$f_2(1270) \rightarrow \pi\pi$ as a Probe of Spin and Vorticity in Heavy-Ion Collisions(2601.22898 [nucl-th])

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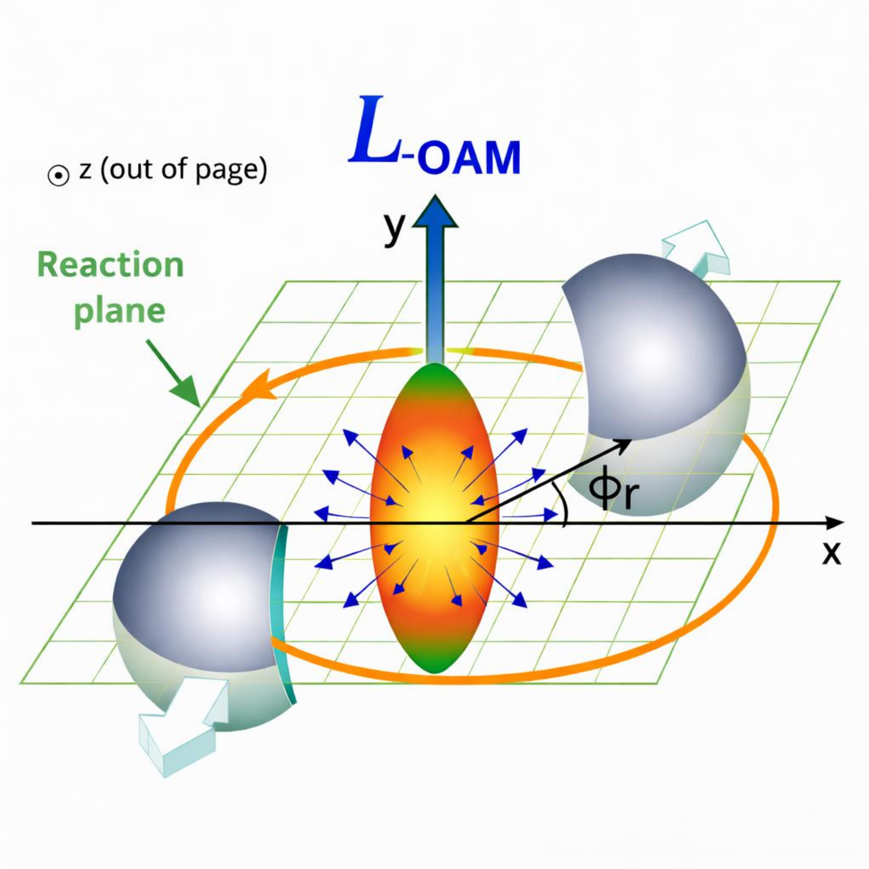
Contents

Introduction

- Global orbital angular momentum(OAM) $10^3 \sim 10^5 \hbar$ is generated in peripheral HIC along the y-axis(perpendicular to the reaction plane)
- OAM $\xrightarrow{\text{LS-coupling}}$ Quark spin polarization $\rightarrow$ Global spin polarization/alignment of hadron
- Spin polarization: $\langle \vec{S} \rangle$
- Spin alignment: ρ_{00}



Introduction

- Λ **Hyperon($J=1/2$) global polarization through $\Lambda \to p+\pi$(Weak decay)**
  - **Au+Au in STAR Collaboration(**10.1038/nature23004**)**

$$\frac{dN}{d\cos\theta}=\frac{1}{2}(1+\alpha_\Lambda P_\Lambda \cos\theta)$$

- **Vector meson ($J=1$) global spin alignment through**
  - $K^* \to K+\pi,\ \phi \to K^+ + K^-$(10.1103/PhysRevLett.125.012301(ALICE), 10.1038/s41586-022-05557-5(STAR))
  - $D^* \to D+\pi$**(Strong decay)(**10.1007/JHEP10(2025)094(ALICE)**)**

$$\frac{dN}{d\cos\theta}=\frac{3}{8\pi}(1-\rho_{00}+(3\rho_{00}-1)\cos^2\theta)$$

  - $J/\psi \to e^+ + e^-$**(EM decay)**

$$\frac{dN}{d\cos\theta}=\frac{3}{16\pi}(1+\rho_{00}+(1-3\rho_{00})\cos^2\theta)$$

Introduction

- Higher spin particle: $f_2(1270) \rightarrow$ Higher density matrix dimension
- $f_2(1270) \rightarrow \pi + \pi$ will provide more information than spin alignment of vector meson.
- General angular distribution will be given as a function of θ, ϕ, ρ_{ij}
- ρ_{ii} and ρ_{20} of f_2 will be given as a function of azimuthal angle in the transverse plane

Effective Lagrangian and partial decay width

$$\mathscr{L}_{f_2\pi\pi} = -\frac{2g_{f_2\pi\pi}}{m_{f_2}}\partial_\mu\pi\cdot\partial_\nu\pi f_2^{\mu\nu}$$

