Structure of the X(3915) in a quark model analysis and its production in heavy ion collisions

[arXiv: 2602.07941 [hep-ph]]

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Collaborated with
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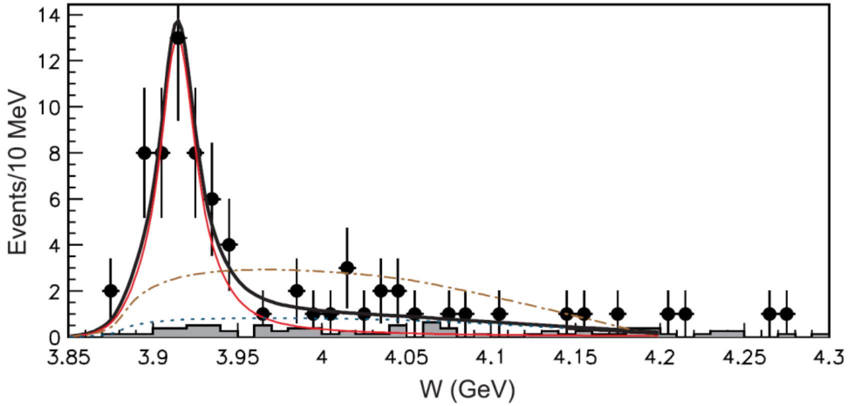
What is X(3915)?

- X(3915) was first observed by Belle Collaboration in 2004 from the analysis of the $\omega J/\psi$ invariant mass in the B meson decay.

Phys. Rev. Lett. 94, 182002 (2005)

- From the W distribution for the final $\gamma\gamma \to \omega J/\psi$ candidate events, a prominent resonance-like peak around 3.92 GeV is observed.



$$M = (3915 \pm 3 \pm 2)\,\mathrm{MeV}/c^2$$
$$\Gamma = (17 \pm 10 \pm 3)\,\mathrm{MeV}$$

Fig 1. W distribution for the final $\gamma\gamma \to \omega J/\psi$ candidate events.

- X(3915) is a charmonium-like resonance with its quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$.
- $X(3915) \to \omega J/\Psi$ is dominant → not a conventional $c\bar{c}$.

Phys. Rev. Lett. 104, 092001 (2010)

Phys. Rev. D 86, 072002 (2012)

What is X(3915)?

- Diquark model proposes that X(3915) is the lightest $\bar{c}c\bar{s}s$ state based on its decay pattern and the mass lying slightly below $D_s\bar{D}_s$ threshold.

Phys. Rev. D 93, 094024 (2016)

- QCD Sum-Rule studies compute masses and decay constants showing X(3915) fits a $\bar{c}c\bar{s}s$ tetraquark.

Eur. Phys. J. C 77, 78 (2017) | Phys. Rev. D 96, 114017 (2017)

- X(3915) provides insight into the internal structure of exotic hadrons, including other hidden charm exotics.
- We will discuss the internal structure of X(3915) as a $\bar{c}c\bar{s}s$ configuration from the perspective of $D_s\bar{D}_s$ threshold.

Quark Model Description

Mar. 8th, 2026

Hamiltonian

$$H = \sum_{i=1}^{n} \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i<j}^{n} \frac{\lambda_i^c}{2} \frac{\lambda_j^c}{2} \left(V_{ij}^C + V_{ij}^{CS} \right),$$

where n goes to 4 for tetraquarks.

- Quark potentials

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{CS} = -\frac{3}{16} \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4} \frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij}) r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

- Additional mass dependences

$$r_{0ij} = 1/\left(\alpha + \beta \frac{m_i m_j}{m_i + m_j} \right),$$

$$\kappa' = \kappa_0 \left(1 + \gamma \frac{m_i m_j}{m_i + m_j} \right).$$

- Our model Hamiltonian well describes hadron spectra

Quark Model Description

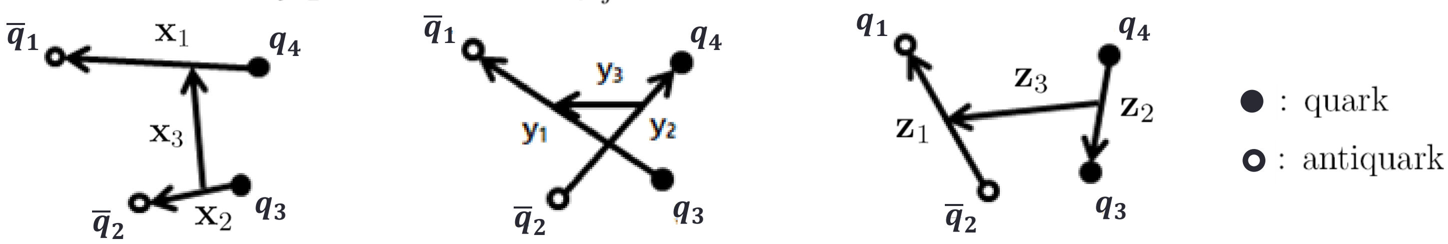
Mar. 8th, 2026

- Our Hamiltonian is based on two-body interactions

$\bar{q}_1 = \bar{c},\ \bar{q}_2 = \bar{s},\ q_3 = c,\ q_4 = s$ for $X(3915)$

$I^G(J^{PC}) = 0^+(0^{++})$.

$$H = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{4}\sum_{i<j}^{n}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}\left(V_{ij}^C + V_{ij}^{CS}\right),$$



- Coordinate Set 1

$$\mathbf{x}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_4),\ \mathbf{x}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_2 - \mathbf{r}_3),\ \mathbf{x}_3 = \frac{1}{\mu}\left(\frac{m_1\mathbf{r}_1 + m_4\mathbf{r}_4}{m_1 + m_4} - \frac{m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_2 + m_3}\right)$$

