Other physics-BSM

Part I continued

Géraldine SERVANT
CERN-Th
The top quark as a link to BSM

Using the top quark to probe BSM physics
The top quark is the heaviest known fundamental particle, $m_t = 173.3 \pm 1.1$ GeV and the only SM fermion to have a natural Yukawa coupling (order 1).

The top dramatically affects the stability of the Higgs mass:

$$m_H^2 = m_{H0}^2 - \frac{3}{8\pi^2} y_t \Lambda^2 + \frac{1}{16\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

It is the main contributor to hierarchy problem

$\rightarrow$ Standard Model is unnatural above 500 GeV

therefore top quark is expected to be a link to BSM
The measurement of its properties (mass, couplings, spin) is used to establish **indirect** evidence for SM and BSM physics: precision EW & QCD, rare decays, anomalous couplings, flavor physics, CP violation.

The top is also a **direct** probe of the EWSB sector and BSM physics exciting the higgs.

Exciting new degrees of freedom e.g. Z', top partners:
What else is special about the top?

The top quark decays before it hadronizes, hence offers the opportunity to study a "bare" quark: spin properties, interaction vertices, top quark mass

\[ \tau_{\text{had}} \approx \Lambda_{QCD}^{-1} \approx 2 \times 10^{-24} \text{s} \]

\[ \tau_{\text{top}} \approx \Gamma_{\text{top}}^{-1} \approx (G_F m_t^3 |V_{tb}|^2 / 8\pi\sqrt{2})^{-1} \approx 5 \times 10^{-25} \text{s} \]

It decays almost exclusively to \( W^+ b \) in the SM as \( |V_{tb}|^2 >> |V_{ts}|^2 , |V_{td}|^2 \)
Two production mechanisms:

- **pair production**

- **single production** (~1/3)
We already knew a lot on top quark from the Tevatron. Tevatron had already set strong constraints on top-philic new physics.

What has been mainly tested at the Tevatron is the $q \bar{q}$ process while new physics contributions to $gg \rightarrow t\bar{t}$ remained unconstrained.

From Tevatron to LHC

85 % of total cross section

90 % of total cross section at 14 TeV (70 % at 7 TeV)
A large effort has been devoted to search for new physics in $t\bar{t}$ resonances in many scenarios for EWSB. New resonances show up, some of which preferably couple to 3rd generation quarks. For instance, there are vector resonances, which can appear in a color singlet or octet states. These resonances can couple to the $Z^\prime$ boson or to the Higgs boson, leading to different decay modes. The figure illustrates the production cross-section of $pp \rightarrow (Z^\prime/g^* \rightarrow t\bar{t})$ as a function of the $t\bar{t}$ invariant mass. The cross-section is shown for various models, including QCD only and models with different couplings to the Higgs boson or other particles. The calculations are performed using LO, CTEQ6L1, and LHC conditions, with $\mu_q = \mu_Z = m_{Z^\prime} = 2$ TeV.
Resonances are excluded in mass regions:
- Narrow Z' mass
- Wide Z' mass
- KK gluon mass

Expected limits:
- CMS TOP-11-010: < 1.1 TeV
- ATLAS CONF-2011-123: < 0.8 TeV
- CMS TOP-11-009: < 1.3 TeV, < 1.7 TeV, < 1.4 TeV
- ATLAS CONF-2012-029: < 0.9 TeV, < 1.0 TeV
- CMS EXO-11-093: < 1.6 TeV, < 2.0 TeV
- CMS EXO-11-006: < 1.6 TeV, < 2.0 TeV, 1.4 < M_{KKg} < 1.5

CMS Preliminary $\sqrt{s} = 7$ TeV

Events / GeV

narrow Z' mass

ATLAS Preliminary $L = 4.4-5.0$ fb$^{-1}$

Data vs. Expected limits:
- Dilepton
- Lepton+jets (low mass)
- Lepton+jets (high mass)
- All-hadronic

narrow Z' mass: $\Gamma(Z') = 0.1 \times M_Z$

Wide Z' mass: $\Gamma(Z') = 1.6$ TeV

KK gluon mass: $\Gamma(Z') = 2.0$ TeV

CMS Preliminary $\sqrt{s} = 7$ TeV

L = 4.4-5.0 fb$^{-1}$

Nothing found so far
If all these particles are too heavy to be accessible at the LHC
-> Effective Field Theory (EFT) approach
EW precision data together with constraints from flavour physics make plausible if not likely that there exists a mass gap between the SM degrees of freedom and any new physics threshold.

In this case, the effects from new physics on processes such as $t\bar{t}$ production can be well captured by higher dimensional interactions among the SM particles.

\[ L = L_{SM} + \frac{g^2}{M^2} \bar{\psi} \psi \bar{\psi} \psi \]

\[ \dim = 6 \]

no bias on what the TeV new physics should be
New interactions are assumed to respect all symmetries of the SM.

\[ L = L_{SM} + \sum_i \frac{c_i}{\Lambda^2} O_i \]

\[ \text{dim} = \leq 4 \]

\[ \geq 6 \text{ operators} \]

**Good news: Only a few operators contribute to top quark physics**
study of new physics in $t\bar{t}$ final state in the most general model-independent approach
There are only 15 relevant operators:

**CP-even**

<table>
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We will only consider those which affect top pair production at tree level by interference with the SM (QCD) amplitudes (we neglect weak corrections)
### Dimension 6 operators for top physics

Zhang & Willenbrock’10, Aguilar-Saavedra ‘10, Degrande & al ‘10

There are only 15 relevant operators:

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We will only consider those which affect top pair production at tree level by interference with the SM (QCD) amplitudes (we neglect weak corrections)
Effective Field Theory for Top Quark Pair production

We calculate top pair production at order $O(1/\Lambda^2)$

$$|M|^2 = |M_{SM}|^2 + 2\Re(M_{SM}M_{NP}^*) + O\left(\frac{1}{\Lambda^4}\right)$$

i.e. we assume new physics manifests itself at low energy only through operators interfering with the SM

We focus on top-philic new physics (and therefore ignore interactions that would only affect the standard gluon vertex $O_G = f_{ABC}G_{\mu\nu}^AG_{\nu\rho}^BG_{\rho\mu}^C$)

We are left with only two classes of dim-6 gauge invariant operators (when working at order $O(1/\Lambda^2)$)
Effective Field Theory for Top Quark Pair production

We are left with only two classes of dim-6 gauge invariant operators (when working at order $O(1/\Lambda^2)$)

- op. with $t$, $\bar{t}$ and one or two gluons (chromomagnetic moment)

$$\mathcal{O}_{hg} = \left[(H\bar{Q})\sigma^{\mu\nu}T^A t\right]G^A_{\mu\nu}$$

- 4-fermion op.

