Across the Tev frontier with the LHC
Cargèse, August 30th, 2012

The “New physics” behind neutrino masses

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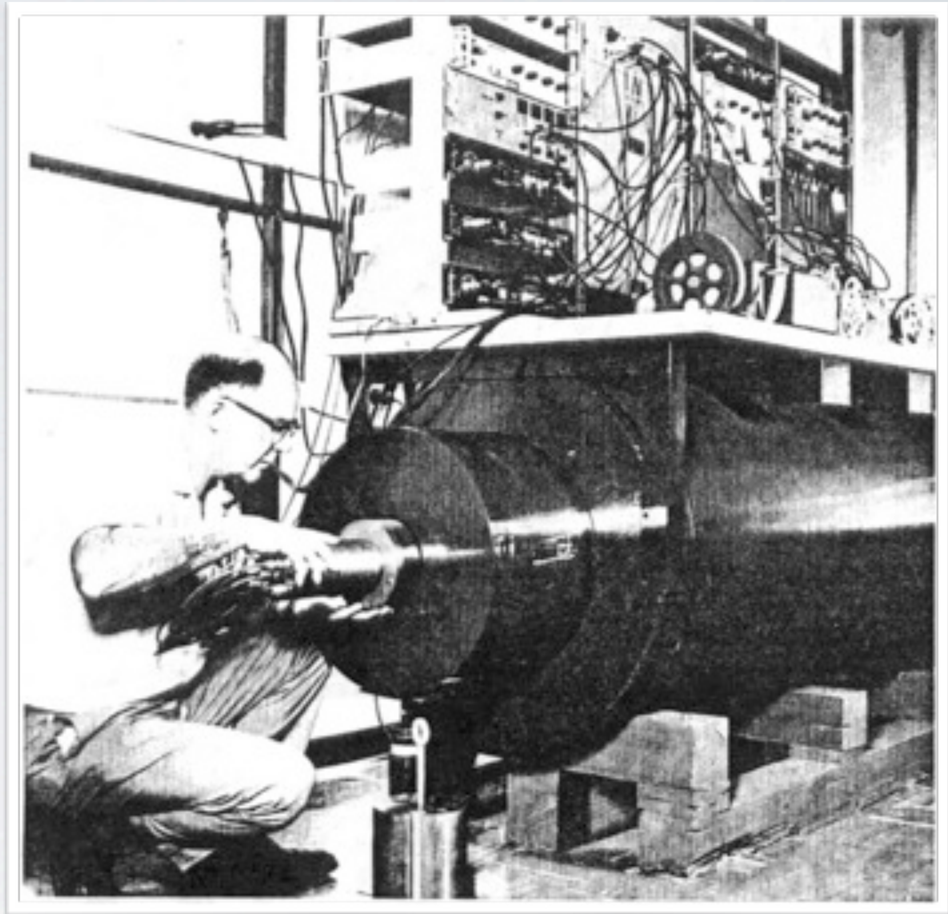
1. Why do neutrinos have masses?

1960's experiment by Raymond Davies at the Homestake mine in South Dakota revealed the "Solar neutrino problem":



Neutrinos, ν_e produced in the sun "vanished" before reaching the earth!

Reason for this: $\nu_e \to \nu_\mu$

$\nu_e \to \nu_\tau$

Neutrino oscillations ➡ neutrino mixing ➡ neutrino masses!

Shock! Standard Model wrong?

Can't be! Many discoveries from it!

Need "New physics" to explain neutrino masses

Invent a new model.

Modify the standard model as little as possible.

2. How do neutrinos get masses?

Dirac or Majorana neutrinos?

Dirac:
Yukawa type coupling with right-handed neutrino(s) and Higgs:

$$\bar{N}_R \phi \psi_L + h.c.$$

Not favorable: Can not explain the small neutrino masses.