- Coordinate Set 2

$$\mathbf{y}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_3),\ \mathbf{y}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_4 - \mathbf{r}_2),\ \mathbf{y}_3 = \frac{1}{\mu}\left(\frac{m_1\mathbf{r}_1 + m_3\mathbf{r}_3}{m_1 + m_3} - \frac{m_2\mathbf{r}_2 + m_4\mathbf{r}_4}{m_2 + m_4}\right)$$

- Coordinate Set 3

$$\mathbf{z}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2),\ \mathbf{z}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_3 - \mathbf{r}_4),\ \mathbf{z}_3 = \frac{1}{\mu}\left(\frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4}\right)$$

where

$$\mu = \left[\frac{m_1^2 + m_4^2}{(m_1 + m_4)^2} + \frac{m_2^2 + m_3^2}{(m_2 + m_3)^2}\right]^{1/2}$$

Under permutations (13)(24)

$\mathbf{x}_1 \to -\mathbf{x}_2$

$\mathbf{x}_2 \to -\mathbf{x}_1$

- Spatial Function

$$\psi^{Spatial}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \left(\frac{2}{\pi}\right)^{\frac{9}{4}} a_1^{\frac{3}{4}} a_1^{\frac{3}{4}} a_3^{\frac{3}{4}} \exp\left[-a_1\mathbf{x}_1^2 - a_1\mathbf{x}_2^2 - a_3\mathbf{x}_3^2\right]$$

Fitting Results

Table I. Measured and fitted masses of mesons.

Mesons	Experimental value (MeV)	Mass (MeV)	Variational parameter (fm^{-2})	Error (%)
D	1864.8	1860.4	$a = 4.4$	0.24
D^*	2007.0	2005.2	$a = 3.5$	0.08
η_c	2983.6	2995.4	$a = 14.3$	0.39
J/Ψ	3096.9	3117.5	$a = 11.0$	0.67
D_s	1968.3	1961.0	$a = 7.0$	0.37
D_s^*	2112.1	2097.6	$a = 5.4$	0.69
K	493.68	503.7	$a = 14.3$	1.23
K^*	891.66	875.0	$a = 2.6$	1.87
B	5279.3	5283.5	$a = 4.3$	0.07
B^*	5325.2	5339.2	$a = 3.9$	0.27
η_b	9398.0	9370.6	$a = 52.7$	0.30
Υ	9460.3	9485.7	$a = 40.8$	0.27
B_s	5366.8	5351.2	$a = 7.4$	0.29
B_s^*	5415.4	5410.3	$a = 6.6$	0.09
B_c	6275.6	6273.2	$a = 21.2$	0.02
B_c^*	...	6357.7	$a = 17.4$	...

Table II. Measured and fitted masses of baryons.

Baryons	Experimental value (MeV)	Mass (MeV)	Variational parameters (fm^{-2})	Error (%)
Λ	1115.7	1112.0	$a_1 = 2.6, a_2 = 2.6$	0.33
Λ_c	2286.5	2265.4	$a_1 = 2.7, a_2 = 3.5$	0.92
Ξ_{cc}	3621.4	3609.6	$a_1 = 7.3, a_2 = 3.0$	0.33
Λ_b	5619.4	5608.8	$a_1 = 2.7, a_2 = 3.9$	0.19
Σ_c	2452.9	2444.6	$a_1 = 2.0, a_2 = 3.5$	0.37
Σ_c^*	2517.5	2528.2	$a_1 = 1.8, a_2 = 3.1$	0.39
Σ_b	5811.3	5825.5	$a_1 = 1.9, a_2 = 3.8$	0.17
Σ_b^*	5832.1	5858.2	$a_1 = 1.9, a_2 = 3.6$	0.40
Σ	1192.6	1199.7	$a_1 = 2.0, a_2 = 2.9$	0.59
Σ^*	1383.7	1405.3	$a_1 = 1.7, a_2 = 2.2$	1.56
Ξ	1314.9	1316.6	$a_1 = 3.2, a_2 = 2.7$	0.13
Ξ^*	1531.8	1532.7	$a_1 = 2.8, a_2 = 2.1$	0.06
Ξ_c	2467.8	2459.5	$a_1 = 3.1, a_2 = 4.4$	0.44
Ξ_c^*	2645.9	2639.9	$a_1 = 2.3, a_2 = 4.0$	0.24
Ξ_b	5787.8	5790.3	$a_1 = 3.1, a_2 = 5.2$	0.03
Ξ_b^*	5945.5	5958.0	$a_1 = 2.4, a_2 = 4.9$	0.04
p	938.27	958.93	$a_1 = 2.2, a_2 = 2.2$	2.20
Δ	1232	1269.5	$a_1 = 1.7, a_2 = 1.7$	3.04

$$V_{ij}^{CS} = -\frac{3}{16}\frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c$$

where

$$r_{0ij} = 1/\left(\alpha + \beta\frac{m_i m_j}{m_i + m_j}\right), \quad \kappa' = \kappa_0\left(1 + \gamma\frac{m_i m_j}{m_i + m_j}\right).$$

$\kappa = 107.7\,\text{MeV fm}, \quad a_0 = 0.0327338\,(\text{MeV}^{-1}\text{fm})^{1/2},$

$D = 950\,\text{MeV},$

$m_u = 320\,\text{MeV}, \quad m_s = 612\,\text{MeV},$

$m_c = 1893\,\text{MeV}, \quad m_b = 5285\,\text{MeV},$

$\alpha = 1.1349\,\text{fm}^{-1}, \quad \beta = 0.0011554\,(\text{MeV fm})^{-1},$

$\gamma = 0.001370\,\text{MeV}^{-1}, \quad \kappa_0 = 230.244\,\text{MeV}.$

Color-Spin States of X(3915)

