Baryon-antibaryon and dibaryon-antidibaryon configurations in a constituent quark model

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Reimei workshop on in-medium modification of vector mesons(2026.03.10)

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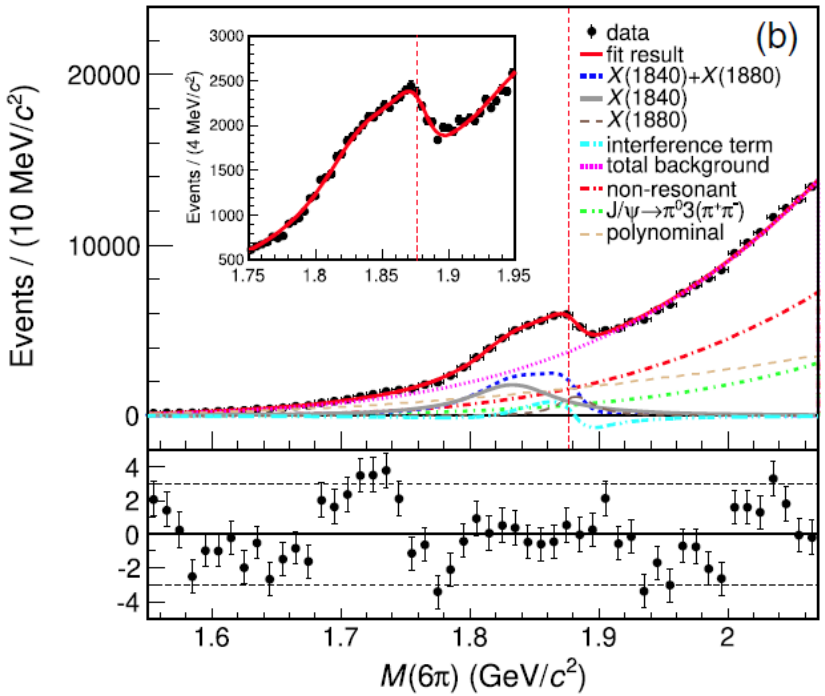
Introduction

$p\bar{p}$ bound state

$$J/\psi \to \gamma 3(\pi^+\pi^-)$$



M. Ablikim et al.(BESIII Collaboration)
Phys. Rev. Lett. 132, 151901(2024)

X(1840)

$$M = 1832.5 \pm 3.1 \pm 2.5 \text{ MeV}/c^2$$

$$\Gamma = 80.7 \pm 5.2 \pm 7.7 \text{ MeV}$$

X(1880)

$$M = 1882.1 \pm 1.7 \pm 0.7 \text{ MeV}/c^2$$

$$\Gamma = 30.7 \pm 5.5 \pm 2.4 \text{ MeV}$$

This indicates the existence of a $p\bar{p}$ bound state.

$q^3\bar{q}^3$ configuration

Calculating the color-color interaction of $q^3\bar{q}^3$ state is challenging because it is difficult to construct the color wave function when quarks and antiquarks are present together.

q^n : totally antisymmetric wave function.

$q^n\bar{q}^m$: antisymmetric for quarks and antiquarks, respectively.

Constructing a total color singlet requires an appropriate connection between the quark and antiquark color state.

Dibaryon case

For a dibaryon,

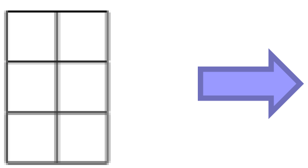
$$3 \otimes 3 \otimes 3 \otimes 3 \otimes 3 \otimes 3 = (\bar{3} \oplus 6) \otimes (\bar{3} \oplus 6) \otimes (\bar{3} \oplus 6)$$

$$= \cdots$$

$$= 1_{(m=5)} \oplus 8_{(m=16)} \oplus 10_{(m=10)} \oplus \overline{10}_{(m=5)} \oplus 27_{(m=9)} \oplus 35_{(m=5)} \oplus 28$$

too complicated

Let's consider the Young diagrams.



$$|C_1\rangle = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array}, \quad |C_2\rangle = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 & 6 \\ \hline \end{array}, \quad |C_3\rangle = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & 6 \\ \hline \end{array}, \quad |C_4\rangle = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & 6 \\ \hline \end{array}, \quad |C_5\rangle = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline \end{array}.$$

We can represent the color state using the Young-Yamanouchi basis.

1. For the Young-Yamanouchi basis, the entries in each row and column are increasing.
2. From one basis, we can generate the remaining basis using the permutation property.

Dibaryon case

For a dibaryon,

$$3 \otimes 3 \otimes 3 \otimes 3 \otimes 3 \otimes 3 = (3 \oplus 6) \otimes (3 \oplus 6) \otimes (3 \oplus 6)$$

$$= \cdots$$

$$= 1_{(m=5)} \oplus 8_{(m=16)} \oplus 10_{(m=10)} \oplus \overline{10}_{(m=5)} \oplus 27_{(m=9)} \oplus 35_{(m=5)} \oplus 28$$

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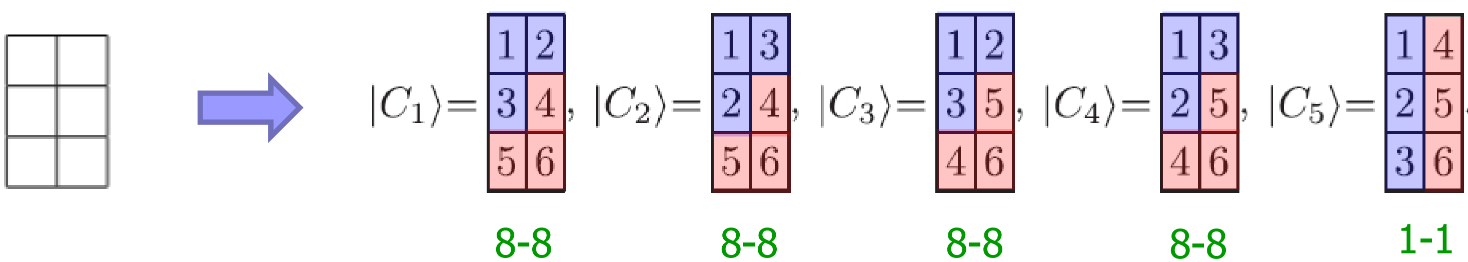
Let's consider the Young diagrams.



$$\lambda_i^c \lambda_j^c = 2(ij) - \frac{2}{3} I$$

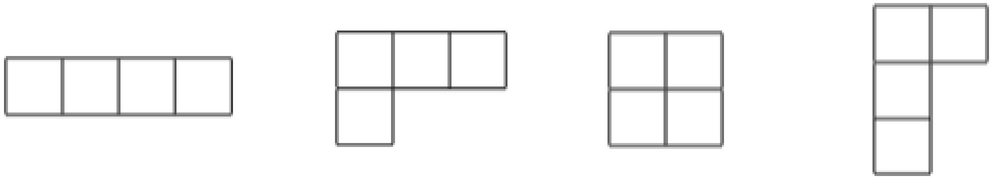
$$\sum_{i<j} \lambda_i \lambda_j = \begin{pmatrix} -16 & 0 & 0 & 0 & 0 \\ 0 & -16 & 0 & 0 & 0 \\ 0 & 0 & -16 & 0 & 0 \\ 0 & 0 & 0 & -16 & 0 \\ 0 & 0 & 0 & 0 & -16 \end{pmatrix}$$

Conclusion: If we can construct the multiplet only using the fundamental representations (or antifundamental representations only), then we can easily calculate the color-color matrices.

color basis of $q^n \bar{q}^n$

$q^n \bar{q}^n$

ex) $n = 4$



$15'$ $\quad$ 15 $\quad$ $\bar{6}$ $\quad$ 3

color basis of $q^n \bar{q}^n$

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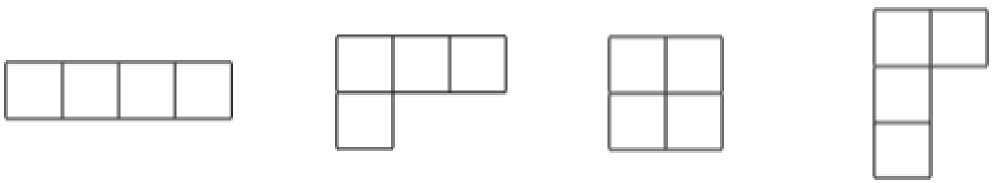
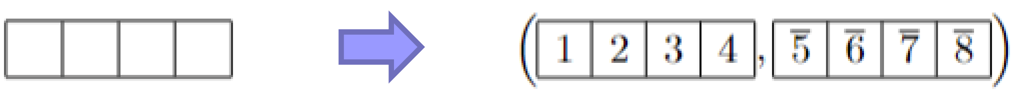
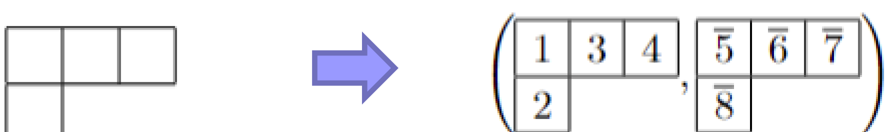


