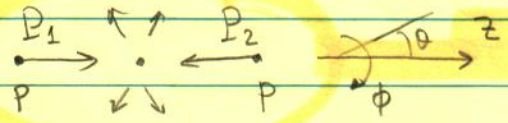


Introduction to Collider Physics

I. Introduction

Collider Experiment: e.g. LHC



"Lab frame" $\vec{P}_1 + \vec{P}_2 = 0$

$$s = (p_1 + p_2)^2 = 4E_1^2$$

$\sqrt{s} = 2E_1 =$ "c.-of.-M. energy"

(e.g. 2 TeV @ the Tevatron

7 TeV $\rightarrow$ 14 TeV @ LHC)

After collision: detect $e^\pm, \mu^\pm, \gamma, q/\bar{q}, g, \tau^\pm(?)$

if within
"acceptance"

$\rightarrow$ hadrons $\rightarrow$ jets

$\left\{ \begin{array}{l} \text{b-tag: } CT = 400-500 \mu\text{s} \\ \text{c-tag: } CT = 100-300 \mu\text{s} \\ \text{g, q, } \gamma: \text{ maybe} \end{array} \right.$

+ stable, el. charged or colored BSM particles

$$\uparrow CT \geq 10 \text{ m} \Rightarrow \tau \geq 3 \cdot 10^{-8} \text{ sec}$$

cf "natural" lifetime $\tau_{\text{nat}} \sim \frac{1}{M} \sim \frac{1}{100 \text{ GeV}} \sim 10^{-27} \text{ sec}$ for BSM.

do not detect $\downarrow$ + stable, el. neutral, uncolored BSM particles

$\uparrow$ ex: WIMP dark matter candidates!

measure $E_i, \vec{p}_i$ $i=1 \dots N_{\text{vis}}$ - visible external particles

$\uparrow$ in some cases, need to use $E^2 - \vec{p}^2 = m^2$.

"Event record": $\mathcal{E} = \{ \{PID_i, p_i^\mu\}_1, \{PID_i, p_i^\mu\}_2, \dots \}$

En.-mom. conservation: $P_1^\mu + P_2^\mu = \sum_{i=1}^N p_i^\mu$

Proton remnant escapes unobserved $\Rightarrow$ useless for $\mu = 3, 0$.

But useful for transverse momentum: $\sum_{i=1}^N \vec{p}_i^\perp = 0$

If this sum is non-zero for visible particles, define

"Missing $\vec{p}_T$ " $\equiv \vec{p}_T = - \sum_{i=1}^{N_{vis}} \vec{p}_i^\perp$

"Missing transverse energy" $\equiv E_T = |\vec{p}_T|$.

This is the total mom. of invisible particles, and is the only information available about them.

The goal is to connect $\mathcal{L}$ to "fundamental theory": $\mathcal{L}$

$\mathcal{L} \rightarrow \mathcal{E}$ is algorithmic (at least if pert. theory applies):

$\mathcal{L} \rightarrow \text{Feynman rules} \rightarrow S \text{ matrix} \rightarrow \frac{d\sigma}{d\Omega}, d\Omega = \prod_{i=1}^N \frac{d^3 p_i}{(2\pi)^3 2E_i}$ "phase space"

$\rightarrow \frac{dN}{d\Omega} = L_{int} \cdot \frac{d\sigma}{d\Omega} \rightarrow \mathcal{E}_{th}$: a realization of this distribution
 (typically generated numerically with a "Monte-Carlo generator")
 $\uparrow$
 "integrated luminosity"
 $\propto$ # of collisions
 reported by acc. operators

Compare $\mathcal{E}_{th}$ with measured $\mathcal{E} \Rightarrow$ theory OK or ruled out!
 (roughly, χ^2 test)

