Cosmology and gravitation: the grand scheme for high energy physics
Part 1

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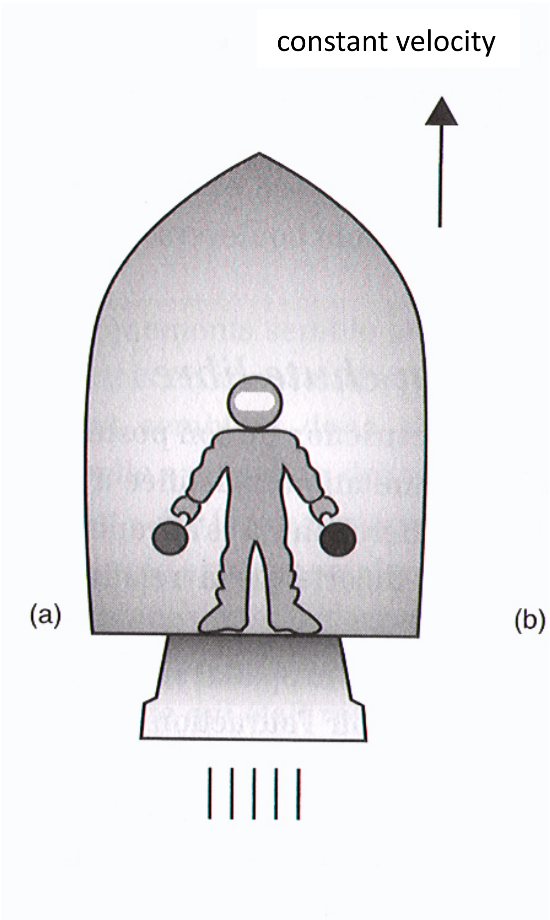
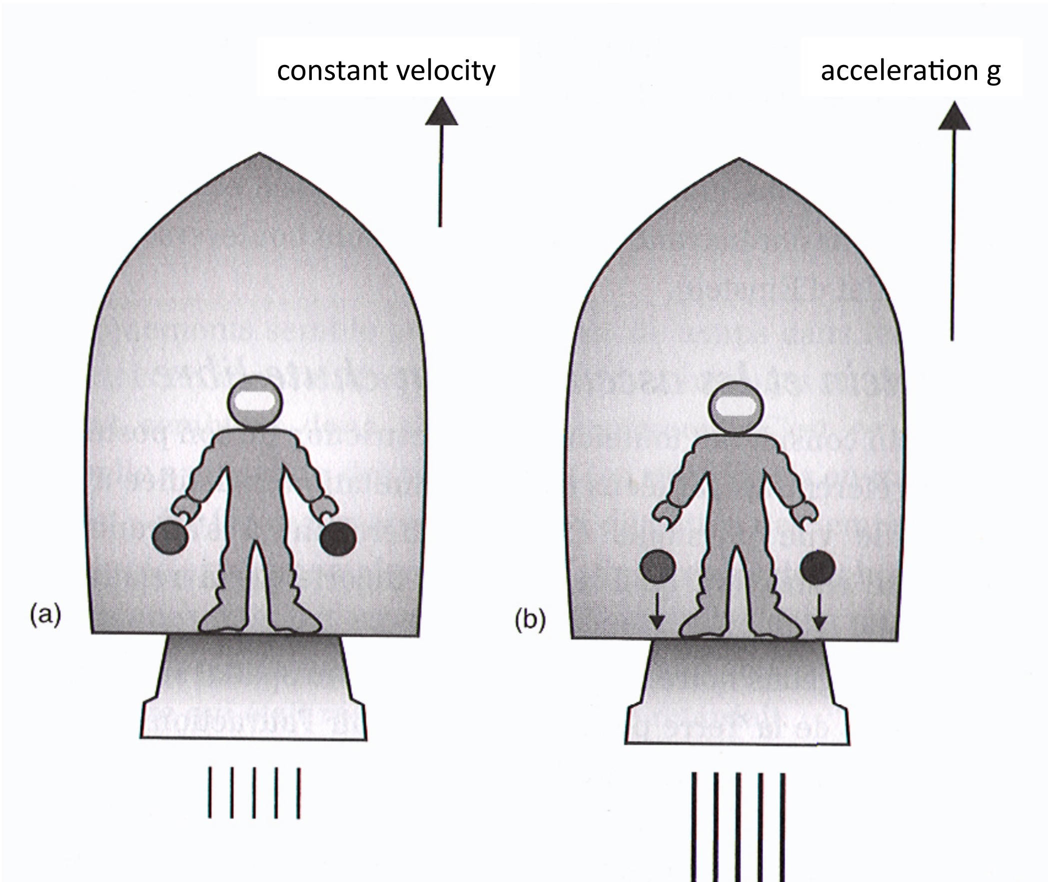
Plan

A not so brief history of modern cosmology
The days where cosmology became a quantitative science: Cosmic Microwave Background
Light does not say it all (1): the violent Universe
Light does not say it all (2): dark matter
Light does not say it all (3) : dark energy
The beginning and the end

A (not so) brief history of modern cosmology

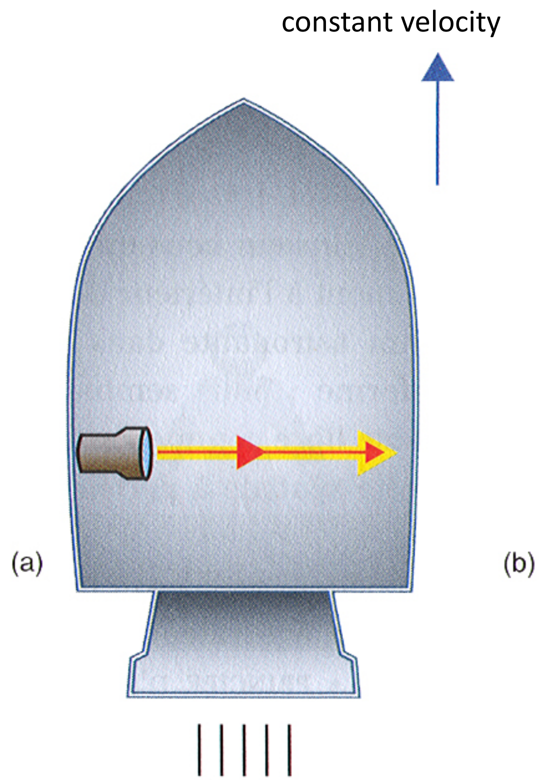
1916, general relativity theory, A. Einstein


constant velocity
(a)
(b)