$15'$ $\quad$ 15 $\quad$ $\bar{6}$ $\quad$ 3



$15'$ $\quad$ $\overline{15}'$

$A^i \times B_j \rightarrow A^i B_i$



15 $\quad$ $\overline{15}$

color basis of $q^n\bar{q}^n$

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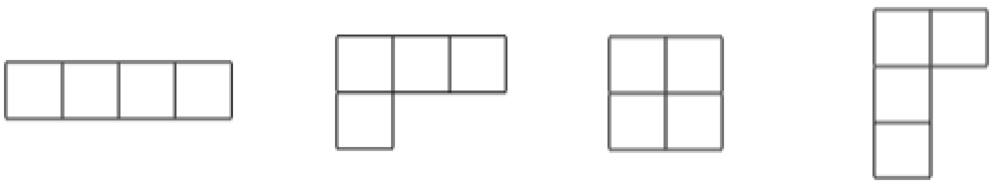

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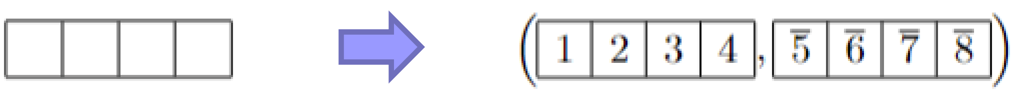

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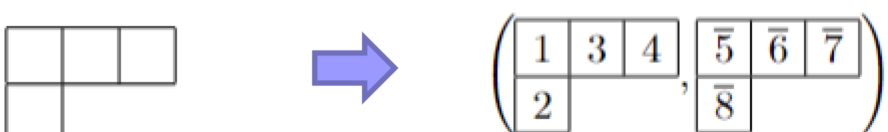
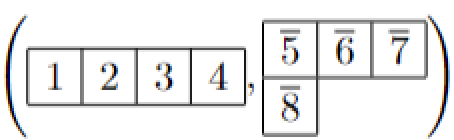


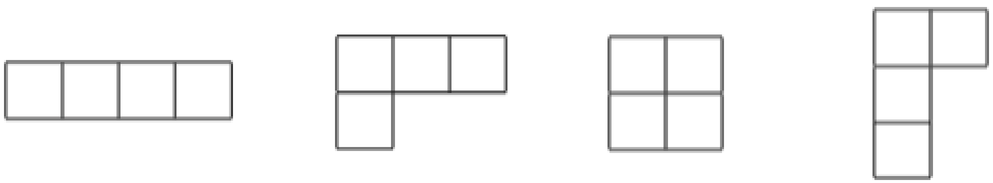
15 $\quad$ $\overline{15}$



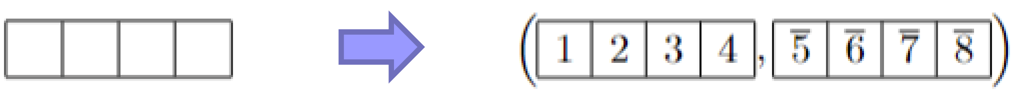
color basis of $q^n \bar{q}^{\mathrm{n}}$

$q^n \bar{q}^n$

ex) $n = 4$

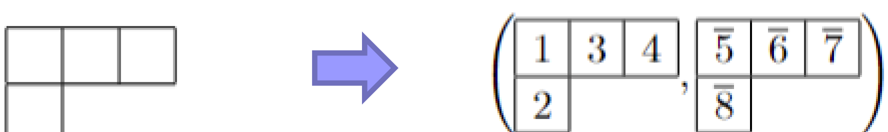


$15'$ 15 $\bar{6}$ 3

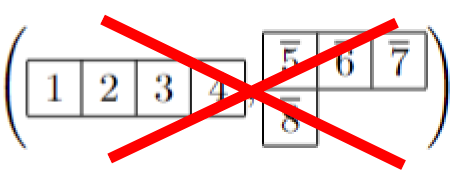


$15'$ $\overline{15}'$

$A^i \times B_j \rightarrow A^i B_i$



15 $\overline{15}$



Young diagrams with different shapes cannot make color singlet.

Number of color basis

The number of color singlet basis of $q^n\bar{q}^n = \sum_Y d_Y^2$

where d_Y is a dimension of the corresponding Young diagram.

The summation is taken for all possible Young diagrams consisting n boxes satisfying the SU(3) symmetry.

ex1) $q^2\bar{q}^2$

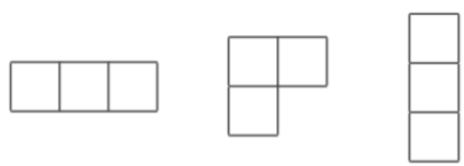
The possible Young diagrams :

$d_{[2]} = 1, d_{[1,1]} = 1$

→ $\sum_Y d_Y^2 = 2$

ex2) $q^3\bar{q}^3$

The possible Young diagrams :



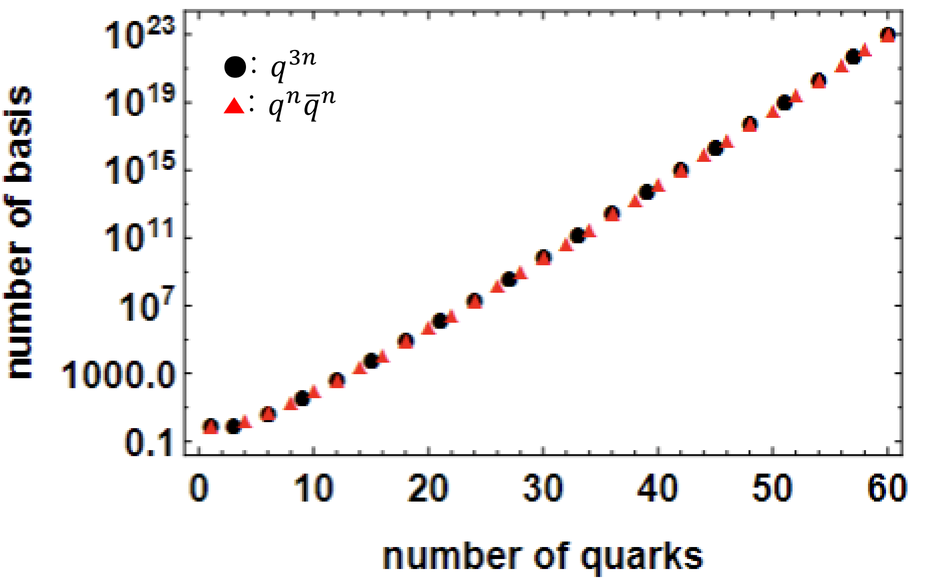
$d_{[3]} = 1, d_{[2,1]} = 2, d_{[1,1,1]} = 1$

→ $\sum_Y d_Y^2 = 6$

Number of basis

q^{3n}	number of basis
n=1	1
2	5
3	42
4	462
5	6006
6	87516
7	1385670
8	23371634
9	414315330
10	7646001090
11	145862174640
12	2861142656400
13	57468093927120
14	1178095925505960
15	24584089974896430
16	521086299271824330
17	11198784501894470250
18	243661974372798631650
19	5360563436201569896300
20	119115896614816702500900

$q^n\bar{q}^n$	number of basis
n=1	1
2	2
3	6
4	23
5	103
6	513
7	2761
8	15767
9	94359
10	586590
11	3763290
12	24792705
13	167078577
14	1148208090
15	8026793118
16	56963722223
17	409687815151
18	2981863943718
19	21937062144834
20	162958355218089
21	1221225517285209
22	9225729232653663
23	70209849031116183
24	537935616492552297
25	4147342550996290153
26	32159907636432567578
27	250717538500344886206
28	1964347085978431234383
29	15462159345628498316319
30	122238900487877503161969