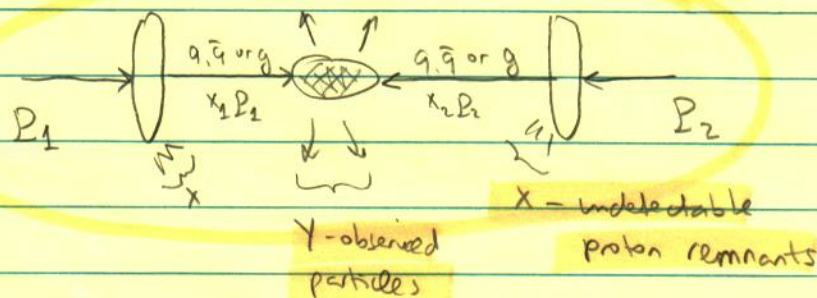
This is all pretty straightforward, but not completely trivial:

- p is composite, $q/\bar{q} \rightarrow$ hadrons - some non-pert. physics enters
- Pert. theory has IR divergences

If deviation from the SM is discovered, we will also need to construct $\mathcal{E} \rightarrow \mathcal{L}$ ("the inverse problem"). There is no algorithmic solution. Of course, one can just try many $\mathcal{L}$'s until get a good fit, but need some intuition to know what to try! As usual, intuition comes from collecting and internalizing a set of examples. We will try to do it in these lectures.

II. Hello World: Z production at the LHC (or Tevatron)

1) Parton model



$$\sigma(p(P_1) + p(P_2) \rightarrow Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{f_1 f_2} f_{f_1}(x_1, Q^2) f_{f_2}(x_2, Q^2) \sigma(f_1(x_1 P_1) + f_2(x_2 P_2) \rightarrow Y)$$

inclusive -
sum over all possible X

"parton distribution functions - pdf's"

$$f_1, f_2 = u, \bar{u}, d, \bar{d}, \dots, g$$

(some refinements are needed - later!)

roughly,
 $\frac{dP_T^2}{dy} = M_{\text{inv}}^2$

"hat" for all parton-frame quantities

"Parton frame" $\hat{x}_1 \hat{P}_1 + \hat{x}_2 \hat{P}_2 = 0$

Parton c.-of-m energy: $\hat{E}_{cm} \equiv \sqrt{\hat{S}} = \sqrt{(\hat{x}_1 P_1 + \hat{x}_2 P_2)^2} = \sqrt{2\hat{x}_1 \hat{x}_2 P_1 \cdot P_2} = \sqrt{\hat{x}_1 \hat{x}_2 S} \leq \sqrt{S}$

Boost lab $\rightarrow$ parton: $\hat{E} = \sqrt{\hat{x}_1 \hat{x}_2 S} = \gamma(1-\beta)\hat{x}_1 \sqrt{S} = \gamma(1+\beta)\hat{x}_2 \sqrt{S}$

$$\Rightarrow \beta = \frac{\hat{x}_1 - \hat{x}_2}{\hat{x}_1 + \hat{x}_2}, \quad \gamma = \frac{\hat{x}_1 + \hat{x}_2}{2\sqrt{\hat{x}_1 \hat{x}_2}}$$

S fixed, $\hat{S}, \beta, \gamma$ vary event-by-event, according to p.d.f.'s and cross sections.

Facts about pdf's: - not calculable: reflect non-pert. physics of confinement

- obtained by fits to exp. data (mainly DIS)
 $\Rightarrow$ error bars, dep. on f_i and x may be large.