10.1103/PhysRevC.72.019903, 10.1103/PhysRevD.47.1043

$$\mathscr{M} = \frac{g_{f_2\pi\pi}}{m_{f_2}}(p_1-p_2)_\mu(p_1-p_2)_\nu\varepsilon^{\mu\nu}(\boldsymbol{p},\lambda),$$

$$\Gamma = \frac{1}{8\pi}\frac{|\boldsymbol{p}_1|}{m_{f_2}^2}\frac{1}{5}\frac{g_{f_2\pi\pi}^2}{m_{f_2}^2}\frac{32}{3}|\boldsymbol{p}_1|^4 = 185.8\times(0.843)\text{ MeV},$$

$$|\boldsymbol{p}_1| = \frac{1}{2}\sqrt{m_{f_2}^2-4m_\pi^2} = 623\text{ MeV}.$$

$$g_{f_2\pi\pi} = 5.89$$

General angular distribution 1(Effective Lagrangian)

① **Initial state:** $\sum_{\lambda=-2}^{2} a_\lambda \varepsilon^{\mu\alpha}(\vec{p},\lambda)$

$$\mathcal{M} = \frac{4g_{f_2\pi\pi}}{m_{f_2}} p_{1\mu} p_{1\alpha} \sum_{\lambda=-2}^{2} a_\lambda \varepsilon^{\mu\alpha}(\boldsymbol{p},\lambda), \quad \rho_{\lambda\lambda'} = a_\lambda a_{\lambda'}^{*}.$$

$$\varepsilon^{\mu\nu}(\mathbf{0},\pm2) = \varepsilon^{\mu}(\mathbf{0},\pm1)\varepsilon^{\nu}(\mathbf{0},\pm1) - \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm i & 0 \\ 0 & \pm i & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\varepsilon^{\mu\nu}(\mathbf{0},\pm1) = \frac{1}{\sqrt{2}}\left(\varepsilon^{\mu}(\mathbf{0},\pm1)\varepsilon^{\nu}(\mathbf{0},0) + \varepsilon^{\mu}(\mathbf{0},0)\varepsilon^{\nu}(\mathbf{0},\pm1)\right) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mp1 \\ 0 & 0 & 0 & -i \\ 0 & \mp1 & -i & 0 \end{pmatrix},$$

$$\varepsilon^{\mu\nu}(\mathbf{0},0) = \frac{1}{\sqrt{6}}\left(\varepsilon^{\mu}(\mathbf{0},+1)\varepsilon^{\nu}(\mathbf{0},-1) + \varepsilon^{\mu}(\mathbf{0},-1)\varepsilon^{\nu}(\mathbf{0},+1)\right) + \sqrt{\frac{2}{3}}\varepsilon^{\mu}(\mathbf{0},0)\varepsilon^{\nu}(\mathbf{0},0) = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

$$|\mathcal{M}|^2 = \frac{128\pi g_{f_2\pi\pi}^2}{15 m_{f_2}^2} |\boldsymbol{p}_1|^4 W(\theta,\phi,\rho_{ij}).$$

General angular distribution 2(Helicity formalism)

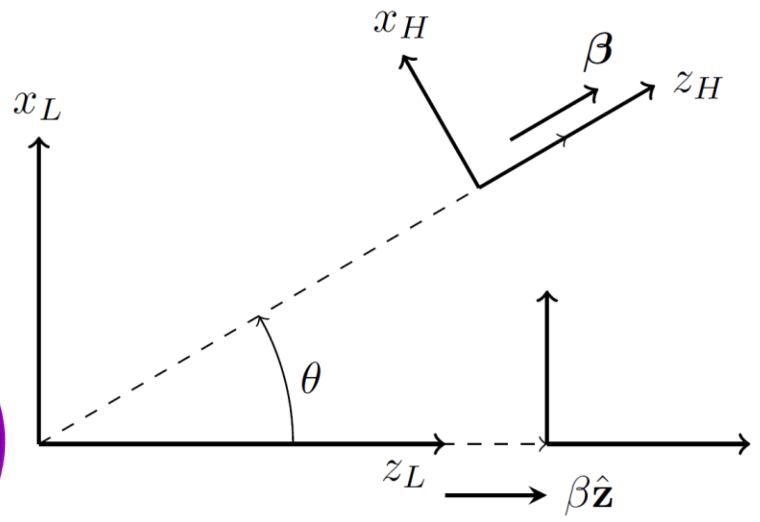
② **Helicity formalism(Wigner D-matrix)**

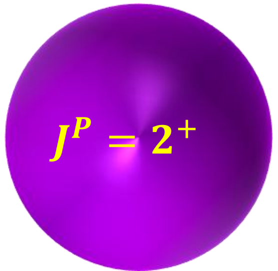
Initial quantization axis(Axis diagonalizing the spin density matrix)

