Long-range $N - J/\psi$ interaction from an operator product expansion perspective

Seokwoo Yeo, In Woo Park and Su Houng Lee, Phys. Rev. D 112, 094044 (2025).

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Talk at Reimei 2026
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(*Speaker)

Lattice QCD study[1]

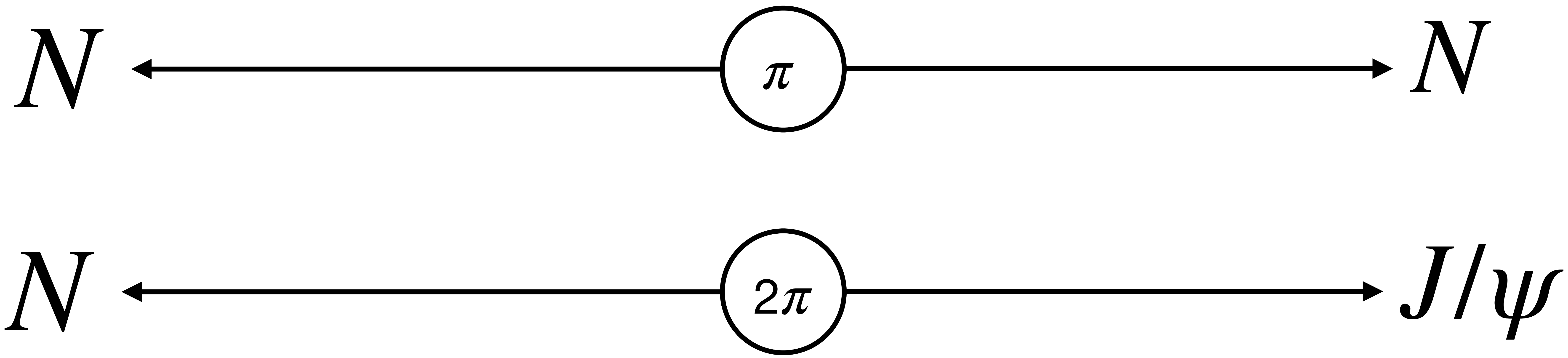
$N - J/\psi$

- attractive at all distances
- long-range tail, which is well explained by two-pion exchange

