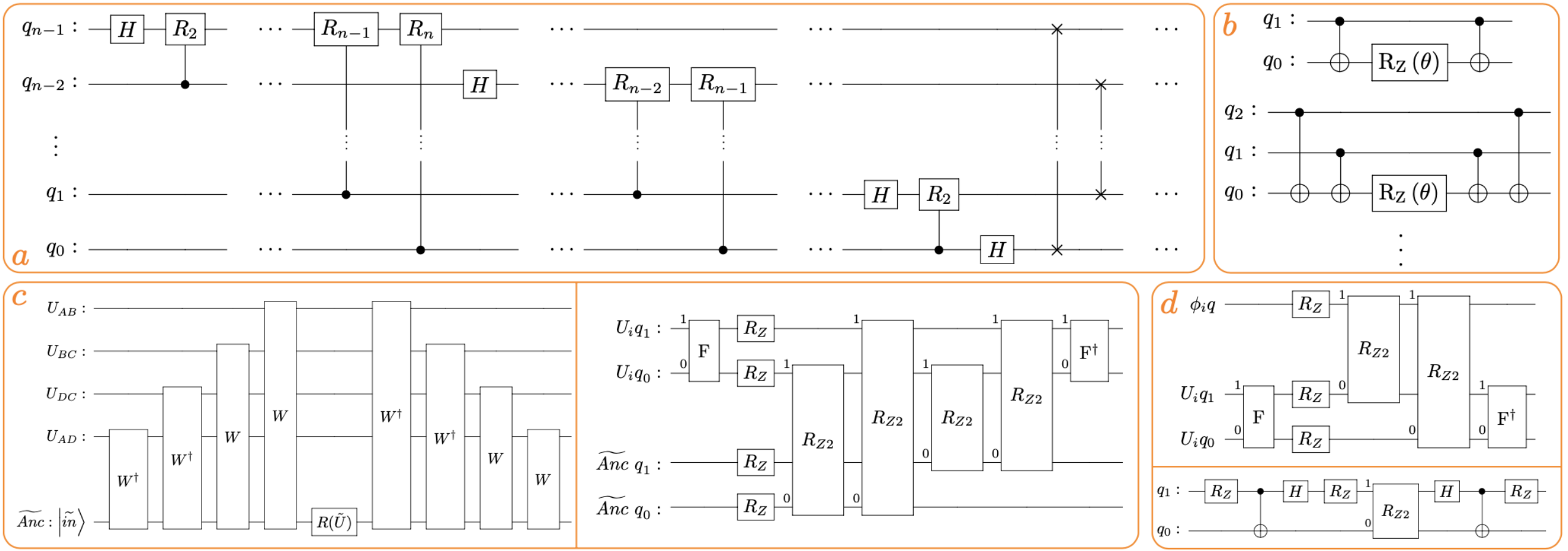
$$R_P = \begin{pmatrix} U_{PQ} & \\ & I \end{pmatrix} = \exp\left(\phi_P^\dagger \phi_P \ln(U_{PQ})\right)$$

Resources for one Trotter step

	2D			3D		
	n=1	n=2	n=3	n=1	n=2	n=3
Qubit:	9	14	19	22	36	50
CZ:	75	244	734	598	1644	4481
Depth:	172	399	894	740	1776	3454