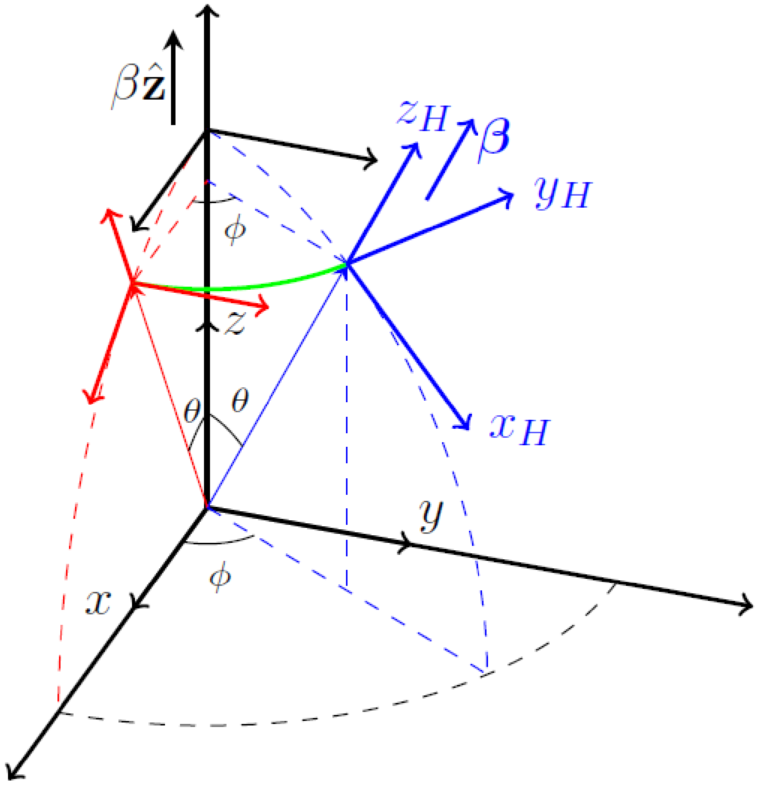
$\longrightarrow$ Rotation$\longrightarrow$ Aligns with momentum(θ, ϕ) of produced particle

Angular distribution of $J \rightarrow 0 + 0$

$$W(\theta,\phi,\rho_{ij}) = \sum_{MM'} Y_J^M(\theta,\phi) Y_J^{M'*}(\theta,\phi)\rho_{MM'}.$$







Result: Angular distribution of $f_2(1270) \to \pi + \pi$

$$
\begin{aligned}
W(\theta,\phi,\rho_{ij}) = \frac{5}{64\pi}\Big(&\rho_{00}(3\cos 2\theta+1)^2 + 6(\rho_{11}+\rho_{-1-1})\sin^2 2\theta + 6(\rho_{22}+\rho_{-2-2})\sin^4\theta \\
&-4\sqrt{6}\sin 2\theta(3\cos^2\theta-1)\left(\mathrm{Re}[\rho_{10}-\rho_{0-1}]\cos\phi - \mathrm{Im}[\rho_{10}-\rho_{0-1}]\sin\phi\right) \\
&-12\sin 2\theta\sin^2\theta\left(\mathrm{Re}[\rho_{21}-\rho_{-1-2}]\cos\phi - \mathrm{Im}[\rho_{21}-\rho_{-1-2}]\sin\phi\right) \\
&+\sqrt{6}(3\sin^2 2\theta - 4\sin^2\theta)\left(\mathrm{Re}[\rho_{20}+\rho_{0-2}]\cos 2\phi - \mathrm{Im}[\rho_{20}+\rho_{0-2}]\sin 2\phi\right) \\
&-12\sin^2 2\theta\left(\mathrm{Re}[\rho_{1-1}]\cos 2\phi - \mathrm{Im}[\rho_{1-1}]\sin 2\phi\right) \\
&+12\sin 2\theta\sin^2\theta\left(\mathrm{Re}[\rho_{2-1}-\rho_{1-2}]\cos 3\phi - \mathrm{Im}[\rho_{2-1}-\rho_{1-2}]\sin 3\phi\right) \\
&+12\sin^4\theta\left(\mathrm{Re}[\rho_{2-2}]\cos 4\phi - \mathrm{Im}[\rho_{2-2}]\sin 4\phi\right)\Big).
\end{aligned}
$$

$$
\begin{aligned}
W(\theta,\rho_{ii}) &= \frac{5}{16}\Big(2\rho_{00} + 3(\rho_{22}+\rho_{-2-2}) + 2\left(-6\rho_{00} + 6(\rho_{11}+\rho_{-1-1}) - 3(\rho_{22}+\rho_{-2-2})\right)\cos^2\theta \\
&\quad +3\left(6\rho_{00} - 4(\rho_{11}+\rho_{-1-1}) + \rho_{22}+\rho_{-2-2}\right)\cos^4\theta\Big). \\
&= \frac{5}{12}\Big(\rho_{11}+\rho_{-1-1} + 2(\rho_{22}+\rho_{-2-2}) + 3(\rho_{11}+\rho_{-1-1}-\rho_{22}-\rho_{-2-2})P_1(\cos\theta)^2 \\
&\quad +\left(\rho_{22}+\rho_{-2-2} - 4(\rho_{11}+\rho_{-1-1}) + 6\rho_{00}\right)P_2(\cos\theta)^2\Big).
\end{aligned}
$$