1. Y. Lyu, T. Doi, T. Hatsuda, and T. Sugiura, Physics Letters B 860, 139178 (2025).

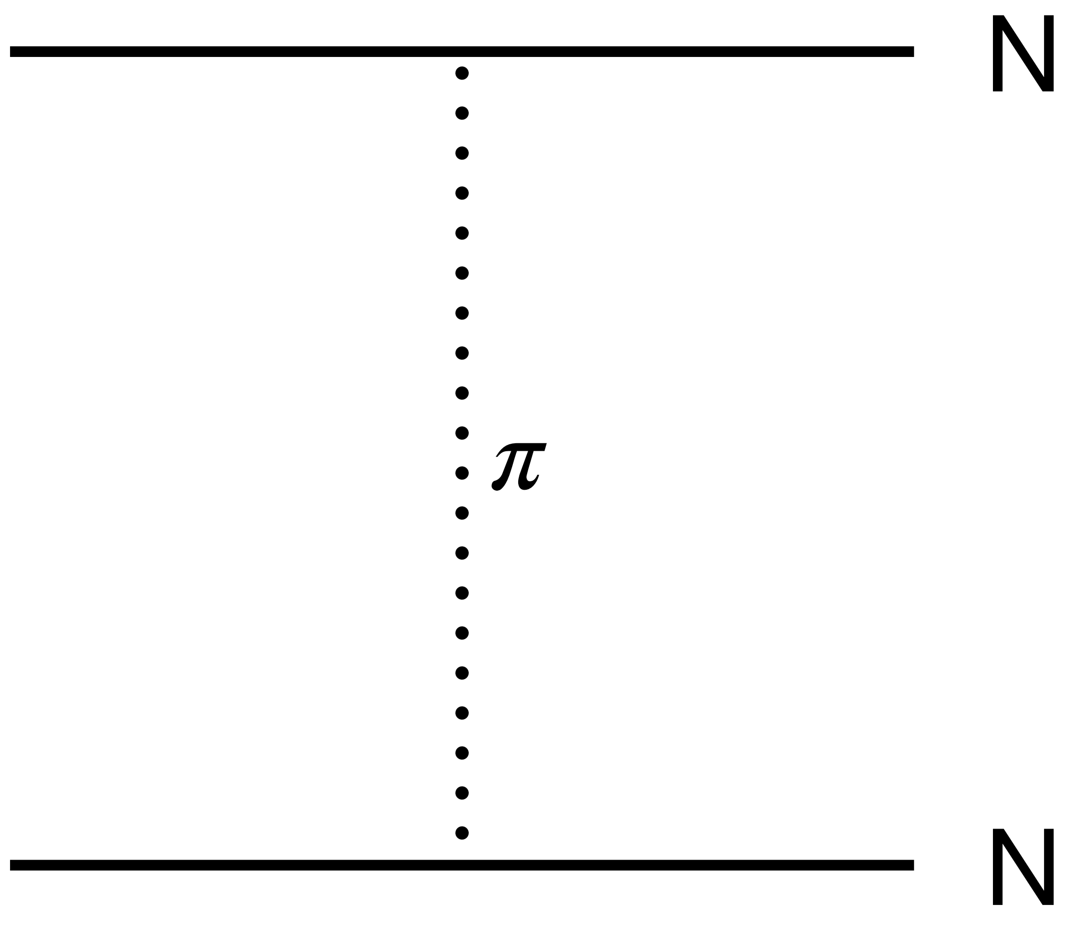
Long-range hadron—hadron interaction

What is the difference?

