Hierarchy in Quiver Models

Leonardo de Lima

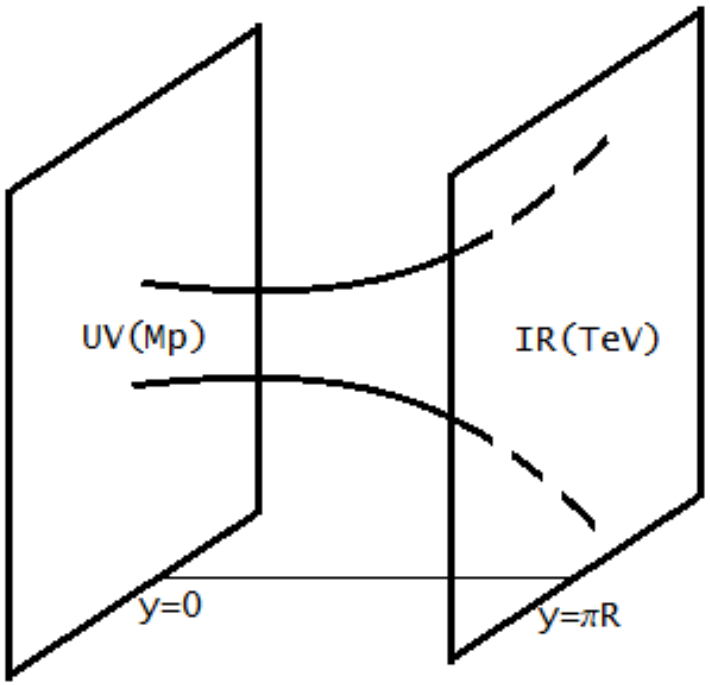
with Gustavo Burdman

and Nayara Fonseca de Sá

São Paulo University

The Randall-Sundrum Model

- Describes a 5-dimensional universe with the geometry of a slice of $AdS_5/\mathbb{Z}_2$:



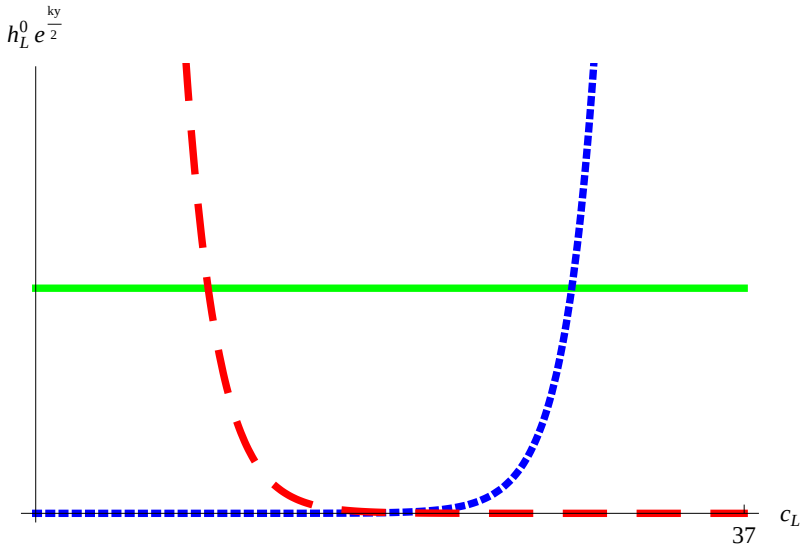
- Metric:
  $g_{MN}dx^M dx^N = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$,
  $0 < y < \pi R$
- Energy scales are naturally warped down by the factor: $e^{-k\pi R}$
- With the Higgs localized at the IR brane, the gauge hierarchy problem is solved for $k\pi R \sim 37$.

The Randall-Sundrum Model

- The fermionic KK modes have the profile:

$$h^{(0)}_{L(R)}(y) = N(c)e^{(1/2\mp c)ky},$$

- Fermion masses are determined by the localization (bulk mass) parameter, of order $\mathcal{O}(1)$.



