HIGGS MECHANISM

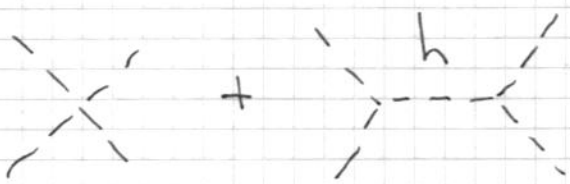
Adding a scalar:

$$\mathcal{L}_{EWSB} = \frac{v^2}{4}\, \mathrm{tr}\left|D_\mu \Sigma\right|^2 \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \dots\right)$$

$$+ \left(\bar{u}_L\ \bar{d}_L\right)\Sigma\left(1 + c\frac{h}{v} + \dots\right)\begin{pmatrix} m_u & \\ & m_d \end{pmatrix}\begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

($a>0$ always by redefinition of h)

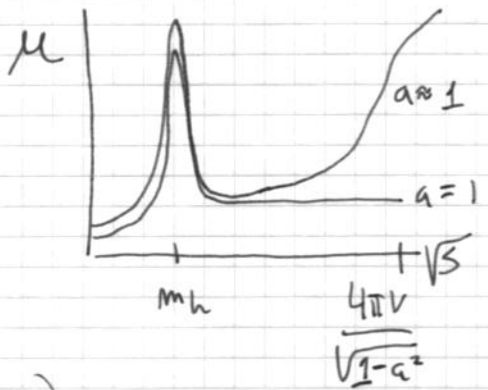
(a) $G_+G_- \to G_3G_3$



$$\xrightarrow{\text{large } s} \frac{s}{v^2}\left(1-a^2\right) + \mathcal{O}\left(\frac{m_h^2}{v^2}\right)$$

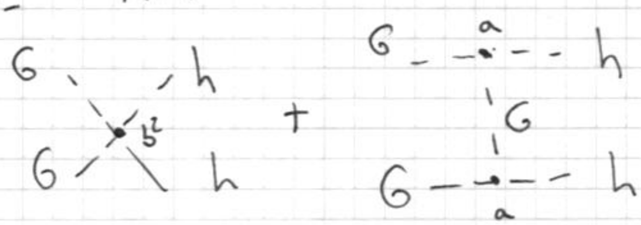
$$\frac{s}{v^2} \qquad -a^2\frac{s^2}{s-m_h^2}\,\frac{1}{v^2}$$

For $\boxed{a=1}$, $\mathcal{M} \to C$



(b) $G_+G_- \to hh$

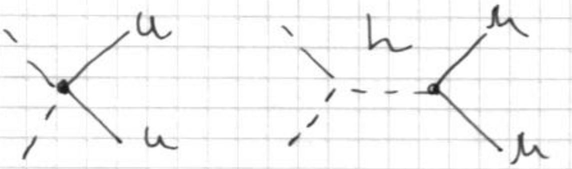
($s+t+u = 2m_h^2$)



$$\longrightarrow \frac{s}{v^2}\left(b-a^2\right) + \mathcal{O}\left(\frac{m_h^2}{v^2}\right)$$

No growing with s for $\boxed{b = a^2 = 1}$

ⓒ $G_+G_- \to ff$



$$\longrightarrow \frac{m_u\sqrt{s}}{v^2}\left(1-ac\right)+\mathcal{O}\left(\frac{m_h^2}{v\cdot E}\right)$$

Not growing with s for $\boxed{c=1}$

$a=b=c$: $\mathcal{L}_{EWSB}=\frac{1}{4}|D_\mu\phi\Sigma|^2$

Cure by adding the radial excitation

$$\Sigma \longrightarrow \phi\Sigma \qquad\qquad \phi\equiv v+h$$

$$\phi\Sigma = \phi\left(\cos G + i\vec{\sigma}\cdot\frac{\vec{G}}{G}\sin G\right)\xrightarrow[\text{FIELD REDEFINITION}]{}\phi+i\vec{\sigma}\cdot\vec{G}$$

such that

$$\frac{v^2}{4}\mathrm{tr}\left|\partial_\mu\phi\Sigma\right|^2=\underbrace{\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}(\partial_\mu G_a)^2}_{\text{perfectly ok } \checkmark}$$

but now a mass term allowed: $m^2\,\mathrm{tr}\left|\phi\Sigma\right|^2=2m^2\phi^2 \Rightarrow$ hierarchy problem

Recovering the SM lagrangian:

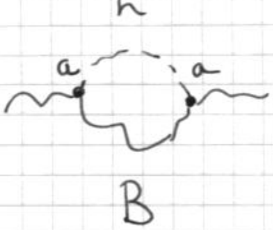
$$H=\Sigma\cdot\begin{pmatrix}0\\ \phi\end{pmatrix} \qquad \text{or equivalently} \qquad \phi\Sigma=\sqrt{2}\big(\underbrace{i\sigma_2H^*}_{\tilde{H}},H\big)$$