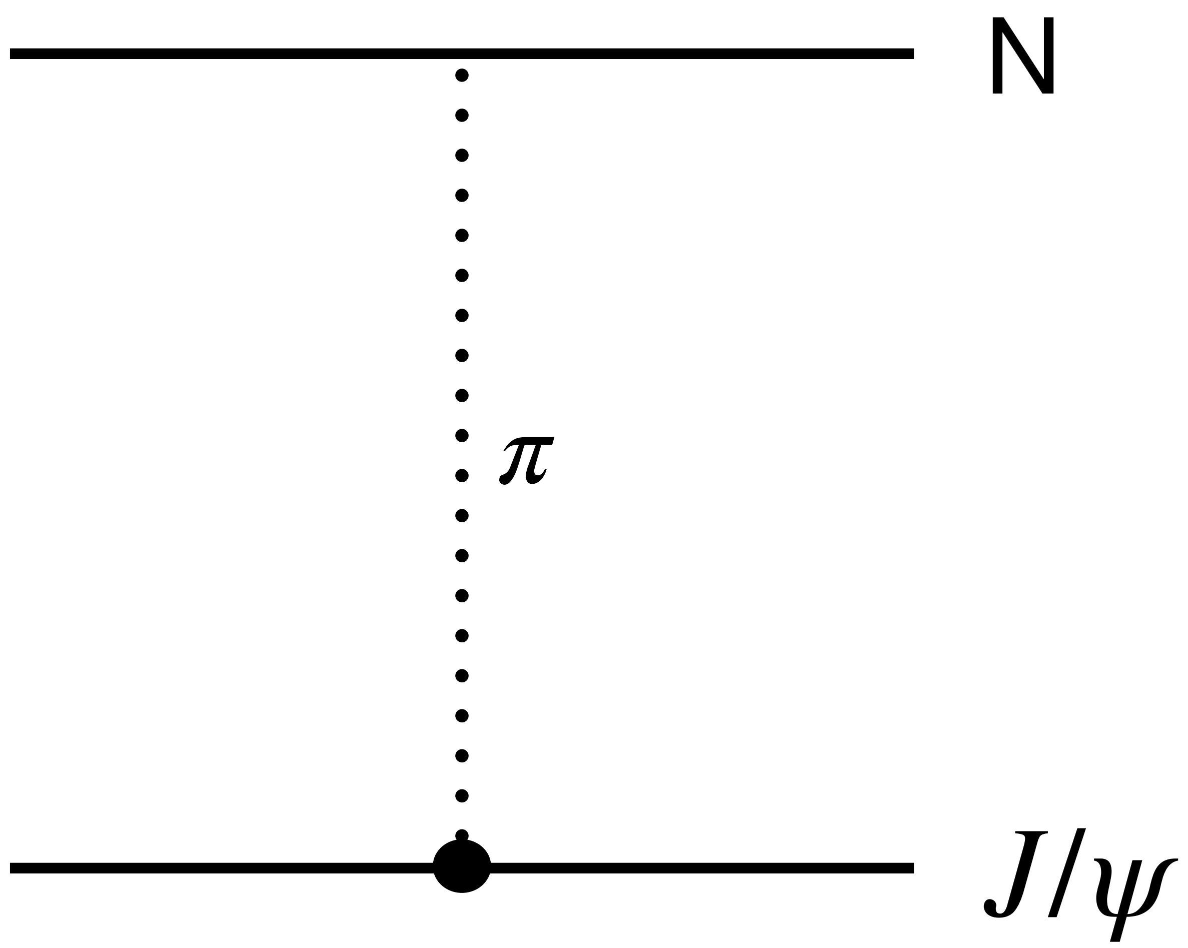


(One-pion exchange is OZI suppressed)

