Meaningful characterization of perturbative theoretical uncertainties

Laura Jenniches

Niels Bohr Institutet, København Universitet

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work together with Alberto Guffanti, Matteo Cacciari, Emanuele Bagnaschi

based on 1105.5152v2 (Cacciari, Houdeau)

Theoretical uncertainty estimates

Why?

Theoretical uncertainty estimates are

- necessary to decide whether a deviation between theory and experiment is significant and
- essential ingredients in the search for new physics.

- Observable (Cross section, inclusive/ differential decay rate ...)

$$\sigma_k(Q) = \sum_{i=0}^{k} \alpha_s^i(\mu_r) c_i(Q, \mu_r),$$

- Uncertainty on $\sigma_k(Q)$

$$\begin{aligned}\Delta_k &\simeq \alpha_s^{k+1} c_{k+1} \\ &\simeq \frac{3}{2}\alpha_s^{k+1} k \beta_0 c_k\end{aligned}$$

- Assumption: c_k''s have the same order of magnitude. Then

$$\sigma_k = \sum_{i=0}^{k} \left(\frac{3}{2}\beta_0\right)^{i-1} (i-1)! \alpha_s^i c_i'.$$

- Ratios r_k independent of expansion parameter

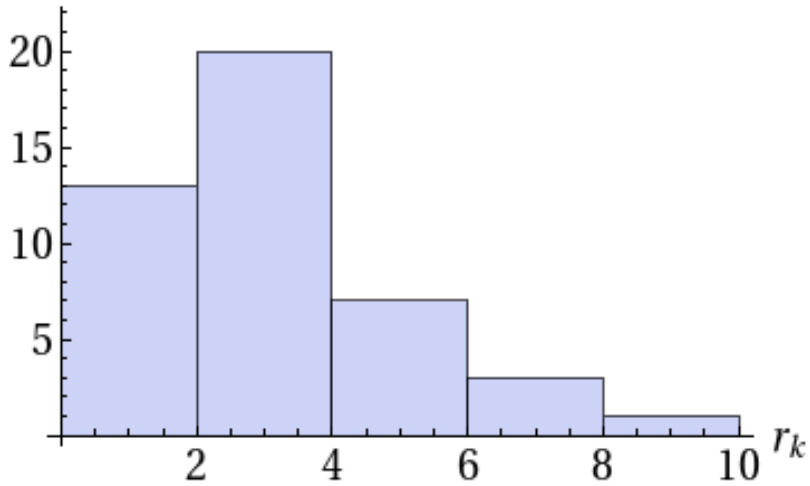
$$r_k = \frac{\alpha_s c_k}{(k-1)c_{k-1}} \simeq \frac{3}{2}\alpha_s \beta_0$$

Bayesian statistics

Definition of probability

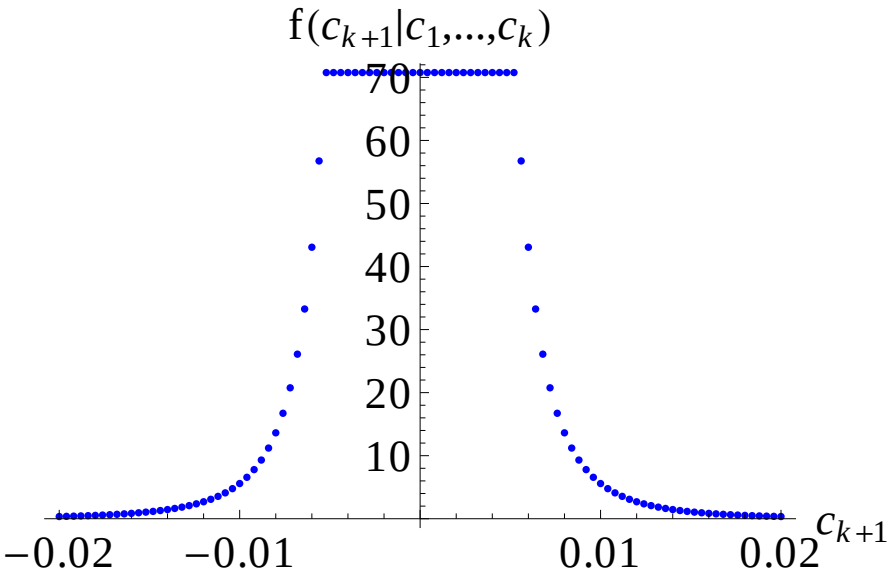
- Frequentist: relative frequency of occurrence of an experiment's outcome
- Bayesian: degree of belief (DoB), combination of a priori probability distribution and data incorporated in a likelihood function

$$f_\varepsilon(\Delta_k|c_1,\ldots,c_k) = \frac{1}{f_\varepsilon(c_1,\ldots,c_k)}\frac{1}{\alpha_s^{k+1}} \times \int d\bar{c}\; f(c_1|\bar{c})\ldots f(c_k|\bar{c}) f(c_{k+1} = \frac{\Delta_k}{\alpha_s^{k+1}}|\bar{c}) f_\varepsilon(\bar{c})$$



