

Intro

Refs: SUSY basics:

WB

Argyres

Terning

OSB:

Intǎlǎzǎn & Seiberg

Poppitz & Trivedi

Terning

GMSB:

~~MSSM~~ MSSM pheno, GWSB, ...

- Many ingredients → SUSY, OSB, ...
→ mediation, ...
- I cannot do justice to any of them, but I will try to give a comprehensive overview.
- Main goal is to give you a taste of everything,
so you can continue learning after the school.
- Before plan of lecture, let's discuss motivation

It's mid-2012, LHC running nearly 2 years,
Higgs-like ptcl discovered, still no sign of SUSY or gmsb

Why study SUSY models of EW scale?

1. Now that we've discovered (probably) the Higgs,
the hierarchy problem is more relevant than ever.
→ SUSY is, now more than ever, the best solution out there.

$$\frac{M_1^2}{M_2^2} \sim 10^{32}$$

2. Some of the pre-LHC motivations are still there:
 - gauge coupling unification
 - uniqueness (?) (Hag-Loparowski-Sokratis)
 - dark matter (?)

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3. Limits on colored sparticles are strong with simplified assumptions, but
- ⇒ EWinos & stops can still be light
 - ⇒ ~~stop~~ and other sparticles can still be light w/ less simplified assumptions (e.g. squashed sp)
 - ⇒ Evidently, SUSY was not just right around the corner — but life is never easy!!

- What does SUSY model of EW scale look like (with $m_t = 125 \text{ GeV}$)?

- If it's MSSM, either very fine tuned (minisplit $\gtrsim 10 \text{ TeV stop}$) or ~~or~~ somewhat fine tuned (maximal mix $\gtrsim 10^4 \text{ GeV}$)
That could be the way Nature is. Look @ nucleophilic
Solve big hierarchy $\sim 10^{32} \text{ } \checkmark$ e.g.
but not little hierarchy $\sim 10^{2-3}$, $m_b - m_d \text{ too}$
- Maybe it's nonminimal (NMSSM, nondec. D terms, ...)
and stops, etc are light
 - bonds on gluons, etc are still nontrivial
 - hard to make work w/ inf. perturbativity
- Maybe we don't understand fine-tuning.
Really need underlying theory for this!!

Intro

No matter what, the basic setup of any SUSY model is the same.

1) SUSY must be broken spontaneously
(typically in a hidden sector)

$$\langle X \rangle = \theta^2 F$$

2) SUSY must be realized softly in the SSM - dimensionful pars only

$$L_{\text{soft}} \sim m^2 \tilde{g}^+ \tilde{g}^- + M_2 \tilde{\chi}_2^0 + \dots$$

This way quadratic div. not reintroduced

$$\delta m_h^2 \propto m^2 \log \Lambda, \text{ not } \Lambda^2.$$

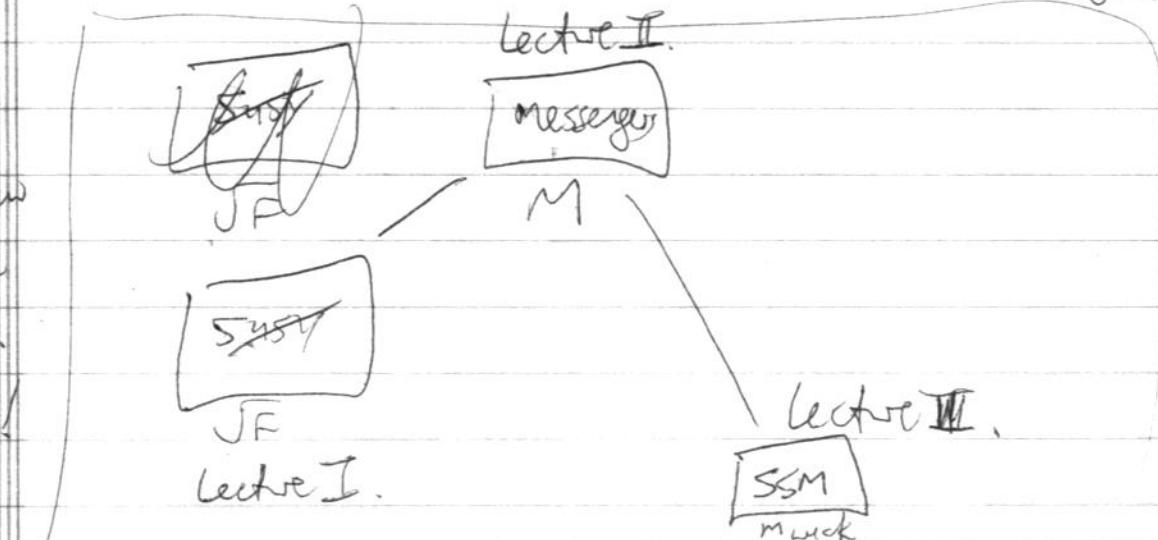
3) This must ~~arise~~ arise from higher dim ops