Isospin argument

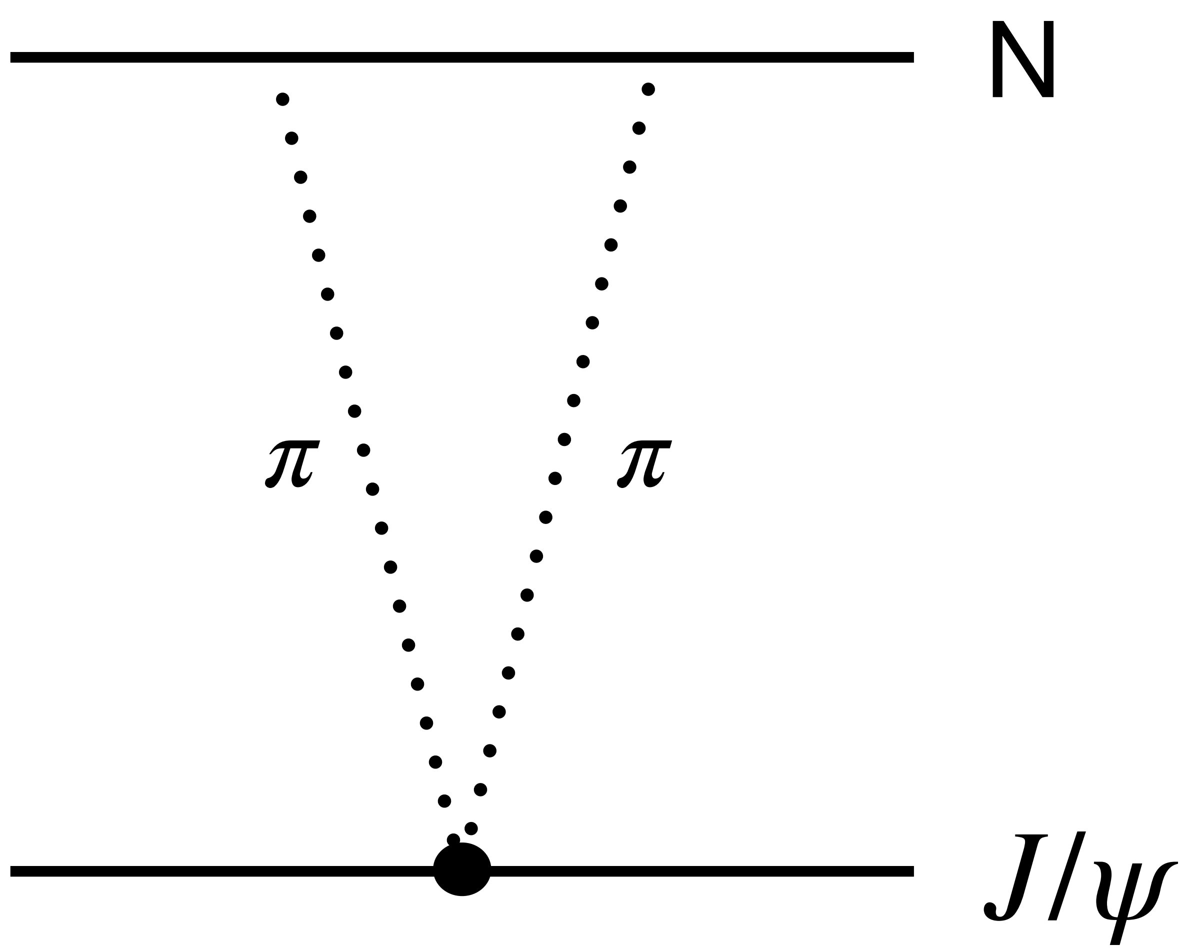


$$\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2} \leftrightarrow \frac{1}{2}$$



$$0 \otimes 1 \leftrightarrow 0$$

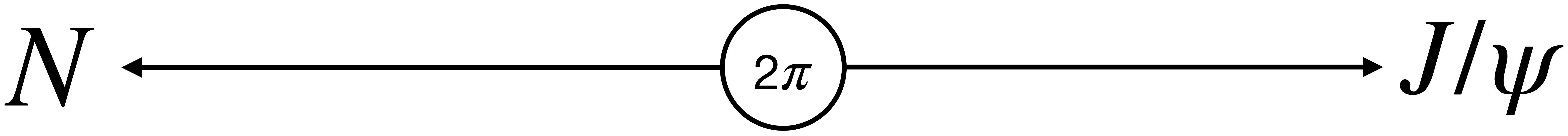
Suppressed



$$0 \otimes 1 \otimes 1 = 0 \oplus 1 \oplus 2 \leftrightarrow 0$$

Isoscalar **two-pion exchange** is leading term in this case

Long-range N—J/psi interaction



I'd like to show you how this TPE contribution can be interpreted in QCD Sum Rules (QSR), whether it shows attraction, and, if so, whether its size is comparable to the lattice result.

Long-range N—J/psi interaction

QSR result ⇔ Lattice result

- Lattice QCD study[1] → attractive at all r; TPE tail.

↓

Extracting TPE induced mass shift (~ a few MeV).

↓

We compare the QCD Sum Rule (QSR) result with the Lattice TPE-induced mass shift.

1. Y. Lyu, T. Doi, T. Hatsuda, and T. Sugiura, Physics Letters B 860, 139178 (2025).

QCD Sum Rules(QSR)

For a current that can probe a specific hadron state, let us examine the structure through the two-current correlation function as follows.

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle T\{j_\mu(x) j_\nu(0)\}\rangle$$

QCD Sum Rules(QSR)

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle T\{ j_\mu(x) j_\nu(0) \} \rangle$$

Analytic property

$$\Pi_{\mu\nu}^{OPE} \Longleftrightarrow \frac{1}{\pi} \int_0^\infty ds \frac{\mathrm{Im}\,\Pi(s)}{s + Q^2}$$

$$\sum_i C_i \langle \mathcal{O}_i \rangle, \quad \langle \mathcal{O}_i \rangle : \langle \bar{q}q \rangle, \langle G^2 \rangle, \langle G^3 \rangle, \ldots$$

QCD Sum Rules(QSR)

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle T\{j_\mu(x)j_\nu(0)\}\rangle$$

Analytic property

$$\Pi^{OPE}_{\mu\nu} \Longleftrightarrow \frac{1}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}\,\Pi(s)}{s+Q^2}$$

In the framework of QSR, how can we interpret the long-range two-pion exchange contribution to the J/psi interaction?

Which operators should be identified as those representing this effect?

In order to reproduce the long-range TPE, it is necessary to isolate the pion-coupled components at the operator level.