$q^3\bar{q}^3$: Baryon-antibaryon

Color basis of $q^3\bar{q}^3$

$$\mathbf{3}_1 \times \mathbf{3}_2 \times \mathbf{3}_3 \times \bar{\mathbf{3}}_4 \times \bar{\mathbf{3}}_5 \times \bar{\mathbf{3}}_6$$
$$= (\mathbf{1}_{123} + \mathbf{8}_{123} + \mathbf{8}_{123} + \mathbf{10}_{123}) \times (\mathbf{1}_{456} + \mathbf{8}_{456} + \mathbf{8}_{456} + \overline{\mathbf{10}}_{456})$$

$$|C_1\rangle = \left(\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}, \begin{array}{|c|}\hline \bar{4} \\ \hline \bar{5} \\ \hline \bar{6} \\ \hline \end{array} \right), \; |C_2\rangle = \left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{4} & \bar{5} \\ \hline \bar{6} \\ \cline{1-1} \end{array} \right), \; |C_3\rangle = \left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{4} & \bar{6} \\ \hline \bar{5} \\ \cline{1-1} \end{array} \right),$$

$$|C_4\rangle = \left(\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{4} & \bar{5} \\ \hline \bar{6} \\ \cline{1-1} \end{array} \right), \; |C_5\rangle = \left(\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{4} & \bar{6} \\ \hline \bar{5} \\ \cline{1-1} \end{array} \right), \; |C_6\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline \end{array}, \begin{array}{|c|c|c|}\hline \bar{4} & \bar{5} & \bar{6} \\ \hline \end{array} \right)$$

There is an additional color basis from the decuplet-antidecuplet.

Color-color matrix of $q^3\bar{q}^3$

$$\lambda_1\lambda_2 = \begin{pmatrix} -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{8}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4}{3} \end{pmatrix}, \quad \lambda_1\lambda_3 = \begin{pmatrix} -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{3} & 0 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & -\frac{5}{3} & 0 & -\sqrt{3} & 0 \\ 0 & -\sqrt{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4}{3} \end{pmatrix}, \quad \lambda_1\lambda_4 = \begin{pmatrix} 0 & -\sqrt{2} & \sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} & -\frac{\sqrt{2}}{3} & 0 \\ -\sqrt{2} & -\frac{5}{2} & \frac{5}{2\sqrt{3}} & \frac{5}{2\sqrt{3}} & -\frac{3}{2} & -\frac{\sqrt{5}}{3} \\ \sqrt{\frac{2}{3}} & \frac{5}{2\sqrt{3}} & -\frac{5}{6} & -\frac{3}{2} & -\frac{1}{2\sqrt{3}} & -\sqrt{\frac{5}{3}} \\ \sqrt{\frac{2}{3}} & \frac{5}{2\sqrt{3}} & -\frac{3}{2} & -\frac{5}{6} & -\frac{1}{2\sqrt{3}} & -\sqrt{\frac{5}{3}} \\ -\frac{\sqrt{2}}{3} & -\frac{3}{2} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{7}{6} & -\sqrt{5} \\ 0 & -\frac{\sqrt{5}}{3} & -\sqrt{\frac{5}{3}} & -\sqrt{\frac{5}{3}} & -\sqrt{5} & -\frac{8}{3} \end{pmatrix},$$

.
.
.

$$\sum_{i<j} \lambda_i\lambda_j = \begin{pmatrix} -16 & 0 & 0 & 0 & 0 & 0 \\ 0 & -16 & 0 & 0 & 0 & 0 \\ 0 & 0 & -16 & 0 & 0 & 0 \\ 0 & 0 & 0 & -16 & 0 & 0 \\ 0 & 0 & 0 & 0 & -16 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16 \end{pmatrix}$$

The sum of color-color matrices is the same as dibaryon.

Three-meson configuration: Color

We can obtain another color basis set changing the order of quarks and antiquarks.

$$
\begin{aligned}
&(3_1 \times \bar{3}_4) \times (3_2 \times \bar{3}_5) \times (3_3 \times \bar{3}_6) \\
&= (1+8)_{14} \times (1+8)_{25} \times (1+8)_{36} \\
&= [1(1_{14}1_{25}) + 8(1_{14}8_{25}) + 8(8_{14}1_{25}) + 1(8_{14}8_{25}) + 8(8_{14}8_{25}) + 8'(8_{14}8_{25}) \\
&\quad + 10(8_{14}8_{25}) + \overline{10}(8_{14}8_{25}) + 27(8_{14}8_{25})] \times (1+8)_{36}
\end{aligned}
$$

Here, we can determine six color singlet basis.

$$
\begin{aligned}
&|C'_1\rangle = 1_{14}1_{25}1_{36}, \quad |C'_2\rangle = 1_{1425}[8_{14}8_{25}]1_{36}, \quad |C'_3\rangle = 8_{1425}[1_{14}8_{25}]8_{36}, \\
&|C'_4\rangle = 8_{1425}[8_{14}1_{25}]8_{36}, \quad |C'_5\rangle = 8_{1425}[8_{14}8_{25}]8_{36}, \quad |C'_6\rangle = 8'_{1425}[8_{14}8_{25}]8_{36}.
\end{aligned}
$$

$$|C'\rangle = U|C\rangle$$

$$
U = \begin{pmatrix}
\frac{1}{3\sqrt{3}} & \frac{2\sqrt{2}}{3\sqrt{3}} & 0 & 0 & \frac{2\sqrt{2}}{3\sqrt{3}} & \frac{\sqrt{10}}{3\sqrt{3}} \\
-\frac{\sqrt{2}}{3\sqrt{3}} & \frac{2}{3\sqrt{3}} & 0 & 0 & -\frac{4}{3\sqrt{3}} & \frac{\sqrt{5}}{3\sqrt{3}} \\
-\frac{\sqrt{2}}{3\sqrt{3}} & -\frac{5}{6\sqrt{3}} & \frac{1}{2} & \frac{1}{2} & \frac{1}{6\sqrt{3}} & \frac{\sqrt{5}}{3\sqrt{3}} \\
-\frac{\sqrt{2}}{3\sqrt{3}} & -\frac{5}{6\sqrt{3}} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{6\sqrt{3}} & \frac{\sqrt{5}}{3\sqrt{3}} \\
\frac{5\sqrt{2}}{3\sqrt{21}} & -\frac{5}{6\sqrt{21}} & \frac{3}{2\sqrt{7}} & -\frac{3}{2\sqrt{7}} & -\frac{5}{6\sqrt{21}} & \frac{\sqrt{5}}{3\sqrt{21}} \\
\frac{\sqrt{10}}{\sqrt{21}} & -\frac{\sqrt{5}}{2\sqrt{21}} & -\frac{\sqrt{5}}{2\sqrt{7}} & \frac{\sqrt{5}}{2\sqrt{7}} & -\frac{\sqrt{5}}{2\sqrt{21}} & \frac{1}{\sqrt{21}}
\end{pmatrix}
$$

Three-meson configuration: Spin

S=0

$$S_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array}, \quad S_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & 5 & 6 \\ \hline \end{array}, \quad S_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & 5 & 6 \\ \hline \end{array}, \quad S_4 = \begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 3 & 4 & 6 \\ \hline \end{array}, \quad S_5 = \begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline 2 & 4 & 6 \\ \hline \end{array}.$$

$$S' = US$$

$$\begin{aligned} S_1' &= (14)_{S=0}(25)_{S=0}(36)_{S=0} \\ S_2' &= (14)_{S=1}(25)_{S=1}(36)_{S=0} \\ S_3' &= (14)_{S=1}(25)_{S=0}(36)_{S=1} \\ S_4' &= (14)_{S=0}(25)_{S=1}(36)_{S=1} \\ S_5' &= (14)_{S=1}(25)_{S=1}(36)_{S=1} \end{aligned}$$

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{4} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{4\sqrt{3}} & \frac{3}{4} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{1}{\sqrt{6}} & \frac{5}{4\sqrt{3}} & \frac{1}{4} & \frac{1}{4} & \frac{\sqrt{3}}{4} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{4\sqrt{3}} & -\frac{1}{4} & \frac{3}{4} & -\frac{\sqrt{3}}{4} \\ 0 & \frac{\sqrt{\frac{3}{2}}}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{\sqrt{\frac{3}{2}}}{2} \end{pmatrix}$$