$$\int d^4 \theta \frac{X^\dagger X}{M^2} g^+ g^- + \int d^2 \theta \frac{X^\dagger W_\alpha^2}{M}$$

M signals presence of heavy states connecting SUSY & SSM - messengers.

In these lectures, we will give an overview of all the ingredients.

Ambitions!!



SUSY

C

- Most ~~ways~~ How do we break SUSY spontaneously?

How do we see this?

Need $\langle \bar{Q} | \psi \rangle \neq 0$

- Order par for SUSY: vacuum energy.
 ↴
 one way:
 mass splittings
 more fundamentally,

$$V = \langle 0 | H | 0 \rangle$$

$$H = \{ Q_i, \bar{Q}_i \}_{i=1}^n$$

$$= \sum_{i=1}^2 (|\bar{Q}_i|_0|^2 + |Q_i|_0|^2) = Q_1 \bar{Q}_1 + Q_2 \bar{Q}_2$$

so $V \neq 0 \Leftrightarrow Q|_0 \neq 0$
 or $\bar{Q}|_0 \neq 0$

- $V = V_F + V_B$ so either $V_B \neq 0$ (FI)

$$= \left(\frac{\partial W}{\partial E} \right)^2 + g^2(\dots)^2 \quad \text{or } \boxed{V_F \neq 0} (10' R)$$

Need

$$\boxed{\frac{\partial W}{\partial E} \neq 0}$$



SUSY

~~NR model~~

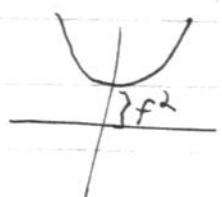
- Here is the simplest model of F-term SUSY:

$$W = fX$$

$$K = X^+X - c \frac{(X^+X)^2}{\pi^2}$$

"Polonyi model"

$$V = \frac{f^2}{1 - c \frac{X^+X}{\pi^2}}$$



Only problem is: not renormalizable.

- Simplest ren'able F-term SUSY: O'R model

$$W = fX + \frac{1}{2}\lambda X\phi^2 + m\phi\tilde{\phi}$$

$$K = K_{can}$$

$$V = \left| f + \frac{1}{2}\lambda\phi^2 \right|^2 + \left| \lambda X\phi + m\tilde{\phi} \right|^2 + m^2|\tilde{\phi}|^2$$

mutually incompatible F-ferms!

SUSY is broken(?)

where is the vacuum?

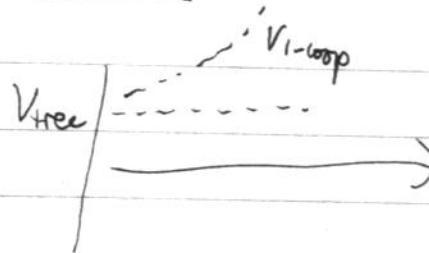
V minimized @ $\phi_2 \tilde{\phi} = 0$ (for $\frac{\partial f}{\partial \phi^2} < 1$).

indep. of X ! \rightarrow pseudomoduli space $\{ \langle X \rangle \}$

\rightarrow general feature of all ren. F-terms
"generalized O'R"

SUSY

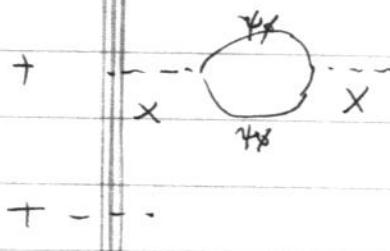
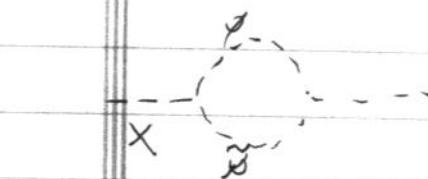
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$$V(X) \sim 5 \times M^4 \log \frac{M}{\Lambda}$$

Coleman-Weinberg

Homework: compute V_{loop} , find vacuum.