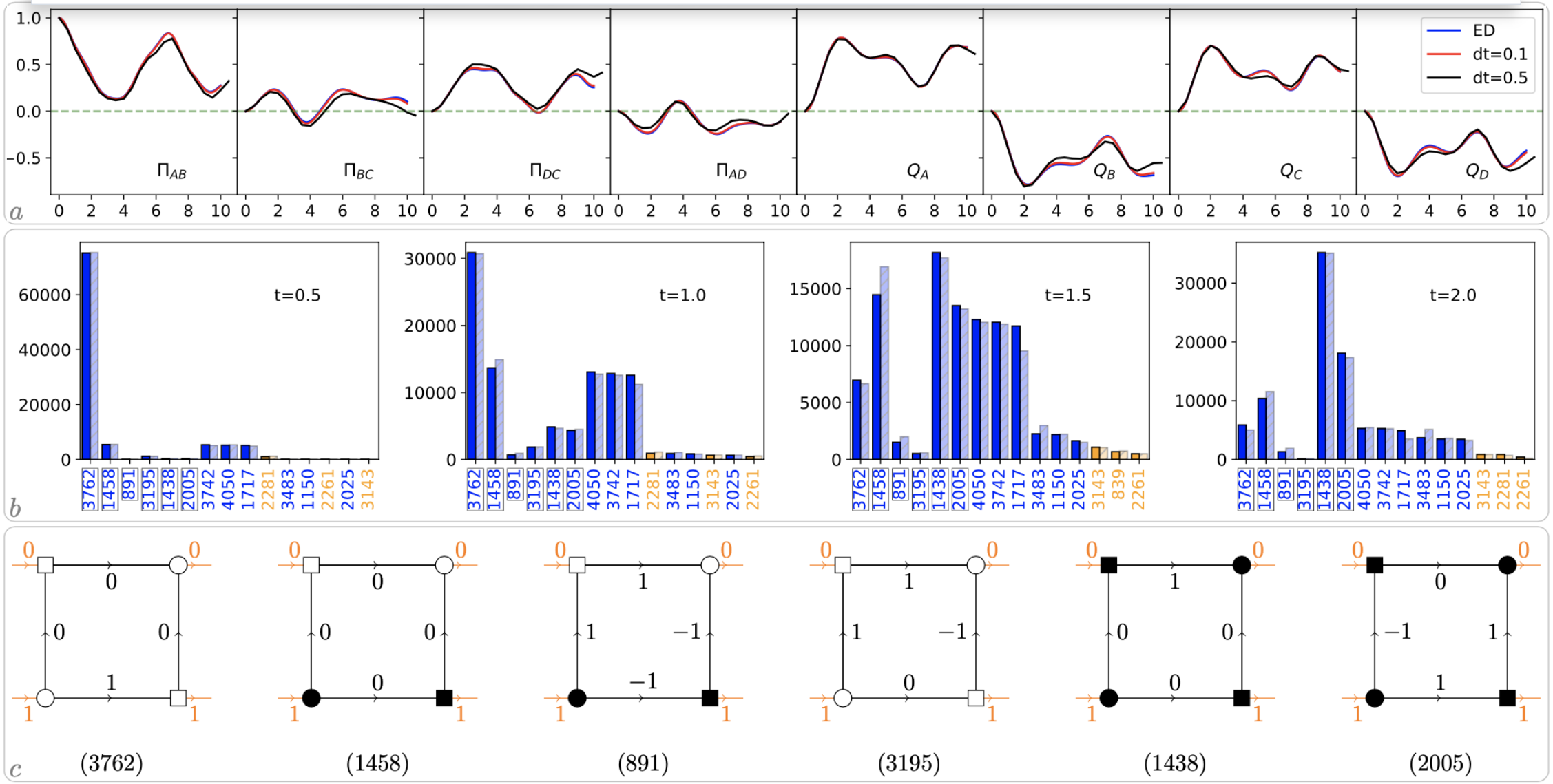
Exact Diagonalization + Qiskit + IBM
Qiskit + IBM
IBM