$$\mathcal{O}^{(8)}_{Qu} = (\bar{Q}\gamma^\mu T^A Q)(\bar{u}\gamma_\mu T^A u),$$

$$\mathcal{O}^{(8)}_{Qd} = (\bar{Q}\gamma^\mu T^A Q)(\bar{d}\gamma_\mu T^A d),$$

$$\mathcal{O}^{(8)}_{tq} = (\bar{q}\gamma^\mu T^A q)(\bar{t}\gamma_\mu T^A t),$$

$$\mathcal{O}^{(8)}_{tu} = (\bar{t}\gamma^\mu T^A t)(\bar{u}\gamma_\mu T^A u),$$

$$\mathcal{O}^{(8)}_{td} = (\bar{t}\gamma^\mu T^A t)(\bar{d}\gamma_\mu T^A d),$$

$$\mathcal{O}^{(8,1)}_{Qq} = (\bar{Q}\gamma^\mu T^A Q)(\bar{q}\gamma_\mu T^A q),$$

$$\mathcal{O}^{(8,3)}_{Qq} = (\bar{Q}\gamma^\mu T^A\sigma^I Q)(\bar{q}\gamma_\mu T^A\sigma^I q),$$

$$\mathcal{O}_d^{(8)} = (\bar{Q}T^A t)(\bar{q}T^A d),$$

however only 7 independent operators

: negligible (QCD is chirality diagonal)
top pair production in EFT at order $O(1/\Lambda^2)$

$$|M|^2 = |M_{SM}|^2 + 2\Re(M_{SM}M_{NP}^*) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

New vertices:

Chromomagnetic operator $\mathcal{O}_{hg} = (H\bar{Q})\sigma^{\mu\nu}T^A t G^A_{\mu\nu}$

Four-fermion operators

Top pair production from gluon fusion: corrections from $c_{hg}$ only

Top pair production from $q$ anti-$q$ annihilation: corrections from both $c_{hg}$ and 4-fermion operators

we assume new physics manifests itself at low energy only through operators interfering with the SM
The new physics and SM contributions for gluon fusion have a common factor

\[
\frac{d\sigma}{dt} (gg \rightarrow tt) = \frac{d\sigma_{SM}}{dt} + \sqrt{2} \alpha_s g_s \frac{v m_t c_{hg}}{s^2} \frac{1}{\Lambda^2} \left( \frac{1}{6 \tau_1 \tau_2} - \frac{3}{8} \right)
\]

\[
\frac{d\sigma_{SM}}{dt} (gg \rightarrow tt) = \frac{\pi \alpha_s^2}{s^2} \left( \frac{1}{6 \tau_1 \tau_2} - \frac{3}{8} \right) (\rho + \tau_1^2 + \tau_2^2 - \frac{\rho^2}{4 \tau_1 \tau_2})
\]

\[\tau_1 = \frac{m_t^2 - t}{s}, \quad \tau_2 = \frac{m_t^2 - u}{s}, \quad \rho = \frac{4m_t^2}{s}\]

\[m_t^2 - t = \frac{s}{2} (1 - \beta \cos \theta)\]

The operator \(O_{ng}\) can hardly be distinguished from the SM in gluon fusion.

Distortions in the shape of the distributions can only come from \(q \bar{q}\) annihilation \(\rightarrow\) small effect at LHC.

Gluon fusion (contribution from one operator only)
q̅ q̅ annihilation (contribution from the 8 operators)

Only two linear combinations of 4-fermion operators actually contribute to the differential cross section after averaging over the final state spins:

\[
\frac{d\sigma}{dt}(q\bar{q} \to t\bar{t}) = \frac{d\sigma_{SM}}{dt} \left( 1 + \frac{c_{Vv} \pm \frac{c'_{Vv}}{2}}{g_s^2} \frac{s}{\Lambda^2} \right) + \frac{1}{\Lambda^2} \frac{s}{9s^2} \left( \frac{c_{Aa} \pm \frac{c'_{Aa}}{2}}{2} \right) s(\tau_2 - \tau_1) + 4g_s c_{hg} \sqrt{2} v m_t
\]

**Even** part in the scattering angle \(\theta\) comes from \(\bar{t}\gamma^\mu T^A t\bar{q} \gamma^\mu T^A q\)

**Odd** part in the scattering angle \(\theta\) comes from \(\bar{t}\gamma^\mu \gamma_5 T^A t\bar{q} \gamma^\mu \gamma_5 T^A q\)

This dependence vanishes after integration over \(t\)

**Vector** combination of the light quarks involving the RH and LH top quarks:

\[
\begin{align*}
C_{Vv} &= C_{Rv} + C_{Lv} \\
c'_{Vv} &= (c_{tu} - c_{td})/2 + (c_{Qu} - c_{Qd})/2 + c_{Qq}^{(8,3)}
\end{align*}
\]

\[\left\{\begin{array}{l}
C_{Rv} = c_{tq}/2 + (c_{tu} + c_{td})/4 \\
C_{Lv} = c_{Qq}^{(8,1)}/2 + (c_{Qu} + c_{Qd})/4
\end{array}\right.\]

**Axial** combination of the light quarks involving the RH and LH top quarks:

\[
\begin{align*}
C_{Aa} &= C_{Ra} - C_{La} \\
c'_{Av} &= (c_{tu} - c_{td})/2 - (c_{Qu} - c_{Qd})/2 - c_{Qq}^{(8,3)}
\end{align*}
\]

\[\left\{\begin{array}{l}
C_{Ra} = -c_{q}/2 + (c_{tu} + c_{td})/4 \\
C_{La} = -c_{Qq}^{(8,1)}/2 + (c_{Qu} + c_{Qd})/4
\end{array}\right.\]
total cross section

Tevatron

$$\sigma (pp \to t\bar{t}) / \text{pb} = 6.15^{+2.41}_{-1.61} + [(0.87^{+0.23}_{-0.16}) c_{V\nu} + (1.44^{+0.47}_{-0.33}) c_{h\gamma} + (0.31^{+0.08}_{-0.06}) c'_{V\nu}] \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2.$$  

LHC 7 TeV

$$\sigma (pp \to t\bar{t}) / \text{pb} = 94^{+22}_{-17} + [(4.5^{+0.7}_{-0.6}) c_{V\nu} + (25^{+7}_{-5}) c_{h\gamma} + (0.48^{+0.068}_{-0.056}) c'_{V\nu}] \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2.$$  

LHC 14 TeV

$$\sigma (pp \to t\bar{t}) / \text{pb} = 538^{+162}_{-115} + [(15^{+2}_{-1}) c_{V\nu} + (144^{+34}_{-25}) c_{h\gamma} + (1.32^{+0.12}_{-0.12}) c'_{V\nu}] \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2.$$  

LO with CTEQ6L1 pdfs  
In fits, we'll use NLO+NLL SM results but in interference, we'll keep LO SM amplitude

u+d (isospin 0)  
chromo magnetic moment  
u-d (isospin 1)
The $p\bar{p} \rightarrow t\bar{t}$ total cross section at Tevatron depends on both $c_{hg}$ and $c_{vv}$ and constrains thus a combination of these parameters.
The $p\bar{p} \rightarrow t\bar{t}$ total cross section at Tevatron depends on both $c_{hg}$ and $c_{vv}$ and constrains thus a combination of these parameters.
The LHC - Tevatron complementarity

- The Tevatron cross section depends on both $c_{hg}$ and $c_{VV}$ and constrains thus a combination of these parameters.
- At the LHC, the $pp \rightarrow tt$ total cross section mostly depends on $c_{hg}$ and can be directly used to constrain the allowed range for $c_{hg}$

Region allowed by the Tevatron at 2

LHC total cross section limits (7 TeV: thin line, 14 TeV: thick line)
CMS Preliminary, $\sqrt{s}=7$ TeV

- CMS $e/\mu$+jets+btag
  TOP-11-003 (L=0.8-1.09/pb)
  - $164 \pm 3^{+12}_{-12}^{+7}$ (val ± stat. ± syst. ± lum)

- CMS dilepton (ee,μμ,μeμ)
  TOP-11-005 (L=1.14/fb)
  - $170 \pm 4^{+16}_{-16}^{+8}$ (val ± stat. ± syst. ± lum)

- CMS all-hadronic
  TOP-11-007 (L=1.09/fb)
  - $136 \pm 20^{+40}_{-40}^{+8}$ (val ± stat. ± syst. ± lum)

- CMS dilepton ($\mu\tau$)
  TOP-11-006 (L=1.09/fb)
  - $149 \pm 24^{+26}_{-26}^{+9}$ (val ± stat. ± syst. ± lum)

- CMS 2010 combination
  arXiv:1108.3773 (L=36/pb)
  - $154 \pm 17^{+6}_{-6}$ (val ± stat. ± syst. ± lum.)