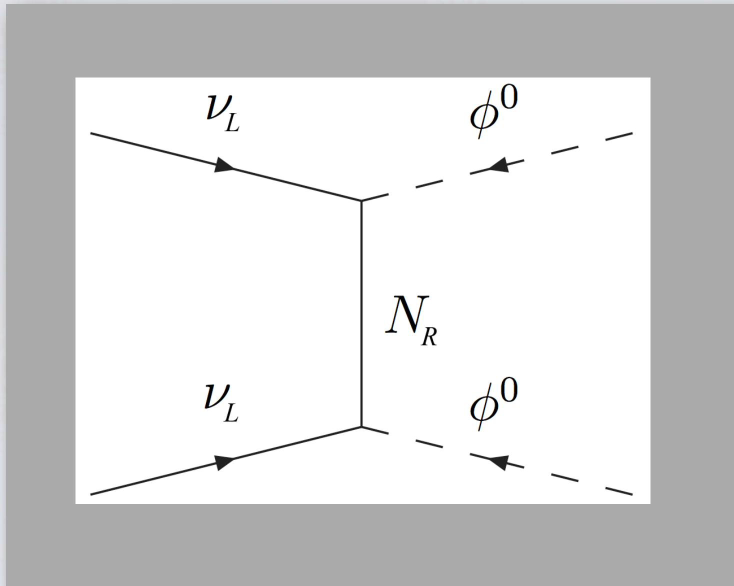
Majorana:
Neutrinos own antiparticles
→ Non-renormalizable dimension 5 operator:

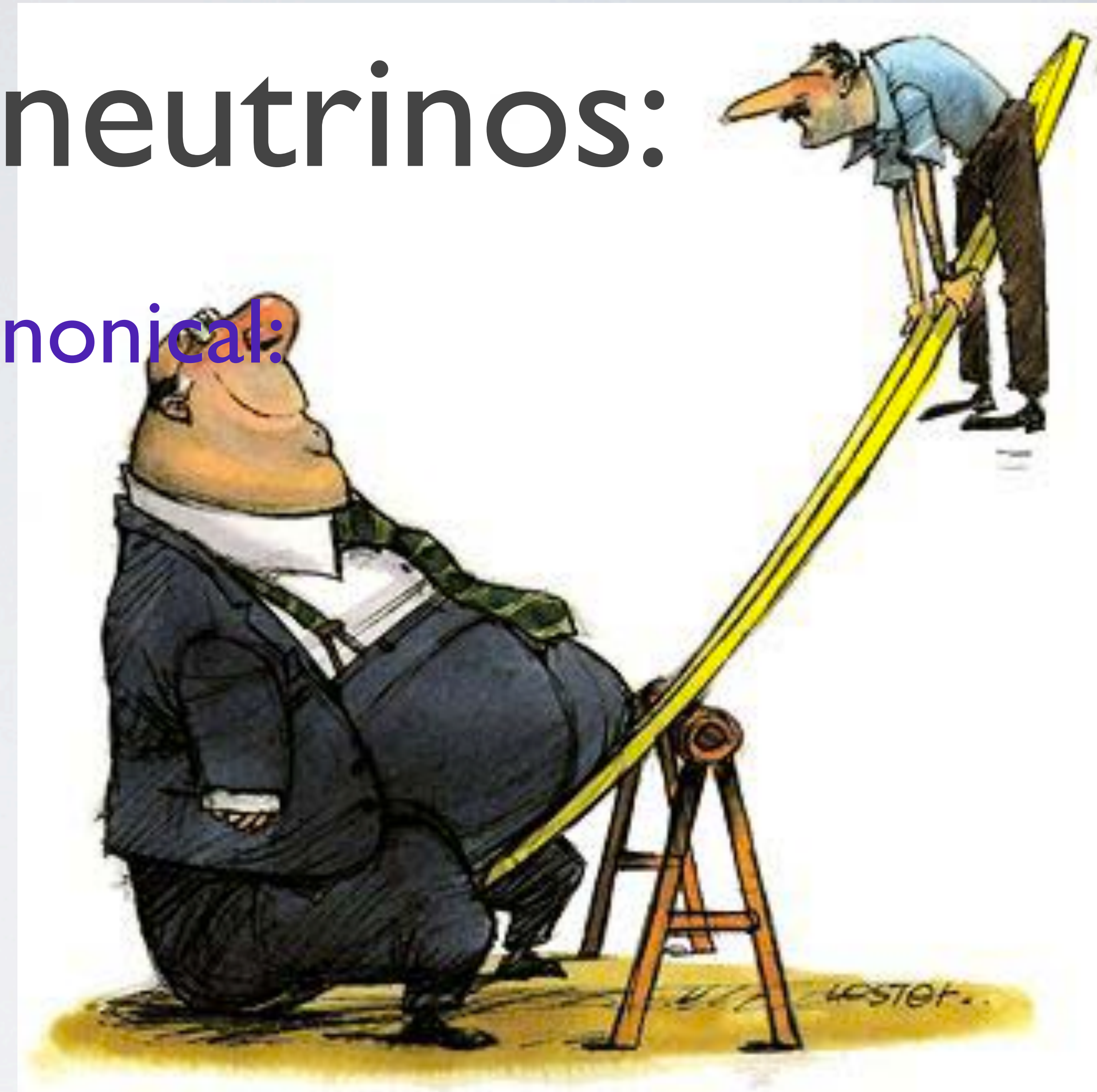
$$(\phi \tau_2 \psi_L) C (\phi \tau_2 \psi_L)$$