An illustration of quantum circuits used



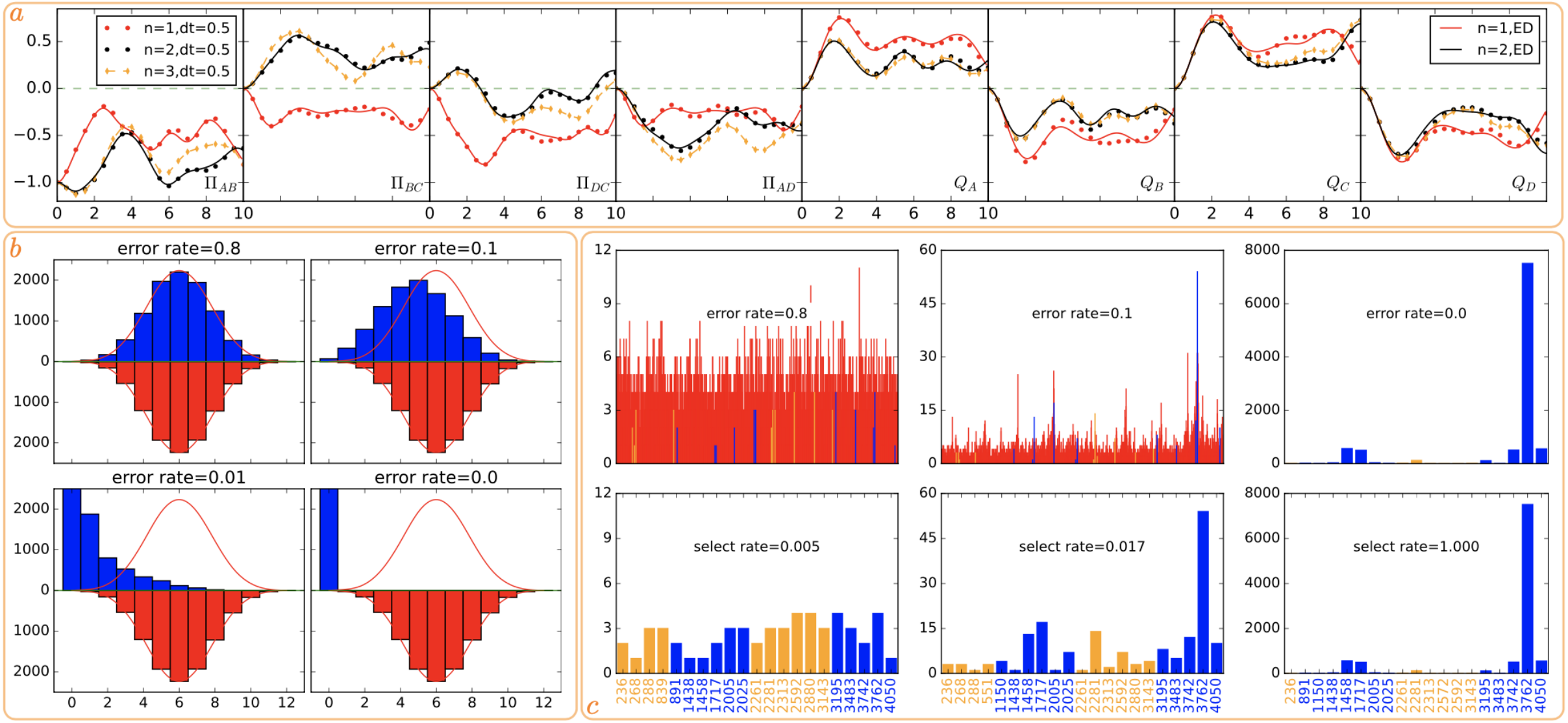
(a) Quantum Fourier Transform. (b) multi-Z rotations. (c) The plaquette rotation in the "stator" method: (L) the four links and the ancilla; (R) the W-operation illustrated with 2-qubit for the gauge degree. (d) The gauge-fermion rotation: (Upper) the unitary transformation which decouples the gauge degree; (Lower) the two-fermion rotation.