- CMS $e/\mu$+jets+btag
  arXiv:1108.3773 (L=36/pb)
  - $150 \pm 9^{+17}_{-17}^{+6}$ (val ± stat. ± syst. ± lum)

- CMS dilepton (ee,μμ,μeμ)
  arXiv:1105.5661 (L=36/pb)
  - $168 \pm 18^{+14}_{-14}^{+7}$ (val ± stat. ± syst. ± lum)

- CMS $e/\mu$+jets
  arXiv:1106.0902 (L=36/pb)
  - $173 \pm 14^{+36}_{-29}^{+7}$ (val ± stat. ± syst. ± lum)

ATLAS Preliminary

15 May 2012

Theory (approx. NNLO)
for $m_t = 172.5$ GeV

- Channel & Lumi.
  - Single lepton: $0.70 \text{ fb}^{-1}$
  - Dilepton: $0.70 \text{ fb}^{-1}$
  - All hadronic: $1.02 \text{ fb}^{-1}$
  - Combination: $177 \pm 3^{+6}_{-7} \pm 7 \text{ pb}$

New measurements

- $\tau_{\text{had}} + \text{jets}$: $1.67 \text{ fb}^{-1}$
  - $200 \pm 19^{+42}_{-27} \pm 7 \text{ pb}$

- $\tau_{\text{had}} + \text{lepton}$: $2.05 \text{ fb}^{-1}$
  - $186 \pm 13^{+20}_{-9} \pm 7 \text{ pb}$

- All hadronic: $4.7 \text{ fb}^{-1}$
  - $168 \pm 12^{+60}_{-57} \pm 6 \text{ pb}$
Constraining Non-resonant New Physics in top pair production

A 10% uncertainty on the total cross section at the LHC already rules out a large region of parameter space

[Degrande et al’10]

yellow region is excluded by Tevatron

green (blue) region excluded by LHC at 7 TeV (14 TeV) after a precision of 10% is reached on $\sigma_{tt}$

A 10% uncertainty on the total cross section at the LHC already rules out a large region of parameter space
Minor effect on shapes of distributions at the LHC

interference only

total

Minor effect on shapes of distributions at the LHC

interference only

total
1) when $O(1/\Lambda^4)$ terms are subdominant

At the Tevatron, our results apply to a region of parameter space bounded by

$$|c_i| \left( \frac{\text{TeV}}{\Lambda} \right)^2 \lesssim 7$$

At the LHC, since the center of mass energy is larger, the reliable region shrinks to

$$|c_{hg}| \left( \frac{\text{TeV}}{\Lambda} \right)^2 \lesssim 3$$

and

$$|c_{Vv}| \left( \frac{\text{TeV}}{\Lambda} \right)^2 \lesssim 2$$

2) For which typical mass scale does the effective field theory treatment apply?

$$\rightarrow \sim 1.5 \text{ TeV}$$

correction to SM cross section at the LHC due to a $W'$ and comparison with EFT computation
Effective Field Theory Approach to the Forward-Backward asymmetry

\[ A_{FB} \equiv \frac{\sigma (\cos \theta_t > 0) - \sigma (\cos \theta_t < 0)}{\sigma (\cos \theta_t > 0) + \sigma (\cos \theta_t < 0)} \]

\[ A_{FB}^{SM} = 0.05 \pm 0.015. \quad A_{FB}^{\text{EXP}} = 0.15 \pm 0.05 \text{(stat)} \pm 0.024 \text{(syst)}, \]

- top quarks are preferentially emitted in the direction of the incoming quark

\[ \frac{d\sigma}{dt} (q\bar{q} \to t\bar{t}) = \frac{d\sigma_{SM}}{dt} \left( 1 + \frac{c_{VV} + c'_{VV}}{g_s^2} \frac{s}{\Lambda^2} \right) + \frac{1}{\Lambda^2} \frac{\alpha_s}{9s^2} \left( \left( c_{AA} + \frac{c'_{AA}}{2} \right) s(\tau_2 - \tau_1) + 4g_{ch}^2 \sqrt{2} m_t \right) \]

\[ \delta A_{FB}^{\text{dim}^6} = \left( 0.0342^{+0.016}_{-0.009} c_{AA} + 0.0128^{+0.0064}_{-0.0036} c'_{AA} \right) \times \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \]  

[C_{AA} and C'_{AA} are only constrained by the asymmetry and not by the total cross section or the invariant mass distribution

Link to axigluon models:

\[ c_{AA}/\Lambda^2 = -2 g_A^q g_A^t / m_A^2 \]

AFB prediction at the Tevatron due to an axigluon and comparison with the EFT computation

![Graph showing A_{FB} vs. M_A]

[Degrande et al’10]
Most general expression at order $O(\Lambda^{-2})$

\[
\delta A(m_{t\bar{t}} < 450\text{ GeV}) = (0.023_{-1}^{+3}c_A + 0.0081_{-4}^{+6}c'_A) \left( \frac{1\text{ TeV}}{\Lambda} \right)^2,
\]

\[
\delta A(m_{t\bar{t}} \geq 450\text{ GeV}) = (0.087_{-9}^{+10}c_A + 0.032_{-3}^{+4}c'_A) \left( \frac{1\text{ TeV}}{\Lambda} \right)^2.
\]

[Degrande et al'10,’11]

\[
\sigma(t\bar{t}) = \sigma_{SM} + \delta\sigma_{int} + \delta\sigma_{quad}
\]

\[
\delta\sigma_{int} + \delta\sigma_{quad} \approx 0
\]

This requires \[A_{new} \sim -2A_{SM}\]

\[
t\bar{t}\text{ tail at LHC}
\]

Including $O(\Lambda^{-4})$ terms can alleviate the tension. See analysis by Aguilar-Saavedra & Perez-Victoria,1103.2765 and Delaunay et al, 1103.2297.
Spin correlations

The three observables $\sigma$, $d\sigma/dm_{t\bar{t}}$ and $A_{FB}$ are unable to disentangle between theories coupled mainly to right- or left-handed top quarks. However, spin correlations allow us to determine which chiralities of the top quark couple to new physics, and in the case of composite models, whether one or two chiralities of the top quark are composite.