QCD Sum Rules(QSR)

The axial current couples to the pion.

$$\langle 0 | A^a_\mu | \pi^b \rangle = i f_\pi q_\mu \delta^{ab}, \text{ where } A^a_\mu = \bar{q}\gamma_5\gamma_\mu\tau^a q\,.$$

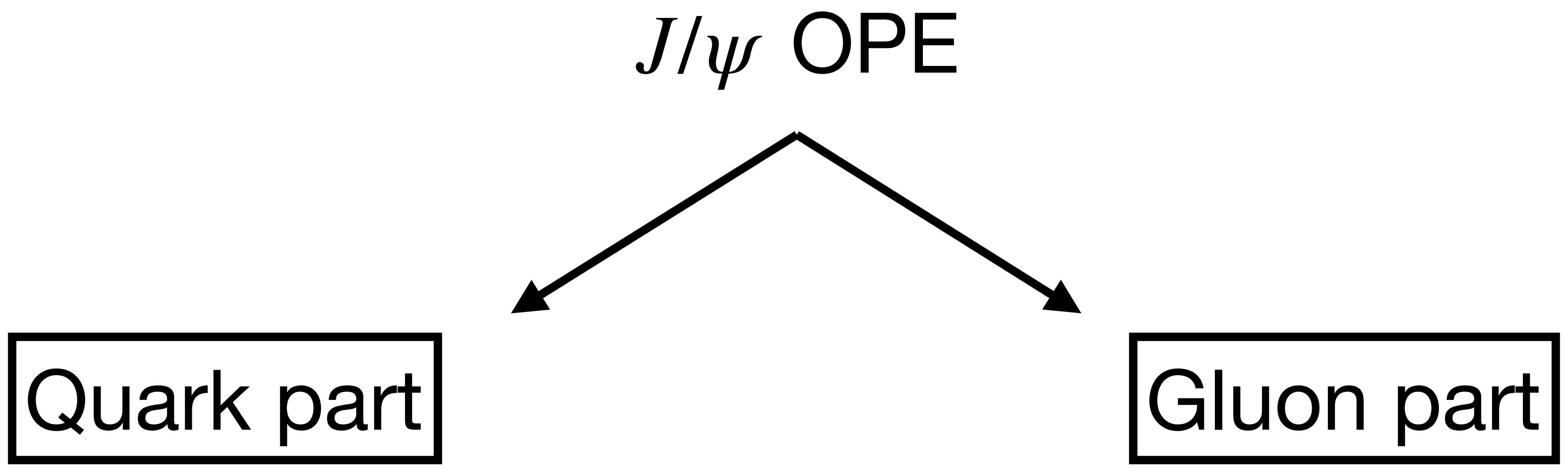
PCAC relates the axial current to the pseudoscalar density.

$$\partial^\mu A^a_\mu = (m_u + m_d)P^a, \text{ where } P^a = \bar{q}\gamma_5\tau^a q\,.$$

⟶ $PP \sim \pi\pi,\ AA \sim (\partial\pi)(\partial\pi)$

Therefore, let us look for such operators in the J/psi OPE.

QCD Sum Rules(QSR)



1. Here, the quark part contains a charm quark. Since the charm quark is too heavy to form a condensate, its contribution is therefore suppressed.

2. The gluon part is composed of $\langle G^2 \rangle$, $\langle G^3 \rangle$, and $\langle (DG)(DG) \rangle$.

→ At first glance, the light quark operators do not seem to appear in the OPE.

QCD Sum Rules(QSR)

J/ψ OPE

Quark part | Gluon part

$$\langle (DG)(DG) \rangle$$

However, by applying the equation of motion to this operator, we can obtain a light-quark operator as follows.

$$\langle (DG)(DG) \rangle \xrightarrow[\text{e.o.m.}]{} \langle (\Sigma \bar{q}_i \gamma_\mu \lambda^a q_i)(\Sigma \bar{q}_j \gamma_\mu \lambda^a q_j) \rangle$$

QCD Sum Rules(QSR)

$$\langle (DG)(DG) \rangle \xrightarrow[\text{e.o.m.}]{} \langle (\Sigma \bar{q}_i \gamma_\mu \lambda^a q_i)(\Sigma \bar{q}_j \gamma_\mu \lambda^a q_j) \rangle$$

However, each bilinear is a vector current, whereas we need operators of the PP or AA type.

Let us perform a Fierz transformation and isolate those terms.