Why do these models break SUSY?

What general principles are broken?

R-symmetry is the key feature

F-t

Nelson & Seiberg: An R-symmetry is necessary for SUSY.

reminder

$$\{Q, \bar{Q}\} = P$$

$Q \rightarrow e^{i\theta} Q$, $\bar{Q} \rightarrow e^{i\theta} \bar{Q}$
compatibility algebra

R-symmetry

doesn't count if supercharges

fields in multiplets have diff R-charges

(ϕ, ψ, F)

$r \ r-1 \ r-2$

$$2 \sim \frac{\partial}{\partial \theta} \Rightarrow \theta \text{ has } R=1$$

Set θ has $R=\infty$.

$$\partial R(W)=2.$$

- SUSY is broken when $\frac{\partial W}{\partial \bar{I}_a} = 0$ cannot be solved.
- If ~~thy~~ has no symmetries, have n eqs for n eqs unknown generically $\exists \bar{J} \neq 0$.
- If ~~thy~~ has non-R global symm, $\bar{I}_i' = \bar{I}_i'(I_a)$ with $W = W(\bar{I}_i')$
Same as previous ~~SL~~.
- If ~~thy~~ has R-symmetry, $R(W)=2$, so

$$W = \bar{I}_1^{\partial W_1} \tilde{W} \left(\frac{\bar{I}_2}{\bar{I}_1^{\partial W_1}}, \dots, \frac{\bar{I}_n}{\bar{I}_1^{\partial W_1}} \right)$$

$$\Rightarrow \tilde{W} = \frac{\partial \tilde{W}}{\partial \bar{I}_1} = 0 \quad n \text{ eqs for } n-1 \text{ unknowns!} \quad \text{SUSY possible}$$

SUSY

Go back to examples:

- Polonyi : $R(X)=2 \quad \checkmark$

- $O'R$: $R(X)=2, R(\emptyset)=0, R(\tilde{\emptyset})=2 \quad \checkmark$

Comments.

- 1) Amusing ~~modification~~ small explicit R $W = F X + \frac{1}{2} \epsilon X^2$ in Poly

$$\langle X \rangle_{\text{SUSY}} \sim -\frac{f}{\epsilon} \quad \text{far away from}$$



Metastable SUSY!

So Nelson & Seiberg can be evaded, but still need approx R-symmetry.

- 2) Need R for SUSY. But need to break R Majora gauge masses

~~What about~~

$$M \neq 0$$

$$R(2) = 1$$

quicky spontaneous $R \rightarrow$ goldstone boson

R must be broken explicitly
(and probably spontaneously)

→ metastab. /
is. rev. table.
↳ loophole:

- 3) In $O'R$, turns out $\langle X \rangle = 0$ R not broken. gravit.

True of all generalized $O'R$ models w/ $R=0, 2$.

Need $R \neq 0, 2$ for $\langle X \rangle \neq 0$ in $O'R \rightarrow$ still no dynan. Example!!!

- 4) Much more can be said about $O'R$, gauging messes, ...
A fruitful & recent area of study!

SUSY - OSB

Motivation for OSB:

$$\text{SUSY} \rightarrow m_A^2 \sim \left(\frac{F}{M}\right)^2 \text{ from } \int \frac{X^+ X^- Q^+ Q^-}{M^2}$$

So for $m_{soft} \sim \text{TeV}$ & $M \sim M_p$ need $\boxed{\sqrt{F} \ll M_p}$

Why the hierarchy in SUSY scale?

c.f. OR
or PBHⁱⁿⁱ ← Can be technically natural, but still would like an explanation

D₀S_B (Witten): SUSY happens due to nonpert've gauge dynamics

$$M_{SO(10)} = M_p e^{-\frac{1}{2} g^2} \rightarrow \sqrt{F}$$

OSB is a very natural paradigm, b/c if ~~SUSY~~
 → SUSY unbroken at tree, it is unbroken all orders in pert'n thy. —non renom. of W.

But, & This leaves only nonpert'ive effe

Nonpert'ive gauge thy challenging, broad.