$\in 2$ of $SU(2)_L$

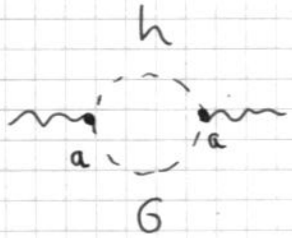
In the limit $g'\to 0$, $m_u=m_d$ custodial invariant: $SU(2)_R$ rotations of $\begin{pmatrix}H\\ \tilde{H}\end{pmatrix}$

2 is a pseudo real representation $[H_i \;\; H_i\epsilon_{ij}]$

Higgs and EWSB

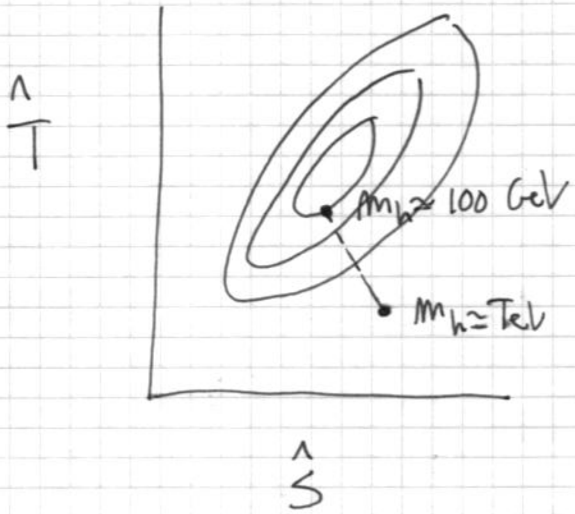
$\hat{T} \simeq$ 

$\simeq + \frac{3 g'^2}{64\pi^2} a^2 \ln \frac{\Lambda^2}{m_h^2}$ cancel Λ-divergence for $a=1$

$\hat{S} \sim$ 

$\simeq \frac{g^2}{192\pi^2} a^2 \ln \frac{\Lambda^2}{m_h^2}$ cancel Λ-divergence for $a=1$

$$\Downarrow$$



Light Higgs fit
EWP data

More Higgs: (T, Y)

If they get VEV

(a) $$\Delta\rho = \rho - 1 \propto (2\overset{\text{isospin}}{T}+1)^2 - 3\overset{\text{hypercharge}}{Y}^2 - 1 \approx 0$$

(b) Must contain a singlet:

$$\varphi = (T_3^a + Y)/2 = 0$$

$$\boxed{T_3^a = -Y}$$

$\hookrightarrow$ $T = \frac{1}{2}$ $Y = 1$ $\Rightarrow$ Higgs doublet

$T = 3$ $Y = 4$ $\rightarrow$ 7-plet

$\vdots$

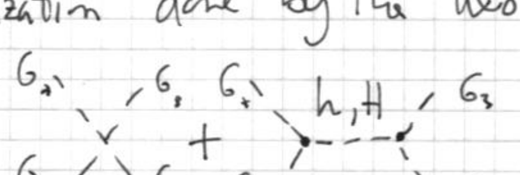
$\Rightarrow$ $\boxed{\text{H doublets} \oplus \text{singlet}}$

~~If they get small (or zero) VEV~~

~~they can still~~

THDM (as in the MSSM): H_1 and H_2

Unitarization done by the two Higgs



$$\xrightarrow[\text{large } s]{} \frac{s}{v^2}\left(1 - a_h^2 - a_H^2\right)$$

$$\boxed{a_h^2 + a_H^2 = 1} \Rightarrow \boxed{a_{h,H} \leq 1}$$

and other constraints

from $GG \to h_1 h_2, H^+H^-, \ldots$

Couplings to fermions:

Danger of FCNC if the two Higgs couple to same fermion

Impose a parity (Z_2) $\quad Y^{(1)}_{ij} H_1 \bar{q}_L^i u_R^j$; $Y^{(2)}_{ij} H_2 \bar{q}_L d_R$ $\rightarrow$ Not diagonal at the same time

type I: $H_1 \bar{q}_L u_R + \tilde{H}_1 \bar{q}_L d_R$ $\qquad$ type II: $H_1 \bar{q}_L u_R + H_2 \bar{q}_L d_R$

$\boxed{H_2 = -}$ $\quad + \ + \ + \quad + \ + \ + \qquad\qquad + \ + \ + \quad - \ + \ -$

THEORY OF A COMPOSITE HIGGS FIELD $\equiv$ QCD:

$SU(3)_c$ with two flavors: $(m_q = 0)$

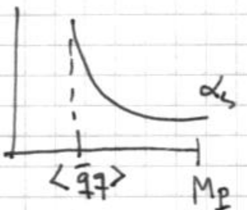
$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