ED & QISKIT Simulation and results



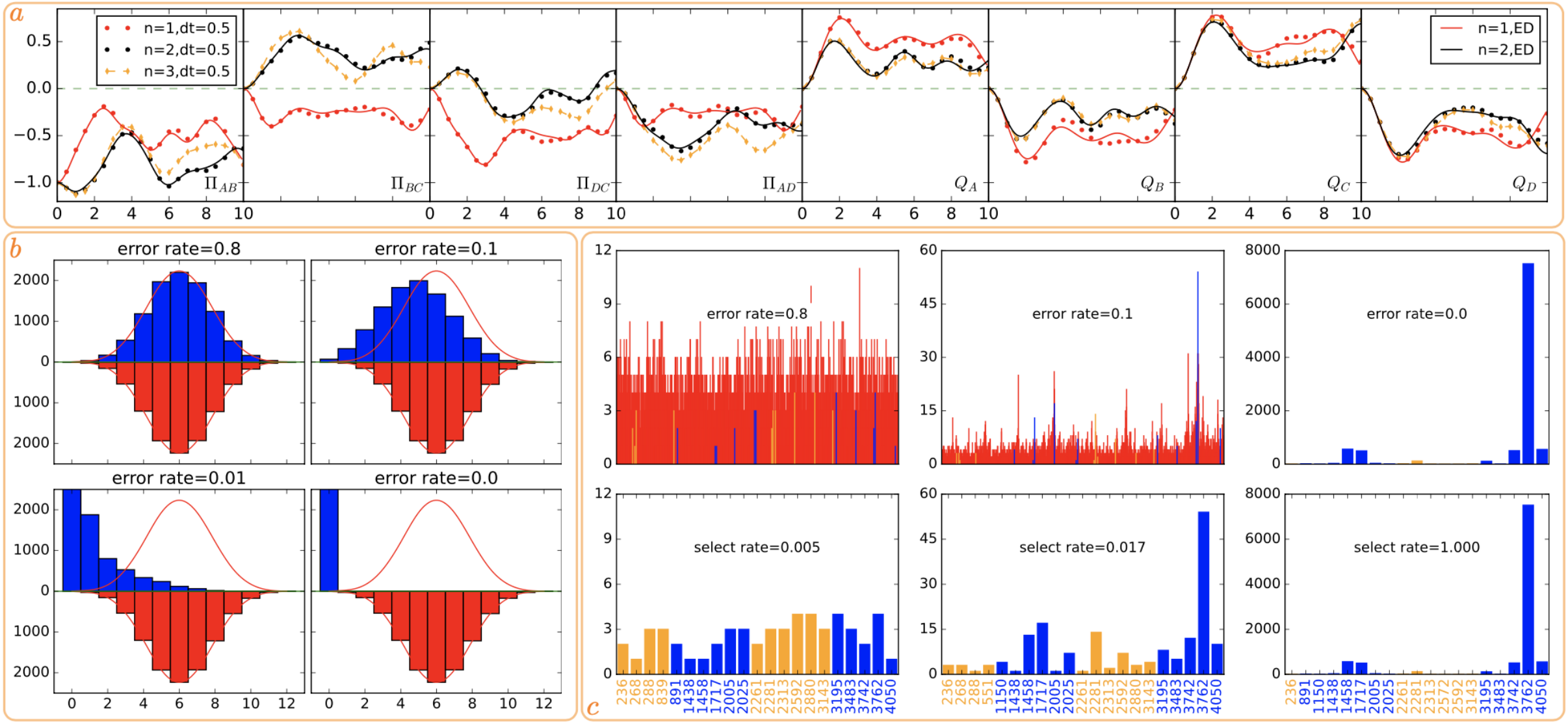
(a) The evolution of the observables over time. Results from ED and Qiskit simulation with dt = 0.1 and 0.5 are plotted in blue, red and black individually, (b) The histogram of the states over time, (c) An illustration of the dominating states.

QISKIT Simulation and results (2D square)



(a) The observables, including electric fields and fermion charges, evolve along time on the 2D square, (b) The distance (or the bit-flip number) plot with varying quantum error rate in the reference/trivial runs, simulated via Qiskit. For a contrast, the worst case, where the measurement is totally random, is plotted in red. (c) The histogram of the final states in the target runs.