Principle of equivalence

Observations made in an accelerating reference frame are indistinguishable from those made in a gravitational field.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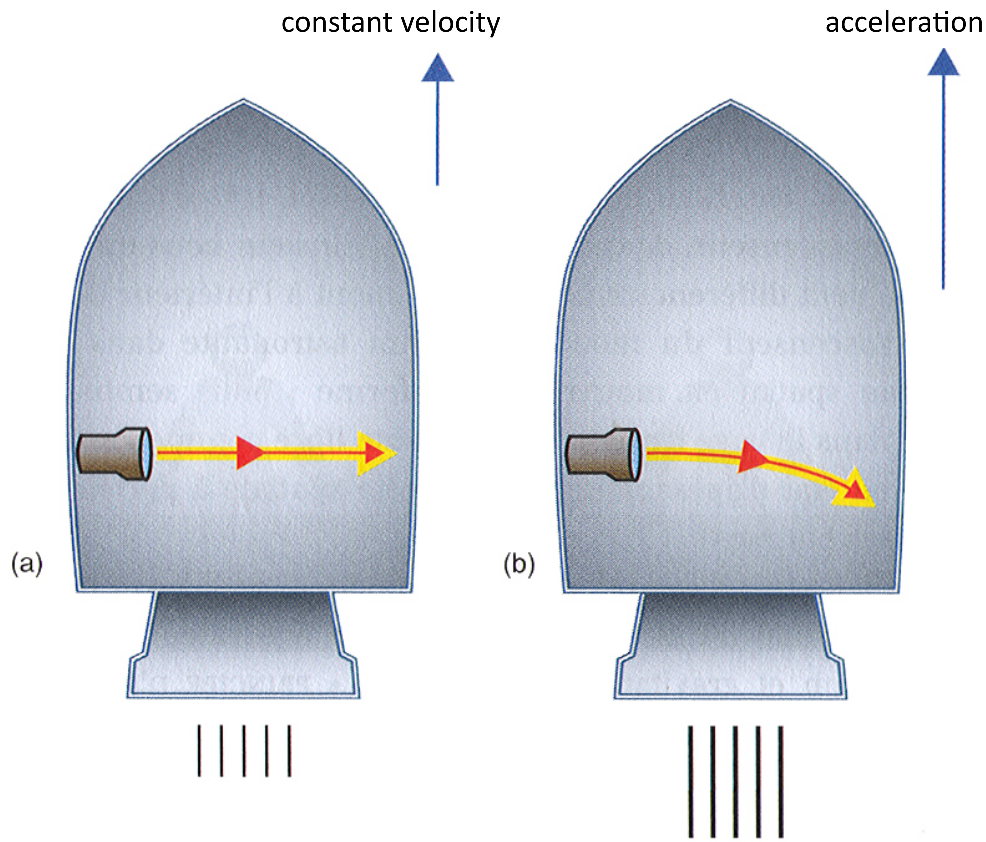

Light rays are bent by gravitational fields: they follow *geodesics*.

Newton's constant

Einstein equations: $R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G_N\, T_{\mu\nu}$

geometric content

energy-momentum content

Invariant spacetime length element: $ds^2 = g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$

$R_{\mu\nu}$ is expressed in terms of $g_{\mu\nu}$ and involves its second spacetime derivatives

$$\Gamma^\rho{}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left[\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right] \quad , \tag{2.2}$$

where $g^{\rho\sigma}$ is the inverse metric tensor: $g^{\rho\sigma} g_{\sigma\tau} = \delta^\rho_\tau$.

In the same way that one defines the field strength by differentiating the gauge field, one introduces the Riemann curvature tensor:

$$R^\mu{}_{\nu\alpha\beta} = \partial_\alpha \Gamma^\mu{}_{\nu\beta} - \partial_\beta \Gamma^\mu{}_{\nu\alpha} + \Gamma^\mu{}_{\alpha\sigma} \Gamma^\sigma{}_{\nu\beta} - \Gamma^\mu{}_{\beta\sigma} \Gamma^\sigma{}_{\nu\alpha} \quad . \tag{2.3}$$

By contracting indices, one then defines the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R

$$R_{\mu\nu} \equiv R^\alpha{}_{\mu\alpha\nu}, \quad , \quad R \equiv g^{\mu\nu} R_{\mu\nu} \quad . \tag{2.4}$$

Hence Einstein equations are field equations!

Of which field?

Weak gravitational field limit i.e. almost flat spacetime : $g_{\mu\nu}(x) \sim \eta_{\mu\nu} + h_{\mu\nu}(x)$

$\ll 1$

graviton field

Note that $g_{00} \sim 1 + 2\,\Phi$

Newtonian potential whch satisfies the Poisson equation:

$$\Delta\Phi = 4\pi G_N\,\rho$$

Einstein applies his equations to the whole Universe!

But what is the « Universe » in 1917?

Our galaxy, the Milky Way, and presumably a void beyond.

Hence, in the days of Einstein, the Universe is a static universe, which is incompatible with Einstein's equations or even with the Poisson equation:

$$\Delta\Phi = 4\pi G_N \rho$$

Einstein proposes the following modification:

$$\Delta\Phi + \lambda\Phi = 4\pi G_N \rho$$

Note: $[\lambda]=L^{-2}$

Solution: $\Phi = 4\pi G_N \rho/\lambda$

modified Einstein's equations: $R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G_N T_{\mu\nu} + \lambda g_{\mu\nu}$

cosmological constant

Solution: $g_{00}=1 \quad g_{ij}=-\delta_{ij}+x_i x_j/(\mathbf{x}^2-r^2)$ $\qquad \lambda \equiv r^{--2} = 4\pi G_N \rho$

Other solutions of the original Einstein's equations:

• Schwarzschild 1915: static solution with spherical symmetry (e.g. outside a star of mass M)

$$ds^2 = (1-2G_N M/r)\, dt^2 - (1-2G_N M/r)^{-1}\, dr^2 - r^2 d\theta - r^2 \sin^2\theta\, d\phi^2$$

What happens if the radius R of the star satisfies $R < 2G_N M \equiv R_S$?

Other solutions of the Einstein's equations with cosmological constant:

• de Sitter 1917: time dependent solution

$$ds^2 = dt^2 - e^{2Ht}\, d\mathbf{x}^2 \qquad H^2 \equiv \lambda/3$$

If there exists a time-dependent solution, why should the Universe be static?

The great debate of 1920

Harlow Shapley

Heber D. Curtis

Harlow Shapley	Heber D. Curtis
The Universe is composed of only one big Galaxy	The Universe is composed of many galaxies like our own.
``Spiral nebulae » are just nearby gas clouds.	They have been identified by astronomers as ``spiral nebulae".
Our Sun is far from the center of this big Galaxy	Our Sun is near the center of our relatively small Galaxy.

<u>The great debate of 1920</u>

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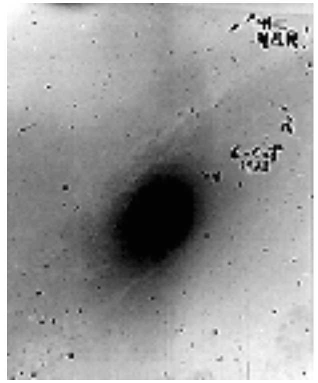
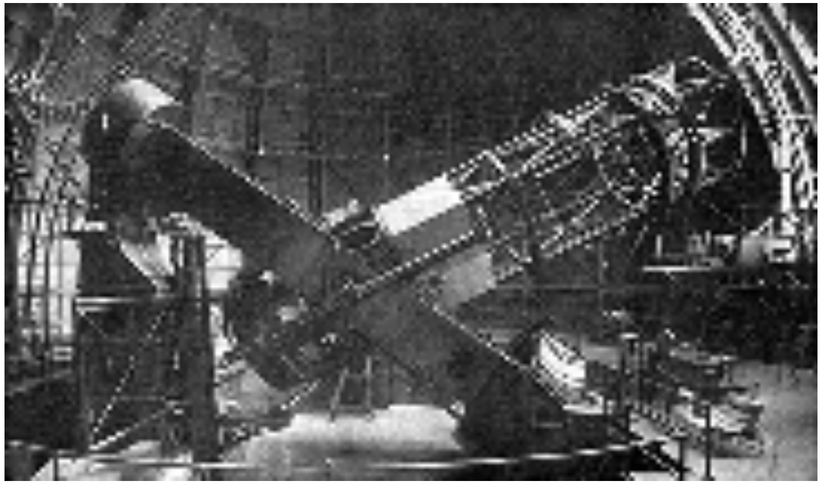
1925