Global symmetry: $SU(2)_L \otimes SU(2)_R \otimes U(1)_B \longrightarrow SU(2)_V \otimes U(1)_B$
(accidental)

Broken by the condensate $\langle \bar{q}_L q_R \rangle = \mathbb{1}\, \Lambda^3_{QCD}$

$$\equiv H(x) \quad \text{Higgs field} \quad 2\times 2 \text{ matrix}$$
$$\in 2$$

Dynamical generation of small scale



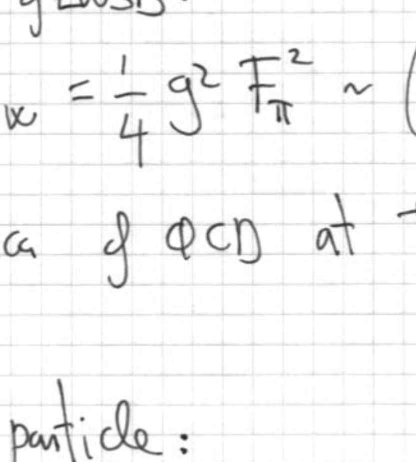
Custodial invariant theory of EWSB:

$$m_W^2 = m_Z^2 \cos^2\theta_W = \frac{1}{4} g^2 F_\pi^2 \sim (50\ \text{MeV})^2$$

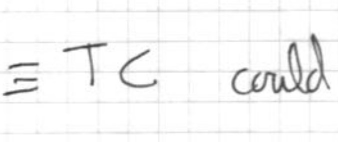
Too small but a replica of QCD at TeV $\equiv$ TC could do the job

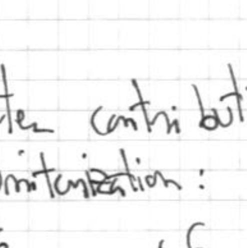
Theory of no Higgs particle:

$\boxed{\text{SPECTRUM}}$



All states contributing to unitarization:





as expected since large coupling theory $(g^2/16\pi^2 \sim 1)$:

$$m_h^2 \sim \lambda v^2 \sim 16\pi^2 v^2 \sim (\text{GeV})$$

But let's do this small change:

$$\boxed{SU(3)_c \longrightarrow SU(2)_c}$$

Now, the global symmetry is enlarged ("SU(2)" "gluons" can't distinguish between 2 and $\bar{2}$)

$$\boxed{SU(2)_L \times SU(2)_R \longrightarrow SU(4) \approx SO(6)}$$

$$\begin{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\ \begin{pmatrix} u_R^c \\ d_R^c \end{pmatrix} \end{pmatrix} = \Psi_L^a = (2_c, 4 \text{ of } SU(4))$$

Condensate:

$$\langle \bar{\Psi}_L^c \Psi_L \rangle \sim \left(\begin{array}{cc|cc} & 1 & & \\ -1 & & & \\ \hline & & & 1 \\ & & -1 & \end{array}\right) \Lambda^3$$

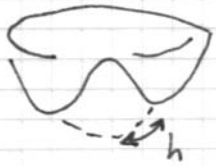
$\downarrow$ if maximal subgroup preserved

breaks to $SP(4) \approx SO(5) \supset SU(2)_L \times SU(2)_R$
(No EWSB)

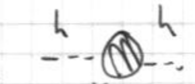
Symmetry breaking: $\underset{15}{SO(6)} \longrightarrow \underset{10}{SO(5)}$

5 Goldstone $= 4 + 1$

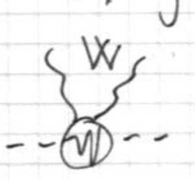
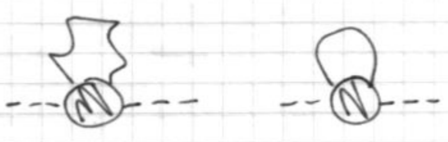
$\hookrightarrow$ 2 of $SU(2)_L$ $\equiv$ Higgs as Goldstone:



$\hookrightarrow$ 1 of $SU(2)$ $\equiv$ singlet

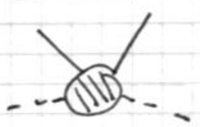
No potencial  $= 0$

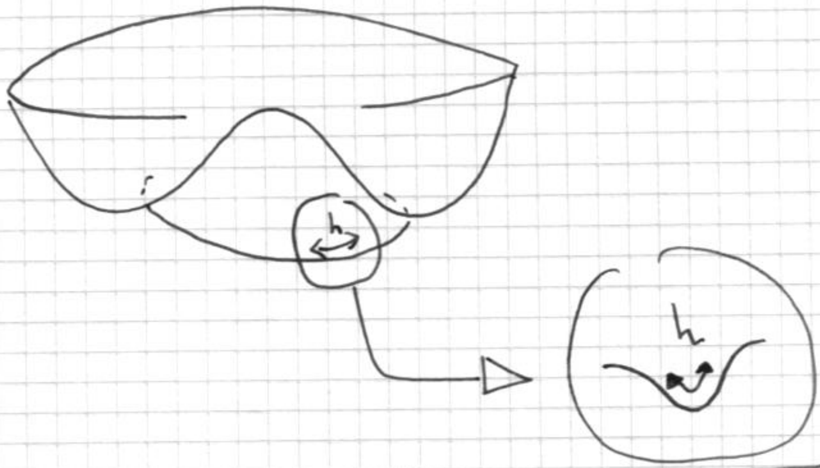
But $SU(4)$ symmetry broken by the gauging of $SU(2)_L \otimes U(1)_Y$

 $\Rightarrow$ 

weak coupling

$$m_h^2 \sim \frac{g^2}{16\pi^2} \Lambda_{QCD'}^2$$

and Top  +  + ... a loop suppressed from heavy resonances (m_ρ^2)



$\boxed{\text{Higgs as PGB}}$

PHENOMENOLOGY OF THE MINIMAL COMPOSITE (PGB) HIGGS (MCHM)

Strong sector at TeV whose condensate breaks:

$$\boxed{\begin{array}{c} \underset{(10\text{ gen})}{SO(5)} \xrightarrow{\langle\Sigma\rangle} \underset{(6\text{ gen})}{SO(4)} \approx SU(2)_L \times SU(2)_R \\ 4 \text{ Goldstones} = \text{Higgs doublet} \end{array}}$$

condensate: 5-vector pointing toward a direction

"H" "higgs field of strong sector" $\langle\Sigma\rangle = (0,0,0,0,1)$

$$i\begin{pmatrix} 0_{4\times4} & \begin{matrix} h^1 \\ h^2 \\ h^3 \\ h^4 \end{matrix} \\ -h^1\ -h^2\ -h^3\ -h^4 & 0 \end{pmatrix}$$

Higgs = angular excitation

$$\Sigma = \langle\Sigma\rangle e^{-i\sqrt{2}\,\widetilde{T^a h^a}/f} \in 5 \text{ of } SO(5)$$

T^a = generators broken: $SO(5)/SO(4)$ coset

$$T^a_{ij} = -\frac{i}{\sqrt{2}}\left(\delta^a_i \delta^5_j - \delta^a_j \delta^5_i\right)$$

$a = 1,2,3,4$

$i,j = 1,2,3,4,5$

$$\boxed{\Sigma = \frac{\sin h/f}{h}\left(h^1, h^2, h^3, h^4, h \cot h/f\right)} \qquad h = \sqrt{\sum_a (h^a)^2}$$

Effective theory of Σ and SM fields:

SPECTRUM: $\equiv$ TeV, — h

$$\mathcal{L}_{Higgs} = \frac{f^2}{2}(D_\mu\Sigma)(D^\mu\Sigma)^T + \dots + \underbrace{V(\sin h/f)}_{\text{at one-loop}}$$

must induce $\langle h\rangle \neq 0 \sim f$

Higgs coupling to WW:

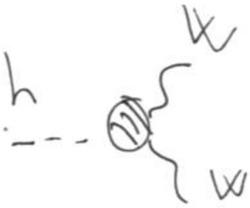
unitary gauge: $\Sigma = \left(0, 0, 0, \sin\frac{h}{f}, \cos\frac{h}{f}\right)$

$h =$ Higgs

$$\mathcal{L}_\Sigma = \frac{1}{2}(\partial_\mu h)^2 + \underbrace{g^2 \frac{f^2}{4} \sin^2\frac{h}{f}}\left[WW + \frac{1}{2\cos^2\theta_W} ZZ\right]$$

$\downarrow$ taylor expanding around $\langle h \rangle$

$$\frac{1}{4} g^2 f^2 \left[\sin^2\frac{\langle h\rangle}{f} + 2\sin\frac{\langle h\rangle}{f}\cos\frac{\langle h\rangle}{f}\cdot\frac{h}{f} + \cdots\right]$$



$$= g\, m_W \cdot \cos\frac{\langle h\rangle}{f} = g^{SM}_{hWW}\cdot a$$

Defining $\frac{v}{f} = \sin\frac{\langle h\rangle}{f} \longrightarrow \boxed{a = \sqrt{1-\frac{v^2}{f^2}}}$

—— f = SO(5) breaking scale

—— v = EWSB scale

Deviation from SM prediction expected for a composite Higgs

$\downarrow$

other states, at TeV, needed to fully unitarize the amplitude $G^+G^- \to G_3 G_3$