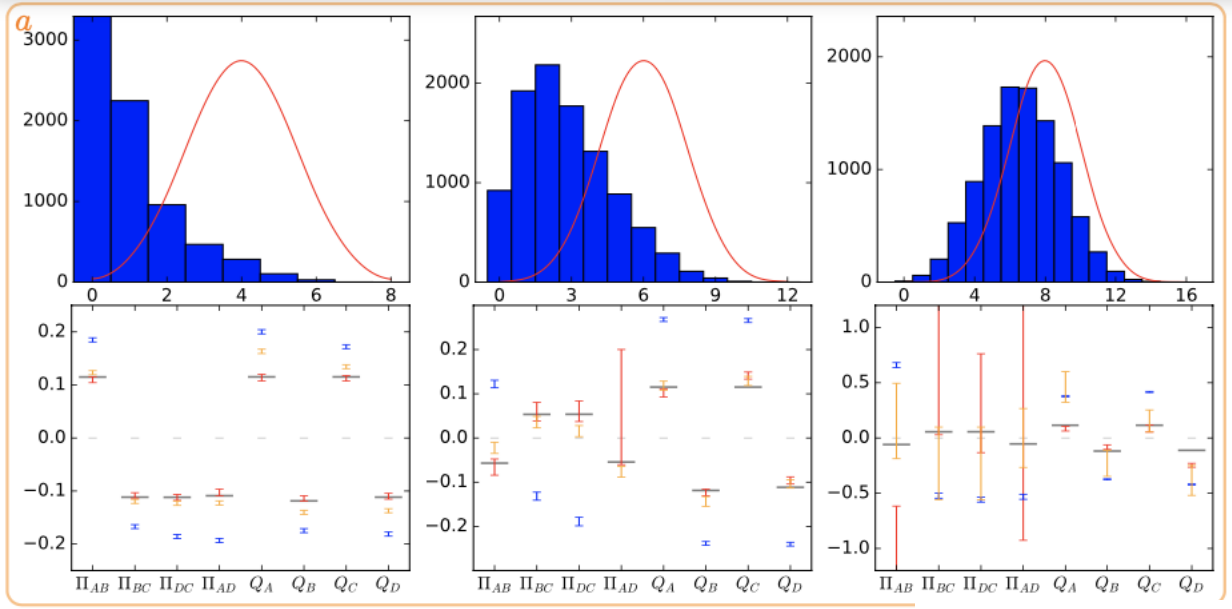
QISKIT Simulation and results (2D square)



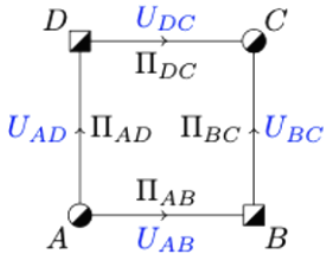
QEM techniques: 1. Post-selection method

2. Calibration method (Operator Decoherence Renormalization method)

IBM computation and results (2D square)