Any general guidelines?

SUSY - OSB

Witten index: $\text{Tr}(-1)^F = 0$ is necessary condition for (0)SB.
 (Put thy on ~~(A)~~ ~~S¹~~ → then spectrum is discrete)
 or $R \times S^3$

$$\text{Tr}(-1)^F = \sum_i n_B(\epsilon_i) - n_{\bar{B}}(\epsilon_i)$$

if $\epsilon_i \neq 0$, $n_B = n_F$ {Q, Q-bar} & E

raising & lowering

$$IBS \stackrel{F}{=} \bar{Q}IBS$$

if $\epsilon_i = 0$, not necessarily true.

- So if $\text{Tr}(-1)^F \neq 0$, there must be zero energy states
 SUSY unbroken, no grounds
- OSB should involve strong gauge dynamics, don't have

physical effects: gaugino condensation
 $\langle \bar{Q}W^\mu W_\mu \rangle$ ←
 $= \langle \bar{\chi}_R \chi_R \rangle = \langle \bar{\chi}_L \chi_L \rangle$

Witten: simplest SUSY gauge theory ^{SUSY} ~~SUSY~~ SYM

has $\boxed{\text{Tr}(-1)^F = N_c}$

So any thy which reduces to $SU(N_c)$ SYM @ low^e
 doesn't break SUSY in g.s. !!

~~(Dynamical)~~ → OSB models must be chiral, or
 have massless matter (e.g. as result of global symmetry)

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SUSY - DSB

Archetype models:

1. 3-2 model of AOS - chiral

2. ITIY - massless matter

I. 3-2 model

	$SU(3)$	$SU(2)$	$U(1)_R$	$U(1)$	
Q	□	□	+1	$\frac{+1}{3}$	$\exists U(1)_f$
\bar{u}	□	1	-8	$-\frac{4}{3}$	chiral
\bar{d}	□	1	-4	$+\frac{2}{3}$	
L	1	□	-3	-1	

$U(1)_R \text{ subj.}$
 $U(1)_{SU(2)}: -\frac{9}{2} \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} = 0$
 $2 \cdot \frac{1}{2} \cdot \frac{1}{2} - 4 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = 0$
anomaly free
global symm

gauge inv'ts: $Q\bar{d}L$, QuL , $(Q\bar{u})(Q\bar{d})$

$R=2, U(1)=0$

Add

$$W_{\text{tree}} = 2Q\bar{d}L$$

renormalizable

preserves $U(1)_R$ above

SUSY vac @ origin

What about nonpert'wely?

$$SU(3) \rightarrow \Lambda_3$$

$$b = 3N_c - N_f \rightarrow 7 \text{ for } SU(3)$$

$$SU(2) \rightarrow \Lambda_2$$

$$\text{both AF.} \rightarrow 4 \text{ for } SU(2)$$

SUSY - OSB

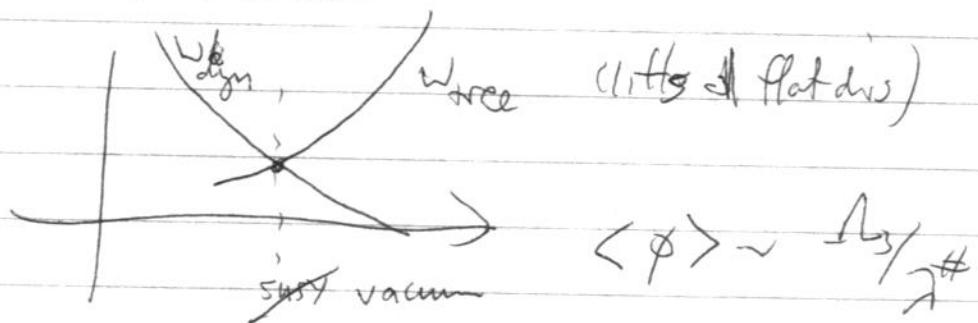
Take $\Lambda_3 \gg \Lambda_2$, so $SU(3)$ dynamics dominates

$$W_{\text{dyn}} = \frac{\Lambda_3^7}{(Q\bar{u})(Q\bar{d})}$$

AOS superpotential
from instantons.

$$W_{\text{full}} = W_{\text{dyn}} + W_{\text{tree}} = \frac{\Lambda_3^7}{(Q\bar{u})(Q\bar{d})} + \mathcal{T} Q\bar{d} L$$

$$W_{\text{tree}} \rightarrow (Q\bar{d})(Q\bar{u}) = 0 \Rightarrow \text{with } W_{\text{dyn}} \\ \text{SUSY is broken!}$$



Homework:

Analyze more thoroughly
the 3d model.