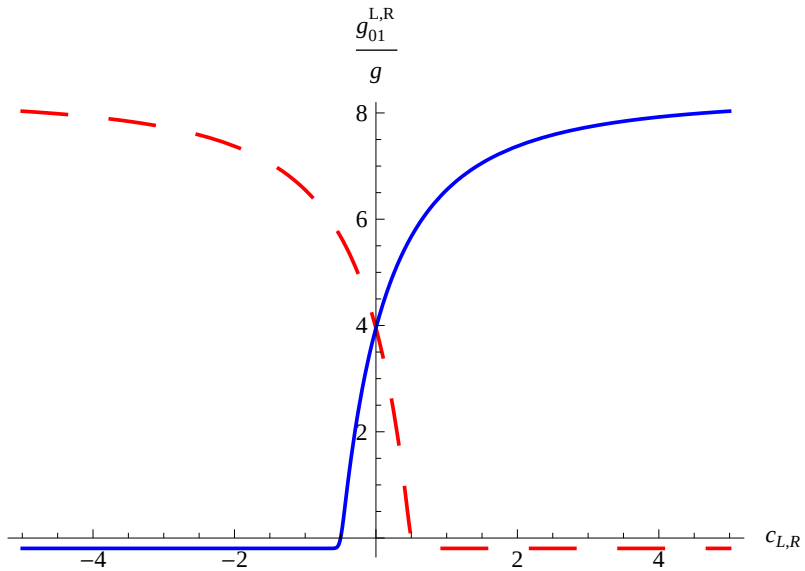
$h_L^{(0)}$ by c_L. Presented are $c_L = 0.5$ (green), $c_L = 1.1$ (red), $c_L = -0.1$ (blue).

The Randall-Sundrum Model

- Gauge couplings come from the integral:

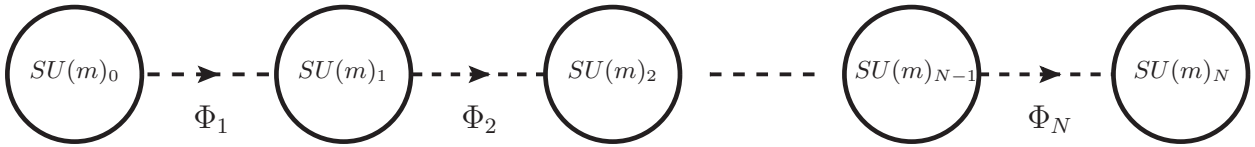
$$g^{L,R}_{(0,1)} = \frac{g}{(\pi R)} \int_0^{\pi R} dy\, e^{ky} f^{(1)}(y) |h^{(0)}_{L,R}(y)|^2$$

- UV couplings are universal, mostly supressing flavor violation (RS-GIM).
- However, it's not enough for Kaon mixing (Csáki, C. *et al.* - 2008).
  $\Rightarrow m_{KK} \gtrsim 30$ TeV.



g^L_{01} (red) and g^R_{01} (blue).

Dimensional Deconstruction



- A four dimensional gauge theory with N+1 gauge groups: $G = G_0 \times G_1 \times \ldots \times G_{N-1} \times G_N$.
- Gauge fields are conected by scalars, called link fields, transforming as: $\Phi_j \to L_{j-1}\Phi_j R_j^\dagger$.
- If all groups and couplings are equal, this linear moose mimics an extra dimension of size $L = (N+1)\ell$ for large N.
- This is the idea of Dimensional Deconstruction (Arkani-Hamed et al. 2001 and Hill et al. 2001).

Dimensional Deconstruction

- The gauge plus links action is:

$$S_4 = \int d^4x \sum_{j=0}^{N} \left\{ -\frac{1}{2g^2} \mathrm{Tr}(F_{\mu\nu,ja}\, F_j^{\mu\nu a}) + \mathrm{Tr}\left[(\mathcal{D}_\mu \Phi_j)^\dagger (\mathcal{D}^\mu \Phi_j) \right] - V(\Phi) \right\}$$

  with $\mathcal{D}_\mu \Phi_j = \partial_\mu\, \Phi_j + i\, A_{\mu,j-1a}\, T^a\, \Phi_j - i\, \Phi_j\, A_{\mu,ja}\, T^a$.

- We assume a potential that gives the Φ_j diagonal vevs, breaking $SU(m)^j_L \times SU(m)^j_R \to SU(m)^j_V$.
- The massive state (KK mode like) tower is truncated at N+1 states.

Dictionary

- This model can be matched onto a discretized Randall-Sundrum model:

$$S_5 = \frac{\ell}{g_5^2}\int d^4x \sum_{j=0}^{N}\left[-\frac{1}{2}\mathrm{Tr}(F_{\mu\nu,j}F_j^{\mu\nu}) + \frac{1}{2}e^{-2k\ell j}\mathrm{Tr}\left(\frac{A_{\mu,j}-A_{\mu,j-1}}{\ell}\right)^2\right]$$