Results: Nucleon-antinucleon

Hamiltonian and Jacobi coordinate

$$H = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{4}\sum_{i<j}^{n}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}\left(V_{ij}^C + V_{ij}^{CS}\right) + \sum_{i<j<k}^{n}\left(AL_{ijk}^{C-C} + BL_{ijk}^{S-S} + CL_{ijk}^{C-S}\right),$$

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{CS} = \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{(r_{0ij}) r_{ij}}\sigma_i \cdot \sigma_j .$$

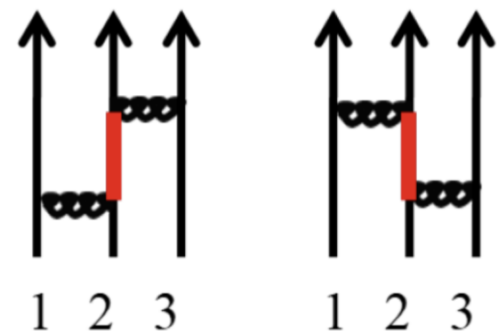
$$\vec{x}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{x}_2 = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

$$\vec{x}_3 = \frac{1}{\sqrt{2}}(\vec{r}_4 - \vec{r}_5)$$

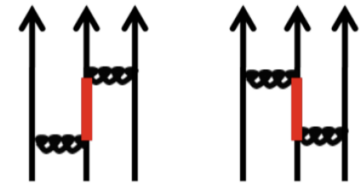
$$\vec{x}_4 = \frac{1}{\sqrt{6}}(\vec{r}_4 + \vec{r}_5 - 2\vec{r}_6)$$

$$\vec{x}_5 = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_5 - \vec{r}_6)$$



$$R = e^{-a_1 x_1^2 - a_2 x_2^2 - a_1 x_3^2 - a_2 x_4^2 - a_3 x_5^2}$$

Three-quark potentials



For antiquarks, $\{\bar{\lambda}^a, \bar{\lambda}^b\} = \frac{4}{3}\delta^{ab} - 2d^{abc}\bar{\lambda}^c$

$[\bar{\lambda}^a, \bar{\lambda}^b] = 2if^{abc}\bar{\lambda}^c$

$$L_{C-C} = \left[\sum_{i<j<k}\frac{4}{3}\left(\frac{\lambda_i^c\lambda_j^c}{m_k}+\frac{\lambda_i^c\lambda_k^c}{m_j}+\frac{\lambda_j^c\lambda_k^c}{m_i}\right)+2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)}{m_i}+\frac{\mathrm{sign}(j)}{m_j}+\frac{\mathrm{sign}(k)}{m_k}\right)\right]$$

$$L_{S-S} = \sum_{i<j<k}\frac{1}{m_i m_j m_k}\left[\frac{4}{3}\left(\frac{\lambda_j^c\lambda_k^c\sigma_j\cdot\sigma_k}{m_i^2}+\frac{\lambda_i^c\lambda_k^c\sigma_i\cdot\sigma_k}{m_j^2}+\frac{\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j}{m_k^2}\right)\right.$$
$$+2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)\sigma_j\cdot\sigma_k}{m_i^2}+\frac{\mathrm{sign}(j)\sigma_i\cdot\sigma_k}{m_j^2}+\frac{\mathrm{sign}(k)\sigma_i\cdot\sigma_j}{m_k^2}\right)$$
$$\left.-2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{1}{m_3^2}\right)\right],$$

$$L_{C-S} = \sum_{i<j<k}\frac{4}{3}\left\{\frac{\lambda_j^c\lambda_k^c}{m_i}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j}+\frac{\sigma_i\cdot\sigma_k}{m_i m_k}\right)+\frac{\lambda_i^c\lambda_k^c}{m_j}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j}+\frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)+\frac{\lambda_i^c\lambda_j^c}{m_k}\left(\frac{\sigma_i\cdot\sigma_k}{m_i m_k}+\frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)\right]$$
$$+2d^{abc}\lambda_i^c\lambda_j^c\lambda_k^c\left[\frac{\mathrm{sign}(i)}{m_i}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j}+\frac{\sigma_i\cdot\sigma_k}{m_i m_k}\right)+\frac{\mathrm{sign}(j)}{m_j}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j}+\frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)+\frac{\mathrm{sign}(k)}{m_k}\left(\frac{\sigma_i\cdot\sigma_k}{m_i m_k}+\frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)\right]\right\}$$

where $\mathrm{sign}(i) = -1$ for antiquarks.

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Flavor, color, spin state(I=0,S=0)

$$F_1^A = \left(\begin{array}{ccc}1&2&3\end{array}, \begin{array}{ccc}\bar{4}&\bar{5}&\bar{6}\end{array}\right),\quad CS_1^A = \left(\begin{array}{c}1\\2\\3\end{array}, \begin{array}{c}\bar{4}\\\bar{5}\\\bar{6}\end{array}\right)$$

$$F_1^B = \left(\begin{array}{cc}1&2\\3&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),\quad F_2^B = \left(\begin{array}{cc}1&2\\3&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),$$

$$F_3^B = \left(\begin{array}{cc}1&3\\2&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),\quad F_4^B = \left(\begin{array}{cc}1&3\\2&\end{array}, \begin{array}{cc}\bar{4}&\bar{6}\\\bar{5}&\end{array}\right),$$

$$CS_1^B = \left(\begin{array}{cc}1&2\\3&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),\quad CS_2^B = \left(\begin{array}{cc}1&2\\3&\end{array}, \begin{array}{cc}\bar{4}&\bar{6}\\\bar{5}&\end{array}\right),$$

$$CS_3^B = \left(\begin{array}{cc}1&3\\2&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),\quad CS_4^B = \left(\begin{array}{cc}1&3\\2&\end{array}, \begin{array}{cc}\bar{4}&\bar{6}\\\bar{5}&\end{array}\right),$$

$$\psi_1 = F_1^A \otimes C_1 \otimes [(12)_{S=1}3]_{S=\frac{3}{2}}[(45)_{S=1}6]_{S=\frac{3}{2}}$$

$$\psi_2 = F_1^A \otimes (C_2 \sim C_5) \otimes [123]_{S=\frac{1}{2}}[456]_{S=\frac{1}{2}}$$

$$\psi_3 = (F_1^B \sim F_4^B) \otimes C_1 \otimes [123]_{S=\frac{1}{2}}[456]_{S=\frac{1}{2}}$$

$$\psi_4 = (F_1^B \sim F_4^B) \otimes (C_2 \sim C_5) \otimes [123]_{S=\frac{3}{2}}[456]_{S=\frac{3}{2}}$$

$$\psi_5 = (F_1^B \sim F_4^B) \otimes (C_2 \sim C_5) \otimes [123]_{S=\frac{1}{2}}[456]_{S=\frac{1}{2}}$$

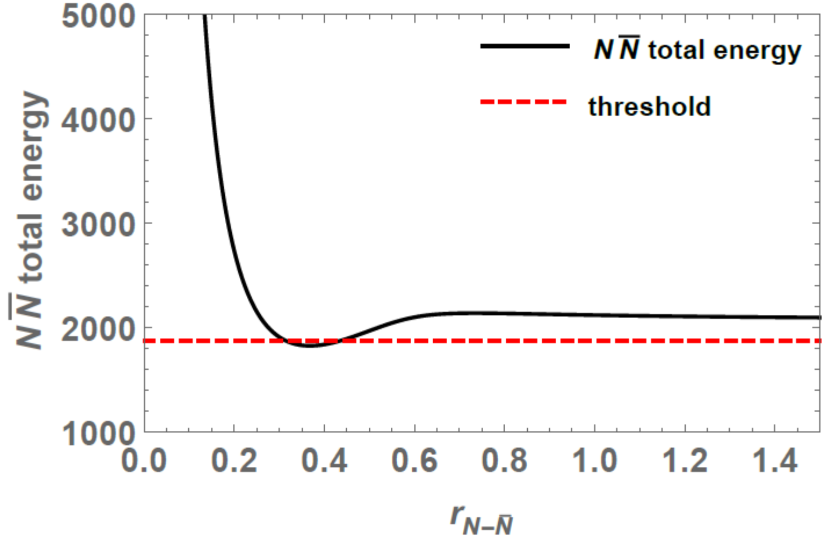
$$\psi_6 = (F_1^B \sim F_4^B) \otimes C_6 \otimes [123]_{S=\frac{1}{2}}[456]_{S=\frac{1}{2}}$$

