Hawking emission from quantum black holes

Elizabeth Winstanley

Consortium for Fundamental Physics
Astro-Particle Theory and Cosmology Group
School of Mathematics and Statistics
University of Sheffield
United Kingdom

Work done in collaboration with: Marc Casals, Sam Dolan, Gavin Duffy, Panagiota Kanti and Piero Nicolini

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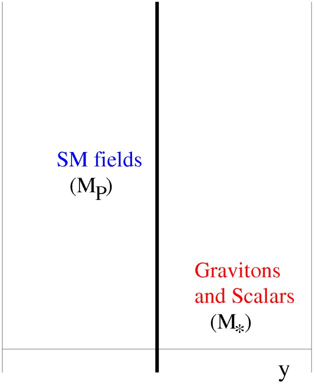
Outline

Brane worlds

Our universe is a brane living in a higher-dimensional bulk

- ADD model - flat compactified extra dimensions
- Standard model physics restricted to the brane
- Only gravity propagates in the bulk
- Fundamental higher-dimensional scale of quantum gravity, M_*, may be as low as the energy of the LHC:

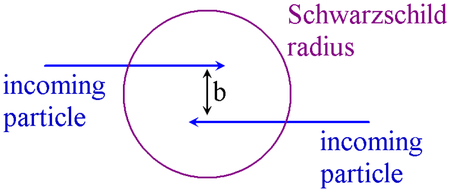
$$M_P^2 \sim M_*^{n+2} R^n$$



[Figure taken from Kanti, arXiv:0802.2218 [hep-th]]

Formation of mini black holes

If $M_* \sim$ few TeV, particle collisions at LHC may produce heavy, quantum gravitational objects



Will a semi-classical black hole be formed?

Compton wavelength of colliding particle of energy $E/2$ must lie within the Schwarzschild radius:

$$4\pi/E < r_h(E)$$

Therefore $E/M_* \gtrsim 10$ in order for black holes to form
[Meade and Randall, `arXiv:0708.3017 [hep-ph]`]

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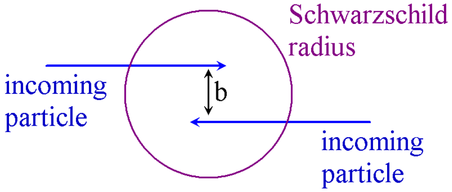

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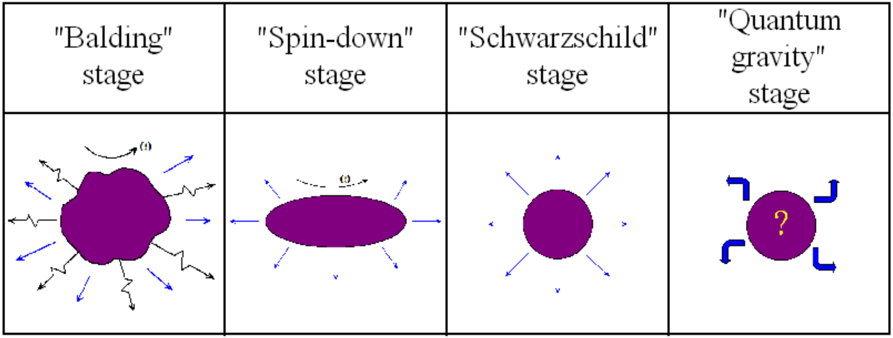
Stages in the evolution of small black holes

Black holes formed will be rapidly rotating, highly asymmetric, and have gauge field hair

Four stages of subsequent evolution:

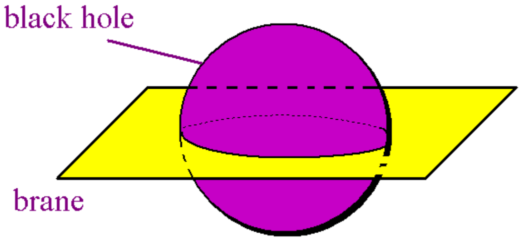


[Giddings and Thomas, `hep-ph/0106219`]

Modelling small black holes at the end of the balding stage

Small black holes in ADD

- Metric of higher-dimensional black holes in general relativity is known [Myers and Perry, *Annals Phys.* **172**, 304 (1986)]
- Take a 'slice' through a higher-dimensional black hole to give a brane black hole



Modelling small black holes in ADD

Myers-Perry higher-dimensional black hole

$$\begin{aligned} ds^2 &= \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}} dt\, d\varphi - \frac{\Sigma}{\Delta_n} dr^2 - \Sigma\, d\theta^2 \\ &\quad - \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\varphi^2 - r^2 \cos^2\theta\, d\Omega_n^2 \end{aligned}$$

where

$$\Delta_n = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \Sigma = r^2 + a^2 \cos^2\theta$$