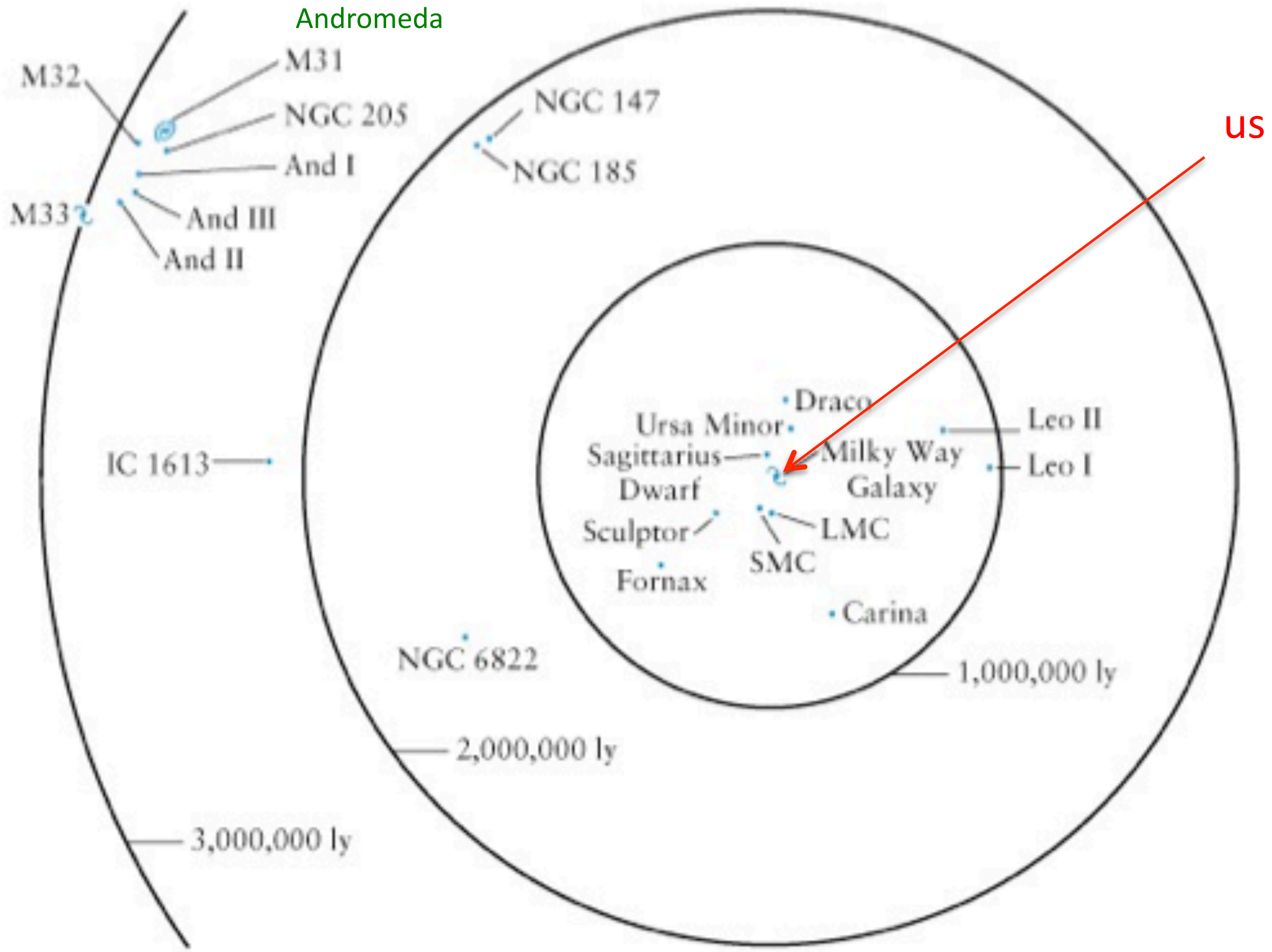
Cepheids, variable stars in Andromeda nebula M31



100 inch Hooker telescope (Mount Wilson)

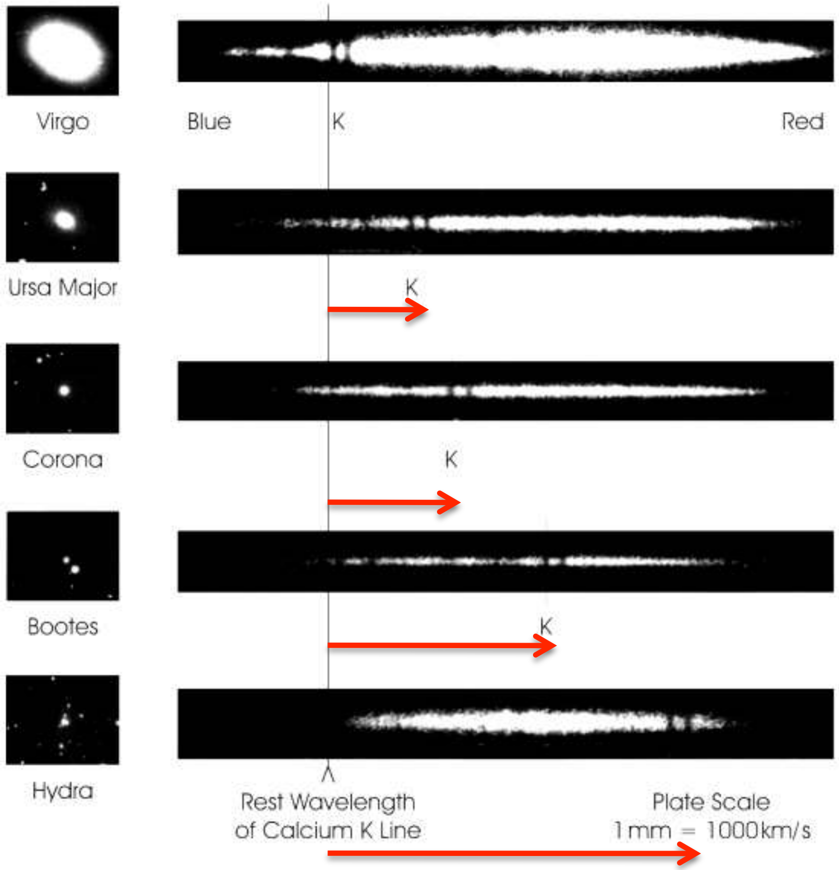
Edwin Hubble shows that the distance to M31 is greater than even Shapley's proposed extent of our Milky Way galaxy: M31 is a galaxy of its own, the Andromeda galaxy.