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_+ d \cos \theta_-} = \frac{1}{4} (1 + C \cos \theta_+ \cos \theta_- + b_+ \cos \theta_+ + b_- \cos \theta_-)$$

$\theta_+$ ($\theta_-$) is the angle between the charged lepton $l^+$ ($l^-$) resulting from the top (antitop) decay and some reference direction $\vec{a}$ ($\vec{b}$).

$$C = \frac{1}{\sigma} \left( \sigma_{RL} + \sigma_{LR} - \sigma_{RR} - \sigma_{LL} \right),$$

$$b_+ = \frac{1}{\sigma} \left( \sigma_{RL} - \sigma_{LR} + \sigma_{RR} - \sigma_{LL} \right),$$

$$b_- = \frac{1}{\sigma} \left( \sigma_{RL} - \sigma_{LR} - \sigma_{RR} + \sigma_{LL} \right).$$

$$C \times \sigma/pb = 2.82^{+1.06}_{-0.72} + \left[ (0.37^{+0.10}_{-0.08}) c_{hq} + (0.50^{+0.13}_{-0.10}) c_{V\nu} \right] \times \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2,$$

$$b \times \sigma/pb = (0.45^{+0.12}_{-0.09}) c_{AV} \times \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2,$$

proportional to $C_{R\nu} - C_{L\nu}$

allows to distinguish between LH and RH quarks
Non-resonant top philic new physics can be probed using measurements in top pair production at hadron colliders

This model-independent analysis can be performed in terms of 8 operators. Observables depend on different combinations of only 4 parameters:

\[ \sigma(gg \to t\bar{t}), \frac{d\sigma(gg \to t\bar{t})}{dt} \leftrightarrow c_{hg} \]

\[ \sigma(q\bar{q} \to t\bar{t}) \leftrightarrow c_{hg}, c_{Vv} \]

\[ \frac{d\sigma(q\bar{q} \to t\bar{t})}{dm_{tt}} \leftrightarrow c_{hg}, c_{Vv} \]

\[ A_{FB} \leftrightarrow c_{Aa} \]

spin correlations \( \leftrightarrow c_{hg}, c_{Vv}, c_{Av} \)
Chromo-magnetic operator $O_{hg}$

1-loop generation of the chromo-magnetic operator

$$(H\bar{Q}t) (H\bar{Q}t) \rightarrow \delta c_{hg}$$
Constraints from higgs searches on top-philic new physics

$$O_{hg} = (\bar{Q}_L H) \sigma^{\mu\nu} T^a_{\mu\nu} t_R G_{\mu\nu}^a.$$  

$$O_{HG} = \frac{1}{2} H^\dagger H G_{\mu\nu}^a G_a^{\mu\nu}.$$  

$$\delta c_{HG} \approx 0.03 \Re c_{hg} - 0.006 c_y$$  

$$c_y = c_H + \frac{v}{\sqrt{2} m_t} \Re (c_{Hy})$$
Using $t\bar{t}h$ to constrain the chromomagnetic operator

---

Constraints from $h$ production

Constraints from $t\bar{t}h$ production

---

$C_y (1 \text{TeV}/\Lambda)^2 = 0$

---

Degrande et al, 1205.1065

---

Using $t\bar{t}h$ to constrain the chromomagnetic operator

---

Constraints from $h$ production

Constraints from $t\bar{t}h$ production

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Degrande et al, 1205.1065

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Degrande et al, 1205.1065

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Using $t\bar{t}h$ to constrain the chromomagnetic operator

---

Constraints from $h$ production

Constraints from $t\bar{t}h$ production

---

Degrande et al, 1205.1065
Let us now imagine the top partners are too heavy to be accessible at the LHC (i.e. $\gtrsim 1.5-2$ TeV), and heavy gluons also too heavy ($\gtrsim 4$ TeV).

Where shall we search for signs of top compositeness?
Enhanced four-top production in composite top models

In models of composite tops, the operators contributing directly to top pair production are subdominant compared to four-top operators (from Naive Dimensional Analysis)

\[ \frac{1}{\Lambda^2} (\bar{t}_R \gamma^\mu t_R) (\bar{t}_R \gamma^\mu t_R) \]

(The dominant operators are those which contain only fields from the strong sector, scale as \( g_\rho^2 \))

4-fermion op. contributing directly to \( t\bar{t} \) production scale at best as \( g_\rho \) while \( O_{hg} \) scales as \( g_\rho^{-1} \)

In this case, a much better probe of the dominant dynamics is the direct production of four top quarks spectacular events with 12 partons in the final state

typical LHC cross sections at 14 TeV: 10 - 100 fb

[ Pomarol, Serra’08 ]
[Lillie, Shu, Tait ’08]
Four-top production in the Standard Model

\[ \frac{\sigma_{\text{LHC}}}{\sigma_{\text{tevatron}}} \sim 7.5 \text{ fb} @ 14 \text{ TeV} \]
\[ \frac{\sigma_{\text{LHC}}}{\sigma_{\text{tevatron}}} \sim 0.2 \text{ fb} @ 7 \text{ TeV} \]
\[ \sigma_{\text{tevatron}} < 10^{-4} \text{ fb} \]

\[ \Rightarrow \text{4 top final state sensitive to several classes of new TeV scale physics} \]
\[ \text{e.g. SUSY (gluino pair production with } \tilde{g} \rightarrow t\bar{t} \chi_0) \]
\[ \text{top compositeness} \]
In the models of interest, 4-top production yields an excess of right-handed tops:

\[
\frac{1}{\sigma} \frac{d\sigma}{d\cos \theta} = \frac{A}{2} (1 + \cos \theta) + \frac{1 - A}{2} (1 - \cos \theta)
\]

- **A**: fraction of RH tops
- **\(\theta\)** is the angle between the direction of the (highest \(p_T\)) lepton in the top rest frame and the direction of the top polarisation.

[Graph showing the distribution of \(\cos[\theta_{l^+ (t_1)}]\) with SM, 4t, and 4q signals.]

[Reference: Pomarol, Serra’08]
Effective field theory approach to BSM: characterizes new physics in a model-independent way, useful to set bounds on non-resonant new physics.

2011 LHC data already rules out large region of parameter space.

New constraints on the 4-fermion and the chromomagnetic operators and more to come.

Complementarity between Higgs, t\(\bar{t}\) and t\(\bar{t}\)H production.

Models of top compositeness can lead to zero signal at 7-8 TeV while non-zero signals (4 top production + top partners production) at 14 TeV.
Other physics - BSM

Part II

Géraldine SERVANT
CERN-Th
The Hierarchy Problem has been the guideline of theorists for over 30 years.

The main goal of the LHC:

Understand why $M_{EW} \ll M_{\text{Planck}}$

However, since LEP II, naturalness arguments have been under high stress and present null LHC searches are confirming theorists’ anxiety.
Part II

- The hierarchy problem associated with the Higgs [R. Rattazzi]
- The SUSY solution [D. Kazakov]
- The extra dimensional solutions
- The 4D strongly interacting solutions
- The Flavour problem [G. Isidori]
- The strong CP problem
- The “why so” puzzles
- The dark matter problem
- The matter antimatter asymmetry problem
- observational facts unexplained by the SM
- fine-tuning problems

charge quantization, gauge coupling unification $\rightarrow$ GUTs
proton stability
fermion mass hierarchy
why 3 generations
Good reason for unification:
Anomaly cancellation in the SM

\[ Q_e = T_3 + Y \]

1. SU(N)–G²: \( T_G = 1 \), so need \( \sum_R \text{Tr} T^A_R = 0 \), trivial for \( N > 1 \)
   - U(1)_Y: \( \sum_{\text{fermions}} Y = (+1/6) \cdot 2 \cdot 3 + (-2/3) \cdot 3 + (+1/3) \cdot 3 \\
     + (-1/2) \cdot 2 + 1 = 0! \) Quarks and leptons cancel separately.