Spin density matrix of $f_2(1270)$ in thermal production

- Spin density matrix in thermal production
- Spin and vorticity in perfect equilibrium
- $\vec{S}$: Spin matrix, $\vec{\Omega}$: Thermal vorticity, T: Temperature

$$S_x = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & \sqrt{\frac{3}{2}} & 0 & \sqrt{\frac{3}{2}} & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad S_y = \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & -i\sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & i\sqrt{\frac{3}{2}} & 0 & -i\sqrt{\frac{3}{2}} & 0 \\ 0 & 0 & i\sqrt{\frac{3}{2}} & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}.$$

$$\rho(\vec{\Omega}, T) = \frac{1}{Z} e^{\frac{\vec{S}\cdot\vec{\Omega}}{T}}$$

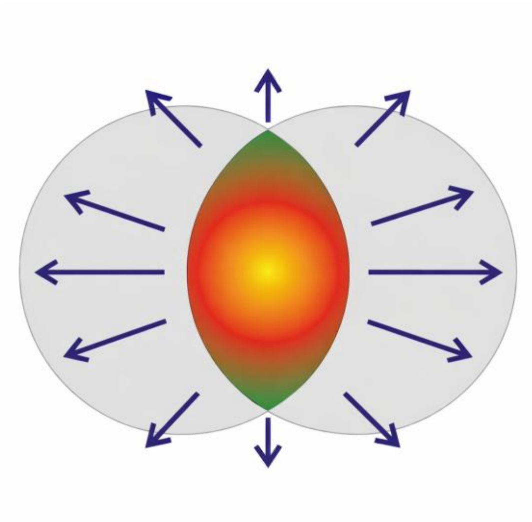
$$Z = 1 + 2\cosh(\Omega) + 2\cosh(2\Omega)$$

Elliptic flow

- Lorentz invariant particle production distribution

$$E_{\vec{p}}\frac{dN}{d^3p} = \frac{dN}{2\pi p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(y, p_T)\cos n(\phi - \Psi_{RP}^{n})\right)$$

  ϕ: Azimuthal angle of particle momentum in the transverse plane
  v_2: Elliptic flow
- Initial spatial deformation → Anisotropy in momentum distribution
- Global OAM gives global vorticity and elliptic flow gives local vorticity



Diagonal element of f_2 spin density matrix

The coefficient: $\rho_{\pm2\pm2}(\Omega_1,\Omega_2,\Omega_3,T)$:

$$\rho_{\pm2\pm2}(\Omega_1,\Omega_2,\Omega_3,T)=\frac{1}{Z\Omega^4}\left(\Omega_1^4\cosh^4\left(\frac{\Omega}{2}\right)+\Omega_2^4\cosh^4\left(\frac{\Omega}{2}\right)+\Omega_1^2\cosh^2\left(\frac{\Omega}{2}\right)\left(\Omega_2^2-2\Omega_3^2\pm2\Omega_3\Omega\sinh(\Omega)+\right.\right. \tag{B4}$$

$$\left.\left.+\left(\Omega_2^2+4\Omega_3^2\right)\cosh(\Omega)\right)+2\Omega_2^2\Omega_3\cosh^2\left(\frac{\Omega}{2}\right)\left(\Omega_3\left(2\cosh(\Omega)-1\right)\pm\Omega\sinh(\Omega)\right)+\Omega_3^3\left(\Omega_3\cosh(2\Omega)\pm\Omega\sinh(2\Omega)\right)\right).$$

The coefficient: $\rho_{\pm1\pm1}(\Omega_1,\Omega_2,\Omega_3,T)$:

$$\rho_{\pm1\pm1}(\Omega_1,\Omega_2,\Omega_3,T)=\frac{1}{2Z\Omega^4}\left(\left(\Omega_3^2+\left(\Omega_1^2+\Omega_2^2\right)\left(2\cosh(\Omega)-1\right)\right)\left(\Omega_1^2+\Omega_2^2\pm2\Omega_3\Omega\sinh(\Omega)+\right.\right. \tag{B5}$$

$$\left.\left.+\left(\Omega_1^2+\Omega_2^2+2\Omega_3^2\right)\cosh(\Omega)\right)\right).$$

The coefficient: $\rho_{00}(\Omega_1,\Omega_2,\Omega_3,T)$:

$$\rho_{00}(\Omega_1,\Omega_2,\Omega_3,T)=\frac{1}{4Z\Omega^4}\left(3\left(\Omega_1^2+\Omega_2^2\right)^2\cosh(2\Omega)+12\Omega_3^2\left(\Omega_1^2+\Omega_2^2\right)\cosh(\Omega)+\left(\Omega_1^2+\Omega_2^2-2\Omega_3^2\right)^2\right). \tag{B6}$$

Thermal vorticity and elliptic expansion

- Elliptical parameterization of interaction region in fireball(Elliptic flow effects)