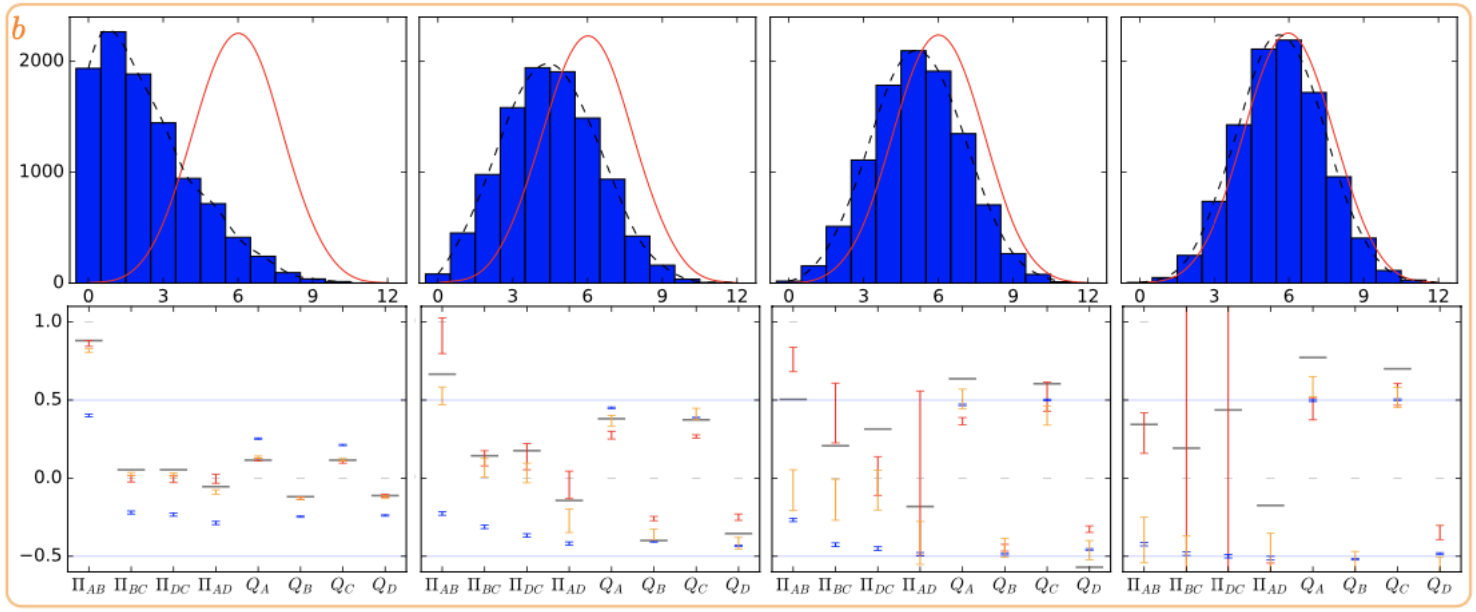


Results for n = 1, 2, 3 (left, middle, right): the distance (upper) and the observables (lower), where the blue, red and orange error bars denote respectively the results from the direct, calibrated and post-selected measurements, and the exact values are drawn as the grey bars.

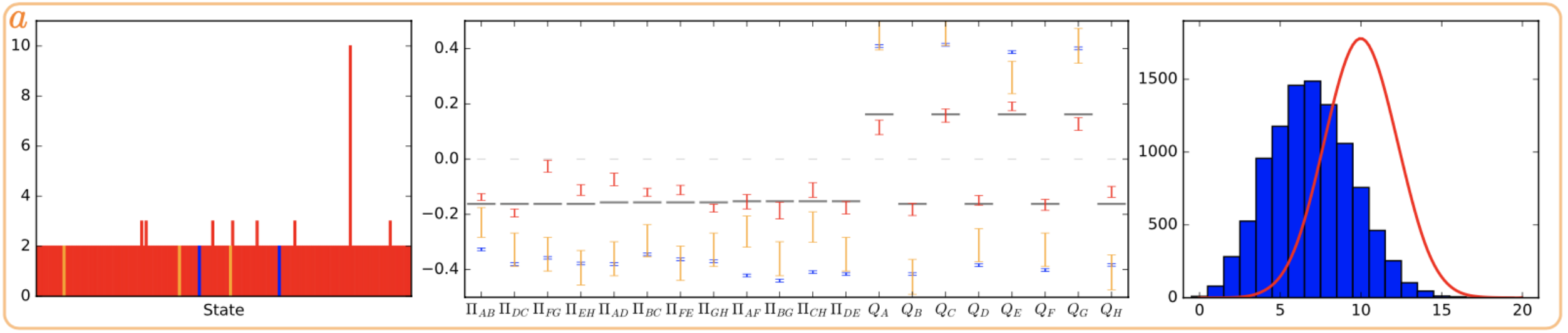


n	1	2	3
Qubit	9	14	19
Depth	172	399	894
CZ	75	244	734

Results for 1, 2, 3, and 4 Trotter steps, of n = 2.