2. SU(3)³ automatic: QCD is vectorlike (# of 3 = # of \( \bar{3} \))

3. SU(2)³ automatic: \( \frac{1}{8} \sum_{\text{doublets}} \text{Tr} \sigma^A \{ \sigma^B, \sigma^C \} = \frac{1}{4} \delta^{BC} \text{Tr} \sigma^A = 0 \)

4. U(1)_Y³: \( \sum_{\text{fermions}} Y^3 = \)
   \( (+1/6)^3 \cdot 2 \cdot 3 + (-2/3)^3 \cdot 3 + (+1/3)^3 \cdot 3 + (-1/2)^3 \cdot 2 + 1^3 = 0 \)
   - Cancellation between quarks and leptons in each generation!

5. SU(3)²–U(1)_Y: \( \propto \sum_{\text{quarks}} Y = 0 \) (just like gravitational anomaly)

6. SU(2)²–U(1)_Y:
   \( \propto \sum_{\text{doublets}} Y \text{Tr} \{ \sigma^B, \sigma^C \} \propto \sum_{\text{doublets}} Y = (+1/6) \cdot 3 + (-1/2) = 0 \)
   - Cancellation between quarks and leptons again!

Highly non-trivial cancellation and suggestive connection of quarks and leptons

The SM as a remnant of a GUT theory?

There are gauge groups for which the anomalies automatically cancel, e.g. SO(10)
Good reason for unification II:
Charge quantization \[ Q_e = T_3 + Y \]

How come is the electric charge quantized?

- Eigen values of the generators of the abelian U(1) are continuous
e.g. in the symmetry of translational invariance of time, there is no restriction in the (energy) eigen values.

- Eigen values of the generators of a simple non-abelian group are discrete
e.g. in SO(3) rotations, the eigen values of the third component of angular momentum can take only integers or 1/2 integers values. In SU(5), since the electric charge is one of the generators, its eigen values are discrete and hence quantized.

simple unification group -> charge quantization

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5) \]  

SM matter content fits nicely into SU(5)

relation between color SU(3) and electric charge.

Quarks carry 1/3 of the lepton charge because they have 3 colors. The SU(5) theory provides a rationale basis for understanding particle charges and the weak hypercharge assignment in the SM
Good reason for unification III

- we observe different couplings but it is a low energy artefact

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5) \]

\[
Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3, 2)_{1/6}, \quad u_R^c = (\bar{3}, 1)_{-2/3}, \quad d_R^c = (\bar{3}, 1)_{1/3},
\]

\[
L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1, 2)_{-1/2}, \quad e_R^c = (1, 1)_{1}
\]

\[
\bar{5} = (1, 2)_{-1/2} \sqrt{3/5} + (\bar{3}, 1)_{1/3} \sqrt{3/5}
\]

\[ 5 = L + d_R^c \]

\[
10 = (5 \times 5)_A = (\bar{3}, 1)_{-2/3} \sqrt{3/5} + (3, 2)_{1/2} \sqrt{3/5} + (1, 1)_{1} \sqrt{3/5}
\]

\[ 10 = u_R^c + Q_L + e_R^c \]

\[
\psi_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^3 & -u^2 & -u_1 & -d_1 \\ -u^3 & 0 & u^1 & -u_2 & -d_2 \\ u^2 & -u^1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L
\]

\[
SU(5) \text{ adjoint rep.}
\]

\[
\frac{1}{\sqrt{5}} \begin{pmatrix} 1/2 \\ 1/2 \\ -1/3 \\ -1/3 \\ -1/3 \end{pmatrix} = \frac{1}{\sqrt{5}} Y
\]

additional U(1) factor that commutes with SU(3)\text{*}SU(2)

\[
\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}
\]

\[
g_5 T^{12} = g' Y \quad g_5 \sqrt{\frac{3}{5}} = g' \quad g_5 = g = g_s
\]

\[
\sin^2 \theta_W = \frac{3}{8} \quad @ \ M_{\text{GUT}}
\]
The evolution of gauge couplings is controlled by the renormalization group equations

\[
\frac{d\alpha(\mu)}{d \log \mu} = \beta(\alpha(\mu))
\]

At one loop:

\[
\beta(\alpha) \equiv \frac{d\alpha(\mu)}{d \log \mu} = \frac{-b}{2\pi} \alpha^2 + \mathcal{O}(\alpha^3)
\]

So couplings vary logarithmically as a function of the mass scale:

\[
\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} + \frac{b}{2\pi} \log \frac{\mu}{\mu_0}
\]

In particular:

\[
\alpha^{-1}_i(M_Z) = \alpha^{-1}_{GUT} - \frac{b_i}{4\pi} \log \left(\frac{M_{GUT}^2}{M_Z^2}\right) + \Delta_i
\]

\(\Delta_i\) : accounts for threshold corrections from the GUT and weak s and the effect of Planck suppressed operators

\(b_i\) : defined by the particle content
**SM beta functions**

\[
b = \frac{11}{3} T_2(\text{spin-1}) - \frac{2}{3} T_2(\text{chiral spin-1/2}) - \frac{1}{3} T_2(\text{complex spin-0})
\]

\[
Tr(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \quad T_2(\text{fund}) = \frac{1}{2} \quad T_2(\text{adj}) = N
\]

**universal contribution coming from complete SU(5) representations**

\(4N_F/3\) in SM in \(4N_F/3 \times 3/2\) in susy

**So in the SM:**

\[
b_3 = \frac{11}{3} \times N_c - \frac{2}{3} \times N_f \left(\frac{1}{2} \times 2 + \frac{1}{2} \times 1 + \frac{1}{2} \times 1\right) = 7
\]

\[
b_2 = \frac{11}{3} \times 2 - \frac{2}{3} \times N_f \left(\frac{1}{2} \times 3 + \frac{1}{2} \times 1\right) - \frac{1}{3} \times \frac{1}{2} = \frac{19}{6}
\]

\[
b_Y = \frac{2}{3} \times N_f \left(\frac{1}{6} \right)^2 \times 2 \times N_c + \left(-\frac{2}{3}\right)^2 \times N_c + \left(\frac{1}{3}\right)^2 \times N_c + \left(-\frac{1}{2}\right)^2 \times 2 + (1)^2
\]

\[-\frac{1}{3} \left(\frac{1}{2}\right)^2 \times 2 = -\frac{41}{6} \quad \rightarrow \quad b_1 = b_Y \times \frac{3}{5} = -\frac{41}{10}
\]