The local group



1929

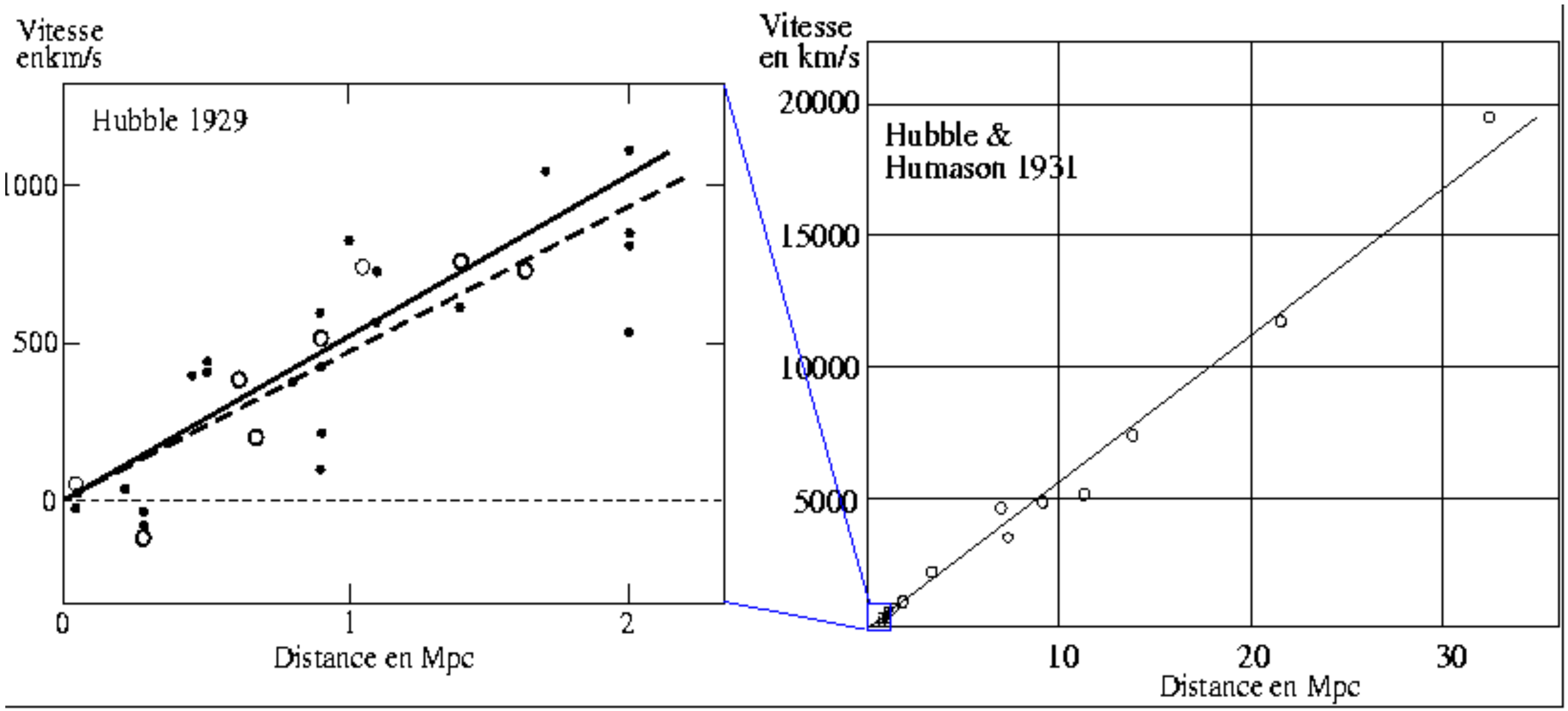
Hubble combines spectroscopic measures with measures of distances To infer his famous law

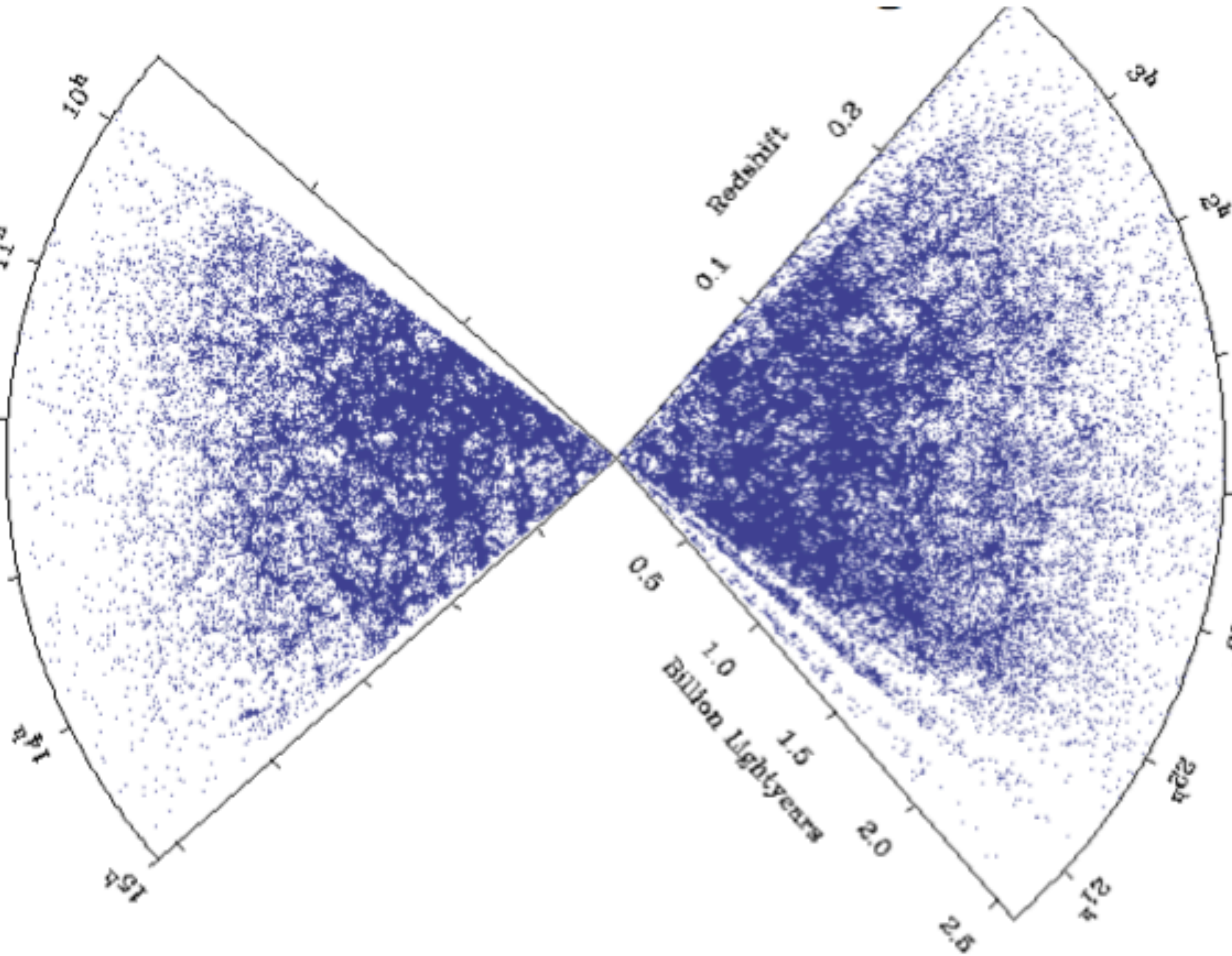


Doppler shift $1+z \equiv \lambda_{obs}/\lambda_{emit} = \dfrac{1+v/c}{\sqrt{1-v^2/c^2}} \sim 1+ v/c$

Galaxies at a distance d recede at a velocity :

$$v = cz \sim H_0 d$$


Vitesse
enkm/s
Hubble 1929
1000
500
0
0
1
2
Distance en Mpc
Vitesse
en km/s
20000
15000
10000
5000
Hubble &
Humason 1931
10
20
30
Distance en Mpc

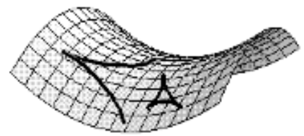


As we get to larger distances, the Universe looks homogeneous and isotropic

Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t)\,\gamma_{ij} dx^i dx^j,$$
$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

k=1 closed space
k=0 flat space
k=-1 open space



$a(t)$ cosmic scale factor describes the expansion of the Universe

Plug this in the left-hand side of Einstein's equations

Assume that the energy-momentum is that of a perfect fluid: in its rest frame

$T_{00} = \rho$ ← energy density

$T_{ij} = -\,p\, g_{ij} = p\, a^2(t)\, \gamma_{ij}$ ← pressure

Energy conservation:

$$\dot{\rho} = -3H(p + \rho)$$

Then Einstein's equations yield :