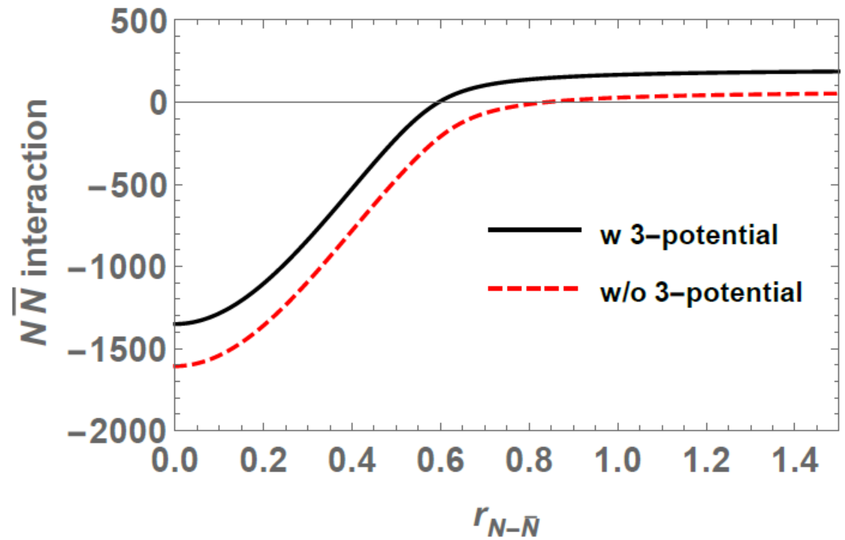
$$|C_1\rangle = \left(\begin{array}{c}1\\2\\3\end{array}, \begin{array}{c}\bar{4}\\\bar{5}\\\bar{6}\end{array}\right),\ |C_2\rangle = \left(\begin{array}{cc}1&2\\3&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),\ |C_3\rangle = \left(\begin{array}{cc}1&2\\3&\end{array}, \begin{array}{cc}\bar{4}&\bar{6}\\\bar{5}&\end{array}\right),$$

$$|C_4\rangle = \left(\begin{array}{cc}1&3\\2&\end{array}, \begin{array}{cc}\bar{4}&\bar{5}\\\bar{6}&\end{array}\right),\ |C_5\rangle = \left(\begin{array}{cc}1&3\\2&\end{array}, \begin{array}{cc}\bar{4}&\bar{6}\\\bar{5}&\end{array}\right),\ |C_6\rangle = \left(\begin{array}{ccc}1&2&3\end{array}, \begin{array}{ccc}\bar{4}&\bar{5}&\bar{6}\end{array}\right)$$

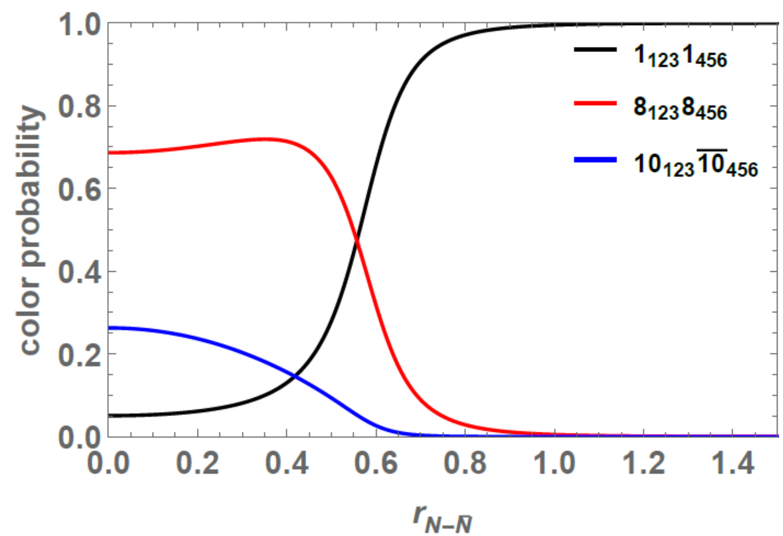
Results: Nucleon-antinucleon(I=0,S=0)

$$r_{N-\overline{N}} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_5 - \vec{r}_6)$$



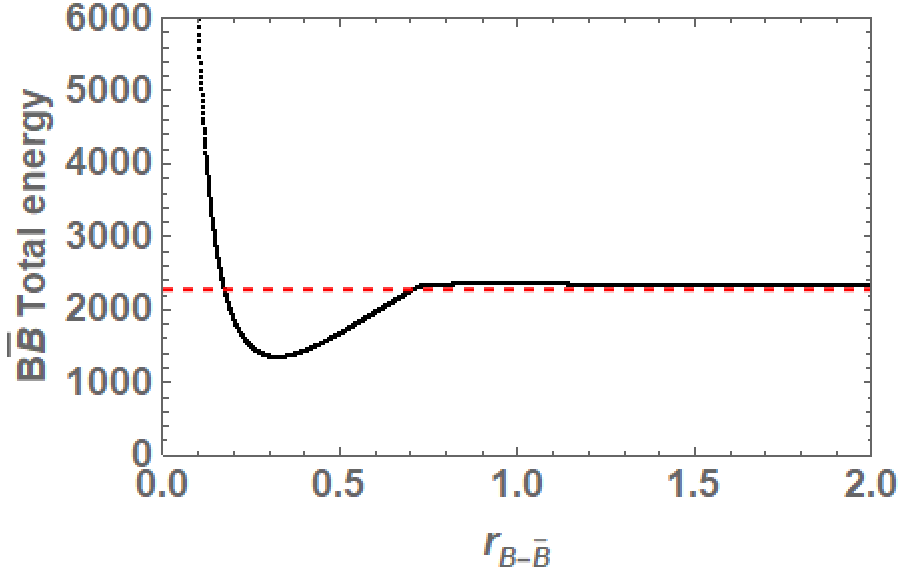


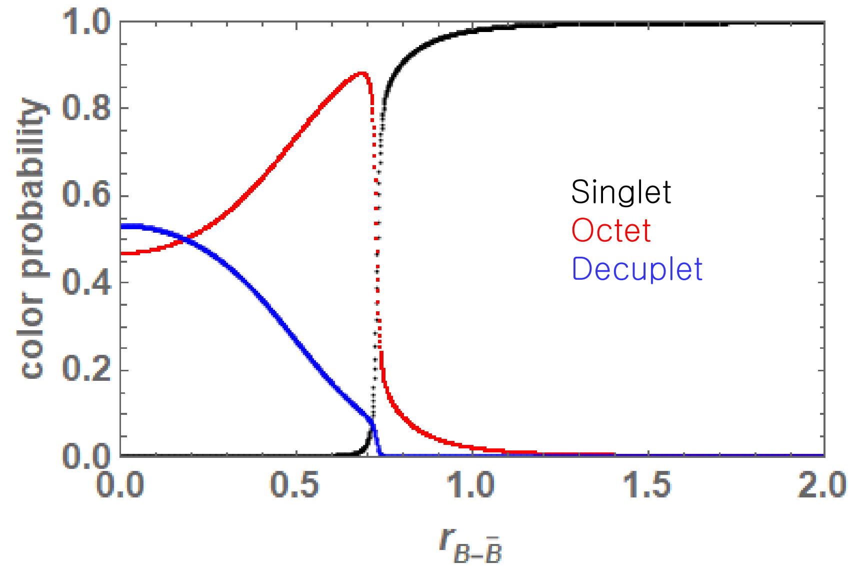
$$V^{B\overline{B}} = H_C^{B\overline{B}} + H_{CS}^{B\overline{B}} - (H_C^B + H_{CS}^B + H_C^{\overline{B}} + H_{CS}^{\overline{B}})$$