For $\bar{c}c\bar{s}s$ with $I^G(J^{PC})= 0^+(0^{++})$

- Color Space

- Decomposition in terms of color SU(3)

$$[\bar{\mathbf{3}}]_{\bar{c}} \otimes [\mathbf{3}]_c \otimes [\bar{\mathbf{3}}]_{\bar{s}} \otimes [\mathbf{3}]_s = ([\mathbf{1}]_{\bar{c}c} \oplus [\mathbf{8}]_{\bar{c}c}) \otimes ([\mathbf{1}]_{\bar{s}s} \oplus [\mathbf{8}]_{\bar{s}s})$$
$$= ([\mathbf{1}]_{\bar{c}c} \otimes [\mathbf{1}]_{\bar{s}s}) \oplus ([\mathbf{1}]_{\bar{c}c} \otimes [\mathbf{8}]_{\bar{s}s}) \oplus ([\mathbf{8}]_{\bar{c}c} \otimes [\mathbf{1}]_{\bar{s}s}) \oplus ([\mathbf{8}]_{\bar{c}c} \otimes [\mathbf{8}]_{\bar{s}s})$$

$$(\bar{c}c)^{\mathbf{1}} \otimes (\bar{s}s)^{\mathbf{1}} , \quad (\bar{c}c)^{\mathbf{8}} \otimes (\bar{s}s)^{\mathbf{8}}$$

- Spin Space

- Decomposition in terms of spin SU(2)

$$[2] \otimes [2] \otimes [2] \otimes [2] = [5] \oplus [3] \oplus [3] \oplus [3] \oplus [1] \oplus [1].$$

Spin 2 | Spin 1 | Spin 0

- For Spin 0, $(\bar{c}c)_0 \otimes (\bar{s}s)_0 , \quad (\bar{c}c)_1 \otimes (\bar{s}s)_1$ ← Satisfying charge conjugation (+)

All possible 4 CS bases satisfy the symmetry of X(3915)

- For Spin 1, $(\bar{c}c)_0 \otimes (\bar{s}s)_1 , \quad (\bar{c}c)_1 \otimes (\bar{s}s)_0 , \quad (\bar{c}c)_1 \otimes (\bar{s}s)_1$ ← Satisfying charge conjugation (+)

If $s \rightarrow q$, CS bases correspond to X(3872) with $I^G(J^{PC})= 0^+(1^{++})$

Only 2 of 6 CS bases satisfy the symmetry of X(3872)

Color-Spin States of X(3915)

- Color-Spin Bases of X(3915)

$$|CS'_1\rangle = \left|(\bar{c}c)_0^{\mathbf{1}}(\bar{s}s)_0^{\mathbf{1}}\right\rangle,\ |CS'_2\rangle = \left|(\bar{c}c)_0^{\mathbf{8}}(\bar{s}s)_0^{\mathbf{8}}\right\rangle$$
$$|CS'_3\rangle = \left|(\bar{c}c)_1^{\mathbf{1}}(\bar{s}s)_1^{\mathbf{1}}\right\rangle,\ |CS'_4\rangle = \left|(\bar{c}c)_1^{\mathbf{8}}(\bar{s}s)_1^{\mathbf{8}}\right\rangle$$

$$|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle,\ |CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$$
$$|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle,\ |CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$$

More appropriate for investigating the properties of $D_s\bar{D}_s$ threshold

- Transformation between CS Basis Sets

$$|CS'_1\rangle = -\frac{1}{2\sqrt{3}}|CS_1\rangle + \frac{1}{6}|CS_2\rangle - \sqrt{\frac{2}{3}}|CS_3\rangle + \frac{\sqrt{2}}{3}|CS_4\rangle$$
$$|CS'_2\rangle = -\sqrt{\frac{2}{3}}|CS_1\rangle + \frac{\sqrt{2}}{3}|CS_2\rangle + \frac{1}{2\sqrt{3}}|CS_3\rangle - \frac{1}{6}|CS_4\rangle$$
$$|CS'_3\rangle = -\frac{1}{6}|CS_1\rangle - \frac{1}{2\sqrt{3}}|CS_2\rangle - \frac{\sqrt{2}}{3}|CS_3\rangle - \sqrt{\frac{2}{3}}|CS_4\rangle$$
$$|CS'_4\rangle = -\frac{\sqrt{2}}{3}|CS_1\rangle - \sqrt{\frac{2}{3}}|CS_2\rangle + \frac{1}{6}|CS_3\rangle + \frac{1}{2\sqrt{3}}|CS_4\rangle$$

$$|CS_1\rangle = -\frac{1}{2\sqrt{3}}|CS'_1\rangle - \sqrt{\frac{2}{3}}|CS'_2\rangle - \frac{1}{6}|CS'_3\rangle - \frac{\sqrt{2}}{3}|CS'_4\rangle$$
$$|CS_2\rangle = \frac{1}{6}|CS'_1\rangle + \frac{\sqrt{2}}{3}|CS'_2\rangle - \frac{1}{2\sqrt{3}}|CS'_3\rangle - \sqrt{\frac{2}{3}}|CS'_4\rangle,$$
$$|CS_3\rangle = -\sqrt{\frac{2}{3}}|CS'_1\rangle + \frac{1}{2\sqrt{3}}|CS'_2\rangle - \frac{\sqrt{2}}{3}|CS'_3\rangle + \frac{1}{6}|CS'_4\rangle$$
$$|CS_4\rangle = \frac{\sqrt{2}}{3}|CS'_1\rangle - \frac{1}{6}|CS'_2\rangle - \sqrt{\frac{2}{3}}|CS'_3\rangle + \frac{1}{2\sqrt{3}}|CS'_4\rangle.$$