$\dot{a} \equiv da/dt$

$$
\begin{aligned}
3\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right) &= 8\pi G_N\rho+\lambda, \\
\dot{a}^2+2a\ddot{a}+k &= -8\pi G_N a^2 p+a^2\lambda,
\end{aligned}
$$

The first equation is known as Friedmann's equation: it gives the rate of expansion of the Universe:

$$
H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{1}{3}(\lambda+8\pi G_N\rho)-\frac{k}{a^2}
$$

At present time $t=t_0$,
$H = H_0$ Hubble constant

- Note $k=0$, $\rho = 0$, one recovers de Sitter solution $\dot{a}^2/a^2 = \lambda/3$ i.e. $a \sim e^{Ht}$
- Note $k=0$, $\lambda=0$, one obtains at present times $\rho = 3H_0^2/(8\pi G_N) \equiv \rho_c \sim 10^{-26}$ kg/m^3

critical energy density

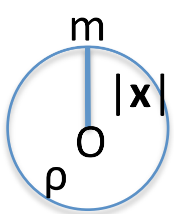
Understanding the Friedmann's equation

Universe at large scale is homogeneous and isotropic: *there is no preferred location nor preferred motion.*

⇒ Most general motion satisfies : **v**(t) = H(t) **x**

But **v** = dx/dt Hence **x** = a(t) **r**, where **r** is a constant for a given body and H(t) = $\dot{a}$/a.

For a particle of mass m at position **x**, the sum of its kinetic and gravitational energy is cst:

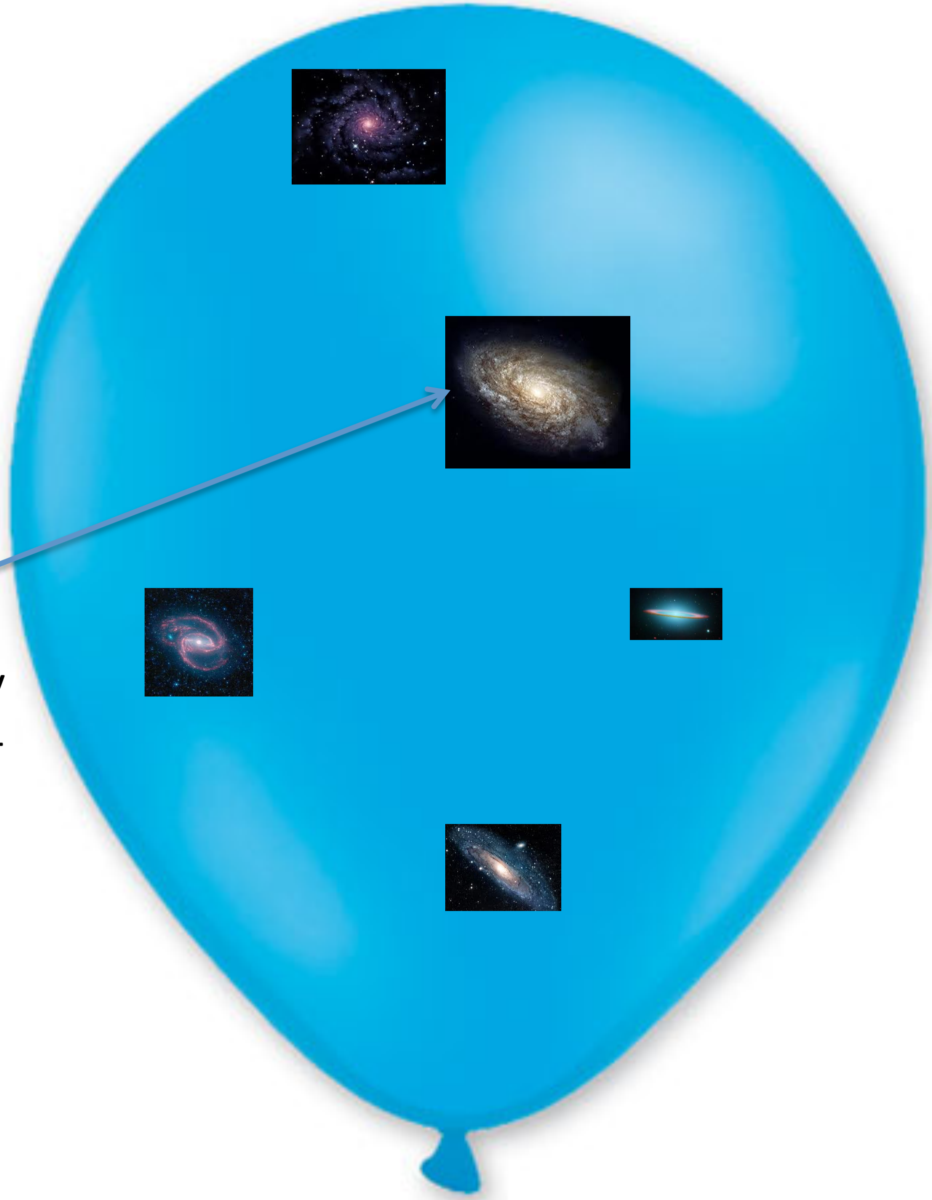


$$\frac{1}{2}m\mathbf{v}^2 - \frac{4\pi}{3}G_N m\rho|\mathbf{x}|^2 = \text{cst}\ .$$

Writing the constant –km$\mathbf{r}^2$/2, one obtains :

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G_N\rho - \frac{k}{a^2}$$

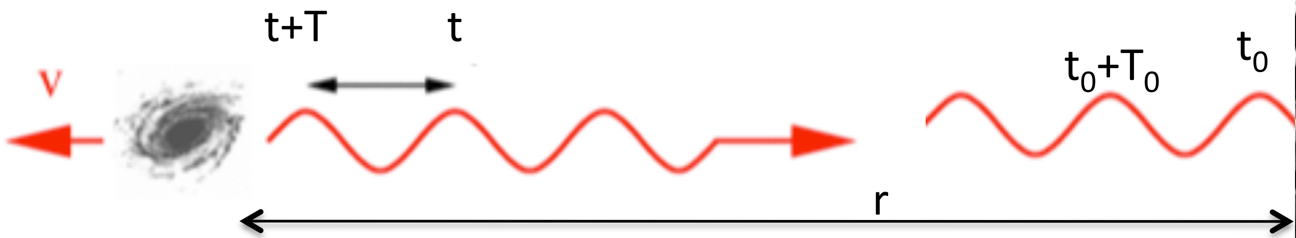
The image of the balloon:



Expansion is not felt inside the galaxy because one has fallen into the gravitational potential well of the galaxy.