**from gauge bosons**

**from Higgs**
$$\frac{1}{\alpha_i^{-1}(M_Z)} = \alpha_{GUT}^{-1} - \frac{b_i}{4\pi} \log \frac{M_{GUT}^2}{M_Z^2} + \Delta_i \quad i = SU(3), SU(2), U(1)$$

$$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z): \text{experimental inputs}$$

$$b_3, b_2, b_1: \text{predicted by the matter content}$$

**3 equations and 2 unknowns** \((\alpha_{GUT}, M_{GUT})\)

**1 consistency relation for unification**

Using

$$\alpha_1 = \frac{5}{3} \frac{1}{\cos^2 \theta_W} \alpha_{em} \quad \text{and} \quad \alpha_2 = \frac{\alpha_{em}}{\sin^2 \theta_W}$$

we obtain:

$$\epsilon_{ijk}(\alpha_i^{-1} - \Delta_i)(b_j - b_k) = 0$$

If the \(\Delta_i\) contributions are universal (\(\Delta_1 = \Delta_2 = \Delta_3\)) or negligible, this translates into

$$\sin^2 \theta_W = \frac{3(b_3 - b_2) + 5(b_2 - b_1) \alpha_{em}(M_Z)}{8b_3 - 3b_2 - 5b_1} \alpha_{s}(M_Z)$$

$$\alpha_{em}(M_Z) \approx 1/128$$

$$\alpha_{s}(M_Z) \approx 0.1184 \pm 0.0007$$

**In the SM:** \(\sin^2 \theta_W \approx 0.207\)

Not so bad ...

to be compared with \(0.2312\pm/-0.0002\)
From the consistency relation, we can define another observable quantity:

\[
B \equiv \frac{b_3 - b_2}{b_2 - b_1} = \frac{\alpha_2^{-1} - \alpha_3^{-1} - (\Delta_2 - \Delta_3)}{\alpha_2^{-1} - \alpha_1^{-1} - (\Delta_2 - \Delta_1)}
\]

Assuming universal contributions, we get:

\[
B = \frac{\sin^2 \theta_w \alpha_{em}^{-1} - \alpha_s^{-1}}{\sin^2 \theta_w \alpha_{em}^{-1} - \alpha_{em}^{-1}} = 0.717 \pm 0.008 \pm 0.03
\]

to be compared with the prediction in the SM:

\[
B_{SM} = 0.528
\]

large (40%) discrepancy! Cannot be accommodated by allowing a 10% theoretical uncertainty due to threshold corrections and higher loop effects.
We can finally derive the values of $M_{GUT}$ and $\alpha_{GUT}$

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \right) \approx 7 \times 10^{14} \text{ GeV}$$

$$\alpha^{-1}_{GUT} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5$$

self-consistent calculation: $M_{GUT} < M_{Pl}$ safe to neglect quantum gravity effects

$\alpha_{GUT} \ll 1$ perturbative

values unchanged when adding universal contributions to the running

Quarks and leptons of the SM contribute universally as they form complete SU(5) multiplets, hence do not affect the relative running and therefore B
Only the Higgs and the SM gauge bosons can affect the relative running (see slide 9)

In the MSSM, extra contributions from the higgsinos and gauginos lead to the prediction $B=0.714$ remarkably close to the experimental value

$$b = \frac{11}{3} T_2(\text{spin-1}) - \frac{2}{3} T_2(\text{chiral spin-1/2}) - \frac{1}{3} T_2(\text{complex spin-0})$$

$$Tr(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \quad T_2(\text{fund}) = \frac{1}{2} \quad T_2(\text{adj}) = N$$

**SM**

**MSSM**

Gauginos

Scalars

$$b_{SU(3)} = 3 \times 3 - \left( \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 1 \times 3 \right) = \boxed{3}$$

$$b_{SU(2)} = 3 \times 2 - \left( \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 3 \right) - \frac{1}{2} - \frac{1}{2} = \boxed{-1}$$

$$b_Y = - \left( \frac{1}{6} \right)^2 3 \times 2 \times 3 + \left( -\frac{2}{3} \right)^2 3 \times 3 + \left( \frac{1}{3} \right)^2 3 \times 3 + \left( \frac{1}{2} \right)^2 2 \times 3 + (1)^2 \times 3 \right) - \left( \frac{1}{2} \right)^2 \times 2 - \left( \frac{1}{2} \right)^2 \times 2 = -11 \quad \boxed{\theta_{T12} = \frac{-33}{5}}$$
Values of \( b \) in various models:

**SM:** 
\[
(b)_{\text{SM}} = \left( \begin{array}{c} 0 \\ -22/3 \\ -11 \end{array} \right) + \left( \begin{array}{c} 4/3 \\ 4/3 \\ 4/3 \end{array} \right) F + \left( \begin{array}{c} 1/10 \\ 1/6 \\ 0 \end{array} \right) N_H ,
\]

**MSSM:** 
\[
(b)_{\text{MSSM}} = \left( \begin{array}{c} 0 \\ -6 \\ -9 \end{array} \right) + \left( \begin{array}{c} 2 \\ 2 \\ 2 \end{array} \right) F + \left( \begin{array}{c} 3/10 \\ 1/2 \\ 0 \end{array} \right) N_H ,
\]

**Split-SUSY:** 
\[
\left. (b)_{\text{split}} \right|_{\tilde{m}} = \left( \begin{array}{c} 0 \\ -6 \\ -9 \end{array} \right) + \left( \begin{array}{c} 4/3 \\ 4/3 \\ 4/3 \end{array} \right) F + \left( \begin{array}{c} 5/10 \\ 5/6 \\ 0 \end{array} \right) ,
\]

**low-\( \mu \) split SUSY:** 
\[
\left. (b)_{\mu-\text{split}} \right|_{\tilde{m}} = \left( \begin{array}{c} 0 \\ -22/3 \\ -11 \end{array} \right) + \left( \begin{array}{c} 4/3 \\ 4/3 \\ 4/3 \end{array} \right) F + \left( \begin{array}{c} 5/10 \\ 5/6 \\ 0 \end{array} \right) ,
\]

light higgs, higgsino & gauginos but heavy sfermions

light higgs, higgsino but heavy sfermions & gauginos
Another interesting observation:

In the SM, one can restore the gauge coupling unification without gauginos and higgsinos but if the third generation is partly composite!

If we substract $H$, $t_R$ and $t^c_R$ from the beta functions, $B$ is approximately within 10% of the experimental value.

The contribution from the partly composite third generation fermion sector restores the low energy prediction to a level that can be explained by threshold and higher loop effects.

\[
\frac{d\alpha_i}{d \ln Q} \in - \frac{b_{\text{comp}}}{2\pi} \alpha_i^2 + \frac{B_{ij}}{2\pi} \frac{\alpha_j^3}{4\pi} + \frac{C_{if}}{2\pi} \frac{\lambda_f^2}{16\pi^2} \]

\[
b_{SU(3)} = b_{SU(3)}^\text{SM} + \frac{2}{3} \left( \frac{1}{2} + \frac{1}{2} \right) = \frac{23}{3}
\]

\[
b_{SU(2)} = b_{SU(2)}^\text{SM} + \frac{1}{3} \times \frac{1}{2} = \frac{10}{3}
\]

\[
b_Y = b_Y^\text{SM} + \frac{2}{3} \left( \left( -\frac{2}{3} \right)^2 \times 3 + \left( -\frac{2}{3} \right)^2 \times 3 \right) + \frac{1}{3} \left( \frac{1}{2} \right)^2 \times 2 = -\frac{44}{9} \Rightarrow b_{T^{12}} = -\frac{44}{15}
\]
A very interesting observation is that there exist another way to achieve beautiful unification: In the SM, one can restore the gauge coupling unification without gauginos and higgsinos but if the third generation is partly composite! The value of $B$ is approximately within 10% of the experimental value while the SM prediction leads to a 40% discrepancy. Remarkably the contribution from the partly composite third generation fermion sector has restored the low energy prediction to a level that can be realistically by threshold and higher loop effects.