Posteriors

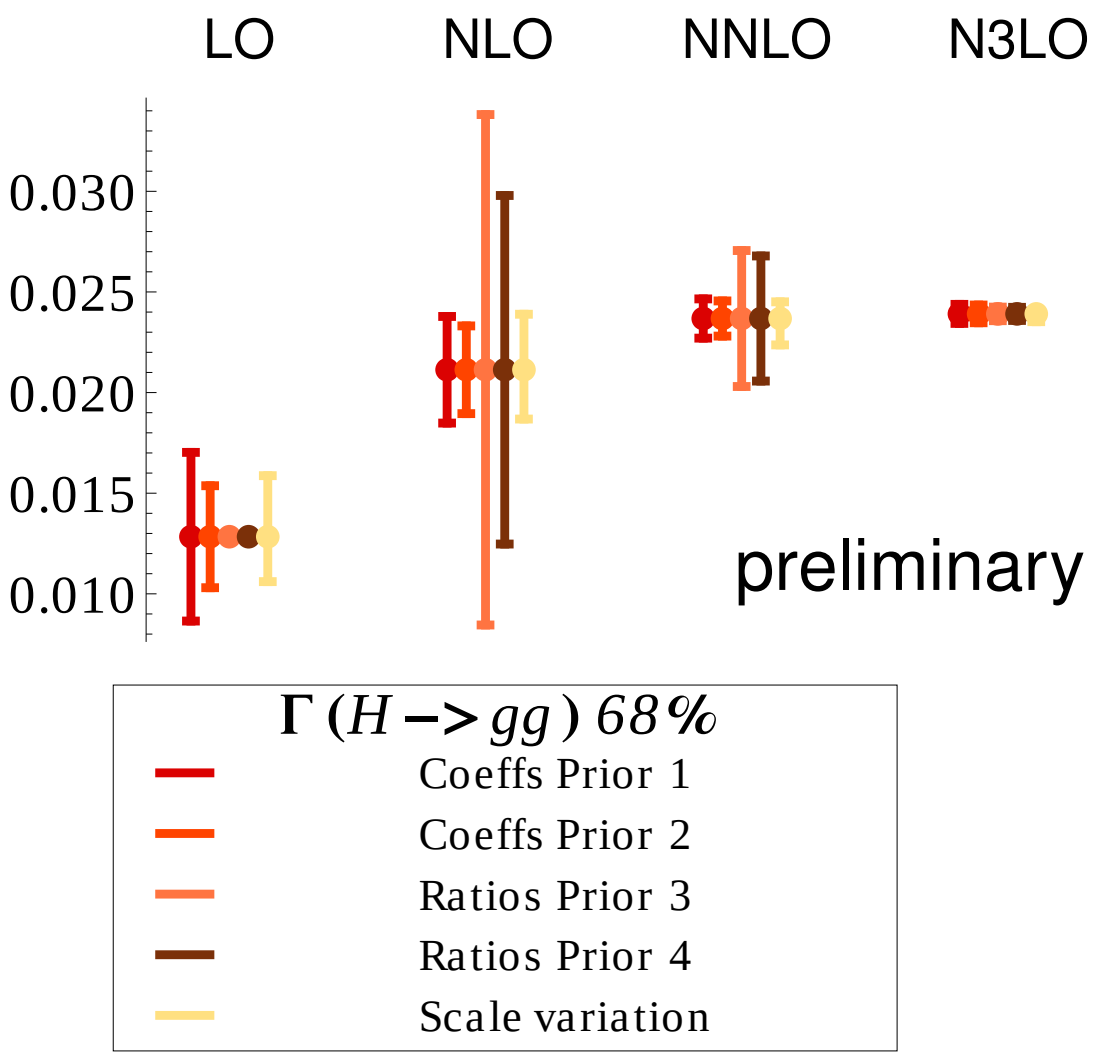
- distribution of c_{k+1}
- required to determine uncertainty Δ_k



Priors

- parameter $\bar{c}$
- $f(c_i|\bar{c})$: e.g. uniform distribution, $c_i \leq \bar{c}$
- $f_\varepsilon(\bar{c})$: e.g. log $\bar{c}$ flat

Apply to data



- modified coefficients

$$\frac{c_k}{(k-1)!\left(\frac{3}{2}\beta_0\right)^{k-1}}$$

- ratios

$$\frac{\alpha_s c_k}{(k-1)c_{k-1}}$$

Further observables:
$e^+e^- \to \gamma \to$ hadr.,
Higgs production,
$H \to b\bar{b}$,
Bjorken sum,
...

Baikov, Chetyrkin