Effective field theory:
→ large cut-off scales
→ naturally small masses.

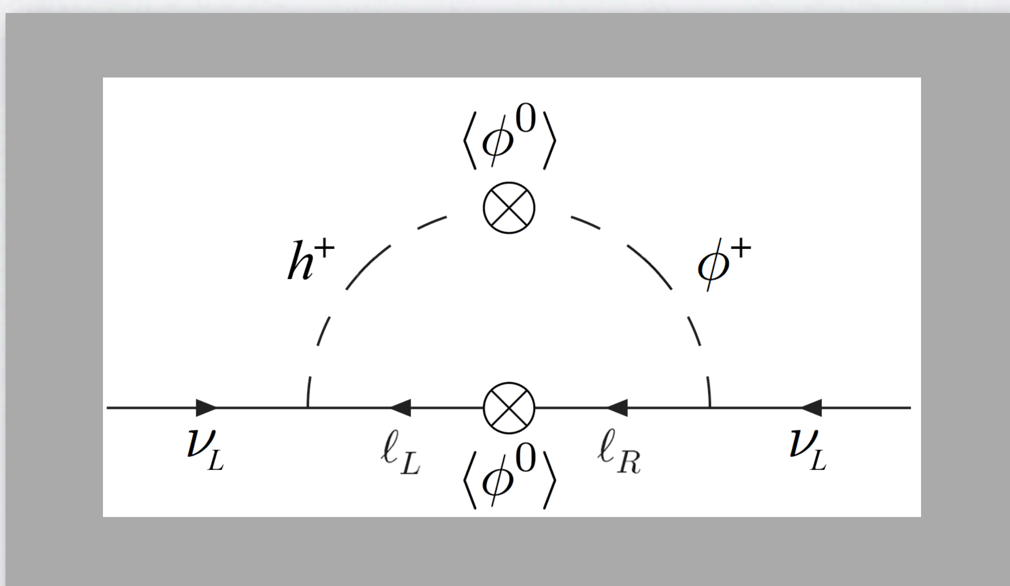
Majorana neutrinos:

See-saw mechanism, Type I, canonical:





Zee model, radiative loop correction:



3. What are the masses?

Nature comes in 3!

$$\nu_e, \; \nu_\mu, \; \nu_\tau$$

My research: Frobenius group!

Convenient Irreducible Representations:

$$\mathbf{1} \quad \mathbf{1}' \quad \bar{\mathbf{1}}' \quad \mathbf{3}_1 \quad \bar{\mathbf{3}}_1 \quad \mathbf{3}_2 \quad \bar{\mathbf{3}}_2$$

Fits exactly with tribimaximal mixing describing neutrino mixing at close approximation.