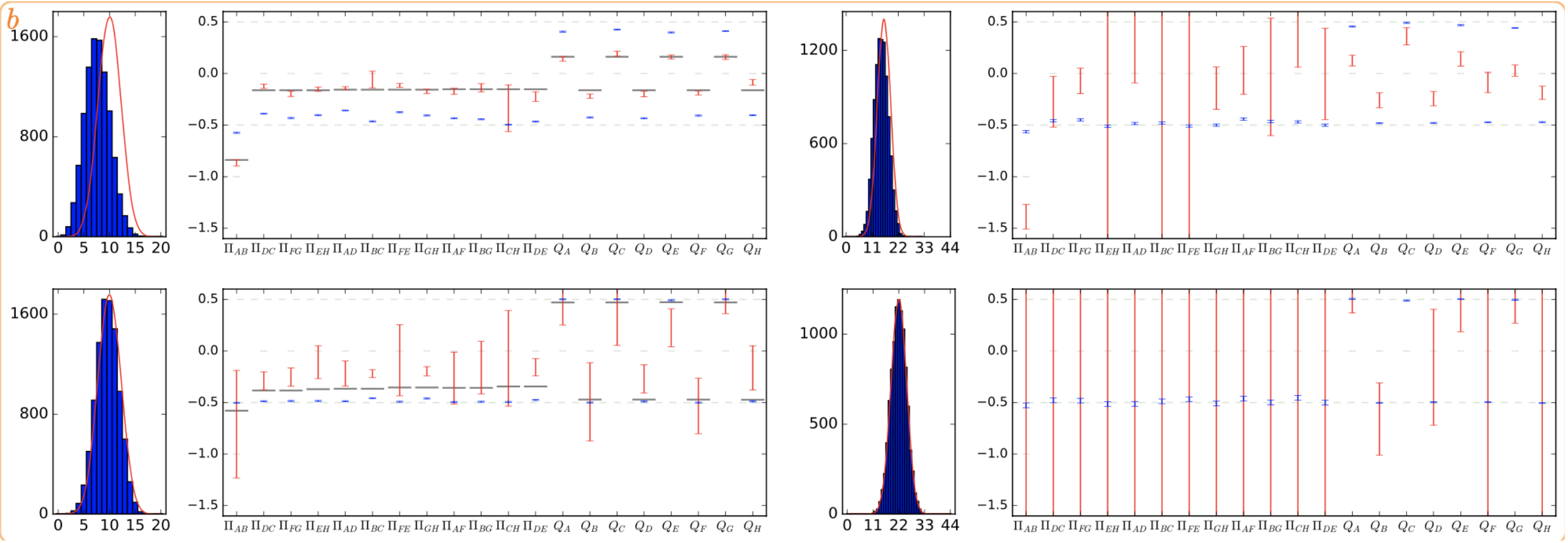
IBM computation and results (3D cube)



n = 1: the histogram of the final states (Left), the distance (Right) and the observables (Middle), where the blue, red and orange error bars denote respectively the results from the direct, calibrated and post-selected measurements, and for comparison, the exact values are drawn as the grey bars.