### Table

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$\sin^2 \theta_W$</th>
<th>$M_{GUT}$</th>
<th>$\alpha^{-1}_{GUT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>$-41/10$</td>
<td>$19/6$</td>
<td>$7$</td>
<td>$0.207$</td>
<td>$7 \times 10^{14}$ GeV</td>
<td>$41.5$</td>
</tr>
<tr>
<td>MSSM</td>
<td>$-33/5$</td>
<td>$-1$</td>
<td>$3$</td>
<td>$0.23$</td>
<td>$2 \times 10^{16}$ GeV</td>
<td>$24.3$</td>
</tr>
<tr>
<td>Split SUSY</td>
<td>$-45/10$</td>
<td>$+7/6$</td>
<td>$+5$</td>
<td>$0.226$</td>
<td>$4 \times 10^{16}$ GeV</td>
<td>$22.24$</td>
</tr>
<tr>
<td>Composite Higgs &amp; topq</td>
<td>$-44/15$</td>
<td>$19/3$</td>
<td>$23/3$</td>
<td>$0.228$</td>
<td>$1.1 \times 10^{15}$ GeV</td>
<td>$45.20$</td>
</tr>
</tbody>
</table>

The region satisfying this bound is shown in light red in Fig. 4-b. naively, the situation looks safer in susy. However, this is because we have imposed an extra symmetry to prevent dangerous contributions coming from dimension-5 and dimension-4 operators.

SUSY pbs: The SM matter fields fall into complete SU(5) multiplets, however the two Higgs doublets of the MSSM do not. Thus if one takes the GUT idea seriously one needs to embed the Higgs fields in an appropriate way. The Higgs boson in the MSSM is predicted to be at $M_H \approx 1.1 \times 10^{15}$ GeV, which is consistent with the experimental bound.

The measured value of $\sin^2 \theta_W$ is $0.23115$. 

**Equation**: 

$$\frac{\alpha^{-1}_{3 \text{ or } 2} - \alpha^{-1}_{1 \text{ or } 4}}{\beta} = \frac{\alpha^{-1}_{3 \text{ or } 2} - \alpha^{-1}_{1 \text{ or } 4}}{b_2 - b_1}$$
The lifetime of a weak scale particle decaying via dim-5 operators leads to decay taking place within a few seconds, which is the timescale for proton decay and is allowed only for 

\[ 0.50 < B < 0.60 \] compared with measured value (red). Right: Predictions of

\[ 0.200 \text{ to } 0.235 \] for each model.

Note that the present low-energy constraints allow

\[ 10^{2} - 10^{3} \] of the order of 10

\[ 10^{2} - 10^{3} \] for each model.

The light red colored region is the one allowed by experimental constraints on the proton 

\[ 10^{-6} \] to detect at high energy

\[ 10^{2} - 10^{3} \] of the order of 10

\[ 10^{2} - 10^{3} \] for each model.

The light red colored region is the one allowed by experimental constraints on the proton 

\[ 10^{2} - 10^{3} \] of the order of 10

\[ 10^{2} - 10^{3} \] for each model.

The light red colored region is the one allowed by experimental constraints on the proton 

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\[ 10^{2} - 10^{3} \] for each model.

The light red colored region is the one allowed by experimental constraints on the proton 

\[ 10^{2} - 10^{3} \] of the order of 10

\[ 10^{2} - 10^{3} \] for each model.
During BBN:
from dim-6 operators is leading to a lifetime

1.4 Yukawa coupling unification

The light red colored region is the one allowed by experimental constraints on the proton

Figure 4: Left: Unification predictions for

\[ \log_{10}[M_{\text{GUT}}/\text{GeV}] \]

\[ \alpha_u^{-1} \]

Experimental constraints lead to:

\[ \tau_p > 5.3 \times 10^{33} \text{ yr} \]

i.e

\[ M_{\text{GUT}} > \left( \frac{\alpha_{\text{GUT}}}{1/35} \right)^{1/2} \times 6 \times 10^{15} \text{ GeV} \]

Naively, the situation looks safer in SUSY. However, this is because we have imposed an extra symmetry to prevent dangerous dimension-5 and dimension-4 operators leading to pb in susy GUTs:

+ doublet-triplet splitting pb...
Astrophysical probes of unification (SUSY GUTs)

The DM LSP can decay, like the proton, via dimension-6 operators, with a lifetime $\tau \sim (m_{DM}/m_p)^5$ shorter than the proton lifetime, of the order of $10^{26}$ sec, which is the timescale probed by indirect detection experiments such as Fermi, PAMELA, HESS...

$$\tau \sim 8\pi \frac{M_{GUT}^4}{m^5} = 3 \times 10^{27} \text{s} \left(\frac{\text{TeV}}{m}\right)^5 \left(\frac{M_{GUT}}{2 \times 10^{16} \text{ GeV}}\right)^4$$
γ -ray Constraints on Decaying Dark Matter

Regions excluded by Fermi and HESS + CTA projections

Similar results obtained for different channels. This is assuming 2-body decay but other decays can be deduced, from a combination of the two-body decays.
The constraints from the Fermi isotropic gamma-ray data exclude decaying dark matter with a lifetime shorter than $10^{26}$ to few $10^{27}$ seconds, depending on its mass and the precise channel.
The strong CP problem

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi}_q (iD - m_q e^{i\theta_q}) \psi_q - \frac{1}{4} G_{\mu\nu} a G^\mu_\alpha G_a^{\nu\alpha} - \Theta \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}_a^{\mu\nu}$$

remove phase of mass term by chiral transformation of quarks

$$\psi_q \rightarrow e^{-i\gamma_5 \theta_q / 2} \psi_q$$

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi}_q (iD - m_q) \psi_q - \frac{1}{4} G G - \left(\Theta - \arg \det M_q\right) \frac{\alpha_s}{8\pi} G \tilde{G}$$

$$-\pi \leq \Theta \leq +\pi$$

induces a sizeable electric dipole moment for the neutron

experimental limit: \( |\Theta| < 10^{-11} \)

Why so small?
The Peccei-Quinn (dynamical) solution

Postulate new global axial $U(1)_{\text{PQ}}$ symmetry

spontaneously broken by $\Phi$, $
\phi(x) = \frac{f_a + \rho(x)}{\sqrt{2}} e^{ia(x)/f_a}$

\[ \mathcal{L}_{\text{KSVZ}} = \left( \frac{i}{2} \overline{\psi} \gamma^\mu \psi + \text{h.c.} \right) + \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(|\Phi|) - h \overline{\psi}_L \psi_R \Phi + \text{h.c.} \]

invariant under $\Phi \to e^{ia} \Phi$, $\psi_L \to e^{ia/2} \psi_L$, $\psi_R \to e^{-ia/2} \psi_R$