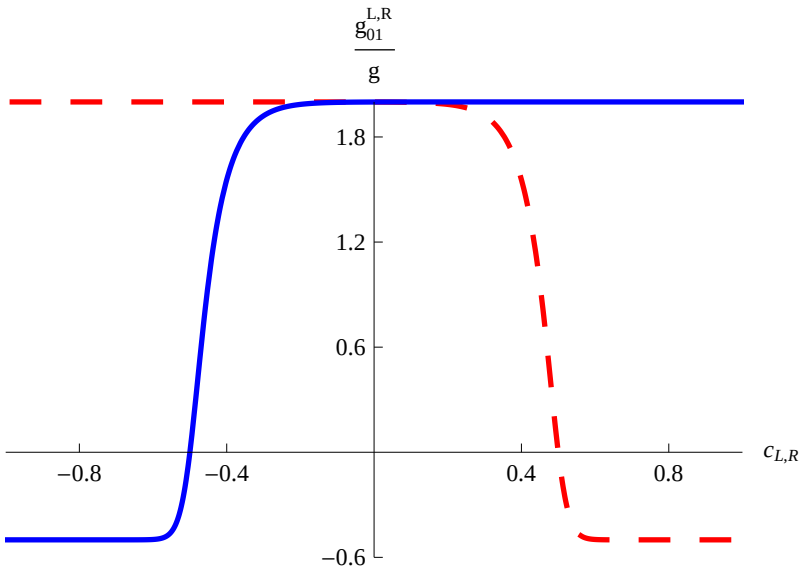
- The AdS_5 warping can be mimicked if the vevs scale as: $v_j = vq^j,\ 0 < q < 1$.
- Setting up a dictionary:

$$\begin{array}{ccc} vq^j & \leftrightarrow & \frac{e^{-k\ell j}}{\ell g_5} \\ \frac{1}{g^2} & \leftrightarrow & \frac{\ell}{g_5^2} \end{array}$$

- The gauge hierarchy is generated if: $q = e^{-k\ell} \sim e^{-\frac{16\ln 10}{N}} \simeq e^{-\frac{37}{N}}$ and $v \sim M_P = 10^{19}$ GeV.
- Fermion zero modes can be localized in quiver space, generating mass hierarchy (if the Higgs is near site N).

Gauge Couplings

- The gauge coupling is now given by the sum: $g_{01}^{L} = \sum_{k=0}^{N} g|h_{k,0}^{L}|^2 f_{k,1}$.

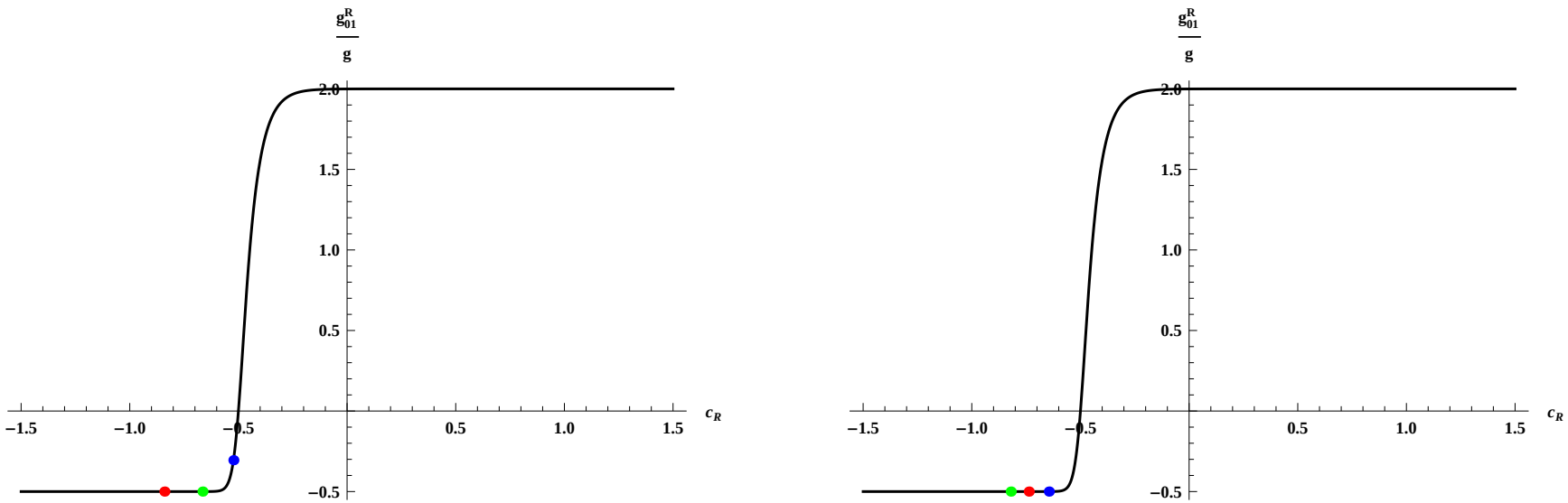


g_{01}^{L} (red) and g_{01}^{R} (blue) for $N = 4$.

- Quick IR coupling saturation for a model with few sites.
- $\Rightarrow$ FCNC are supressed.