C. Hartmann and A. Zee, Nucl. Phys. B 853, 105 (2011) [arXiv:1106.0333[hep-ph]]

C. Hartmann, Phys. Rev. D 85, 013012 (2012) [arXiv:1109.5143[hep-ph]]

Back-up slides

The Frobenius group T_{13}

Subgroup of SU(3)

Semi-direct product: $T_{13} = Z_{13} \rtimes Z_3$

Presentation:

$$\langle a, b \mid a^{13} = b^3 = I, bab^{-1} = a^3 \rangle$$

$$\boxed{a_1 = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^3 & 0 \\ 0 & 0 & \rho^9 \end{pmatrix}}$$

$$\boxed{b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}} \qquad \boxed{\rho = e^{\frac{2\pi i}{13}}}$$

$$\boxed{a_2 = \begin{pmatrix} \rho^2 & 0 & 0 \\ 0 & \rho^6 & 0 \\ 0 & 0 & \rho^5 \end{pmatrix}}$$

Irreducible Representations:

$$\mathbf{1} \quad \mathbf{1}' \quad \bar{\mathbf{1}}' \quad \mathbf{3}_1 \quad \bar{\mathbf{3}}_1 \quad \mathbf{3}_2 \quad \bar{\mathbf{3}}_2$$

Implement Frobenius group with see-saw mechanism

→ Predictions!

$$M_{\nu A} \sim \begin{pmatrix} \alpha & 0 & \beta \\ 0 & \alpha - \frac{\beta^2}{\alpha} & 0 \\ \beta & 0 & \alpha \end{pmatrix}$$

Depends on only two parameters!

Experimental data:

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Parameter	Best fit $\pm 1\sigma$	2 σ	3σ
$\Delta m_{12}^2 [10^{-5} eV^2]$	7.62 ± 0.19	7.27-8.01	7.12-8.20
$\lvert\Delta m_{32}^2\rvert [10^{-3} eV^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	2.34-2.69 -(2.25-2.59)	2.26-2.77 -(2.15-2.68)

Implement Frobenius group with see-saw mechanism

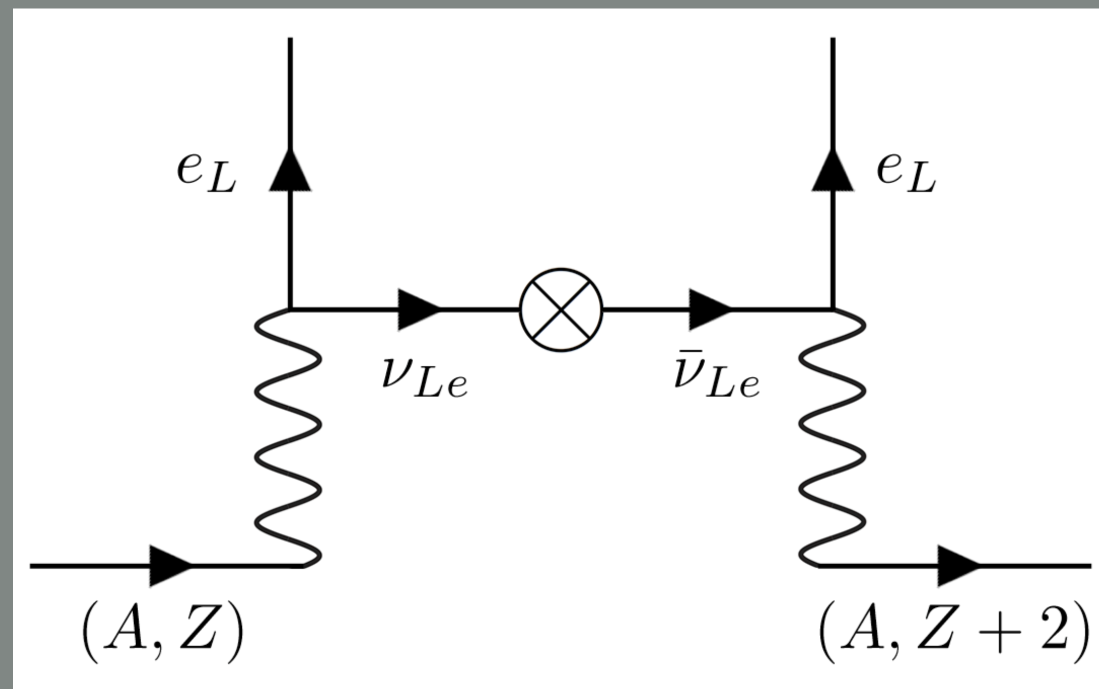
→ Predictions!