New heavy colored quarks with coupling to $\Phi$ generate a $aG\tilde{G}$ term

\[ \Theta \text{ is promoted to a field } a(x) \quad - \frac{\alpha_s}{8\pi} \overline{\Theta} \text{ Tr}(G\tilde{G}) \to - \frac{\alpha_s}{8\pi} \frac{a(x)}{f_a} \text{ Tr}(G\tilde{G}) \]

\[ \mathcal{L}_{\text{KSVZ}} = \left( \frac{i}{2} \overline{\psi} \gamma^\mu \psi + \text{h.c.} \right) + \frac{1}{2} (\partial_\mu a)^2 - m \overline{\Psi} e^{iy_{5a}} \frac{f_a}{f_a} \Psi, \text{ where } m = hf_a/\sqrt{2} \]

axions couple to QCD sector

Peccei & Quinn calculated the axion potential and showed that at the minimum $\langle a \rangle = 0$ thus $\overline{\Theta} = 0$

$f_a$: free parameter

strong CP pb solved whatever the scale $f_a$ is
mass vanishes if $m_u$ or $m_d = 0$

\[
m_A = \frac{f_\pi}{f_A} \frac{\sqrt{m_u m_d}}{m_u + m_d} m_\pi \approx \frac{6 \, \mu\text{eV}}{f_\pi/10^{12} \, \text{GeV}}
\]

axions couple to gluons, mix with pions and therefore couple to photons

photonic coupling

\[
ga = \frac{\alpha}{2\pi f_a} \left( \frac{E}{N} - 1.92 \right)
\]

can be detected when they convert into photons due to magnetic field

thermally produced in stars:

\[
f_\pi = 93 \, \text{MeV}
\]
\[
m_\pi = 135 \, \text{MeV}
\]
Axion as Dark Matter

U(1)_{PQ} phase transition in the early universe: the axion field sits at $a \sim \Theta f_a$ (flat potential)

Scalar field evolution in the expanding universe

\[ \frac{d^2 \langle a_{\text{phys.}} \rangle}{dt^2} + 3 \frac{\dot{R}(t)}{R(t)} \frac{d \langle a_{\text{phys.}} \rangle}{dt} + m_a^2(t) \langle a_{\text{phys.}} \rangle = 0 \]

acquires a mass $m_a \sim \frac{\Lambda_{QCD}^2}{f}$ at a temperature $T^* \sim \Lambda_{QCD}$

classical field oscillations start when $m_a(T^*) \sim H(T^*) \sim \frac{\Lambda_{QCD}^2}{M_{\text{Planck}}}$

energy density of the universe due to axions:

$\rho_a(T^*) \sim m_a^2(T^*) f^2$

redshifts like cold dark matter $\rho_a(t) \sim m_a(t)/R^3(t)$

$\rho_a = \rho_a(T^*) \left[ \frac{m_a}{m_a(T^*)} \right] \left[ \frac{R^3(T^*)}{R^3} \right] \sim \frac{\Lambda_{QCD}^3 T^3}{m_a M_{\text{Planck}}}$

bound on the axion mass not to overclose the universe:

$m_a \geq (10^{-5} - 10^{-6}) \text{ eV}$

$\rho_{DM} \sim 0.3 \text{ GeV cm}^{-3} = \frac{1}{2} m_a^2 \Theta^2 f_a^2 \sim \frac{1}{2} \Theta^2 m_\pi^2 f_\pi^2 \quad \rightarrow \quad \Theta \sim 10^{-19}$
Constraints on axions

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[GeV] $f_a$
Give up on the hierarchy problem. Focus on dark matter, gauge coupling unification and strong CP problem -> no new physics at the weak scale

Solution to strong CP pb: postulate new $U(1)_{\text{PQ}}$ symmetry & new heavy fermions

$$\Psi \rightarrow e^{i\gamma_5 \alpha} \Psi$$

$$\langle A \rangle = T^2 f_a$$

$$M_\Psi = \lambda_\Psi \langle A \rangle$$

$$a = \sqrt{2} \ \text{Im} \ A$$

These new fermions affect the running as well as modify the axion-photon coupling

$$\frac{g_a \gamma_i}{m_a} = \frac{\alpha_{\text{em}}}{2\pi f_\pi m_\pi} \sqrt{\left(1 + \frac{m_d}{m_u}\right) \left(1 + \frac{m_u}{m_d} + \frac{m_u}{m_s}\right)} \left[ \frac{E}{N} - \frac{2}{3} \left( \frac{4 + m_u/m_d + m_u/m_s}{1 + m_u/m_d + m_u/m_s} \right) \right] = 2.0 \left( \frac{E}{N} - 1.92 \right) \frac{10^{16} \text{GeV} \mu\text{eV}}{10^1 \mu\text{eV}}$$

-> get a bound on the axion-photon coupling from requiring unification
Figure 4:
Left: The prediction of each unified model for GUT and for $E/N$, the coefficient entering the axion coupling. The colors indicate the unified mass $M_{\text{GUT}}$. The thick dots are the points identified in fig. 2 as suggested by gauge/gravity unification, see eq. (6).

Right: The same points expressed in terms of the intermediate scale $M_Y$ with colors indicating the value of $1/\alpha_{\text{GUT}}$. For guidance, we have also translated the intermediate mass $M_Y$ into the corresponding value of the axion mass $m_a$, under the assumption $M = f_a$.

This could not occur in a supersymmetric theory where, in the presence of just one scalar, holomorphy implies $Q_PQ_P = 1$. In a non-supersymmetric theory the assumption $Q_PQ_P = 1$ can be justified by assuming that all fermions sit in the same multiplet within a more fundamental description, but we will not try to construct explicit examples. It is also worth remarking that, under the assumption $Q_PQ_P = 1$, the relation between the axion coupling and the $\alpha$-function coefficients is preserved, regardless of the dynamics of the PQ breaking sector and, in particular, regardless of the number and PQ charges of the SM singlets in that sector.

Equations (13) and (14) provide the link between unification and axion phenomenology, which is the key feature of unificaxion. Gauge coupling unification selects a special range for $\beta$ which, in turn, determines the measurable quantity $g_a/m_a$. The prediction for $g_a/m_a$ is obtained only by the request of unification, with no need to specify the particular particle content of the model or their interactions. Fig. 4 shows the correlations between $E/N$, $M_{\text{GUT}}$, $1/\alpha_{\text{GUT}}$, and $M_Y$. In particular, fig. 4b illustrates how the prediction of unificaxion for $E/N$, which is directly related to $g_a/m_a$ through eq. (13), depends on the $1/\alpha_{\text{GUT}}$.
fine-tuning problems

- The hierarchy problem associated with the Higgs [R. Rattazzi]
- The SUSY solution [D. Kazakov]
- The extra dimensional solutions
- The 4D strongly interacting solutions
- The Flavour problem [G. Isidori]
- The strong CP problem
- The “why so” puzzles
  - charge quantization
  - gauge coupling unification
  - GUTs
  - proton stability
  - fermion mass hierarchy
  - why 3 generations

Note: The number of generations may also be determined by the anomaly cancellation conditions ... in extra-dimensional theories, see e.g [Dobrescu & Popppitz hep-ph/0102010]

- The dark matter problem
- The matter antimatter asymmetry problem

observational facts unexplained by the SM