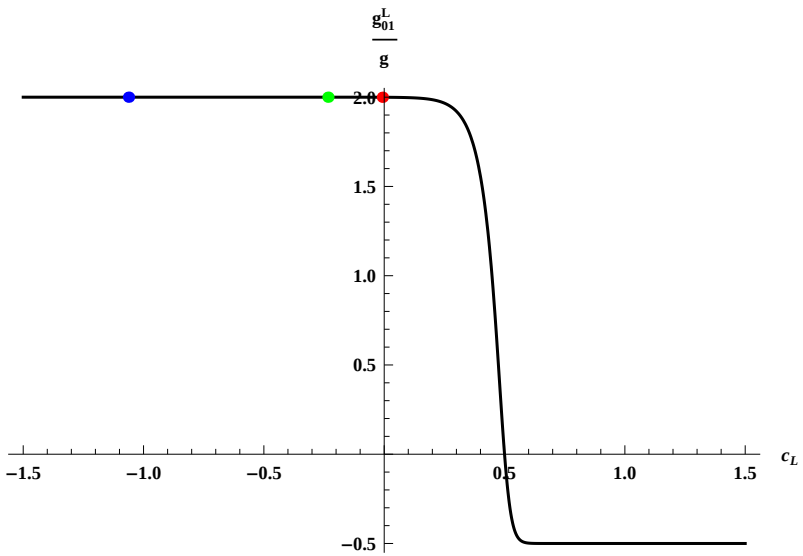
Flavor Violation

- We wish to find points in parameter space that have minimal flavor violation, and generate good masses and mixings.
- A scan (genetic algorithm) was made for the quark localizations and Yukawa parameters.
- One such solution is presented below:



g_{01}^R/g for up type (left) and down type (right) quarks, with $N = 4$.

Flavor Violation



g^L_{01}/g for quark doublets, with $N = 4$.

- For this particular solution, we get:
  $m_u = (172, 1.31, 2.34 \times 10^{-3})$ GeV $m_d = (4.64, 1.08 \times 10^{-1}, 4.08 \times 10^{-3})$ GeV
  $$CKM = \begin{pmatrix} 0.975 & 0.220 & 0.004 \\ 0.220 & 0.975 & 0.041 \\ 0.006 & 0.041 & 0.999 \end{pmatrix}$$

Flavor Violation

- Using this solution, we can compute e.g. the contribution to the neutral mesons mass splitting Δm, obtaining:

$$\Delta m_D \simeq 2.2 \times 10^{-20} \left(\frac{1\ \text{TeV}}{M_{KK}}\right)^2 \text{GeV} \ll 1.57 \times 10^{-14}\ \text{GeV}$$

$$\Delta m_K \simeq 2.6 \times 10^{-20} \left(\frac{1\ \text{TeV}}{M_{KK}}\right)^2 \text{GeV} \ll 3.48 \times 10^{-15}\ \text{GeV}$$

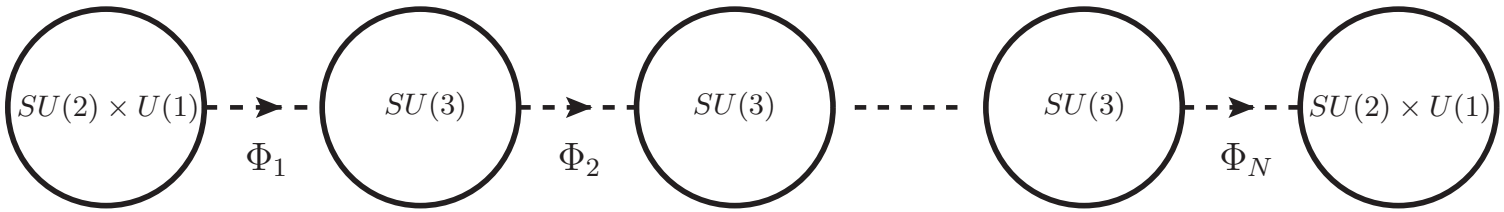
$$\Delta m_{B_s} \simeq 6.7 \times 10^{-18} \left(\frac{1\ \text{TeV}}{M_{KK}}\right)^2 \text{GeV} \ll 1.17 \times 10^{-11}\ \text{GeV}$$

Conclusions

- We obtain a model that solves the hierarchy problems with minimal flavor violation.
- The Higgs doublet can be dynamically localized.
- Phenomenological analysis is still being carried out...

A Toy Model

- We take the (electroweak) quiver:



- With only an $SU(2) \times U(1)$ subgroup of $SU(3)$ gauged at the tips, there remains an IR localized $SU(2)$ doublet that can't be eaten, the Higgs.
- This pseudo Nambu-Goldstone boson is naturally light (maybe too light).
- In the continuum limit, this corresponds to Gauge-Higgs unification.