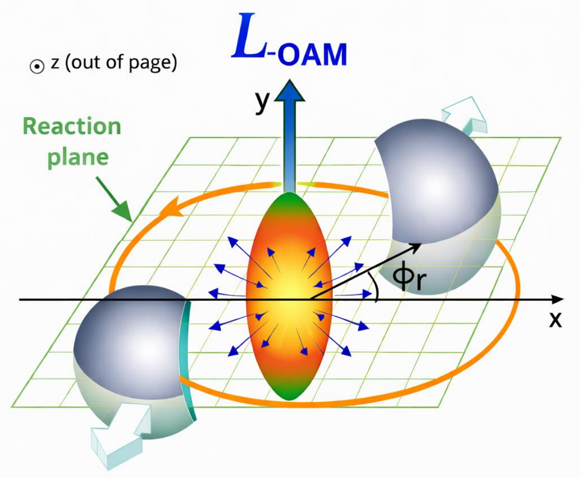
$$x = r_{max}\sqrt{1-\epsilon}\cos\phi_r\,, y = r_{max}\sqrt{1+\epsilon}\sin\phi_r\,, z = \sqrt{\tau_f^2 + x^2 + y^2}\sinh\eta$$

- ϕ_r: Azimuthal angle in the transverse plane, η: Spacetime rapidity
- Flow velocity and thermal vorticity (T: Freeze-out temperature = 165 MeV)

$$u^\mu = \left(\frac{t}{N}, \frac{\sqrt{1+\delta}}{N}x, \frac{\sqrt{1-\delta}}{N}y, \frac{z}{N}\right), N = \sqrt{\tau_f^2 - \delta(x^2 - y^2)},$$

$$\Omega_{\mu\nu} = -\frac{1}{2T}\left(\partial_\mu u_\nu - \partial_\nu u_\mu\right) - \frac{1}{2T^2}\left(u_\mu \partial_\nu T - u_\nu \partial_\mu T\right)$$

  - ϵ: Size deformation(Elongation of fireball)
  - δ: Transverse flow anisotropy
  - τ_f: Lifetime of a system
  - $r_{\max}$: Transverse size of fireball



Thermal vorticity and parameter

- **Local thermal vorticity 3-vector** $\Omega_i = \frac{1}{2}\varepsilon_{ijk}\Omega_{jk}$

$$\Omega_1 = \frac{yz}{TN^3}\left(1-\delta-\sqrt{1-\delta}\right), \Omega_2 = \frac{zx}{TN^3}\left(\sqrt{1+\delta}-1-\delta\right), \Omega_3 = \frac{xy\sqrt{1-\delta^2}}{TN^3}\left(\sqrt{1+\delta}-\sqrt{1-\delta}\right)$$

- **Thermal parameters for 3 different centrality classes**(10.1103/PhysRevC.100.054907, 10.1103/PhysRevC.105.064901)

c%	ϵ	δ	τ_f (fm)	$r_{\max}$ (fm)
0-15	0.055	0.12	7.666	6.540
15-30	0.097	0.26	6.258	5.417
30-60	0.137	0.37	4.266	3.779

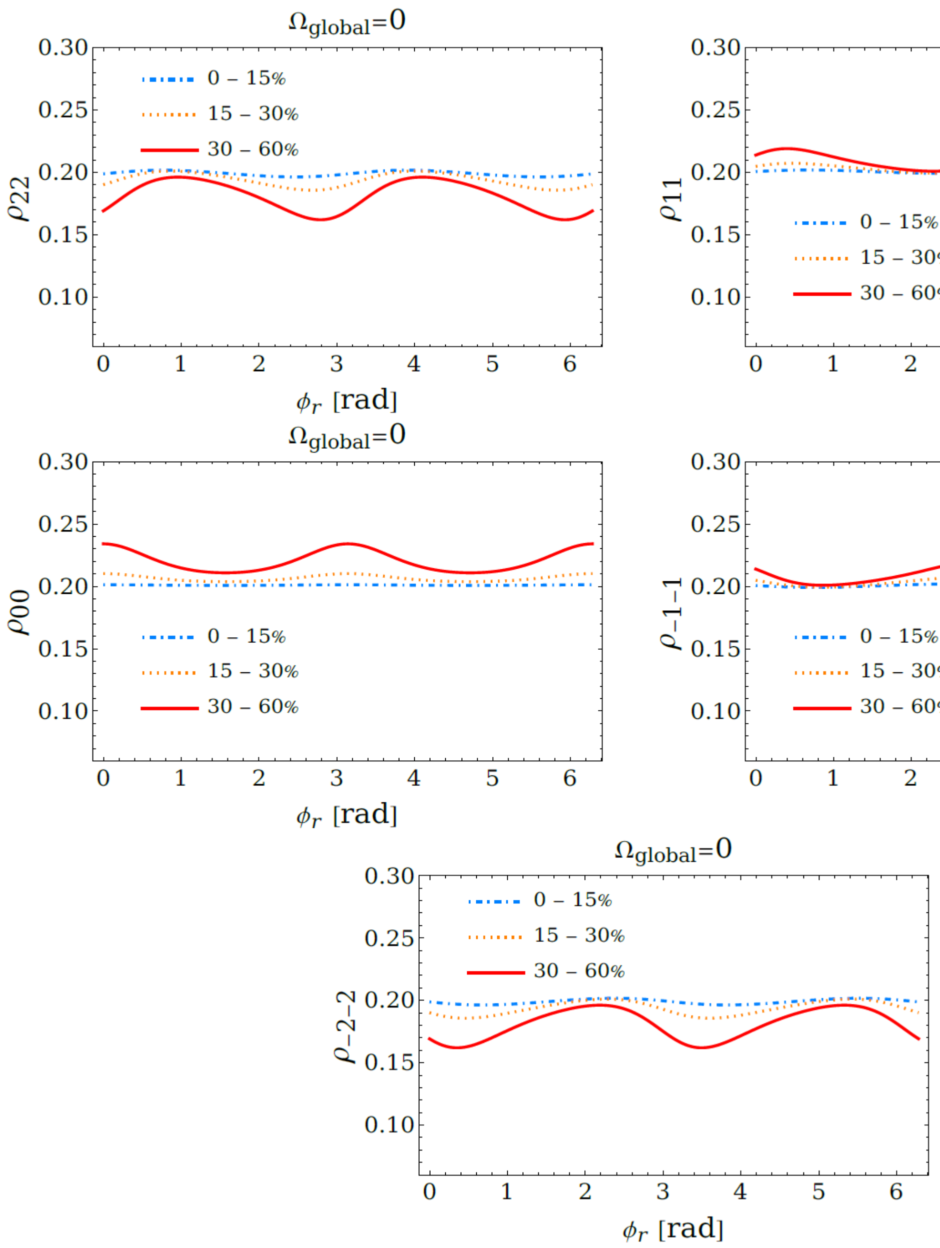
Spin density matrix as a function of ϕ_r