Black hole mass M and angular momentum J:

$$M = \frac{(n+2)\, A_{n+2}\mu}{16\pi G_{4+n}}, \qquad J = \frac{2aM}{n+2}$$

Modelling small black holes in ADD

Slice of Myers-Perry black hole

$$
\begin{aligned}
ds^2 &= \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}} dt\, d\varphi - \frac{\Sigma}{\Delta_n} dr^2 - \Sigma\, d\theta^2 \\
&\quad - \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\varphi^2
\end{aligned}
$$

where

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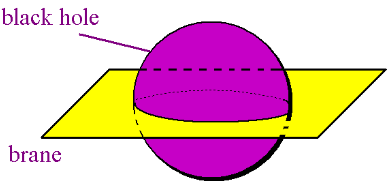
\Delta_n = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \Sigma = r^2 + a^2 \cos^2\theta
$$

and n is the number of extra dimensions.

Usual Kerr black hole

Set $n = 0$ in the above metric

Hawking radiation on the brane and in the bulk



Hawking temperature

$$T_H = \frac{(n+1)r_h^2 + (n-1)a^2}{4\pi(r_h^2 + a^2)r_h}$$

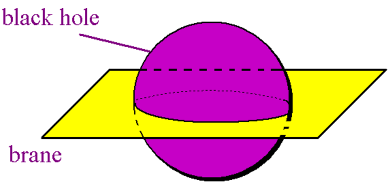
Particles on the brane

- Standard model particles: fermions, gauge bosons, Higgs
- Also gravitons and scalars
- Live on the brane "slice" of the black hole geometry

Particles in the bulk

- Gravitons and scalars
- Will be invisible
- Live on the higher-dimensional black hole geometry

Hawking radiation on the brane and in the bulk



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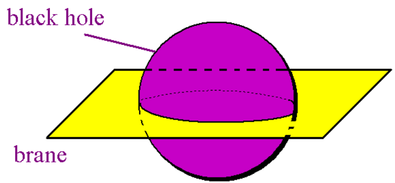

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Quantum fields on black hole space-times

Quantum field theory in curved space-time

- Black hole geometry is fixed and classical
- Quantum fields (scalars, fermions, gauge bosons, gravitons) propagate on this background

Quantum field modes

- "Master" equation for fields of spin 0, $\frac{1}{2}$, 1 and 2 on Kerr
  [Teukolsky, *Phys. Rev. Lett.* **29** 1114 (1972); *Astrophys. J.* **185** 635 (1973)]
- Expand field Ψ in terms of modes of frequency ω:

$$\Psi = \sum_{\omega \ell m} R_{s\omega\ell m}(r) S_{s\omega\ell m}(\theta) e^{-i\omega t} e^{im\varphi}$$

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Computing Hawking radiation

Differential emission rates, integrated over all angles:

$$\frac{d^2}{dt\,d\omega}\begin{pmatrix} N \\ E \\ J \end{pmatrix} = \frac{1}{4\pi}\sum_{\text{modes}} \frac{|\mathcal{A}_{s\omega\ell m}|^2}{e^{\tilde{\omega}/T_H} \mp 1}\begin{pmatrix} 1 \\ \omega \\ m \end{pmatrix}$$

where $\tilde{\omega} = \omega - m\Omega_H$

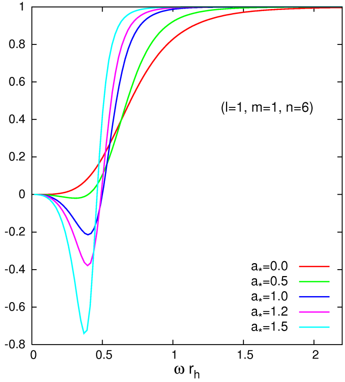
Grey-body factor $|\mathcal{A}_{s\omega\ell m}|^2$

- Emitted radiation is not precisely thermal
- Interaction of emitted quanta with gravitational potential around the black hole
- For an outgoing wave from the event horizon of the black hole:

$$|\mathcal{A}_{s\omega\ell m}|^2 = 1 - |\mathcal{R}_{s\omega\ell m}|^2 = \frac{\mathcal{F}_{\text{infinity}}}{\mathcal{F}_{\text{horizon}}}$$