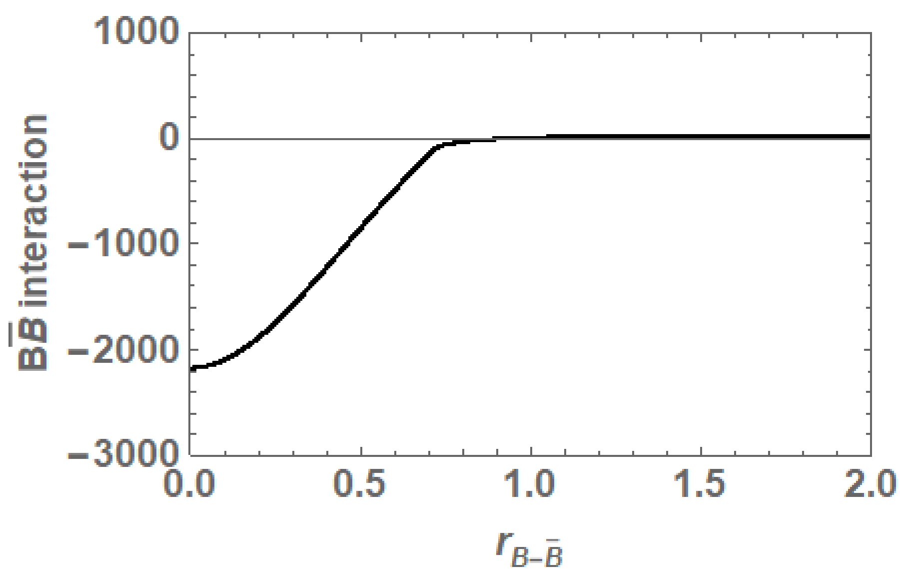


There is a possible bound state for I=0, S=0.

Results: $\Lambda\bar{\Lambda}\ (I = 0, S = 0)$

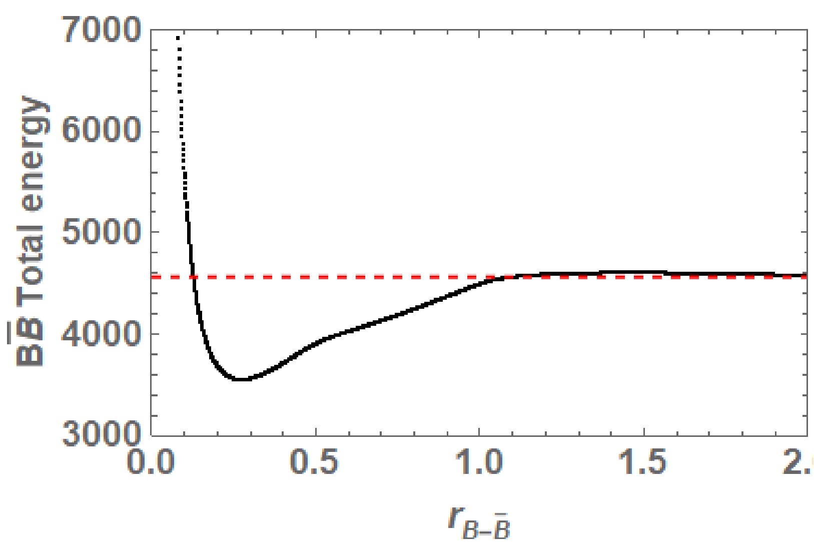


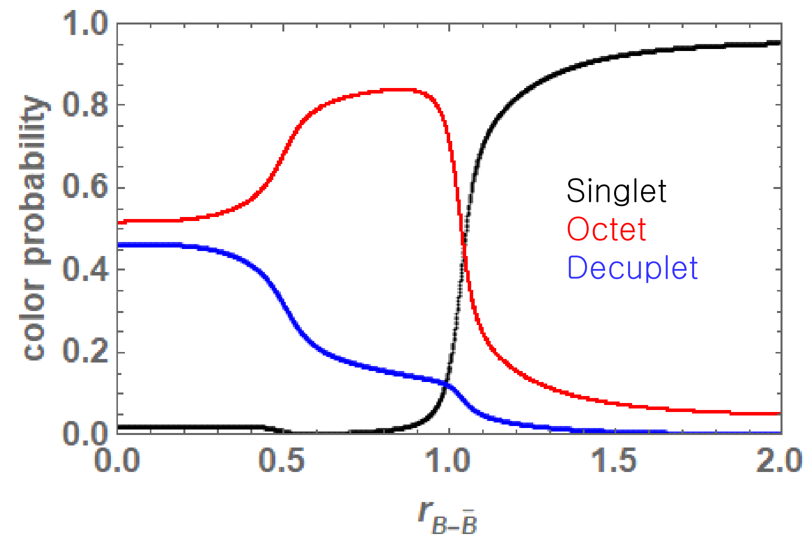


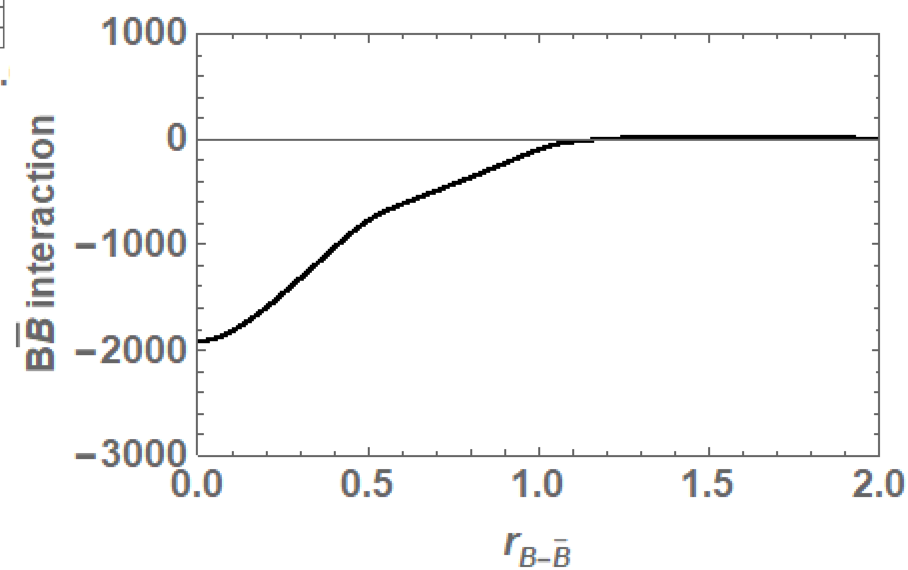


$$V^{B\overline{B}} = H_C^{B\overline{B}} + H_{CS}^{B\overline{B}} - (H_C^B + H_{CS}^B + H_C^{\overline{B}} + H_{CS}^{\overline{B}})$$

Results: $\Lambda_c\bar{\Lambda}_c(I=0, S=0)$







$$V^{B\overline{B}} = H_C^{B\overline{B}} + H_{CS}^{B\overline{B}} - (H_C^B + H_{CS}^B + H_C^{\overline{B}} + H_{CS}^{\overline{B}})$$

$q^6\bar{q}^6$: Dibaryon-Antidibaryon

Color basis : $q^6\bar{q}^6$

$$
\begin{aligned}
&(\mathbf{3}\times\mathbf{3}\times\mathbf{3}\times\mathbf{3}\times\mathbf{3}\times\mathbf{3})\times(\bar{\mathbf{3}}\times\bar{\mathbf{3}}\times\bar{\mathbf{3}}\times\bar{\mathbf{3}}\times\bar{\mathbf{3}}\times\bar{\mathbf{3}})\\
&\quad=(\mathbf{1}_{(m=5)}+\mathbf{8}_{(m=16)}+\mathbf{10}_{(m=10)}+\overline{\mathbf{10}}_{(m=5)}+\mathbf{27}_{(m=9)}+\mathbf{35}_{(m=5)}+\mathbf{28})\\
&\quad\times(\mathbf{1}_{(m=5)}+\mathbf{8}_{(m=16)}+\overline{\mathbf{10}}_{(m=10)}+\mathbf{10}_{(m=5)}+\mathbf{27}_{(m=9)}+\overline{\mathbf{35}}_{(m=5)}+\overline{\mathbf{28}})
\end{aligned}
$$

$$
C_1=\left(\begin{array}{cc}1&2\\3&4\\5&6\end{array},\begin{array}{cc}\bar{7}&\bar{8}\\\bar{9}&\overline{10}\\\overline{11}&\overline{12}\end{array}\right),\ C_2=\left(\begin{array}{cc}1&2\\3&4\\5&6\end{array},\begin{array}{cc}\bar{7}&\bar{9}\\\bar{8}&\overline{10}\\\overline{11}&\overline{12}\end{array}\right),\cdots,
$$