Hubble expansion and redshift

Light ray follows $ds^2 = 0$ i.e. $$c^2 dt^2 = a^2(t) \frac{dr^2}{1 - kr^2}$$ (along θ,ϕ cst)



$$\int_t^{t_0} \frac{c\,dt}{a(t)} = \int_0^r \frac{dr}{\sqrt{1-kr^2}}$$

$$\int_{t+T}^{t_0+T_0} \frac{c\,dt}{a(t)} = \int_0^r \frac{dr}{\sqrt{1-kr^2}}$$

Since $T_0, T \ll t_0, t$ $$\frac{cT_0}{a_0} = \frac{cT}{a(t)}$$ i.e. $$\frac{\lambda_0}{\lambda} = \frac{a_0}{a(t)}$$

$1+z = a_0/a(t)$

Solving the equations with $p = w\rho$ (and $k = \lambda = 0$): $\dot{\rho}/\rho = -3(1+w)\dot{a}/a \Longrightarrow \rho \propto a(t)^{-3(1+w)}$

$$\dot{\rho} = -3H(p+\rho)$$

$\dot{a}^2/a^2 = 8\pi\rho G_N/3 \Longrightarrow a(t) \sim t^{\nu}$ with $\nu = 2/3(1+w)$

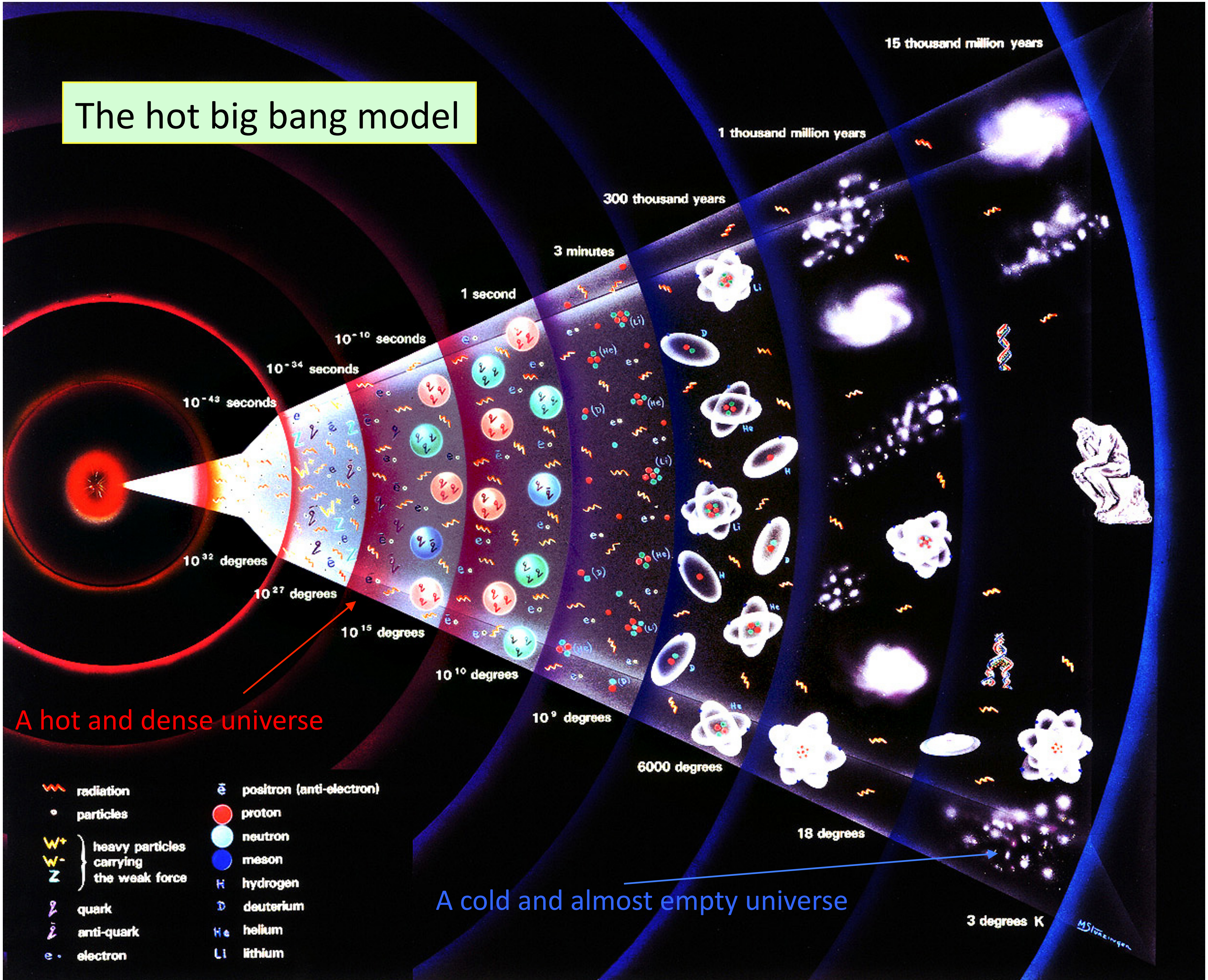
Non-relativistic matter dominated : negligible pressure $p \sim 0$ i.e. $w=0$,

$$\rho \propto a(t)^{-3},\ a(t) \sim t^{2/3}$$

Radiation dominated : $p = \rho/3$ i.e. $w = 1/3$,

$$\rho \propto a(t)^{-4-},\ a(t) \sim t^{1/2}$$

The hot big bang model

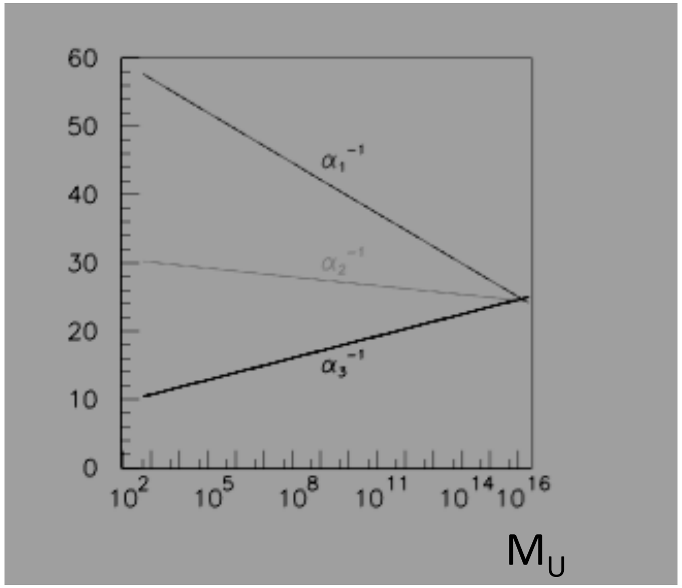


Starting in the late 40's with ideas by Gamow and collaborators,
nucleosynthesis of light elements (deuterium, helium, lithium…)
In the early Universe…

50’s and 60’s: particle zoo : to each increase in energy its set of new particles

End of 70’s : the grand desert

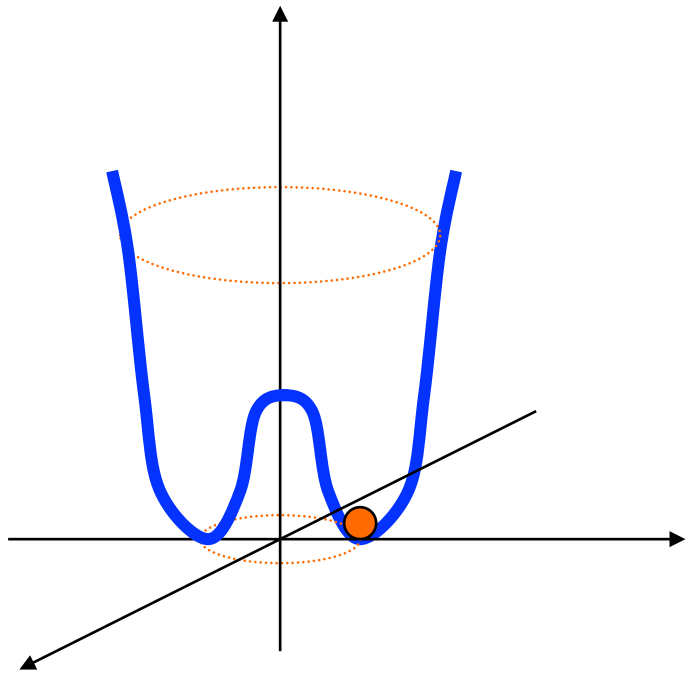
Idea : there exists an underlying theory which is simpler or more predictive and which operates at much higher energy.