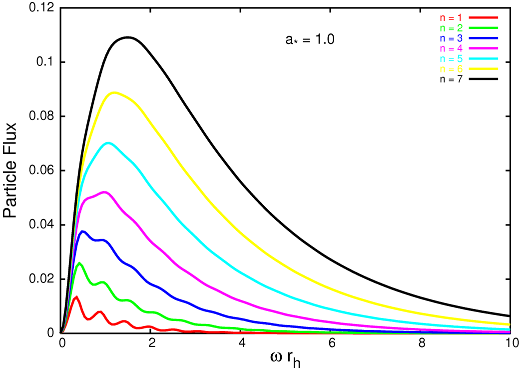
Grey-body factors and emission spectra

Grey-body factors for gauge boson emission and $n = 6$



[Casals et al, hep-th/0511163]

Fermion emission spectra for a rotating black hole, integrated over all angles



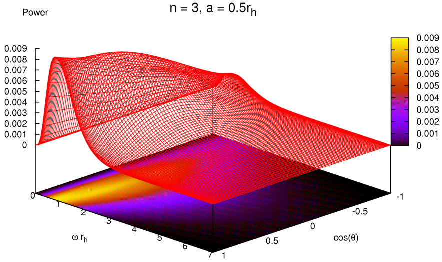
[Casals et al, hep-th/0608193]

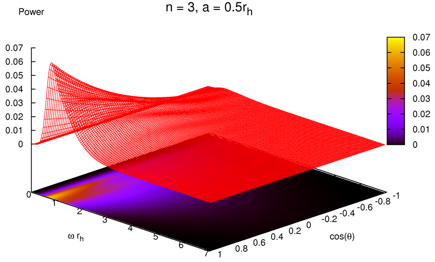
Angular distribution of energy flux

Differential energy emission rate:

$$\frac{d^3E}{dt\,d\omega\,d(\cos\theta)} = \frac{1}{4\pi}\sum_{\text{modes}} \frac{\omega\left|\mathcal{A}_{s\omega\ell m}\right|^2}{e^{\tilde{\omega}/T_H}\mp 1}\left[S_{|s|\omega\ell m}(\theta)^2 + S_{-|s|\omega\ell m}(\theta)^2\right]$$

Energy emission for positive helicity fermions and gauge bosons for $n = 3$ and $a_* = 0.5$





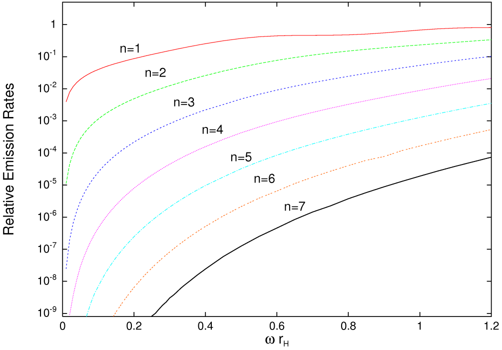
[Casals et al arXiv:0907.1511 [hep-th]]

"Black holes radiate mainly on the brane"

[Emparan, Horowitz and Myers, hep-th/0003118]

Bulk/brane energy emission ratios for scalar fields from a non-rotating black hole

- As a function of frequency
- Total bulk energy emission/total brane energy emission



$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$
0.40	0.24	0.22	0.24	0.33	0.52	0.93

[Harris and Kanti, hep-ph/0309054]

Beyond the semi-classical approximation

The semi-classical approximation

- Treats the black hole geometry as fixed and classical
- Is tractable and yields results which can be used in simulations
- But, by definition, quantum gravity effects are important!

Non-commutative-geometry-inspired black holes - NCBHs

- Use smeared mass density inspired by non-commutative geometry as source for classical metric
- Metric:

$$ds^2 = f(r)\,dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega_{n+2}^2$$

$$f(r) = 1 - \left(\frac{r_h}{r}\right)^{n+1} \to 1 - \frac{\mu}{r^{n+1}}\gamma\left(\frac{n+3}{2}, \frac{r^2}{4\vartheta}\right)$$

[Nicolini et al, gr-qc/0510112]

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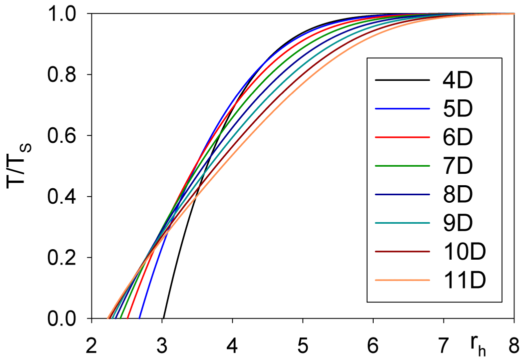
[Nicolini et al, gr-qc/0510112]