(Let us call this the Fierz projection)

$$\left\langle \left(\sum_{u,d,s} \bar{q}_i \gamma_\mu \lambda^a q_i \right) \left(\sum_{u,d,s} \bar{q}_j \gamma_\mu \lambda^a q_j \right) \right\rangle \rightarrow 2 \underbrace{\langle (\bar{q}_i \gamma_5 q_j)(\bar{q}_j \gamma_5 q_i) \rangle}_{\text{PP}} + \underbrace{\langle (\bar{q}_i \gamma_5 \gamma_\mu q_j)(\bar{q}_j \gamma_5 \gamma_\mu q_i) \rangle}_{\text{AA}}$$

We interpret the effect of these terms in nuclear matter as the J/psi mass shift induced by two-pion exchange

QSR result

$$\left\langle \left(\sum_{u,d,s} \bar{q}_i \gamma_\mu \lambda^a q_i \right) \left(\sum_{u,d,s} \bar{q}_j \gamma_\mu \lambda^a q_j \right) \right\rangle \to 2\langle (\bar{q}_i \gamma_5 q_j)(\bar{q}_j \gamma_5 q_i) \rangle + \langle (\bar{q}_i \gamma_5 \gamma_\mu q_j)(\bar{q}_j \gamma_5 \gamma_\mu q_i) \rangle$$

PP AA

$\Delta m_{J/\psi} = -0.36$ MeV (for $\kappa = 2,\ \alpha_{s,IR} = 0.7$)

The parameters will be explained later.

QSR result

$$\left\langle \left(\sum_{u,d,s} \bar{q}_i \gamma_\mu \lambda^a q_i \right) \left(\sum_{u,d,s} \bar{q}_j \gamma_\mu \lambda^a q_j \right) \right\rangle \to 2\langle (\bar{q}_i \gamma_5 q_j)(\bar{q}_j \gamma_5 q_i) \rangle + \langle (\bar{q}_i \gamma_5 \gamma_\mu q_j)(\bar{q}_j \gamma_5 \gamma_\mu q_i) \rangle$$

PP AA

$$\boxed{\Delta m_{J/\psi} = -0.36 \text{ MeV}}$$

(for $\kappa = 2,\ \alpha_{s,IR} = 0.7$)

The parameters will be explained later.

If we consider the effect of the entire light-quark operators without performing the Fierz projection, we find a repulsive interaction. This is consistent with the usual expectation from vector meson exchange. That is, this approach seems to give the best scenario, at least among the following four cases.

W/O Fierz proj.	R	R	A	A
W/ Fierz proj.	R	A	R	A

Comparison

QSR ⟷ **Lattice-TPE potential**

$$\Delta m_{J/\psi} = -0.36 \text{ MeV}$$

- First, we compute the effective mass shift from the lattice TPE potential, then compare it with the QSR result.

Lattice-TPEP induced mass shift

- We adopt Lattice TPE potential

$$V(r) = -\alpha \frac{e^{-2m_\pi r}}{r^2}$$

with potential strengths $\alpha_{J/\psi}^{(S=3/2)} = 22$ and $\alpha_{J/\psi}^{(S=1/2)} = 23$ MeV · fm^2

- The energy shift of the J/ψ in a nuclear matter

$$\Delta E_V = \int_{r_{min}}^{r_{max}} d^3x V(r)\rho(r),$$ where $\rho(r)$ is the nucleon density.

1. $r_{min} = 1.0$ fm
2. $r_{max} = 1.8$ fm

→ The spin-averaged energy shift is $\Delta m_{J/\psi}^{\text{lat}} = -5$ MeV.

→ We take a few MeV as the reference scale.

Result & Discussion

$\Delta m_{J/\psi}^{QSR} = -0.36$ MeV ⟷ $\Delta m_{J/\psi}^{\text{lat}} = -5$ MeV

This value is too small compared with a few MeV.

We extract the two-pion coupled operator. But, we need to discuss about what long-range means in this context.