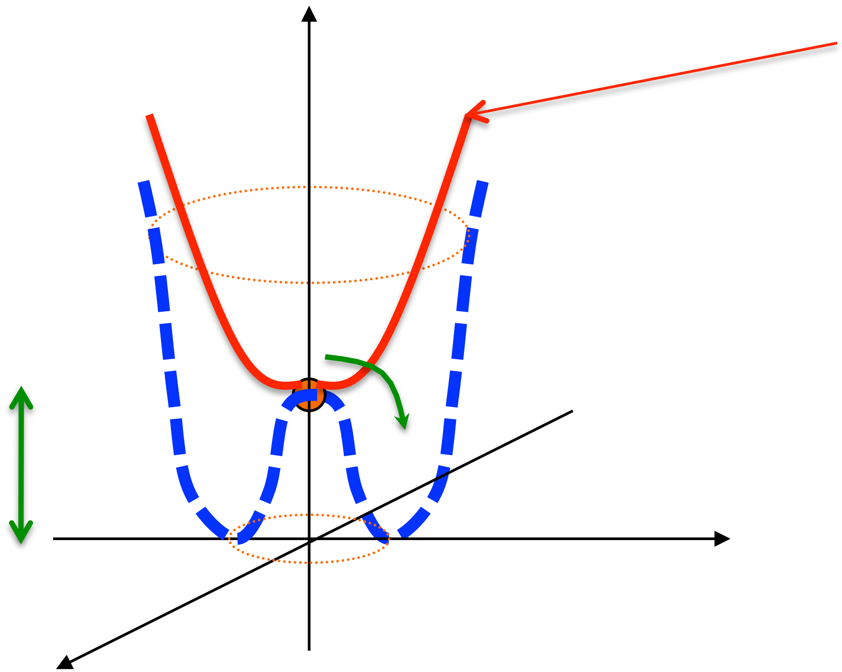
Corollary : the primordial universe (high temperature) is simple and

its evolution can be predicted with a restricted number of parameters.

Inflation scenario proposed first in the context of the phase transition associated with grand unification (Guth, 81)



potential at
high T



energy stored
typically $\rho_0 \sim M_U^4 \sim (10^{16}\ \text{GeV})^4$

Einstein's equations : $T_{\mu\nu} \sim \rho_0\, g_{\mu\nu}$

$R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G_N\, \rho_0\, g_{\mu\nu} \Longrightarrow H^2 = 8\pi G_N\, \rho_0 /3 = \text{constant } H_{vac}{}^2 \Longrightarrow$ de Sitter solution

$a(t) \sim \exp\,(H_{vac} t)$

Why is an early phase of exponential expansion (inflation) a welcome feature?

It solves a certain number of problems:

- flatness problem
- horizon problem
- monopole problem

Flatness problem

One may rewrite the Friedmann equation $H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{1}{3}(\lambda + 8\pi G_N \rho) - \frac{k}{a^2}$

as $$\frac{\rho_T(t)}{\rho_c(t)} - 1 = \frac{k}{\dot{a}^2}$$ with $\rho_T(t) = \rho + \lambda/8\pi G_N$

If the Universe is radiation-dominated, $a(t) \sim t^{1/2}$ and, since $\dot{a} \sim t^{-1/2} \sim a^{-1}$

$$\frac{\rho_T(t)}{\rho_c(t)} - 1 = \left[\frac{\rho_T(t_U)}{\rho_c(t_U)} - 1\right]\left(\frac{a(t)}{a(t_U)}\right)^2 = \left[\frac{\rho_T(t_U)}{\rho_c(t_U)} - 1\right]\left(\frac{kT_U}{kT}\right)^2$$