- available at

durpdg.dur.ac.uk/HEPDATA/PDF

[slides, comments on relative importance of q , valence/sea quarks]

2) Z prod.: total cross-section

Parton-level, tree-level: $\sigma(qg \rightarrow Z) = \sigma(\bar{q}g \rightarrow Z) = 0$

$$\sigma(q\bar{q} \rightarrow Z) = \frac{4}{3} \pi^2 \frac{\Gamma(Z \rightarrow q\bar{q})}{M_Z} \delta(s - M_Z^2) \quad (\text{see P\&S, p.151})$$

$$\Rightarrow \sigma_{\text{tot}}(pp \rightarrow Z + X) = \frac{4}{3} \pi^2 \frac{\Gamma_Z}{M_Z} \int_0^1 dx_1 \int_0^1 dx_2 \sum_q 2 f_q(x_1) f_{\bar{q}}(x_2) \times \text{Br}(Z \rightarrow q\bar{q}) \times \delta(x_1 x_2 s - M_Z^2).$$

x_2 integral: $\int_0^1 dx_2 f_{\bar{q}}(x_2) \delta(x_1 x_2 s - M_Z^2) = f_{\bar{q}}\left(\frac{M_Z^2}{s x_1}\right) \cdot \frac{1}{x_1 s} \cdot \theta(x_1 s - M_Z^2).$

$$\Rightarrow \sigma = \frac{4\pi^2}{3} \frac{\Gamma_Z}{M_Z} \cdot \frac{1}{s} \sum_q \int_{M_Z^2/s}^1 \frac{dx_1}{x_1} \cdot 2 f_q(x_1) f_{\bar{q}}\left(\frac{M_Z^2}{s x_1}\right) \cdot \text{Br}(Z \rightarrow q\bar{q}).$$

↑
if $p\bar{p}$ collision, $2f_q f_{\bar{q}} \rightarrow f_q f_{\bar{q}} + f_{\bar{q}} f_q$

since $f_{\bar{q}}^p = f_q^{\bar{p}}$ and vice versa!

Note: $x_{\text{min}} = \frac{M_Z^2}{s} \approx 2.5 \times 10^{-3}$ @ Tevatron, 1.6×10^{-4} @ LHC-7
or 4.0×10^{-5} @ LHC-14

In general, $x_{\text{min}} \approx \frac{E_{\text{min}}^2}{s}$ for a process requiring $\sqrt{s} \geq E_{\text{min}}$.

"typical" x : $f(x) \cdot f\left(\frac{x_{\text{min}}}{x}\right)$ maximal if $x \sim \sqrt{x_{\text{min}}}$

(rough rule-of-thumb; depends on β and other details!)

For example, $u\bar{u}$ contribution:

$$x u(x) = \begin{cases} -1.2 - 0.9 \log_{10} x, & 10^{-2} < x < 10^{-1} \\ 0.7, & 10^{-2} < x < 0.1 \\ -0.2 - 0.9 \log_{10} x, & 0.1 < x < 0.6 \\ 0, & x > 0.6 \end{cases}$$

Tevatron: $\int_{x_{\text{min}}}^1 \frac{dx_1}{x_1} u(x_1) u\left(\frac{x_{\text{min}}}{x_1}\right) \approx 600$, $\int_{x_{\text{min}}}^1 \frac{dx_1}{x_1} \bar{u}(x_1) \bar{u}\left(\frac{x_{\text{min}}}{x_1}\right) \approx 5$ - valence-quarks dominated!

Plug in numbers $\Rightarrow \sigma(p\bar{p} \rightarrow Z) \approx 3 \text{ nb}$, $u\bar{u}$ alone, LO

Actual $\sigma_{\text{NLO}} = 8.2 \text{ nb}$ [slide] not bad!

LHC 14: $\sigma(p\bar{p} \rightarrow Z) \approx 60 \text{ nb}$ ($u\bar{u}$ alone, LO) $\sigma_{\text{NLO}} = 60 \text{ nb}$ (luck!)
sea quarks dominate: $x \approx \sqrt{4 \times 10^{-5}} \approx 6 \times 10^{-3}$ - in the "sea" range.

[data-slide]

HW: Do it for LHC-7

Note: cross section grows with $\sqrt{s}$, due to growth of pdf's at low x .