513 color singlet bases

$$
C_{26}=\left(\begin{array}{ccc}1&2&3\\4&5&\\6&&\end{array},\begin{array}{ccc}\bar{7}&\bar{8}&\bar{9}\\\overline{10}&\overline{11}&\\\overline{12}&&\end{array}\right),\ C_{27}=\left(\begin{array}{ccc}1&2&3\\4&5&\\6&&\end{array},\begin{array}{ccc}\bar{7}&\bar{8}&\overline{10}\\\bar{9}&\overline{11}&\\\overline{12}&&\end{array}\right),\cdots,
$$

$$
C_{282}=\left(\begin{array}{cccc}1&2&3&4\\5&&&\\6&&&\end{array},\begin{array}{cccc}\bar{7}&\bar{8}&\bar{9}&\overline{10}\\\overline{11}&&&\\\overline{12}&&&\end{array}\right),\ C_{283}=\left(\begin{array}{cccc}1&2&3&4\\5&&&\\6&&&\end{array},\begin{array}{cccc}\bar{7}&\bar{8}&\bar{9}&\overline{11}\\\overline{10}&&&\\\overline{12}&&&\end{array}\right),\cdots,
$$

$$
C_{382}=\left(\begin{array}{ccc}1&2&3\\4&5&6\end{array},\begin{array}{ccc}\bar{7}&\bar{8}&\bar{9}\\\overline{10}&\overline{11}&\overline{12}\end{array}\right),\ C_{383}=\left(\begin{array}{ccc}1&2&3\\4&5&6\end{array},\begin{array}{ccc}\bar{7}&\bar{8}&\overline{10}\\\bar{9}&\overline{11}&\overline{12}\end{array}\right),\cdots,
$$

$$
C_{407}=\left(\begin{array}{cccc}1&2&3&4\\5&6&&\end{array},\begin{array}{cccc}\bar{7}&\bar{8}&\bar{9}&\overline{10}\\\overline{11}&\overline{12}&&\end{array}\right),\ C_{408}=\left(\begin{array}{cccc}1&2&3&4\\5&6&&\end{array},\begin{array}{cccc}\bar{7}&\bar{8}&\bar{9}&\overline{11}\\\overline{10}&\overline{12}&&\end{array}\right),\cdots,
$$

$$
C_{488}=\left(\begin{array}{ccccc}1&2&3&4&5\\6&&&&\end{array},\begin{array}{ccccc}\bar{7}&\bar{8}&\bar{9}&\overline{10}&\overline{11}\\\overline{12}&&&&\end{array}\right),\ C_{489}=\left(\begin{array}{ccccc}1&2&3&4&5\\6&&&&\end{array},\begin{array}{ccccc}\bar{7}&\bar{8}&\bar{9}&\overline{10}&\overline{12}\\\overline{11}&&&&\end{array}\right),\cdots,
$$

$$
C_{513}=\left(\begin{array}{cccccc}1&2&3&4&5&6\end{array},\begin{array}{cccccc}\bar{7}&\bar{8}&\bar{9}&\overline{10}&\overline{11}&\overline{12}\end{array}\right).
$$

Multiplicity of $q^6\bar{q}^6$

Symmetry:
{1,2,3,4,5,6}{7,8,9,10,11,12}

I_1 : isospin of quarks
I_2 : isospin of antiquarks

Isospin	Spin	Multiplicity
$I_1 = 3, I_2 = 3$	0	1
$I_1 = 3, I_2 = 2$	1	1
$I_1 = 3, I_2 = 1$	2	1
	0	1
$I_1 = 3, I_2 = 0$	3	1
	1	1
$I_1 = 2, I_2 = 2$	2	2
	1	4
	0	3
$I_1 = 2, I_2 = 1$	3	2
	2	5
	1	8
	0	3
$I_1 = 2, I_2 = 0$	4	1
	3	2
	2	5
	1	4
	0	2

Isospin	Spin	Multiplicity
$I_1 = 1, I_2 = 1$	4	2
	3	6
	2	16
	1	16
	0	11
$I_1 = 1, I_2 = 0$	5	1
	4	2
	3	7
	2	9
	1	13
	0	3
$I_1 = 0, I_2 = 0$	6	1
	5	1
	4	4
	3	6
	2	9
	1	7
	0	7

Total isospin I:

$$I = 6 \;\rightarrow\; \{(I_1, I_2) = (3,3)\}$$
$$I = 5 \;\rightarrow\; \{(I_1, I_2) = (3,3), (3,2), (2,3)\}$$
$$I = 4 \;\rightarrow\; \{(I_1, I_2) = (3,3), (3,2), (3,1), (2,3), (2,2), (1,3)\}$$
$$I = 3 \;\rightarrow\; \{(I_1, I_2) = (3,3), (3,2), (3,1), (3,0), (2,3), (2,2), (2,1), (1,3), (1,2), (0,3)\}$$
$$I = 2 \;\rightarrow\; \{(I_1, I_2) = (3,3), (3,2), (3,1), (2,3), (2,2), (2,1), (2,0), (1,3), (1,2), (1,1), (0,2)\}$$
$$I = 1 \;\rightarrow\; \{(I_1, I_2) = (3,3), (3,2), (2,3), (2,2), (2,1), (1,2), (1,1), (1,0), (0,1)\}$$
$$I = 0 \;\rightarrow\; \{(I_1, I_2) = (3,3), (2,2), (1,1), (0,0)\}$$

Possibility of $q^6\bar{q}^6$

$I_1 = 0, I_2 = 0, S = 0$ case

$$-\sum_{i<j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j = \begin{pmatrix} 32 & 0 & -16\sqrt{\frac{14}{5}} & 0 & 0 & 0 & 0 \\ 0 & \frac{16}{3} & -6\sqrt{\frac{6}{5}} & -\frac{50\sqrt{2}}{3} & 0 & 0 & 0 \\ -16\sqrt{\frac{14}{5}} & -6\sqrt{\frac{6}{5}} & 18 & -6\sqrt{15} & 0 & 0 & -6\sqrt{10} \\ 0 & -\frac{50\sqrt{2}}{3} & -6\sqrt{15} & \frac{86}{5} & -24\sqrt{\frac{3}{5}} & -24\sqrt{\frac{3}{5}} & -\frac{18\sqrt{6}}{5} \\ 0 & 0 & 0 & -24\sqrt{\frac{3}{5}} & 24 & 0 & -24\sqrt{\frac{2}{5}} \\ 0 & 0 & 0 & -24\sqrt{\frac{3}{5}} & 0 & 24 & -24\sqrt{\frac{2}{5}} \\ 0 & 0 & -6\sqrt{10} & -\frac{18\sqrt{6}}{5} & -24\sqrt{\frac{2}{5}} & -24\sqrt{\frac{2}{5}} & \frac{112}{15} \end{pmatrix}$$

{67.0514, 46.1272, -41.6309, 28.4456, 24., 5.22455, -1.21771}

Lowest threshold :

1. 2 baryons + 2 antibaryons → -32
2. 1 baryon + 1 baryon + 3 mesons → -64
3. 6 mesons → -96

For a nucleon: -8
For a pion: -16

Multiplicity of $q^4 s^2 \bar{q}^4 \bar{s}^2$

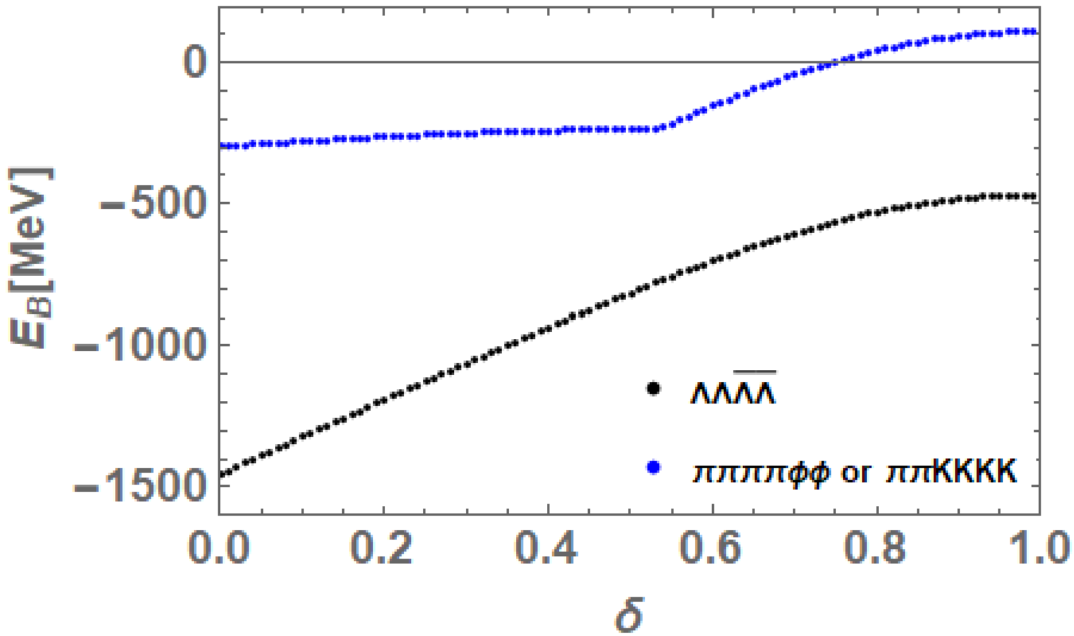
Symmetry:
{1,2,3,4}{5,6}{7,8,9,10},{11,12}