- Color-Spin Interaction Factor

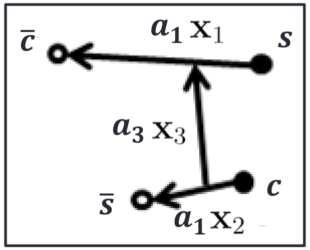
$$V_{ij}^{CS} = -\frac{3}{16}\frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\lambda_i^c\lambda_j^c \longrightarrow K = -\sum_{i<j}^{n}\frac{1}{m_i m_j}\lambda_i^c\lambda_j^c\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$$

H. Yun, S. Noh, A. Park, S. Cho, S. H. Lee, et al, Phys. Rev. C 107, 014906 (2023)

Color-Spin Structure of X(3915)

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^-(3922.0)$	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$



$a_3 \to 0$

- Color-Spin Interaction Factor

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$$K_{X(3915)} = \begin{pmatrix} \frac{16}{3}\left(\frac{1}{m_{\bar c} m_s} + \frac{1}{m_{\bar s} m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar c}-m_s)(m_{\bar s}-m_c)}{3m_{\bar c}m_{\bar s}m_c m_s} & \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} \\ 0 & -\frac{16}{m_{\bar c}m_s} - \frac{16}{m_{\bar s}m_c} & \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & 0 \\ \frac{8\sqrt{2}(m_{\bar c}-m_s)(m_{\bar s}-m_c)}{3m_{\bar c}m_{\bar s}m_c m_s} & \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & -\frac{8m_c m_s+(28m_c+2m_s+8m_{\bar s})m_{\bar c}+(2m_c+28m_s)m_{\bar s}}{3m_{\bar c}m_{\bar s}m_c m_s} & -\frac{4m_c m_s-14m_{\bar c}m_c-14m_{\bar s}m_s+4m_{\bar c}m_{\bar s}}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} \\ \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & 0 & -\frac{4m_c m_s-14m_{\bar c}m_c-14m_{\bar s}m_s+4m_{\bar c}m_{\bar s}}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & \frac{2}{m_{\bar c}m_s} + \frac{2}{m_{\bar s}m_c} \end{pmatrix}$$

where $|CS_1\rangle = |(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\rangle$, $|CS_2\rangle = |(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\rangle$, $|CS_3\rangle = |(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\rangle$, $|CS_4\rangle = |(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\rangle$

$m_s = m_{\bar s} = 0.5$ GeV, $m_c = m_{\bar c} = 1.5$ GeV

$$K_{X(3915)} = \begin{pmatrix} 14.2 & 0 & -6.70 & 23.2 \\ 0 & -42.7 & 23.2 & 0 \\ -6.70 & 23.2 & -50.4 & 29.8 \\ 23.2 & 0 & 29.8 & 5.33 \end{pmatrix}$$

Color-Spin Structure of X(3915)

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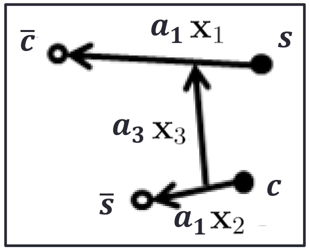

$a_3 \to 0$

- Color-Spin Interaction Factor

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$$K_{X(3915)} =$$

$$\begin{pmatrix} \frac{16}{3}\left(\frac{1}{m_{\bar{c}}m_s}+\frac{1}{m_{\bar{s}}m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} \\ 0 & -\frac{16}{m_{\bar{c}}m_s}-\frac{16}{m_{\bar{s}}m_c} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & 0 \\ \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{[illegible]m_cm_s+[illegible]2m_s+8m_{\bar{s}})m_{\bar{c}}+(2m_c+28m_s)m_{\bar{s}}}{3m_{\bar{c}}m_{\bar{s}}m_cm_s} & -\frac{4m_cm_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} \\ \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & 0 & -\frac{4m_cm_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{2}{m_{\bar{c}}m_s}+\frac{2}{m_{\bar{s}}m_c} \end{pmatrix}$$

$\bar{D}_s$ D_s

where $|CS_1\rangle = |(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\rangle$, $|CS_2\rangle = |(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\rangle$, $|CS_3\rangle = |(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\rangle$, $|CS_4\rangle = |(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\rangle$

- Color-Spin Part of Hamiltonian ($a_3 = 0.001$)

$$\left\langle \sum_{i<j}^{4} V_{ij}^{CS} \right\rangle = \begin{pmatrix} 74.99 & 0 & -0.41 & 0.36 \\ 0 & -224.95 & 0.36 & 0 \\ -0.41 & 0.36 & -10.39 & 0.88 \\ 0.36 & 0 & 0.88 & 28.12 \end{pmatrix}$$

$$V_{ij}^{CS} = -\frac{3}{16}\frac{\hbar^2c^2\kappa'}{m_im_jc^4}\frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c$$

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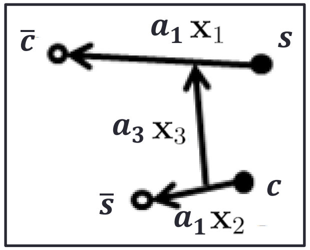


$a_3 \to 0$

- Color-Spin Interaction Factor

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$$K_{X(3915)} =$$

$$\begin{pmatrix} \frac{16}{3}\left(\frac{1}{m_{\bar c} m_s} + \frac{1}{m_{\bar s} m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar c}-m_s)(m_{\bar s}-m_c)}{3m_{\bar c}m_{\bar s}m_c m_s} & \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} \\ 0 & -\frac{16}{m_{\bar c} m_s} - \frac{16}{m_{\bar s} m_c} & \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & 0 \\ \frac{8\sqrt{2}(m_{\bar c}-m_s)(m_{\bar s}-m_c)}{3m_{\bar c}m_{\bar s}m_c m_s} & \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m\cdots}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & \cdots m_c m_s + \cdots \frac{\cdots 2m_s+8m_{\bar s})m_{\bar c}+(2m_c+28m_s)m_{\bar s}}{3m_{\bar c}m_{\bar s}m_c m_s} & -\frac{4m_c m_s-14m_{\bar c}m_c-14m_{\bar s}m_s+4m_{\bar c}m_{\bar s}}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} \\ \frac{4\sqrt{2}(m_{\bar c}+m_s)(m_{\bar s}+m_c)}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & 0 & -\frac{4m_c m_s-14m_{\bar c}m_c-14m_{\bar s}m_s+4m_{\bar c}m_{\bar s}}{\sqrt{3}m_{\bar c}m_{\bar s}m_c m_s} & \frac{2}{m_{\bar c} m_s} + \frac{2}{m_{\bar s} m_c} \end{pmatrix}$$