Replace Z with heavy Z' (same couplings):

$$\sigma(p\bar{p} \rightarrow Z') = \frac{8}{3} \pi^2 \frac{\Gamma_{Z'}}{M_{Z'}} \cdot \frac{1}{s} \cdot \underbrace{\left[\int_{M_{Z'}^2/s}^1 \frac{dx}{x} f_u(x) f_{\bar{u}}\left(\frac{M_{Z'}^2}{sx}\right) + \dots \right]}_{F_u(M_{Z'}^2/s)}$$

Very roughly, $F_u \approx 0.09 \left(\frac{\sqrt{s}}{M_{Z'}} \right)^3$ ($M_{Z'} \sim 0.1 - 1 \text{ TeV}$)

Generally $\Gamma_{Z'} \propto M_{Z'} \Rightarrow \sigma \propto M_{Z'}^{-3}$.

L2

11/01/11

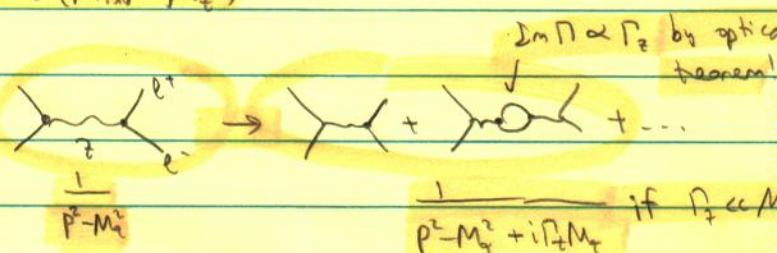
3) Z decays and observables Z decays "promptly" ($c\tau \ll 1 \mu\text{m}$) $\Rightarrow$ see decay products

Decays: $Z \rightarrow q\bar{q} \rightarrow \text{jets} \leftarrow$ invisible: $b(Z) \ll b(\text{jets})$ (plot)
 $Z \rightarrow e^+e^-, \mu^+\mu^- \leftarrow$ easily visible! (event display)

"Invariant mass" $M_{\text{inv}} = \sqrt{(p_{e^+} + p_{e^-})^2}$ is frame-independent
 $\Rightarrow$ compute in parton frame, measure in lab frame.

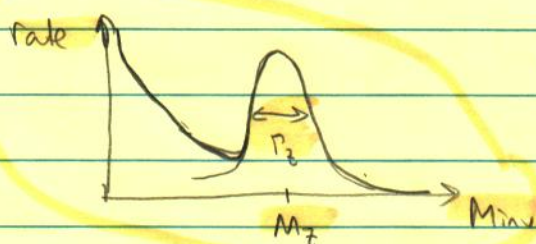
LO: $M_{\text{inv}} = M_Z$, $\frac{db}{dM_{\text{inv}}} \propto \delta(M_{\text{inv}} - M_Z)$

NLO: Breit-Wigner peak



"narrow-width approximation"

$$\Rightarrow \frac{db}{dM_{\text{inv}}} \propto \frac{1}{(M_{\text{inv}} - M_Z)^2 + M_Z^2 \Gamma_Z^2}$$

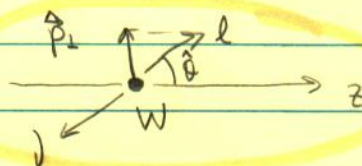


"inv. mass peak" or "bump" -
 Signature of unstable new particle

[plot]

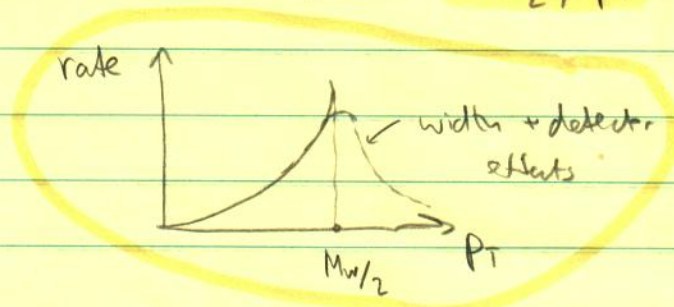
What if $Z \rightarrow W$? $W \rightarrow l\nu$ - p_ν unknown!