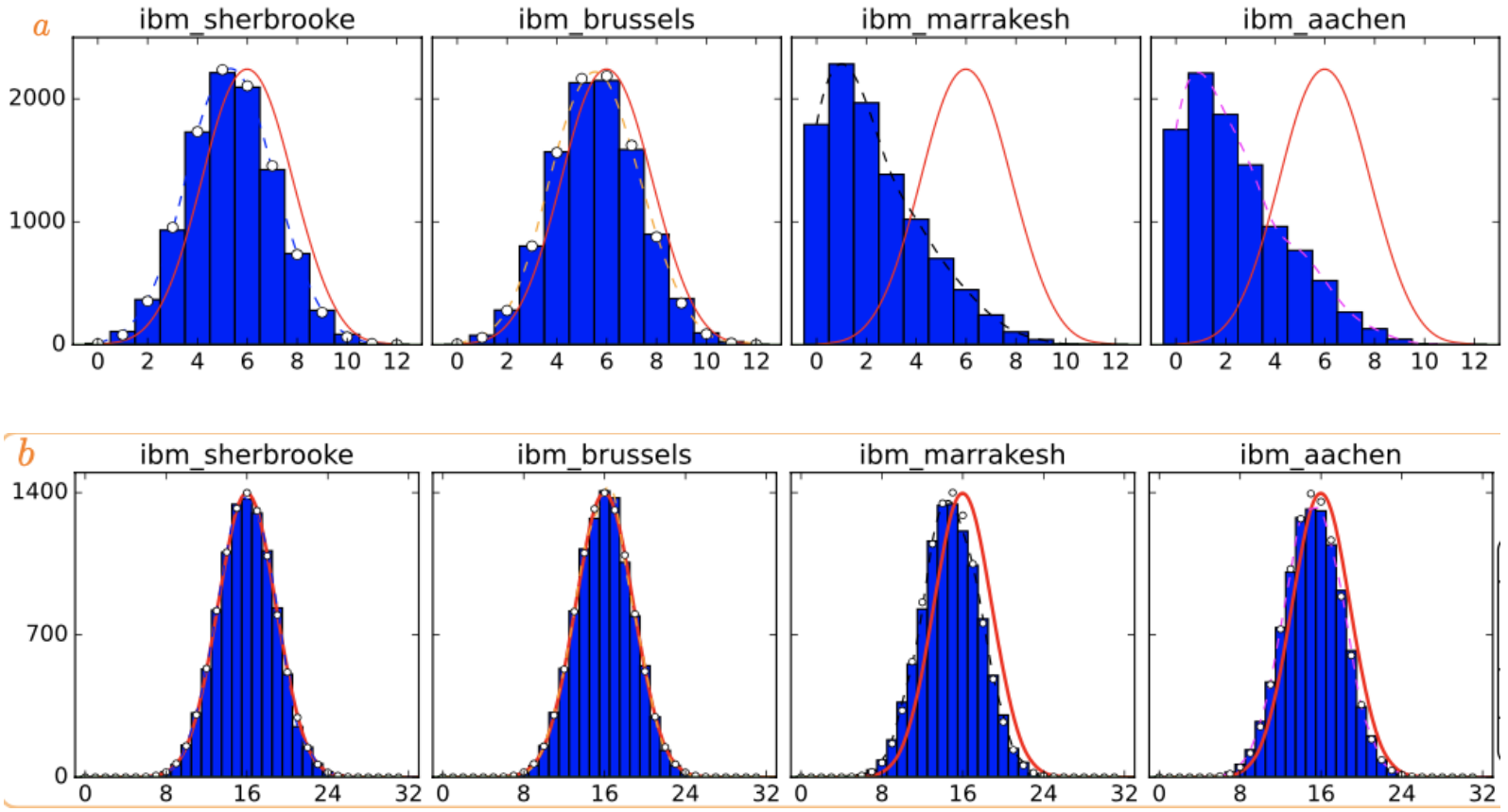
n	Qubit	Depth	CZ
1	22	740	598
2	36	1776	1644
3	50	3454	4481
1	22	1441	1263

IBM computation and results (3D cube)



Computations with initial state of ΠAB = −1: (Left) n = 1 with one Trotter step (upper) and two Trotter steps (lower); (Right) n = 2 (upper) and n = 3 (lower) with one Trotter step.

IBM results across platforms



IBM	sherbrooke	brussels	marrakesh	aachen
Type	Eagle r3 127 150K	Eagle r3 127 220K	Heron r2 156 195K	Heron r2 156 250K
Depth	560	548	389	383
Gates	(ecr,257)	(ecr,257)	(cz,244)	(cz,244)

2D

3D

IBM	sherbrooke	brussels	marrakesh	aachen
Type	Eagle r3 127 150K	Eagle r3 127 220K	Heron r2 156 195K	Heron r2 156 250K
Depth	2293	2282	1591	1600
Gates	(ecr,1679)	(ecr,1679)	(cz,1652)	(cz,1652)

n = 2 and one Trotter step: (a) 2D Square (b) 3D Cube

Summary

- We have demonstrated that QED on 2+1D and 3+1D can be implemented into qubit, with the quantum algorithm similar to the scalar field theory – "Standard approach".

- Gauss's law constraint dissolves automatically when the implementation is completely gauge invariant.

- The advantage of the approach is that the setup can be easily extended to a large representation dimension N of the gauge field.

- We studies two QEM methods in details – Post-selection is highly disfavoured, and the calibration method works making good use the spreading distribution of the states.

- We can obtain a clear signal on Heron platform for the 2+1D setup with a few Trotter steps. For the 3+1D setup, we can see an obvious improvement from Eagle to Heron platforms.

Our scalable quantum simulation may provide a new route toward understanding thermalisation and constrained dynamics in higher-dimensional QED.