My results:

Hierarchy	Normal		Inverted
$\alpha\,[10^{-2}eV]$	-2.80 ±0.07	-2.28 ±0.06	1.70 ±0.05
$\beta\,[10^{-2}eV]$	2.21 ±0.06	2.72 ±0.07	3.42±0.08
$m_1\,[10^{-2}eV]$	-0.59 ±0.01	0.45 ±0.01	5.12 ±0.13
$m_2\,[10^{-2}eV]$	-1.05 ±0.02	0.98 ±0.01	-5.19 ±0.13
$m_3\,[10^{-2}eV]$	-5.01 ±0.13	-5.00±0.13	-1.72 ±0.05

C. Hartmann (2011) Phys. Rev. D **85**, 013012 (2012) [arXiv:1109.5143[hep-ph]]

Neutrinoless double beta decay



$$|m_{ee}| = |m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2| = \frac{1}{3}|2m_1 + m_2|$$

Hierarchy	$\lvert m_{ee}\rvert [10^{-2} eV]$
Normal	0.74 ± 0.01 0.62 ± 0.01
Inverted	1.68 ± 0.05

Oscillation probability
-depends on mass differences

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

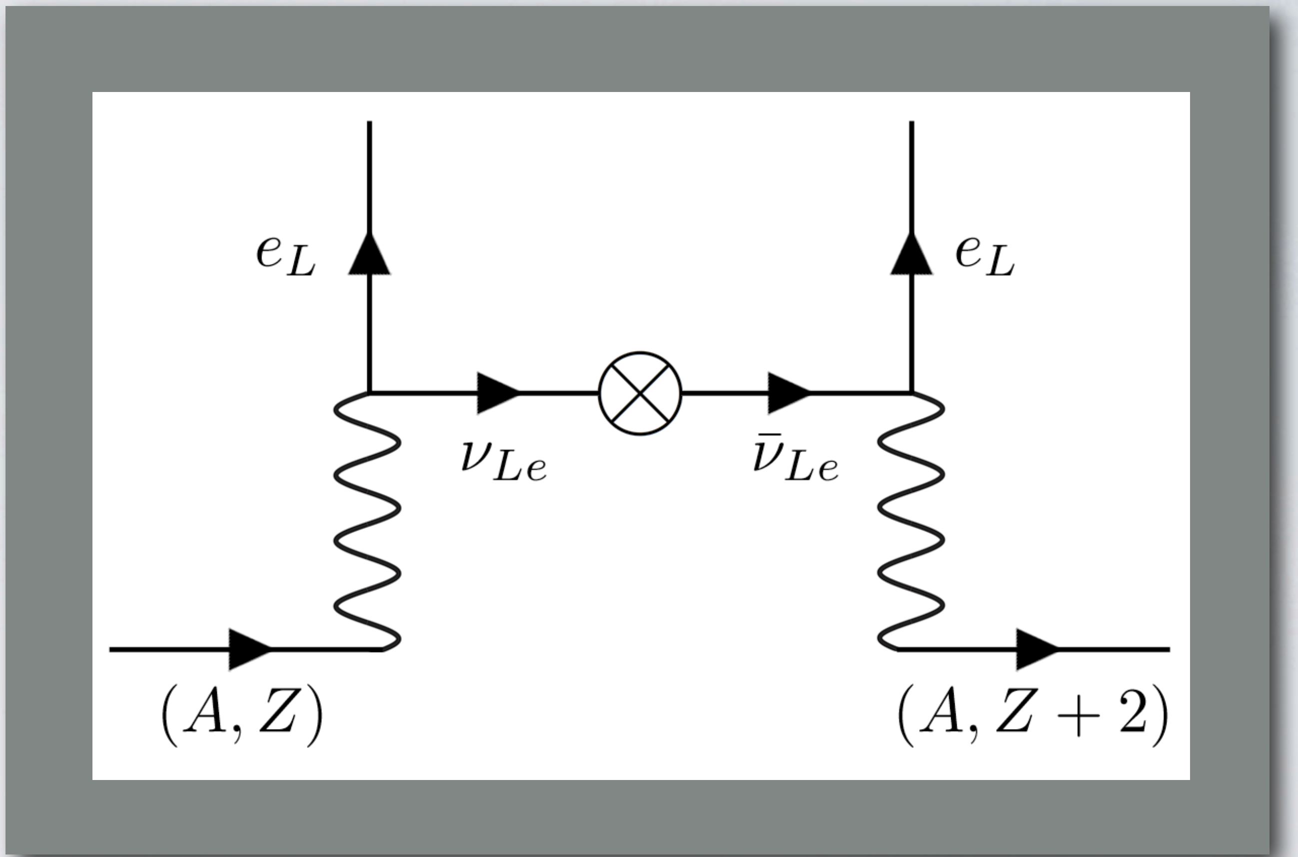
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha,\beta} - 4 \sum_{j<i=1} \mathrm{Re}[U^*_{\beta j} U_{\alpha j} U_{\beta i} U^*_{\alpha i}] \sin^2 \left(\frac{\Delta m^2_{ij} L}{4E} \right) \pm 2 \sum_{j<i=1} \mathrm{Im}[U^*_{\beta j} U_{\alpha j} U_{\beta i} U^*_{\alpha i}] \sin^2 \left(\frac{\Delta m^2_{ij} L}{2E} \right)$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