• Lepton p_T : partonic c-of-m frame $\equiv W$ rest frame



$$p_T = |\vec{p}_\perp| = |\hat{\vec{p}}_\perp| = \hat{E} \sin \hat{\theta} = \frac{1}{2} M_W \sin \hat{\theta} \Rightarrow p_T^{\max} = \frac{1}{2} M_W$$

$$\frac{db}{dp_T} = \frac{db}{d \cos \hat{\theta}} \frac{d \cos \hat{\theta}}{dp_T} = \frac{p_T}{\sqrt{\left(\frac{M_W}{2}\right)^2 - p_T^2}} \frac{db}{d \cos \hat{\theta}}$$



"Jacobean peak" [slide]

• "Transverse mass" $\vec{p}_\perp = -\vec{p}_{\perp e} \equiv \vec{p}_{\perp l}$

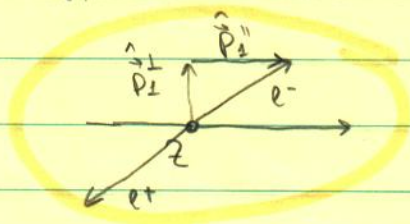
$$M_T^2 \equiv (|\vec{p}_\perp^l| + |\vec{p}_\perp^e|)^2 - (\vec{p}_\perp^l + \vec{p}_\perp^e)^2$$

HW: Show that $0 \leq M_T^2 \leq M_W^2$

M_T distribution also has a Jacobean peak [slide]

$\Rightarrow$ Tevatron $\frac{\Delta M_W}{M_W} \approx \frac{31 \text{ MeV}}{80.4 \text{ GeV}} \sim 4 \times 10^{-4}$ best in the world!

Okay, back to the Z . Again, parton frame $\rightarrow Z$ at rest.



"Pseudo-rapidity" $\hat{\eta}$:

$$\hat{p}_\perp^+ = \hat{p}_\perp^+ \sinh \hat{\eta}_1$$

$$\hat{E}_1 = \sqrt{(\hat{p}_\perp^+)^2 + (\hat{p}_\perp^+)^2} = \hat{p}_\perp^+ \cosh \hat{\eta}_1$$

$\hat{\eta}_2 = -\hat{\eta}_1$ $\vec{p} \rightarrow (p^-, p^+, \phi) \rightarrow (\eta, p^+, \phi)$ - canonical description

Parton $\rightarrow$ lab frame: $p^\perp, \phi$ invariant

HW: Show that $\eta_1 = \hat{\eta}_1 + \eta_z$

where $\eta_z = \frac{1}{2} \log \left(\frac{\sum x_i^2}{M_z^2} \right)$ - independent of $\hat{\eta}_1$, same for all particles - additive Lor. tr. for η_z
 $= \tanh^{-1} \beta$

Likewise $\eta_2 = \hat{\eta}_2 + \eta_z = -\hat{\eta}_1 + \eta_z$

$$\Rightarrow \eta_z = \frac{1}{2} (\eta_1 + \eta_2)$$

So, measure $\eta_1, \eta_2 \Rightarrow$ β velocity in lab frame $\Rightarrow$ p.d.f. information.

[slide: NNLO calculation vs. data]

Lab-frame η observable: $p_z'' = p_z^\perp \sinh \eta_1$

$$\Rightarrow \tan \theta_1 = \frac{1}{\sinh \eta_1}$$

$$\eta_{\max} = \frac{1}{2} \log \left(\frac{\sum x_i^2}{M_z^2} \right) = 3.0 \quad \text{TeVatron} \quad (\theta_{\min} = 5.7^\circ)$$
$$5.0 \quad \text{LHC} \quad (\theta_{\min} = 0.8^\circ)$$

Detectors designed roughly to cover out to this η .

