Naturalness

- Argument always uses a cut-off reg^n
- but in pert QFT everyone uses Dim Reg
- in DR   log div $= 1/\epsilon$
  lin div $= 1/\epsilon$
  quadr. $= 1/\epsilon$

So, where's the problem?

1 Toy model [Collins' book], SM as an eff. theory

ϕ_ℓ (light) ϕ_h (heavy)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_\ell)^2 - \frac{1}{2}m^2\phi_\ell^2 + \frac{1}{2}(\partial_\mu \phi_h)^2 - \frac{1}{2}M^2\phi_h^2$$
$$- \left[\frac{\lambda_1}{4!}\phi_\ell^4 + \lambda_3\frac{\phi_h^4}{4!} + \frac{\lambda_2}{2!2!}\phi_\ell^2\phi_h^2\right]\mu^{4-d}$$

+ counterterms

$$\hookrightarrow \frac{1}{2}(Z_\ell - 1)(\partial_\mu \phi_\ell)^2 + \ldots$$

$$-\frac{1}{2}\left[m^2(Z_m - 1) + M^2 Z_{mM}\right]\phi_\ell^2 .$$

Renormalized theory! $Z_i = 1 + z_i^{(1)}\frac{1}{\epsilon} + \ldots$

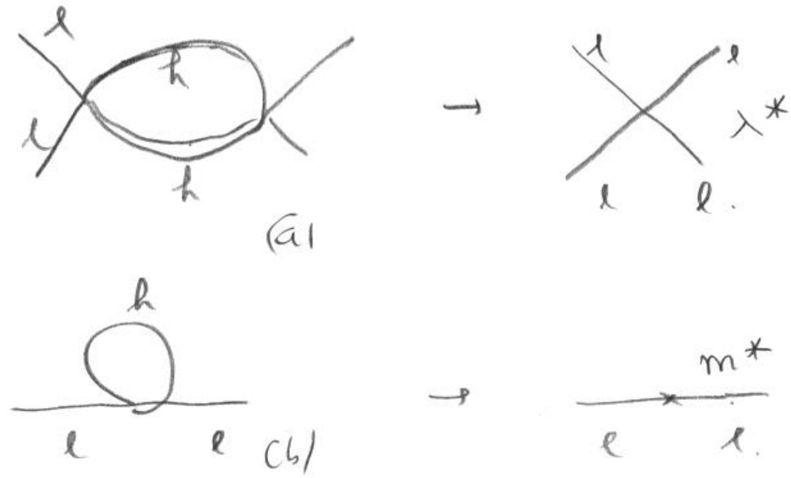
Let's say we use the $\overline{MS}$ scheme, so all $1/\epsilon$ cancel, but not more.

2 At low energies, much below M, we should be able to use effective theory

$$\mathcal{L}_{eff} = \frac{1}{2}z^2(\partial_\mu \phi^*)^2 - \frac{1}{2}m^{*2}\phi^{*2} - \frac{\lambda^*}{4!}\mu^{4-d}\phi^{*4}.$$

$z, \phi^*, m^*, \lambda^*$ should incorporate ϕ_h effects full — eff.

1)



a): $\frac{\lambda_2^2}{256\pi^2}\left[\frac{1}{\epsilon} + \gamma_E + \ln\frac{M^2}{4\pi\mu^2}\right]$

b): $\frac{\lambda_2 M^2}{32\pi^2}\left[\frac{1}{\epsilon} + \gamma_E + 1 - \ln\frac{M^2}{4\pi\mu^2}\right]$

a) $z^2\lambda^* = \lambda_1 - \frac{\lambda_2^2}{256\pi^2}\left[\gamma_E + \ln\frac{M^2}{4\pi\mu^2}\right]$

Relation between renor<u>m</u>alized parameters

b) $zp^2 - zm^{*2} = p^2 - m^2 + \frac{\lambda_2 M^2}{32\pi^2}\left[1 - \gamma_E + \ln\frac{M^2}{4\pi\mu^2}\right]$

$\Rightarrow$ $\boxed{z = 1}$

$$\boxed{m^{*2} = m^2 - \frac{\lambda_2 M^2}{32\pi^2}\left[1 - \gamma_E + \ln\frac{M^2}{4\pi\mu^2}\right]}$$

} Matching condition

Hierarchy problem returns, but now as tuning of m^2, M^2, renormalized para's

(What if there is no UV complete theory?)