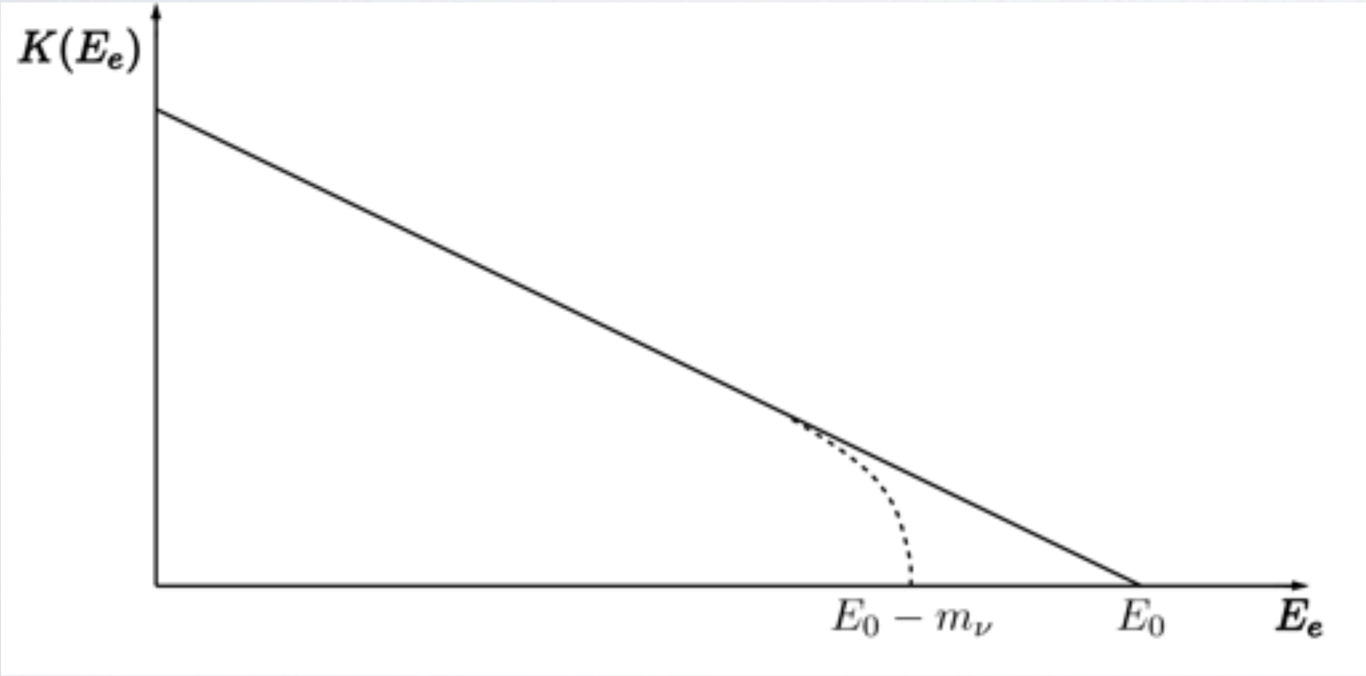
Mass limits from experiments

Neutrinoless double beta decay -Majorana neutrinos



$$|m_{ee}| = |m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2|$$
$$= \frac{1}{3}|2m_1 + m_2|$$