Parameter set

$$\alpha_s^2 \langle (\Sigma \bar{q}_i \gamma_\mu \lambda^a q_i)(\Sigma \bar{q}_j \gamma_\mu \lambda^a q_j) \rangle$$

We need to consider the coupling constant.

$$\alpha_{s,IR} = ?$$

Studies on the infrared value of the coupling constant show that there is no consensus on its value. In fact, since this lies in the nuclear physics regime, the issue is rather subtle.

Infrared coupling

1. $\alpha_{\mathrm{s,IR}}$ can be $\uparrow$ at $r >$ 1fm

We choose $\alpha_{\mathrm{s,IR}} =$ 0.7,1.0, and 1.5

Parameter set

When estimating the numerical value of the four-quark operator, one usually employs the vacuum saturation approximation as follows.

$$\langle \mathcal{O}_{4q} \rangle \rightarrow \kappa \langle \mathcal{O}_{2q} \rangle \langle \mathcal{O}_{2q} \rangle$$

Its violation is parametrized by k, and people often use values around k=2–4, or even as large as 6—7.

$$\kappa = ?$$

Deviation parameter

2. We take $\kappa_{\Delta\rho}$ = 2—5, for three reasons:

(Starting from $\kappa_{\Delta\rho} = 1$)

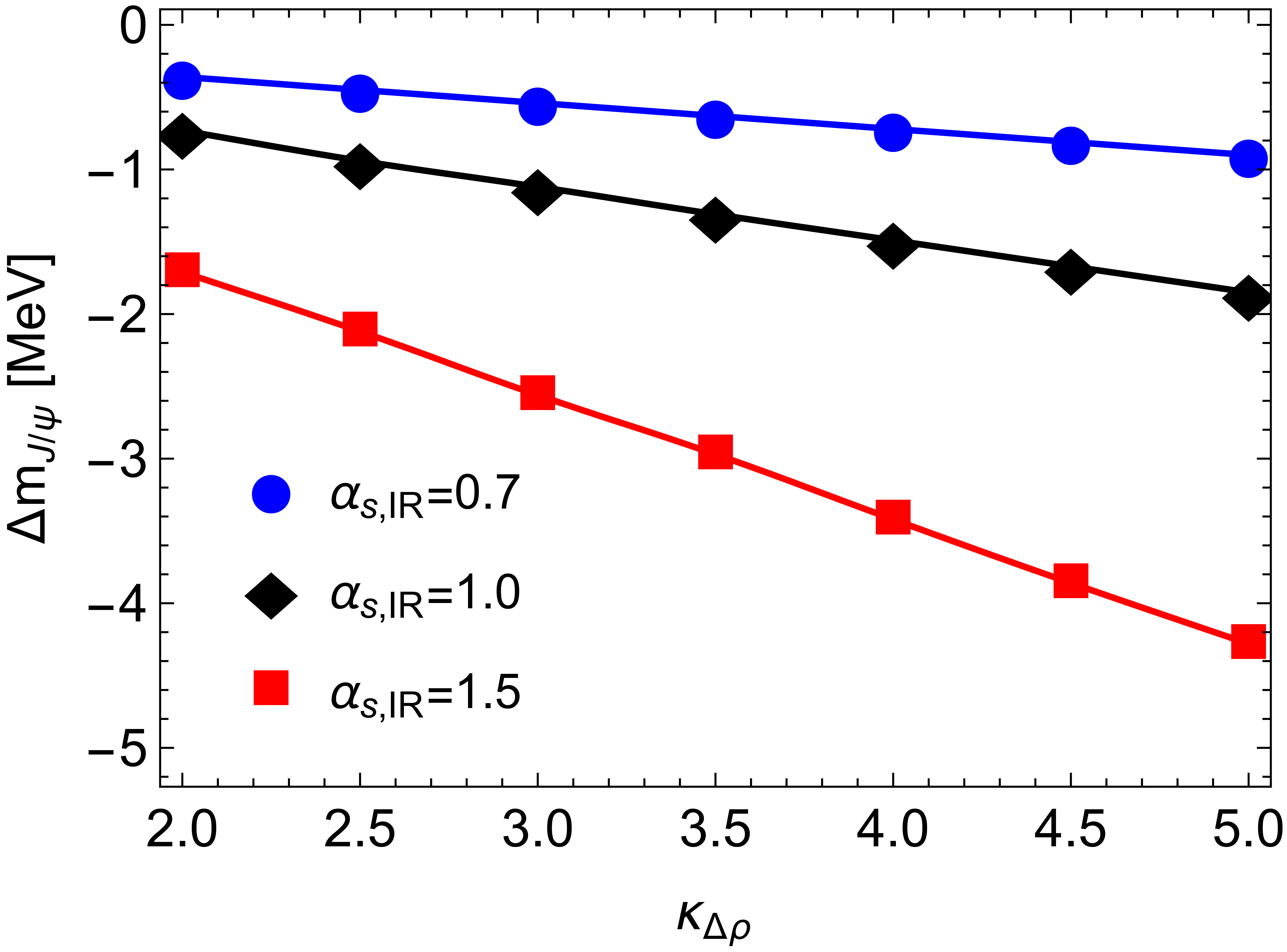
- χ-even $\simeq$ χ-odd (J. Kim *et al.*[1])
  ⟶ $\kappa_{\Delta\rho}$= 2
- perturbative k-factor in QCD(analogy)
  ⟶ another factor of about 2
- Intermediate states in medium
  ⟶ weaken vac. dominance