$\bar{D}_s$ D_s

where $|CS_1\rangle = |(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\rangle$, $|CS_2\rangle = |(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\rangle$, $|CS_3\rangle = |(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\rangle$, $|CS_4\rangle = |(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\rangle$

- Color-Spin Part of Hamiltonian ($a_3 = 1.0 * 10^{-10}$)

$$\left\langle \sum_{i<j}^{4} V_{ij}^{CS} \right\rangle = \begin{pmatrix} 74.99 & 0.0 & -0.0002 & 0.0009 \\ 0.0 & -224.95 & 0.0009 & 0.0 \\ -0.0002 & 0.0009 & -9.38 & 0.0011 \\ 0.0009 & 0.0 & 0.0011 & 28.12 \end{pmatrix}$$

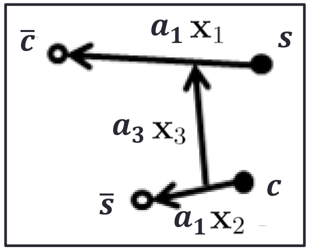
$$V_{ij}^{CS} = -\frac{3}{16} \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4} \frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij}) r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c$$

Color-Spin Structure of X(3915)

Mar. 8th, 2026

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^-$(3922.0)	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$



$a_3 \to 0$

- Color-Spin Interaction Factor

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$$K_{X(3915)} =$$

$$\begin{pmatrix} \frac{16}{3}\left(\frac{1}{m_{\bar{c}}m_s} + \frac{1}{m_{\bar{s}}m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\ 0 & -\frac{16}{m_{\bar{c}}m_s} - \frac{16}{m_{\bar{s}}m_c} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 \\ \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{8m_c m_s+(28m_c+2m_s+8m_{\bar{s}})m_{\bar{c}}+(2m_c+28m_s)m_{\bar{s}}}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{4m_c m_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\ \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 & -\frac{4m_c m_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{2}{m_{\bar{c}}m_s} + \frac{2}{m_{\bar{s}}m_c} \end{pmatrix}$$

where $|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle$, $|CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$, $|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle$, $|CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$

- In threshold limit (ignoring $1/(m_{\bar{c}}m_c),\ 1/(m_{\bar{s}}m_s),\ 1/(m_{\bar{c}}m_{\bar{s}}),\ 1/(m_c m_s)$ terms)

$$K_{X(3915)} = \begin{pmatrix} \frac{32}{3m_c m_s} & 0 & 0 & 0 \\ 0 & -\frac{32}{m_c m_s} & 0 & 0 \\ 0 & 0 & -\frac{4}{3m_c m_s} & 0 \\ 0 & 0 & 0 & \frac{4}{m_c m_s} \end{pmatrix}$$

Color-Spin Structure of X(3915)

Mar. 8th, 2026

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^-(3922.0)$	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$

Table IV. Probability amplitudes for each CS basis in the ground state wave function of X(3915).

CS basis set	Amplitudes
$\{\lvert CS_1\rangle, \lvert CS_2\rangle, \lvert CS_3\rangle, \lvert CS_4\rangle\}$	$\{0.0, 1.0, 0.0, 0.0\}$
$\{\lvert CS_1'\rangle, \lvert CS_2'\rangle, \lvert CS_3'\rangle, \lvert CS_4'\rangle\}$	$\{0.03, 0.22, 0.08, 0.67\}$

where $\lvert CS_1\rangle = \lvert(\bar{c}s)^{\mathbf{1}}_1(\bar{s}c)^{\mathbf{1}}_1\rangle$, $\lvert CS_2\rangle = \lvert(\bar{c}s)^{\mathbf{1}}_0(\bar{s}c)^{\mathbf{1}}_0\rangle$, $\lvert CS_3\rangle = \lvert(\bar{c}s)^{\mathbf{8}}_1(\bar{s}c)^{\mathbf{8}}_1\rangle$, $\lvert CS_4\rangle = \lvert(\bar{c}s)^{\mathbf{8}}_0(\bar{s}c)^{\mathbf{8}}_0\rangle$

$\lvert CS_1'\rangle = \lvert(\bar{c}c)^{\mathbf{1}}_0(\bar{s}s)^{\mathbf{1}}_0\rangle$, $\lvert CS_2'\rangle = \lvert(\bar{c}c)^{\mathbf{8}}_0(\bar{s}s)^{\mathbf{8}}_0\rangle$, $\lvert CS_3'\rangle = \lvert(\bar{c}c)^{\mathbf{1}}_1(\bar{s}s)^{\mathbf{1}}_1\rangle$, $\lvert CS_4'\rangle = \lvert(\bar{c}c)^{\mathbf{8}}_1(\bar{s}s)^{\mathbf{8}}_1\rangle$

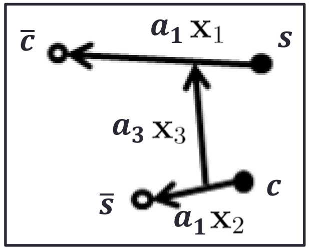
$\bar{D}_s$ D_s

- Well-separated $D_s\bar{D}_s$ configuration

$$\psi^{Spatial}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = N \exp\left[-a_1\mathbf{x}_1^2 - a_1\mathbf{x}_2^2 - a_3\mathbf{x}_3^2\right]$$

$$\psi^{Spatial}(\mathbf{x}_1, \mathbf{x}_2) = N' \exp\left[-a_1\mathbf{x}_1^2 - a_1\mathbf{x}_2^2\right]$$

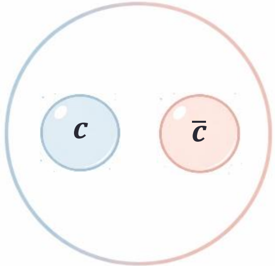
where N and N' are the normalization constants.