Need to take into account D terms!

$\langle \phi \rangle \gg \Lambda_3$ if $\mathcal{T} \ll 1$.
Theory analysis is reliable.

SUSY-OSB

2. ITIY model

Example w/ massless matter

	$SU(2)$	$[SU(4) \cong SU(6)]$	$U(1)_R$
Q	\square	$\square = 4$	0
S	1	$\square = 6$	2
$V \equiv \frac{Q \cdot Q}{n}$	1	\square	0

$-1 \cdot \frac{1}{2} \cdot 4 + 1 \cdot 2 = 0 \checkmark$

$W = \lambda S Q \cdot Q_{\text{free}}$

$\exists R \checkmark$

massless matter \checkmark

$W = \lambda S \cdot \vec{V}_{\text{free}}$

$N_f = N_c$: diff. of SQCD dynamics than 3-2 case!

Here instead of ADS s'pot'l, one has "Quantum modified moduli space"

- Classically, $\vec{Q} = 0, \vec{V}$ anything
- In W₀ thy,
- $\vec{Q} \sim$ In terms of gauge w/ \vec{V} : have $\vec{V}^2 = 0$ constraint not all \vec{V} indep.
- $W=0 \quad \vec{W} \neq 0$
- rank $V = 2$ b/c V is product of 2x4 metric 4x4 matrix

- In quantum thy, b/c $N_f = N_c$, have $\vec{V}^2 = n^2$ in $W=0$ thy

SUSY DSB

Now $\frac{\partial \mathcal{W}}{\partial S} = 0$ cannot be satisfied!

Is there a SUSY vacu^e?
Is ~~SUSY~~?

→ Study thy for $S \ll n$. Then Q remain light through strong SUSY
LEET is

$$W_{\text{eff}} = 2n \vec{S} \cdot \vec{V}, \quad \vec{V}^2 = n^2$$

$$\vec{V} = (\sqrt{n^2 - r^2}, \vec{r})$$

$$\vec{S} = (S_0, \vec{s})$$

$$K \approx K_{\text{can}} \approx S_0 \vec{S}_0 +$$

$$+ \alpha \vec{V}^2$$

$$W_{\text{eff}} \approx S_0 \sqrt{n^2 - r^2} + 2n \vec{S} \cdot \vec{V}$$

Kähler is smooth w/c of
def. mod. space $\xrightarrow[\text{origin!}]{\text{SUSY}}$
remove

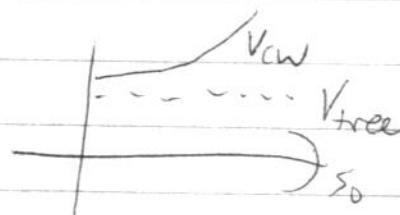
$$\approx [2n^2 S_0 - \frac{1}{2} 2 \vec{S} \vec{V}^2 + 2n \vec{S} \cdot \vec{V}] + \dots$$

→ We recognize the O'R model!!!

SUSY w/ P.M.S.

$$\langle S_0 \rangle = 0.$$

R is not broken.



We see that boring nondyn. O'R model can emerge from DSB!!

→ motivates study of generalized O'R for its own sake.

SUSY-OSB

To

~~To recap so far:~~ $N_f = 5 + \text{Witten index}$

\Rightarrow OSB reg. $\nabla U(1)_F$ &
[chiral or massless]

$3-2$ & $ITIY$ are standard examples
 $(N_f < N_c)$ $(N_f = N_c)$

These are restrictive conditions. "OSB is a special place"

- What if we relax reg of OSB in g.s.?

- Our vac. need only be metastable. In fact, we've seen that ^{realistic} vac. are prob. only metastable anyway.

\rightarrow Many more things are possible!!

Now only need $\text{approx } U(1)_R$; and vectorlike thysc allowed.