$$|m_{ee}| \leq 0.38\,\mathrm{eV}$$



Tritium beta decay - electron spectrum near endpoint

$$m_\beta = (|U_{ei}|^2 m_i^2)^{1/2} \leq 2.2eV$$

Majorana neutrinos

Charged lepton masses generated from dimension 4 operator:

$$\boxed{\mathcal{O}_4 = \bar{l}_R \phi \psi}$$

Neutrino masses generated from dimension 5 operator:

$$\boxed{\mathcal{O}_5 = (\xi_1 \tau_2 \psi) C (\xi_2 \tau_2 \psi)}$$

$$\mathcal{L}_{mass} = -\bar{l}_{L\alpha}(M_l)_{\alpha\beta} l_{R\beta} - \frac{1}{2}\nu^T_{L\alpha}(M_\nu)_{\alpha\beta} C \nu_{L\beta} + h.c$$

$$U_L^\dagger M_l U_R \equiv D_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$$U_\nu^T M_\nu U_\nu \equiv D_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$\boxed{\begin{aligned} l_{L\alpha} &\equiv (U_L)_{\alpha i} l_{Li} \\ \nu_{L\alpha} &\equiv (U_\nu)_{\alpha i} \nu_{Li} \end{aligned}}$$

The lepton mixing matrix

$$\begin{pmatrix} \nu_{L\alpha} \\ l_{L\alpha} \end{pmatrix} = \begin{pmatrix} U_\nu \nu_{Li} \\ U_L l_{Li} \end{pmatrix} = U_L \begin{pmatrix} U_L^{-1} U_\nu \nu_{Li} \\ l_{Li} \end{pmatrix}$$

$$\boxed{U = U_L^\dagger U_\nu}$$

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} \bar{l}_L \gamma^\mu U \nu_L W_\mu^\dagger$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$U_{TB} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Kronecker products and possible representations

$\mathbf{1}' \otimes \mathbf{1}' = \bar{\mathbf{1}}'$
$\bar{\mathbf{1}}' \otimes \bar{\mathbf{1}}' = \mathbf{1}'$
$\mathbf{1}' \otimes \bar{\mathbf{1}}' = \mathbf{1}$
$\mathbf{3}_1 \otimes \mathbf{3}_1 = \bar{\mathbf{3}}_1 \oplus \bar{\mathbf{3}}_1 \oplus \mathbf{3}_2$
$\mathbf{3}_2 \otimes \mathbf{3}_2 = \bar{\mathbf{3}}_2 \oplus \bar{\mathbf{3}}_1 \oplus \bar{\mathbf{3}}_2$
$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_1 = \mathbf{1} \oplus \mathbf{1}' \oplus \bar{\mathbf{1}}' \oplus \mathbf{3}_2 \oplus \bar{\mathbf{3}}_2$
$\mathbf{3}_2 \otimes \bar{\mathbf{3}}_2 = \mathbf{1} \oplus \mathbf{1}' \oplus \bar{\mathbf{1}}' \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_1$
$\mathbf{3}_1 \otimes \mathbf{3}_2 = \bar{\mathbf{3}}_2 \oplus \mathbf{3}_1 \oplus \mathbf{3}_2$
$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_2 = \bar{\mathbf{3}}_2 \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_1$
$\mathbf{3}_2 \otimes \bar{\mathbf{3}}_1 = \mathbf{3}_2 \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_1$

Case	ψ	ϕ	$\bar{l}_R$
1	$\mathbf{3}_1$	$\mathbf{3}_1$	$\mathbf{3}_1$
3	$\mathbf{3}_1$	$\mathbf{3}_1$	$\bar{\mathbf{3}}_2$
22	$\mathbf{3}_1$	$\bar{\mathbf{3}}_1$	$\mathbf{1}, \mathbf{1}', \bar{\mathbf{1}}'$
25	$\mathbf{3}_2$	$\bar{\mathbf{3}}_2$	$\mathbf{1}, \mathbf{1}', \bar{\mathbf{1}}'$