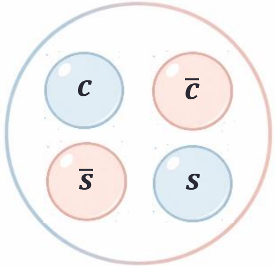
Possible Structures of X(3915)

- Three possible structures for X(3915)

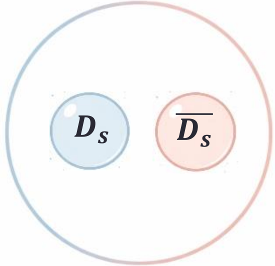
- $c\bar{c}$ charmonium state



- $c\bar{c}s\bar{s}$ tetraquark state



- $D_s\bar{D}_s$ hadronic molecular state



- We studied the possibility of experimentally discriminating among these three different candidate structures.

p_T Distributions for Candidate Structures

- $c\bar{c}$ charmonium state

$$\frac{d^2N_{X_{c\bar{c}}}}{d^2\vec{p}_T} = \frac{g_X}{V}\int d^3\vec{r}d^2\vec{p}_{\bar{c}T}d^2\vec{p}_{cT}W_{2p}(\vec{r},\vec{k}) \times \frac{d^2N_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}}\frac{d^2N_c}{d^2\vec{p}_{cT}}\delta^{(2)}(\vec{p}_T-\vec{p}_{\bar{c}T}-\vec{p}_{cT})$$

X(3915) as 2p wave $c\bar{c}$ state
→ 2p-Wigner function

$I^G(J^{PC})= 0^+(0^{++})$

where

$$\vec{R} = \frac{m_{\bar{c}}\vec{r}_{\bar{c}} + m_c\vec{r}_c}{m_{\bar{c}}+m_c}, \qquad \vec{r} = \vec{r}_{\bar{c}} - \vec{r}_c,$$

$$\vec{K} = \vec{p}'_{\bar{c}T} + \vec{p}'_{cT}, \qquad \vec{k} = \frac{m_c\vec{p}'_{\bar{c}T} - m_{\bar{c}}\vec{p}'_{cT}}{m_{\bar{c}}+m_c}$$

$$W_{2p}(\vec{r},\vec{k}) = \frac{15}{32}e^{-\frac{r^2}{\sigma^2}-k^2\sigma^2}\left(\frac{r^6}{\sigma^6} - \frac{11}{2}\frac{r^4}{\sigma^4} + \frac{15}{2}\frac{r^2}{\sigma^2} + \frac{15}{2}k^2\sigma^2 - \frac{11}{2}k^4\sigma^4 + k^6\sigma^6 + r^2k^2\left(3-\frac{r^2}{\sigma^2}-k^2\sigma^2\right) + 2(\vec{r}\cdot\vec{k})\left(-7+2\frac{r^2}{\sigma^2}+2k^2\sigma^2\right) - \frac{15}{4}\right).$$

where $\sigma^2 = 1/(\mu\omega)$ relating the oscillator frequency and the reduced mass

$$\frac{d^2N_{X_{c\bar{c}}}}{d^2\vec{p}_T} = \frac{g_X}{V}(2\sqrt{\pi}\sigma)^3\int d^2\vec{p}_{\bar{c}T}d^2\vec{p}_{cT}\frac{d^2N_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}}\frac{d^2N_c}{d^2\vec{p}_{cT}} \times \frac{4}{15}\sigma^2k^2e^{-\sigma^2k^2}\left(k^2\sigma^2-\frac{5}{2}\right)^2\delta^{(2)}(\vec{p}_T-\vec{p}_{\bar{c}T}-\vec{p}_{cT})$$

p_T Distributions for Candidate Structures

- $c\bar{c}s\bar{s}$ tetraquark state

$$\frac{d^2N_{X_{c\bar{c}s\bar{s}}}}{d^2\vec{p}_T} = \frac{g_X}{V^3}(2\sqrt{\pi})^9(\sigma_1\sigma_2\sigma_3)^3 \int d^2\vec{p}_{sT}d^2\vec{p}_{\bar{s}T}d^2\vec{p}_{cT}d^2\vec{p}_{\bar{c}T} \times \frac{d^2N_s}{d^2\vec{p}_{sT}}\frac{d^2N_{\bar{s}}}{d^2\vec{p}_{\bar{s}T}}\frac{d^2N_c}{d^2\vec{p}_{cT}}\frac{d^2N_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}}\delta^{(2)}(\vec{p}_T - \vec{p}_{sT} - \vec{p}_{\bar{s}T} - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \times \exp\left(- \sigma_1^2k_1^2 - \sigma_2^2k_2^2 - \sigma_3^2k_3^2\right),$$