"Heavier" final state (i.e. larger $M_{\text{inv}}^2 = (\sum_{i=1}^N p_i)^2$)

$\Rightarrow$ smaller $\eta_{\max}$ - more "central"!

Of course, individual particles may have larger η , may not be detected.

IV. Radiative Corrections and Infrared Divergences

Naively, subleading orders in pert. theory are only important if you need precision. In reality though, certain aspects of beyond-LO physics must be understood even to develop qualitative intuition about collider processes. The reason is IR divergences.

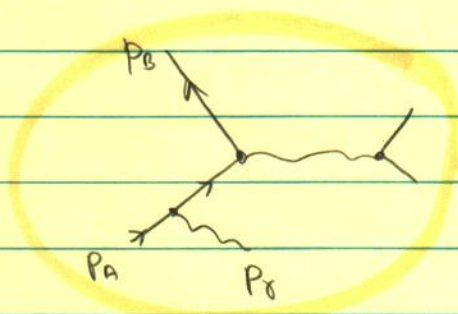
Let's start with a QED example: $e^+e^- \rightarrow \mu^+\mu^-$

LO: $|M|^2 \propto \alpha^2$

NLO: $\left(\text{tree} \right)^* \cdot \left(\text{loop} \right) + \text{h.c.} \propto \alpha^3$ ("loop corr.")

$\left| \text{tree} + \text{real} \right|^2 \propto \alpha^3$ ("real emission")

Let's focus on real emission, one diagram:



$$= \bar{U}_B \gamma^\mu \frac{\not{p}_A - \not{p}_0}{(p_A - p_0)^2} (\not{\epsilon}) U_A \cdot \epsilon_\nu(p_0) \cdot A_\mu$$

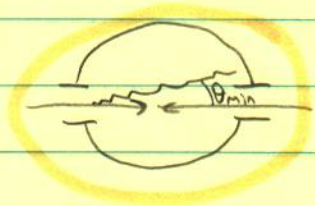
$$p_A = (E, 0, 0, E) \quad p_0 = (zE, \vec{p}_\perp, \sqrt{(zE)^2 - \vec{p}_\perp^2})$$

$$(p_A - p_0)^2 = -2p_A \cdot p_0 = -2zE^2 \left(1 - \sqrt{1 - \frac{\vec{p}_\perp^2}{(zE)^2}} \right)$$

$M \rightarrow \infty$ when $z \rightarrow 0$ "soft singularity"

or $|\vec{p}_\perp| \rightarrow 0$ "collinear singularity"

Physically, soft + collinear photons are unobservable:



- small- θ γ 's escape through holes
- small-E γ 's drowned in "noise"

Define Q :

$p_{\perp} > Q$ - observe $2 \rightarrow 3$

$p_{\perp} < Q$ - observe $2 \rightarrow 2$

~~Small $p_{\perp}$~~ $(p_A - p_B)^2 \rightarrow 0 \Rightarrow p_A - p_B \approx \sum_s \bar{u}_s(p_A - p_B) u_s(p_A - p_B)$

~~At z~~

$$(p_A - p_B)^2 \approx -\frac{p_{\perp}^2}{z}$$

$$\Rightarrow \mathcal{M} = -\frac{z}{p_{\perp}^2} \sum_s \left[e \bar{u}_s(p_A - p_B) \gamma^\mu u_A(p_B) \right] \cdot \mathcal{M}(p_A - p_B, p_B \rightarrow p_1, p_2)$$