What if there is no ϕ_h?

(Bardeen, Nagoya

$\phi \to \alpha\, \phi$, $x \to {}^{1}/_{\alpha}\, x$ exact symmetry of SM if

i) tree level only
ii) no $\mu^2 \phi^2$ term. (soft breaking

Λ reg^n breaks scale inv. explicitly
→ hence the quadr. counterterm)

Landau pole of higgs propagator

Renormalons & the heavy quark pole mass

Consider a perturbative series

$$R = \sum_{n=0}^{\infty} r_n \alpha^{n+1}$$

Q1 Does it converge? In QFT: no (most likely)

R is "asymptotic series"

Q2 Can we assign a value? yes.

A1: Terms $r_i \alpha^{i+1}$ decrease as $i \uparrow$, then increase again. Sum to where $\frac{r_{i+1}\alpha^{i+2}}{r_i \alpha^{i+1}} \approx 1$

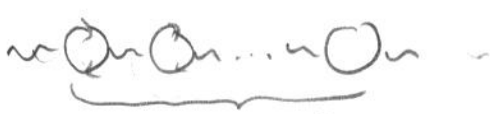
A2 Borel tfm

$$B[R](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!} \quad \text{, then}$$

$$R = \int_0^{\infty} dt \, e^{-t/\alpha} \, B[R](t)$$

Problem if $r_n \sim n!$ **Q3** Does this happen?

Yes, for diagrams

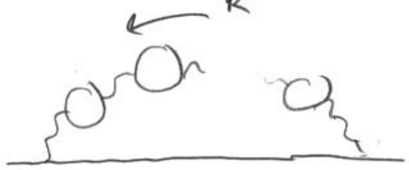


n fermion bubbles

"Renormalon" behavior

↳ not a particle.

Case: fermion self-energy



$$\Sigma(p) = \sum_{n=0}^{\infty} \alpha_s \int_0^{\infty} \frac{d k^2}{k^2} F(k) \left[\beta_0^f \alpha \ln\left(\frac{k^2}{Q^2}\right) \right]^n$$

$$\beta_0^f = \frac{1}{2} \frac{N_f}{3\pi}$$

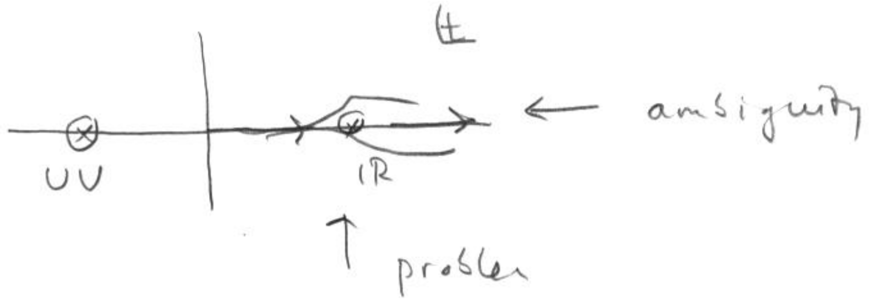
For large k^2 : (after k^2 integral) $\Sigma(p) \sim n!$ UV renormalon

" small k^2 also IR " "

i) Sum over n, change $\beta_0^f \to -\beta_0 = -\left(\frac{11 C_A - 2N_f}{12\pi}\right)$

IR renormalon : $\sum_{n=0}^{\infty} \frac{n!}{n!} t^n c^n = \frac{1}{1-ct}$

UV " $\sum_{n=0}^{\infty} (-1)^n \frac{n! t^n d^n}{n!} = \frac{1}{1+dt}$



$$m_p = \int_0^{\infty} dt\, e^{-t/\alpha_s} B[m](t)$$

$$\delta m_p = O(\Lambda_{QCD})$$