- $D_s\bar{D}_s$ hadronic molecular state

$$\frac{d^2N_{X_{D_s\bar{D}_s}}}{d^2\vec{p}_T} = \frac{g_X}{V}(2\sqrt{\pi}\sigma_h)^3 \int d^2\vec{p}_{D_sT}d^2\vec{p}_{\bar{D}_sT}e^{-\sigma_h^2k_h^2} \times \frac{d^2N_{D_s}}{d^2\vec{p}_{D_sT}}\frac{d^2N_{\bar{D}_s}}{d^2\vec{p}_{\bar{D}_sT}}\delta^{(2)}(\vec{p}_T - \vec{p}_{\bar{D}_sT} - \vec{p}_{D_sT})$$

p_T Distributions for Candidate Structures

- p_T distribution of charm and strange quarks at mid-rapidities in central collisions at $\sqrt{s_{NN}} = 5.02$ TeV, obtained from the analysis of production of ϕ and D^0 mesons

$$\frac{d^2N_c}{d^2\vec{p}_{cT}} = \begin{cases} 1.63\ (\mathrm{GeV}^{-2}) e^{-0.27(p_{cT}/p_{0T})^{2.03}}, & p_{cT} \le 1.80\ \mathrm{GeV} \\ 7.95\ (\mathrm{GeV}^{-2}) e^{-3.49(p_{cT}/p_{0T})^{3.59}} + \frac{90112\ (\mathrm{GeV}^{-2})}{(1.0+(p_{cT}/p_{0T})^{0.50})^{14.19}}, & p_{cT} > 1.80\ \mathrm{GeV} \end{cases}$$

$$\frac{d^2N_s}{d^2\vec{p}_{sT}} = \begin{cases} \frac{V}{(2\pi)^3} m_T e^{-m_T/T_{eff}}, & p_{sT} \le 1.50\ \mathrm{GeV} \\ 21.95\ (\mathrm{GeV}^{-2}) e^{-0.17(p_{sT}/p_{0T})^{3.23}} + \frac{80112\ (\mathrm{GeV}^{-2})}{(1.0+(p_{sT}/p_{0T})^{0.65})^{10.29}}. & p_{sT} > 1.50\ \mathrm{GeV} \end{cases}$$

S. Cho and S. H. Lee, arXiv: 2510.18673

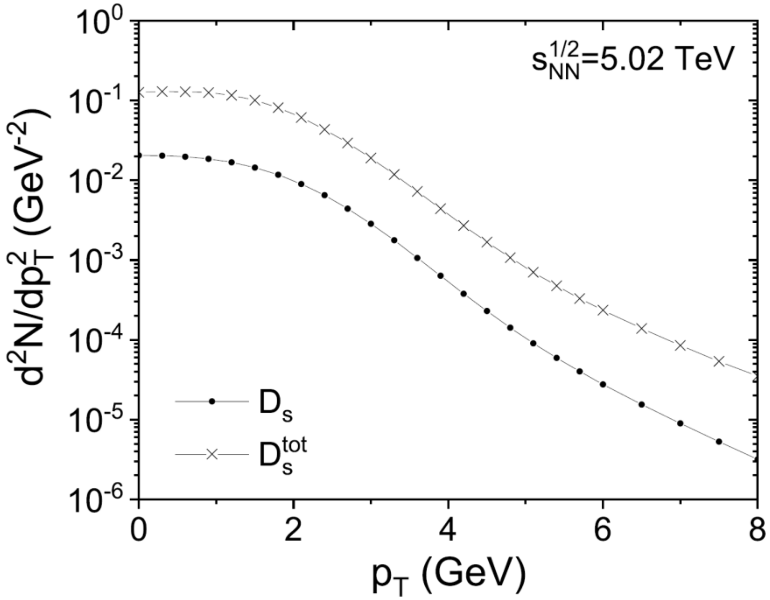


Fig. 2 Transverse momentum distributions of the D_s meson at mid-rapidity at $\sqrt{s_{NN}} = 5.02$ TeV with and without feed down contributions, denoted by D_s and D_s^{tot} ,respectively.

Results from Coalescence Model

Mar. 8th, 2026

Fig. 3(a) Transverse momentum distributions of X(3915) as $c\bar{c}$, $c\bar{c}s\bar{s}$, $D_s\bar{D}_s$ states.

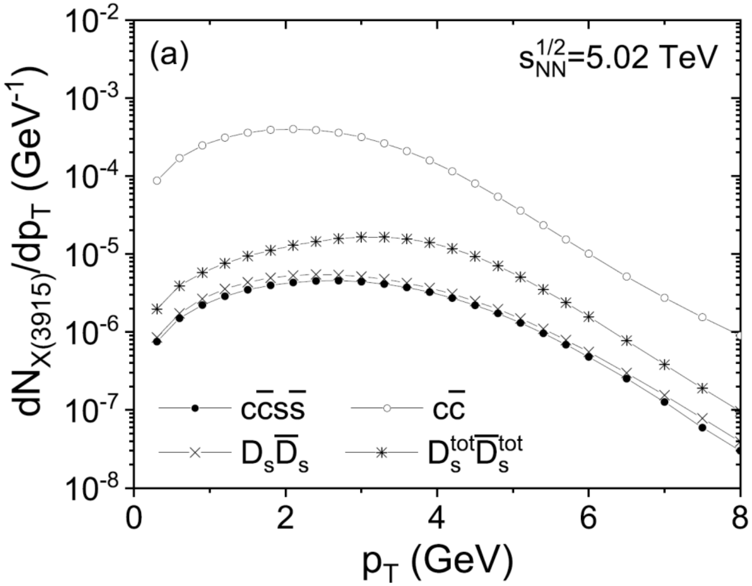


Fig. 3(b) Transverse momentum distribution ratios between X(3915) and D_s meson.