$e \rightarrow e\gamma$ amp., ind. of the rest - "factorization"

$$db_{2 \rightarrow 3} \approx \frac{2\hat{s}}{2s} d\eta_\gamma \left(\frac{z}{p_{\perp}^2}\right)^2 \underbrace{\sum_s |\mathcal{M}(e \rightarrow e\gamma)|^2}_{\frac{4e^2 p_{\perp}^2}{2(1-z)} \left[\frac{1+(1-z)^2}{z}\right]} \cdot \underbrace{|\mathcal{M}_{2 \rightarrow 2}(\hat{s} = (1-z)s)|^2}_{db_{2 \rightarrow 2}^{LO}(\hat{s})} d\eta_1 d\eta_2 \frac{1}{2\hat{s}}$$

- P&S pp. 576-578

$$d\eta_\gamma = \frac{d^3 p_\gamma}{(2\pi)^3 2E_\gamma} = \frac{p_{\perp} dp_{\perp}}{8\pi^2} \frac{dz}{z}$$

$$\Rightarrow b_{2 \rightarrow 3} \approx \int_{z_{min}}^1 \frac{dz}{z} \int_Q^E \frac{p_{\perp} dp_{\perp}}{8\pi^2} \cdot (1-z) \cdot \left(\frac{z}{p_{\perp}^2}\right)^2 \cdot \frac{4e^2 p_{\perp}^2}{2(1-z)} \cdot \left[\frac{1+(1-z)^2}{z}\right] \cdot b_{2 \rightarrow 2}^{LO}(\hat{s})$$

$$z_{min} = \frac{Q}{E}$$

1

2 log divergences!

$$\approx \frac{2}{\pi} \log^2 \frac{\sqrt{s}}{Q} \cdot b_{2 \rightarrow 2}^{LO}(s) + (1 \text{ or } 0 \text{ logs}).$$

9

"Sudakov double log"

$$b_{2 \rightarrow 2} \approx \int_0^1 \frac{dz}{z} \int_0^Q \frac{dp_\perp}{p_\perp} (\dots) b_{2 \rightarrow 2}^{LO}(\hat{s}) = \int_0^1 dz \frac{2}{2\pi} \left[\frac{1+(1-z)^2}{z} \right] \log \frac{Q}{m_e}.$$

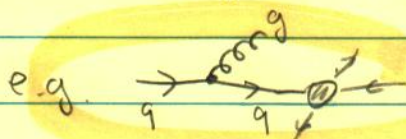
$0 \rightarrow m_e$, if you're careful!

$$\times b_{2 \rightarrow 2}^{LO}(\hat{s} = s(1-z)).$$

This looks like parton model! e beam is really $e\gamma$ beam, with fraction of γ 's depending on Q . Of course, Q is unphysical, so the total rate is independent of it:

$$\frac{d}{dQ} (b_{2 \rightarrow 2} + b_{2 \rightarrow 3}) = 0, \quad b_{2 \rightarrow 2} = b_{2 \rightarrow 2}^{LO} \left(1 + c \frac{2}{\pi} + \dots \right) \quad \text{"IR-safe"}$$

Same thing happens in QCD:



$p_{T,q} < Q \Rightarrow$ correction to p.d.f.'s (Altarelli-Parisi eq's) $\Rightarrow f(x, Q)$

$p_{T,q} > Q \Rightarrow$ extra jet

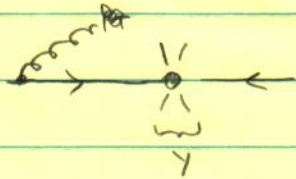
Insert parton model

< note: $m_e \rightarrow m_{had}$ cuts off collinear divergence

If final state Y with inv. mass $M_{inv}(Y)$ is produced, then

exclusive: $b(Y + \text{jet}, p_T > Q) \approx \frac{2}{\pi} \log^2 \frac{M_{inv}(Y)}{Q} \cdot b_{LO}(Y \text{ with no jet})$ - lots of low- p_T jets!

inclusive: $b(Y) + b(Y + \text{jet}) = b_{LO}(Y) \left(1 + c \frac{2}{\pi} \right)$ (no logs!)