I_1 : isospin of quarks
I_2 : isospin of antiquarks

Isospin	Spin	Multiplicity
$I_1 = 2, I_2 = 2$	4	2
	3	10
	2	30
	1	38
	0	23
$I_1 = 2, I_2 = 1$	5	2
	4	13
	3	50
	2	104
	1	130
	0	58
$I_1 = 2, I_2 = 0$	5	1
	4	11
	3	36
	2	79
	1	84
	0	45

Isospin	Spin	Multiplicity
$I_1 = 1, I_2 = 1$	6	2
	5	16
	4	77
	3	222
	2	415
	1	461
	0	212
$I_1 = 1, I_2 = 0$	6	1
	5	13
	4	59
	3	171
	2	301
	1	335
	0	144
$I_1 = 0, I_2 = 0$	6	2
	5	12
	4	55
	3	133
	2	241
	1	241
	0	121

Binding energy: $q^4 s^2 \bar{q}^4 \bar{s}^2 (I_1 = 0, I_2 = 0, S = 0)$



$$\delta = 1 - \frac{m_u}{m_s}$$

$$E_B = V_{CS}^{q^4 s^2 \bar{q}^4 \bar{s}^2} - V_{CS}^{\text{lowest threshold}}$$

$$V_{CS} = -A \sum_{i<j} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$$

$$\equiv -\frac{B}{m_u^2} \sum_{i<j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$$

where $B/m_u^2 = 18.125$ MeV

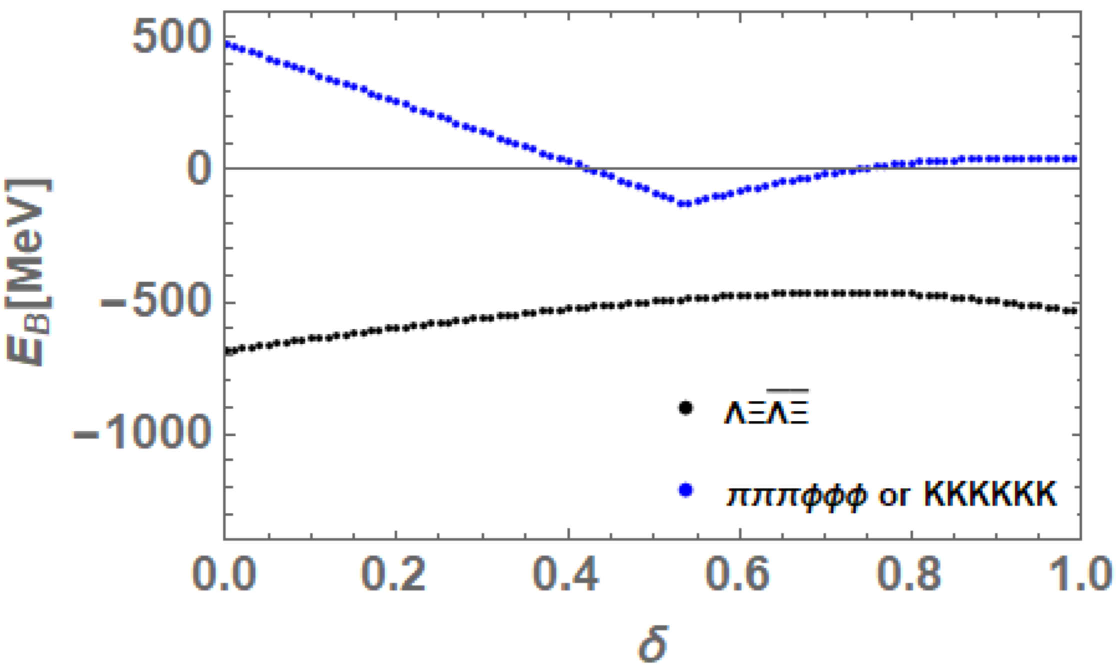
Multiplicity of $q^3 s^3 \bar{q}^3 \bar{s}^3$

Symmetry:
{1,2,3}{4,5,6}{7,8,9},{10,11,12}

Isospin	Spin	Multiplicity
$I_1 = \frac{3}{2}, I_2 = \frac{3}{2}$	6	1
	5	3
	4	12
	3	32
	2	52
	1	60
	0	30
$I_1 = \frac{3}{2}, I_2 = \frac{1}{2}$	5	4
	4	18
	3	52
	2	103
	1	117
	0	52

Isospin	Spin	Multiplicity
$I_1 = \frac{1}{2}, I_2 = \frac{1}{2}$	6	1
	5	7
	4	38
	3	120
	2	229
	1	263
	0	122

Binding energy: $q^3 s^3 \bar{q}^3 \bar{s}^3 (I_1 = 1/2, I_2 = 1/2, S = 0)$



$$\delta = 1 - \frac{m_u}{m_s}$$

$$E_B = V_{CS}^{q^4 s^2 \bar{q}^4 \bar{s}^2} - V_{CS}^{\text{lowest threshold}}$$

$$V_{CS} = -A \sum_{i<j} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$$

$$\equiv -\frac{B}{m_u^2} \sum_{i<j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$$

where $B/m_u^2 = 18.125$ MeV

Summary

- We have investigated the baryon-antibaryon and dibaryon-antidibaryon configurations in a constituent quark model
- There is a possibility of deeply bound state for baryon-antibaryon configuration.
- $q^4 s^2 \bar{q}^4 \bar{s}^2 (H\bar{H})$ has a possibility of multiquark configuration.

Thank you

$q^6\bar{q}^3$: Dibaryon-Antibaryon

Color basis : $q^6\bar{q}^3$

$$(\mathbf{3}\times\mathbf{3}\times\mathbf{3}\times\mathbf{3}\times\mathbf{3}\times\mathbf{3})\times(\overline{\mathbf{3}}\times\overline{\mathbf{3}}\times\overline{\mathbf{3}})$$

$$=(\mathbf{1}_{(m=5)}+\mathbf{8}_{(m=16)}+\mathbf{10}_{(m=10)}+\overline{\mathbf{10}}_{(m=5)}+\mathbf{27}_{(m=9)}+\mathbf{35}_{(m=5)}+\mathbf{28})\times(\mathbf{1}+\mathbf{8}_{(m=2)}+\overline{\mathbf{10}})$$

$$C_1=\left(\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline 5&6\\\hline\end{array},\begin{array}{|c|}\hline \bar{7}\\\hline \bar{8}\\\hline \bar{9}\\\hline\end{array}\right),\cdots$$

$$C_6=\left(\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array},\begin{array}{|c|c|}\hline \bar{7}&\bar{8}\\\hline \bar{9}\\\cline{1-1}\end{array}\right),\ C_7=\left(\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array},\begin{array}{|c|c|}\hline \bar{7}&\bar{9}\\\hline \bar{8}\\\cline{1-1}\end{array}\right),\cdots$$

$$C_{38}=\left(\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline 5\\\cline{1-1} 6\\\cline{1-1}\end{array},\begin{array}{|c|c|c|}\hline \bar{7}&\bar{8}&\bar{9}\\\hline\end{array}\right),\cdots$$

There are $(5\times1+16\times2+10\times1)=47$ color singlet basis.