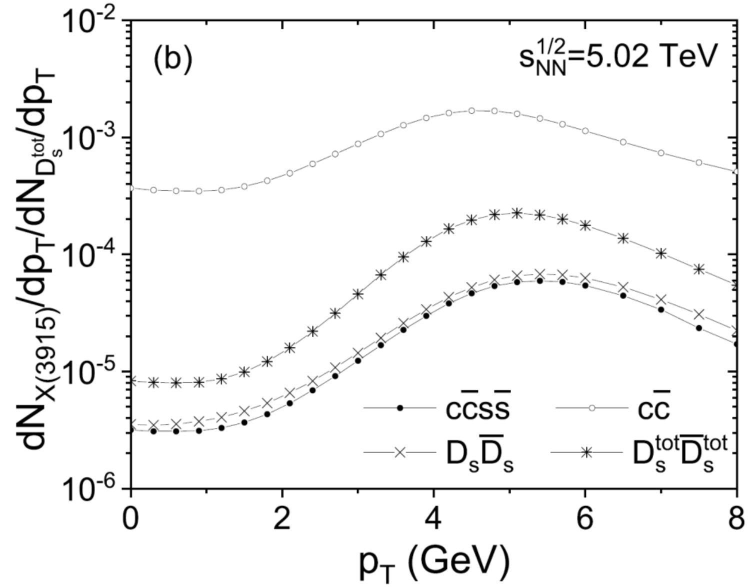


Table IV. Yields of X(3915) for various possible states.

	$c\bar{c}$	$c\bar{c}s\bar{s}$	$D_s\bar{D}_s$	$D_s^{tot}\bar{D}_s^{tot}$	Ther.
Yields $(\times 10^{-4})$	12.0	0.162	0.194	0.567	6.38

m_c=1500 MeV → $c\bar{c}$ is closer to J/ψ

Suppressed by coalescence in heavy ion collisions

Different from most exotic hadrons except D_{s0}^* (2317)

- The results would be helpful in identifying the internal structure of X(3915) from the measurement of its production in heavy ion collisions.

Summary

- X(3915) can provide a valuable insight for understanding other hidden-charm exotics.

- Our quark model analysis suggests that X(3915) is more consistent with a molecular structure, originating from the long-range interaction that extend beyond our quark model.

- p_T distribution and yields from coalescence model would be helpful in specifying the internal structure of X(3915).

- Continued theoretical and experimental studies are essential.

Back-up Slide

- Color octet-octet coupled singlet basis

$$|\mathbf{8}\otimes\mathbf{8}\rangle_{\mathbf{1}} = \sum_a (T^a)_{ij}\,(T^a)_{kl}\,|q_i\bar{q}_j q_k\bar{q}_l\rangle$$

where T^a are the SU(3) Gell-Mann matrices.

- Using the standard Fierz identity for the generators

$$S_{ijkl} \equiv \sum_a (T^a)_{ij}\,(T^a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)$$

- Applying the charge conjugation

$$\begin{aligned}\mathbf{C}\left(S_{ijkl}\,|q_i\bar{q}_j q_k\bar{q}_l\rangle\right) &= S_{ijkl}\,|\bar{q}_i q_j \bar{q}_k q_l\rangle \\ &= S_{jilk}\,|q_i\bar{q}_j q_k\bar{q}_l\rangle\end{aligned}$$

Now the problem reduces to show that $S_{jilk} = S_{ijkl}$.

Since $\delta_{jk}\delta_{li} = \delta_{il}\delta_{kj}$ and $\delta_{ji}\delta_{lk} = \delta_{ij}\delta_{kl}$,

$$S_{jilk} = \frac{1}{2}\left(\delta_{jk}\delta_{li} - \frac{1}{3}\delta_{ji}\delta_{lk}\right) = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) = S_{ijkl}$$

Back-up Slide

- Transverse momentum distributions for the three different candidate structures in the non-relativistic limit.

- $\bar{c}c$ charmonium state

$$\frac{d^2 N_{X_{c\bar{c}}}}{d^2\vec{p}_T} = \frac{g_X}{V}\int d^3\vec{r}\, d^2\vec{p}_{\bar{c}T}\, d^2\vec{p}_{cT}\, W_{2p}(\vec{r},\vec{k}) \times \frac{d^2 N_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}}\frac{d^2 N_c}{d^2\vec{p}_{cT}}\delta^{(2)}(\vec{p}_T - \vec{p}_{\bar{c}T} - \vec{p}_{cT})$$

where

$$\vec{R} = \frac{m_{\bar{c}}\vec{r}_{\bar{c}} + m_c\vec{r}_c}{m_{\bar{c}} + m_c}, \qquad \vec{r} = \vec{r}_{\bar{c}} - \vec{r}_c,$$

$$\vec{K} = \vec{p}_{\bar{c}T}' + \vec{p}_{cT}', \qquad \vec{k} = \frac{m_c\vec{p}_{\bar{c}T}' - m_{\bar{c}}\vec{p}_{cT}'}{m_{\bar{c}} + m_c}$$

Considering X(3915) as 2p wave $\bar{c}c$ state, k=1, l=1 in 3D harmonic oscillator wave function.

$$\psi_{11m}(\vec{r}) = \frac{1}{(\pi\sigma^2)^{1/4}}\frac{4}{\sqrt{15}}\frac{r}{\sigma^2}\left(\frac{5}{2} - \frac{r^2}{\sigma^2}\right)e^{-\frac{r^2}{2\sigma^2}}Y_{1m}(\theta,\phi)$$

Using the m-averaged density for 2p states,

$$\rho(\vec{r},\vec{r}') = \frac{1}{3}\sum_m \psi_{11m}(\vec{r})\psi_{11m}^*(\vec{r}')$$

$$W(\vec{r},\vec{k}) = \int d^3\vec{q}\rho\left(\vec{r}+\frac{\vec{q}}{2},\vec{r}-\frac{\vec{q}}{2}\right)e^{i\vec{k}\cdot\vec{q}}$$

- Transverse momentum distributions for the three different candidate structures in the non-relativistic limit.

- $\bar{c}c$ charmonium state

where

where $\sigma^2 = 1/(\mu\omega)$ relating the oscillator frequency and the reduced mass