Based on the arguments above, we obtain $\kappa_{\Delta\rho} = 5$.

Thus, we adopt $\kappa_{\Delta\rho}$= 2—5 as a reasonable range.

1. J. Kim and S. H. Lee, Phys. Rev. D 103, L051501 (2021),
J. Kim and S. H. Lee, Phys. Rev. D 105, 014014 (2022).

Result & Discussion



- $\alpha_{\rm s,IR}$=0.7 → $|\Delta m|$ = 0.36−0.90 MeV
- $\alpha_{\rm s,IR}\uparrow \rightarrow |\Delta m| \uparrow$
- $\kappa \uparrow \rightarrow |\Delta m| \uparrow$
- $\alpha_{\rm s,IR}$=1.5 → $|\Delta m|$ = 1.71−4.28 MeV

With physically relevant values of $\kappa_{\Delta\rho}$ and $\alpha_{\rm s,IR}$, QSR estimates few-MeV, same order as lattice TPE

S. Yeo, I.W. Park, S.H. Lee, Phys. Rev. D 112, 094044 (2025).

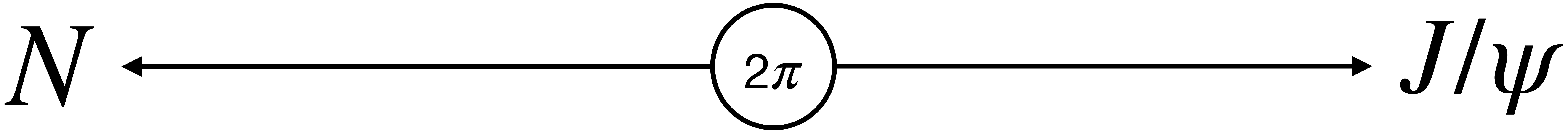
Result & Discussion(additional)

- w/o Fierz → Repulsion
  The Fierz projection matters

- $\alpha_{\rm s,IR} \downarrow\ \to |\Delta m| \downarrow$
  Implying TPE↓ at short distance

- $m_q \uparrow\ \to |\Delta m| \downarrow$
  Chromoelectric polarizability
  Potential strength(in TPEP) trend
  $\alpha_\phi(=91) > \alpha_{J/\psi}(=22) > \alpha_\Upsilon\ \left[\text{MeV} \cdot \text{fm}^2\right]$

Summary



- Lattice[1]: TPE model describes long-range interaction well.
- Extracting TPE induced mass shift (~ a few MeV).
- **QSR[2](This work)**: reproduces long-range TPE; attractive mass shift (~ a few MeV); **consistent** with the lattice result within reasonable parameters.
  (→ shows small but finite effect of partial χ-symmetry restoration in heavy-quark systems.)

1. Y. Lyu, T. Doi, T. Hatsuda, and T. Sugiura, Physics Letters B 860, 139178 (2025).
2. S. Yeo, I.W. Park, S.H. Lee, Phys. Rev. D 112, 094044 (2025).