Simplest example of this is ISS.

$$3. \text{ SUGO w/ } N_c + 1 \leq N_f < \frac{3}{2} N_c$$

$$W_{\text{tree}} = m Q \tilde{Q}$$

• Witten index $\Rightarrow 8 N_c$ SUSY vac. of SU

Add mass terms for
the quarks.
That's all!!

• R-symmetry $\langle \tilde{Q} \tilde{Q} \rangle \neq 0$ $\nabla U(\frac{3N_c}{2})$
Both reg's for OSB are violated! (arbitrary)

$$0 \cdot \frac{1}{2} \cdot 2 N_f + 1 \cdot N_c \neq 0$$

SUSY-DSB

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Forge ahead anyway. What happens @ low energy?

For $N_f > N_c$, have Seiberg duality. (D)

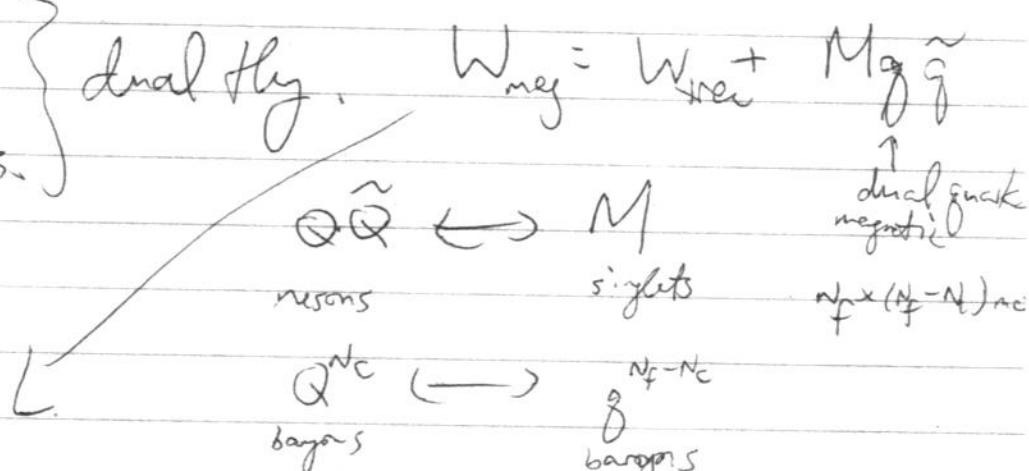
$N_c < N_f < \frac{3}{2}N_c$ — "free magnetic phase"

"electric" (large) N_f flavors
"microscopic" \downarrow

"magnetic" (small) $N_f - N_c$
"macroscopic" N_f flavors
 $N_f N_c$ singlets.

$$b_{\text{mag}} = 3(N_f N_c) - N_f \\ = 2N_c - 3N_c < 0$$

Theory is IR free



$$W_{\text{mag}} = \text{Tr}(mM) + Mg\tilde{g}$$

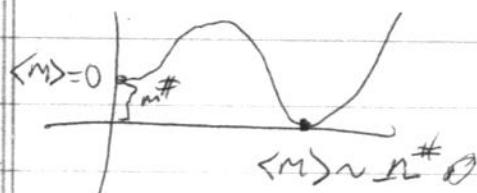
IR free
can reflect gauge dynamics
analyze as WE model.

$$\frac{\partial W_{\text{mag}}}{\partial M} = m \underbrace{I_{N_f \times N_f}}_{\text{rank } N_f} + g \underbrace{\tilde{g}\tilde{g}}_{\text{rank } N_f - N_c} \neq 0$$

O'R? "SUSY by rank condition"

at β_{int}^* another generalized O'R model!!

$$R=0, 2 \rightarrow \langle M \rangle = 0 \quad \text{(approx R not b, not)}$$



• lifetime governed by $\epsilon = \frac{n}{n_0} \ll 1$.

~~SUSY~~ SUSY -OSB

Recap of lecture I

- OSB is fundamental idea to explain $\sqrt{F} \ll M$ and ultimately $M_{\text{weak}} \ll M_{\text{pl}}$.
- Dynamics of SQCD is fundamental to study of OSB
- Several general criteria exist (Nelson-Seiberg, Witten index, ...)
- OSB in ground state \Rightarrow chiral or massless matter models are somewhat "cooked up"
- Metastable OSB opens up many more possibilities, might be "generic"!
- O'R models can arise from OSB. (Sawin 2 out of 3 examples)
R-symmetry breaking is a challenge.