Hawking radiation from non-rotating, neutral NCBHs

Fluxes of scalar particles and energy

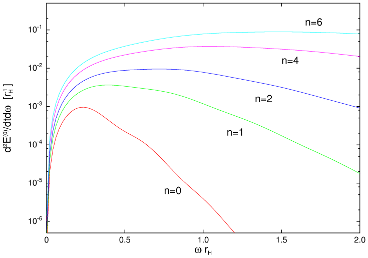
$$\frac{d^2}{dt\, d\omega}\begin{pmatrix} N \\ E \end{pmatrix} = \frac{1}{4\pi}\sum_{\text{modes}} \frac{\left|\mathcal{A}_{\omega\ell m}\right|^2 e^{-\omega^2/2}}{e^{\omega/T_H}-1}\begin{pmatrix} 1 \\ \omega \end{pmatrix}$$

- Temperature of NCBHs much lower than standard BHs
- Quantum back-reaction negligible
- Decay time $\sim 10^{-16} s$ [Casadio and Nicolini, arXiv:0809.2471]



Brane scalar field energy emission

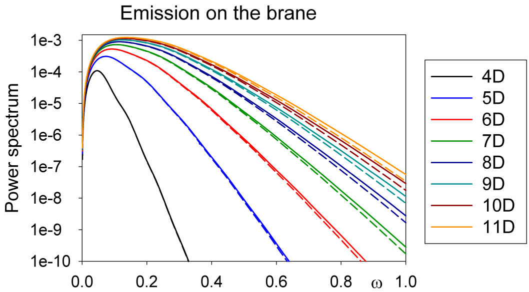
Standard BH



Brane emission from a Schwarzschild-Tangherlini BH, for $n = 0 \ldots 6$

[Harris and Kanti, `hep-ph/0309054`]

NCBH

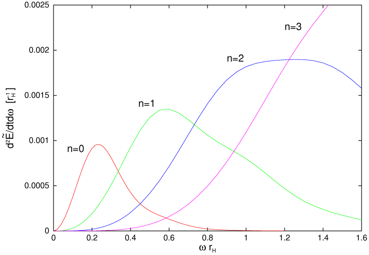


Brane emission from a higher-dimensional NCBH, for $n = 0 \ldots 7$

[Nicolini and EW, `arXiv:1108.4419 [hep-th]`]

Bulk scalar field energy emission

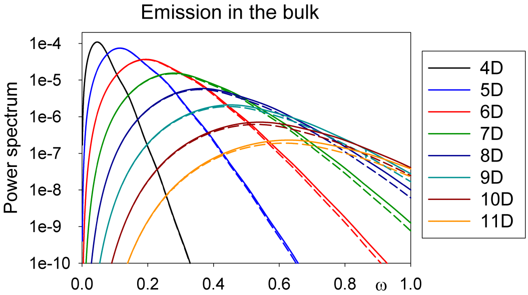
Standard BH



Bulk emission from a Schwarzschild-Tangherlini BH, for $n = 0 \dots 3$

[Harris and Kanti, hep-ph/0309054]

NCBH



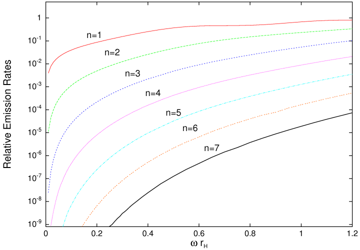
Bulk emission from a higher-dimensional NCBH, for $n = 0 \dots 7$

[Nicolini and EW, arXiv:1108.4419 [hep-th]]

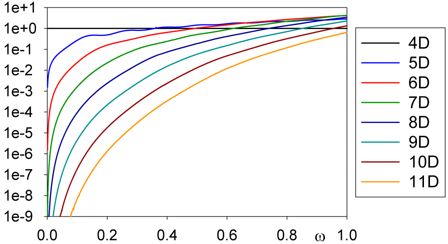
Bulk/brane emission

Ratios of bulk/brane energy emission for scalar fields

Standard BH



NCBH



	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
Standard BH	0.40	0.24	0.22	0.24	0.33	0.52	0.93
NCBH	0.27	0.08	0.03	0.009	0.003	0.001	0.0003

[Harris and Kanti, hep-ph/0309054]

[Nicolini and EW, arXiv:1108.4419 [hep-th]]

Conclusions

Standard Hawking radiation scenario

- Treats black hole geometry as purely classical, with quantum radiation on it
- Two phases of evolution:
  - Spin-down phase
  - Schwarzschild phase
- Both studied in great detail and well understood
- Notable exception is graviton emission in the spin-down phase
- Used in simulations of semi-classical BHs at the LHC

Conclusions

Beyond the semi-classical approximation?

- Use an effective metric incorporating some features of quantum BHs
- Many different approaches to this, including NCBHs
- Much lower temperatures than standard BHs
- Distinctive signatures in emission spectra
- NCBHs radiate almost entirely on the brane!