- Average over spacetime rapidity $-4 \leq \eta' \leq 4$

$$\langle \rho_{ij}(\phi_r) \rangle = \frac{1}{\eta - \eta_0} \int_{\eta_0}^{\eta} d\eta' \rho_{ij}(\phi_r, \eta')$$

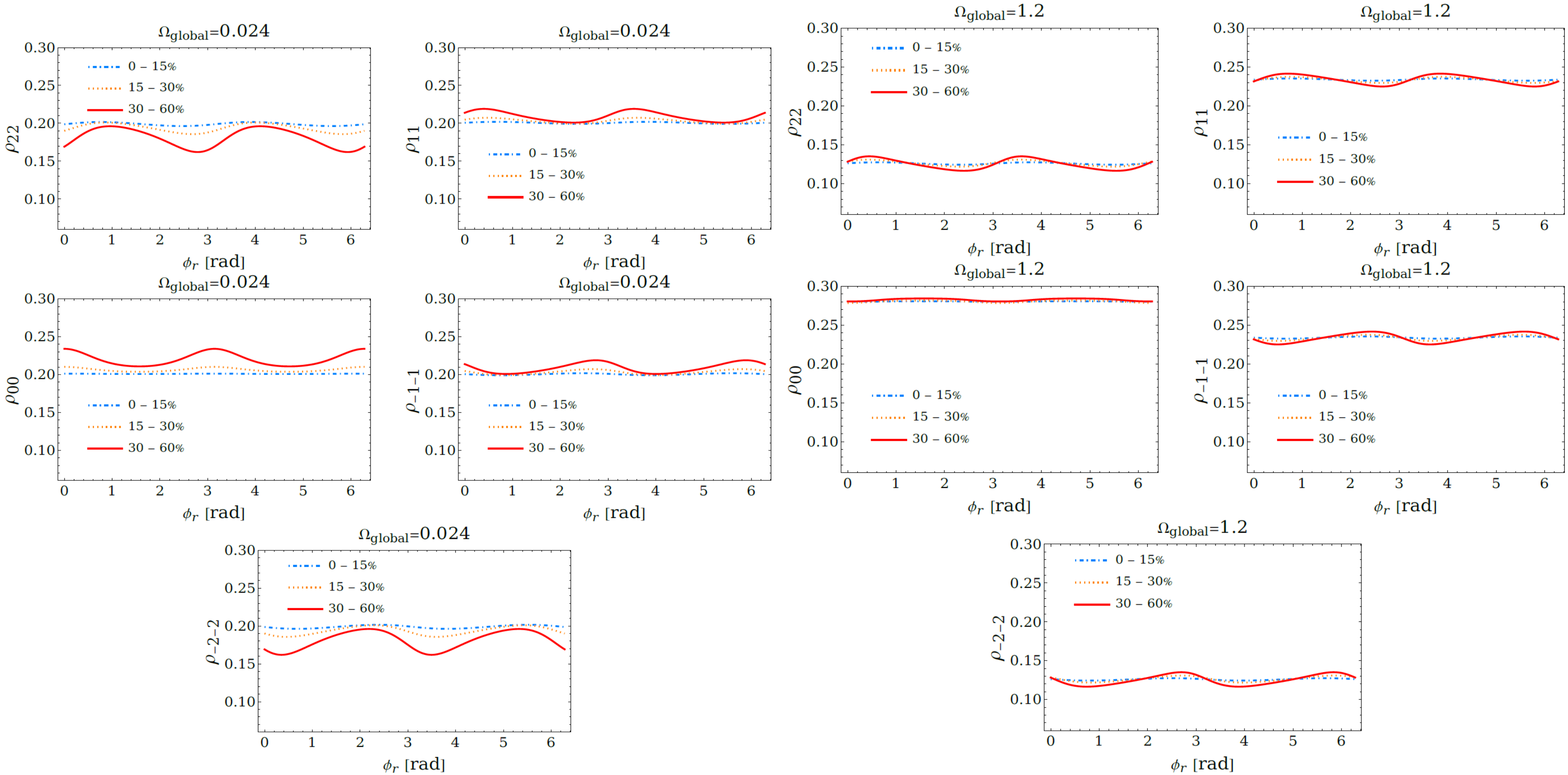
- Perfect equilibrium between spin and vorticity
- $\langle \rho_{|i| \neq |j|}(\phi_r) \rangle = 0$ except $\langle \rho_{20} \rangle = \langle \rho_{0-2} \rangle$
- Calculated $\langle \rho_{ii}(\phi_r) \rangle$ and $\langle \rho_{20}(\phi_r) \rangle$ for 3 cases in 3 different centrality classes(0~15%, 15~30%, 30~60%)
    1. No global vorticity
    2. Addition of global vorticity $\Omega_2(\mathrm{x}, \mathrm{y}, \mathrm{z}) \rightarrow \Omega_2(\mathrm{x}, \mathrm{y}, \mathrm{z}) + \Omega_{\mathrm{global}}(= 0.024)$ 10.1103/PhysRevC.95.054902
    3. Addition of global vorticity $\Omega_2(\mathrm{x}, \mathrm{y}, \mathrm{z}) \rightarrow \Omega_2(\mathrm{x}, \mathrm{y}, \mathrm{z}) + \Omega_{\mathrm{global}}(= 1.2)$