Charged lepton masses generated from dimension 4 operator:

$$\boxed{\mathcal{O}_4 = \bar{l}_R \phi \psi}$$

Neutrino masses generated from dimension 5 operator:

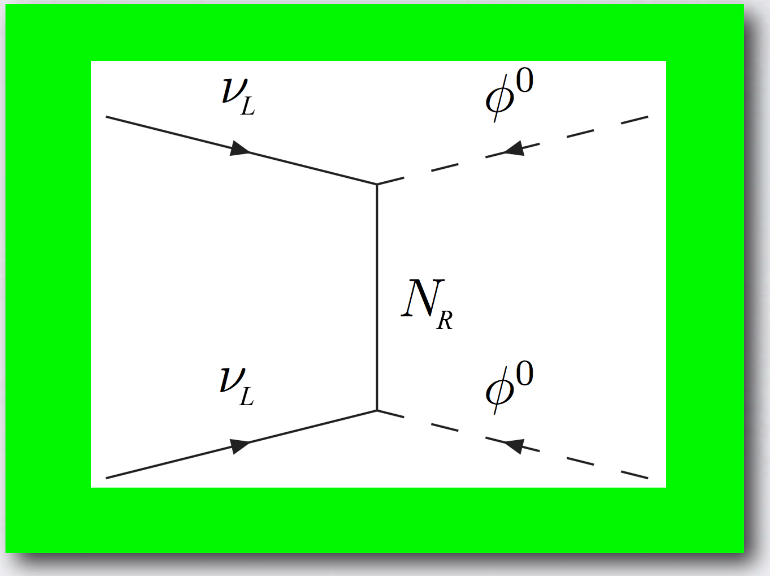
$$\boxed{\mathcal{O}_5 = (\xi_1 \tau_2 \psi) C (\xi_2 \tau_2 \psi)}$$

C. Hartmann and A. Zee, Nucl. Phys. B 853, 105 (2011) [arXiv:1106.0333[hep-ph]].

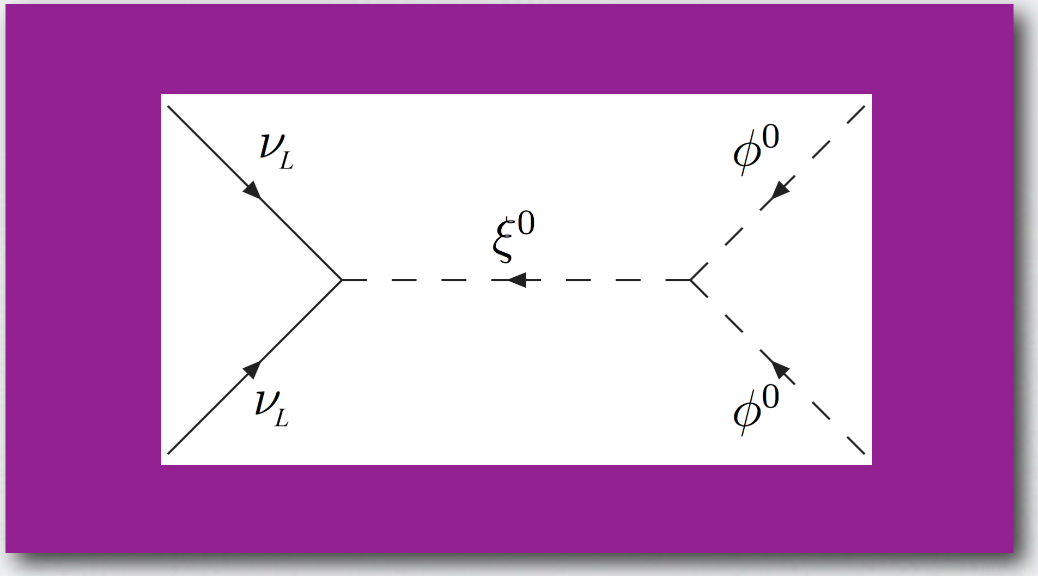
See-saw mechanism

$$\mathcal{O}_5 = \frac{1}{M}(\phi\tau_2\psi)^T C(\phi\tau_2\psi)$$

Type I:



Type II:



Type III: Type I with $N \leftrightarrow \Sigma^0$