Soft/Collinear Divergences

$P_{T,j} < Q$ - pdf correction

$P_{T,j} > Q$ - $pp \rightarrow Y + \text{jet}$

$$\frac{\sigma(pp \rightarrow Y + \text{jet})}{\sigma(pp \rightarrow Y)} \sim \frac{2s}{\pi} \log^2 \frac{M_{\text{inv}}^2(Y)}{Q^2}$$

where $M_{\text{inv}}(Y) = (\sum_{i=1}^n p_i)^2$

Ex.: $pp \rightarrow Z + j$ $\frac{2s}{\pi} \log^2 \frac{M_Z^2}{Q^2} \sim 1$ for $Q \sim 10 \text{ GeV}$

$\Rightarrow$ Z prod. is typically accompanied by 1 or more 10 GeV jets!

slide: p_T^2 distribution, jet mult.

Inclusive cross section: ("parton model @ NLO")

$$\sigma_{\text{inc}}(pp \rightarrow Y + \geq 0 \text{ jets}) = \int dx_1 dx_2 f_1(x_1, Q^2) f_2(x_2, Q^2)$$

$$\times \left(b_0(f_1 f_2 \rightarrow Y) + \underbrace{b_{\text{NLO}}(f_1 f_2 \rightarrow Y) + b_0(f_1 f_2 \rightarrow Y + j, p_{T,j} > Q)}_{\mathcal{O}(\alpha_s) \cdot b_0} + \dots \right)$$

$\frac{db_{\text{inc}}}{dQ} = 0$ but f 's involve "resummation" of leading logs to

all orders in α_s , while partonic σ is fixed-order

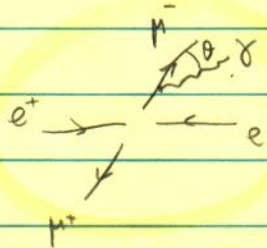
$\Rightarrow$ cancellation is inexact!

Choice of Q : minimize log's in partonic x-section $\Rightarrow Q^2 = M_{\text{inv}}^2(Y)$

Estimate Q dependence: vary up/down by a factor of 2.

Final State Radiation

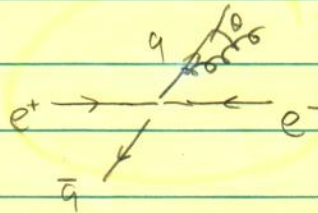
$$e^+e^- \rightarrow \mu^+\mu^-\gamma$$



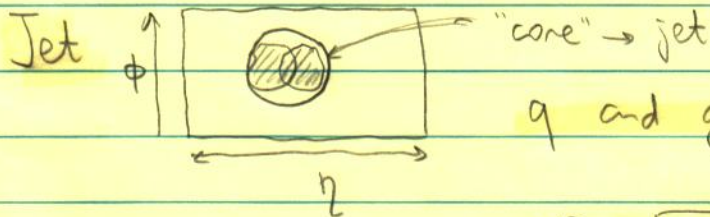
sing.: $E_\gamma \rightarrow 0$ (unobs.)
 $\theta \rightarrow 0$ (cut off by M_μ)

$$\sigma(2 \rightarrow 3, E_\gamma > E_{\min}) \approx \sigma_{\text{LO}}(2 \rightarrow 2) \cdot \frac{2}{\pi} \log \frac{E_\mu}{E_{\min}} \log \frac{E_\mu}{m_\mu} \quad \text{"Sudakov double-log"}$$

$$e^+e^- \rightarrow q\bar{q}g$$



Same story, but collinear sing. is cut off in a different way:



$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < \Delta R_{\min} - 1 \text{ jet} \\
> \Delta R_{\min} - 2 \text{ jets}$$

n -jet rates depend strongly on the details of jet-finding algorithm, very hard to predict;
 inclusive rates are much safer ("IR-safe observable")

Multiple splittings likely $\rightarrow$ PYTHIA, HERWIG, SHERPA
 "showering codes".