Result 1: Diagonal elements, $\Omega_{\mathrm{global}} = 0$

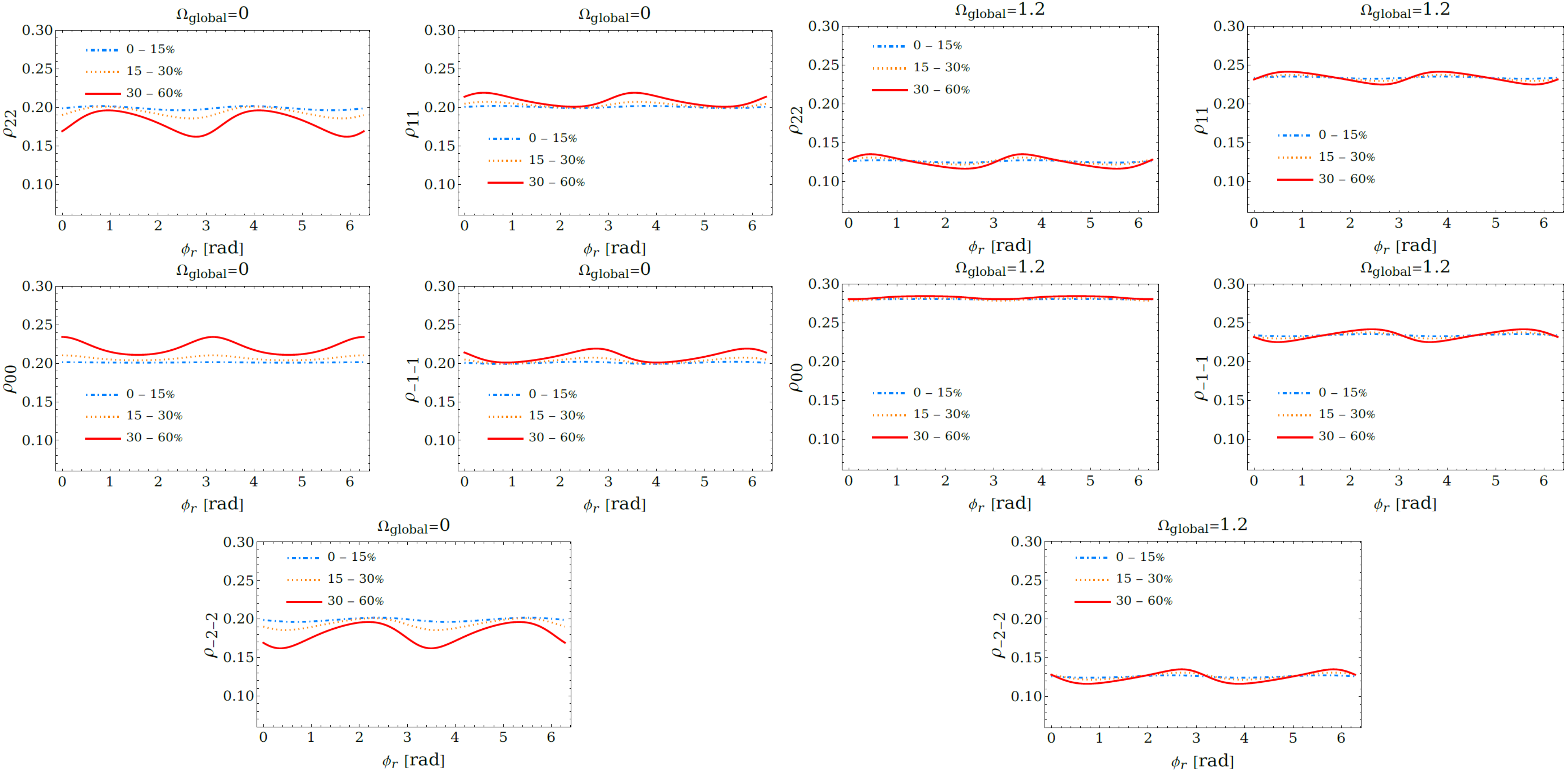


- **Density matrix becomes more oscillatory as centrality increases**

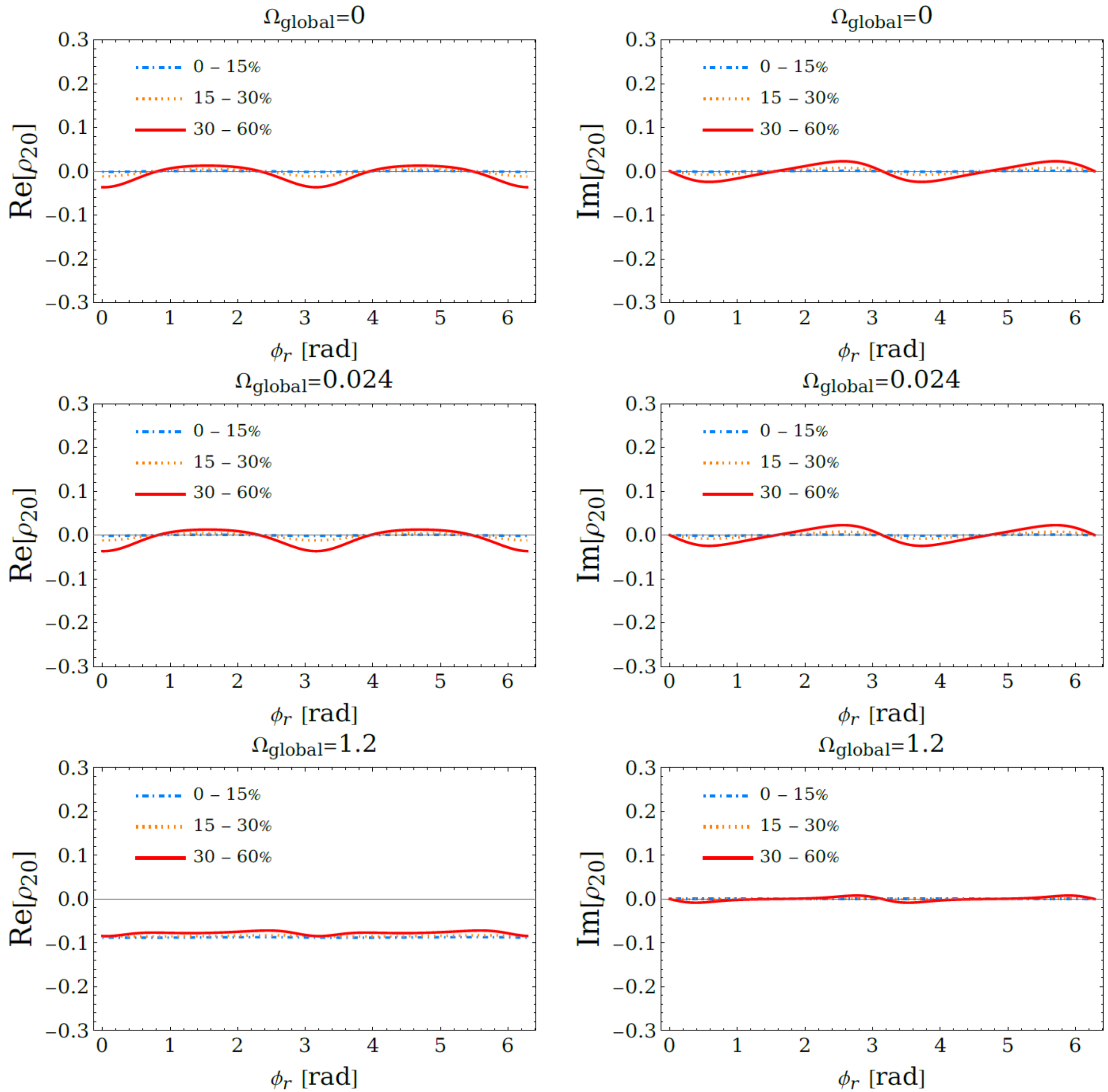
Result 2: Diagonal elements, $\Omega_{global} = 0.024$ & 1.2



Comparison between $\Omega_{global} = 0$ & 1.2



Result 3: ρ_{20}, $\Omega_{\text{global}} = 0$ & 0.024 & 1.2



Summary

- General angular distribution of $f_2 \rightarrow \pi + \pi$ has been calculated

- f_2 spin density matrix element plotted as a function of azimuthal angle ϕ_r

- The oscillatory increases with centrality

- ρ_{00} increases with the addition of global vorticity