10^{16} GeV

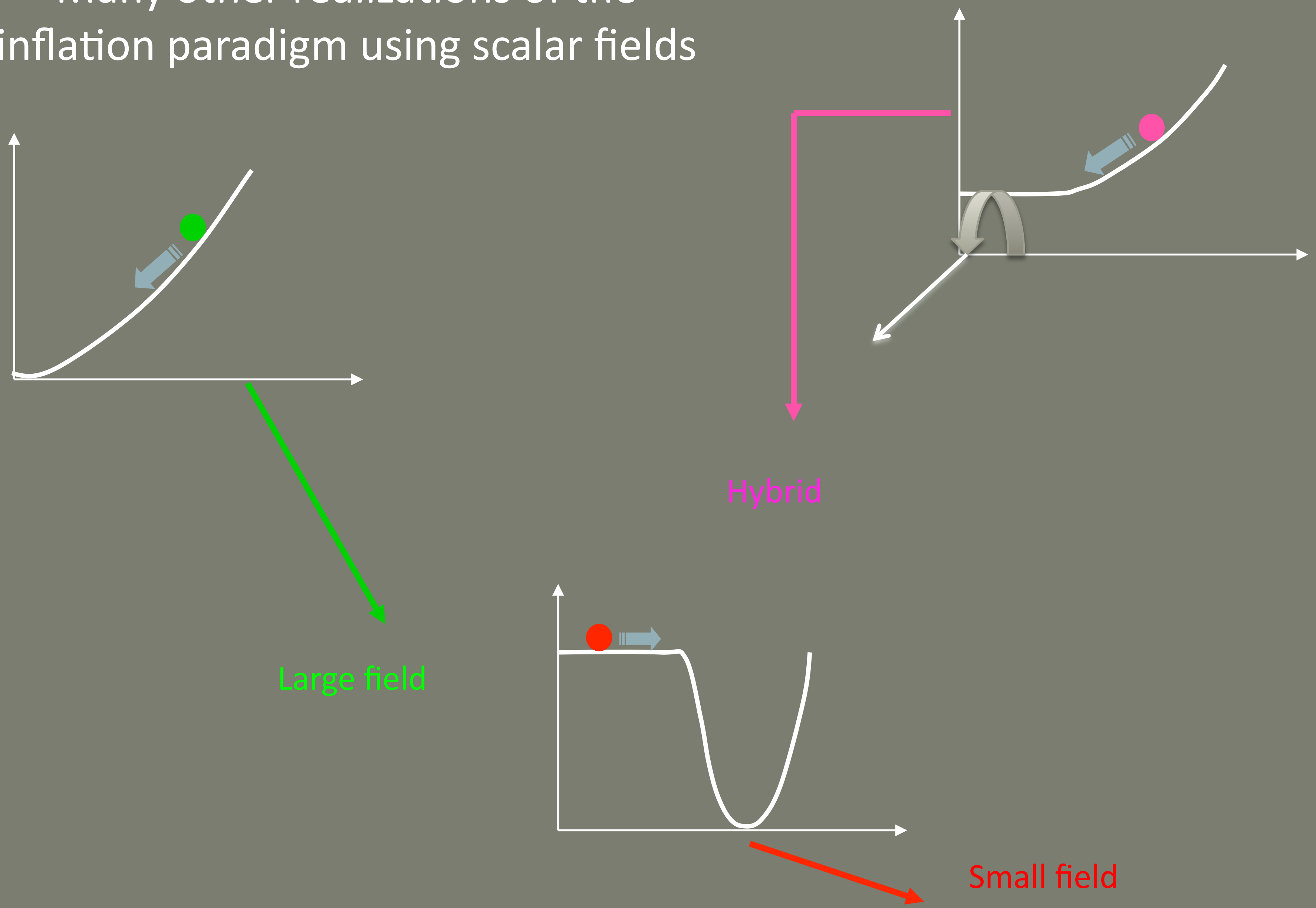
10^{-4} eV

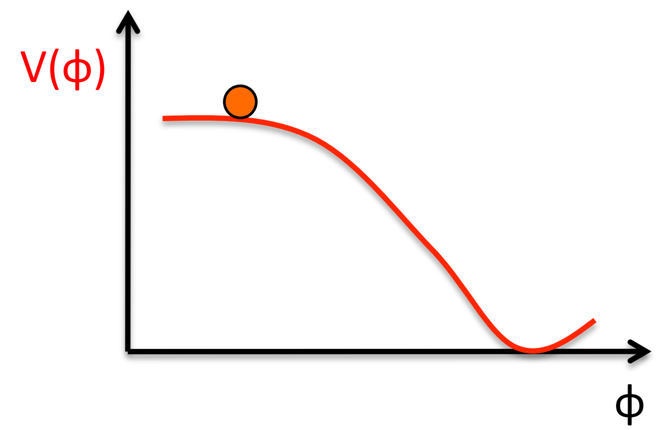
At $t=t_0$ (now) small 10^{-58}smaller!

Factor 10^{58}!

If the Universe undergoes a de Sitter phase, $\rho_T(t) / \rho_{c(t)} \propto e^{-Ht}$

Many other realizations of the inflation paradigm using scalar fields





Motion of a scalar field ϕ in its potential V(ϕ):

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,,$$
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,.$$

$$\underset{\dot{\rho} = -3H(p+\rho)}{\Longrightarrow} \quad \ddot{\phi} + 3H\dot{\phi} = -V'(\phi).$$

friction term due to expansion of the Universe

$$3H\dot{\phi} \simeq -V'(\phi)$$

In the case of slow roll, $\ddot{\phi} \ll 3H\dot{\phi}$

$$H^2 \simeq \frac{\rho}{3m_P^2} \simeq \frac{V}{3m_P^2}$$

Slow roll conditions:

$$\epsilon \equiv \frac{1}{2}\left(\frac{m_P V'}{V}\right)^2 \ll 1\,.$$

$$\eta \equiv \frac{m_P^2 V''}{V} \ll 1 \quad ,$$

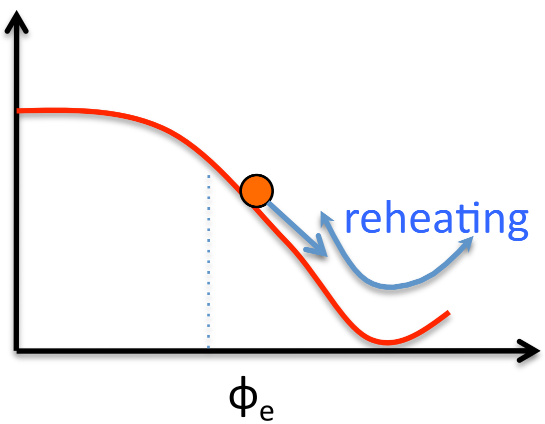
Number of Hubble times elapsed during inflation: 60 needed in order to solve the problems

$$N(t) = \int_t^{t_e} H(t)dt \; .$$

Since $dN = -Hdt = -Hd\phi/\dot{\phi}$,

$$N(\phi) = \int_{\phi_e}^{\phi} \frac{1}{m_p^2} \frac{V}{V'} d\phi \; .$$

End of inflation, when the slow roll conditions are no longer valid



Quantum fluctuations of the scalar fields during de Sitter phase induce scalar fluctuations of the metric

$$ds^2 = a^2 \left[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\delta_{ij}dx^i dx^j\right]$$

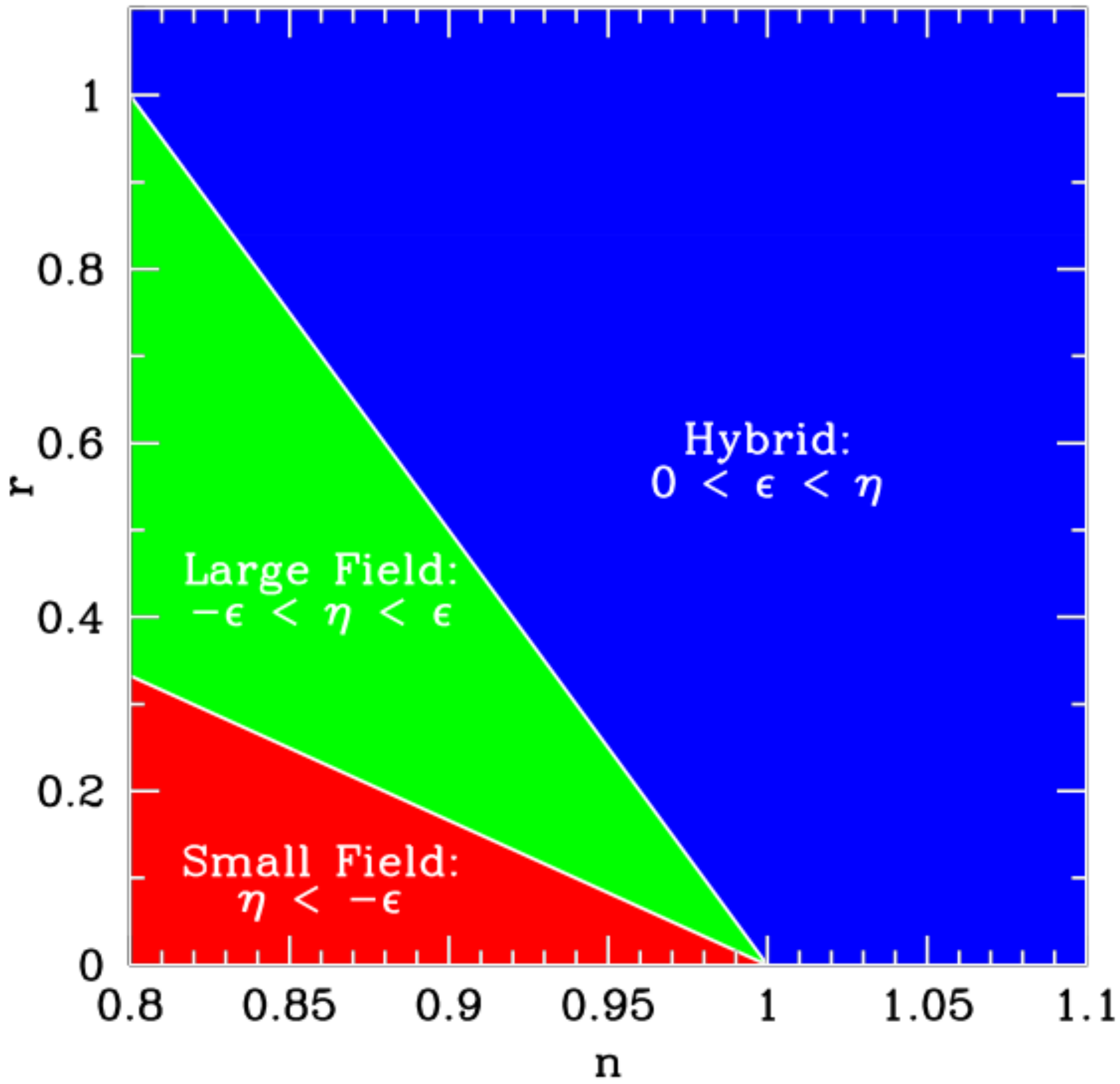
Correlation function in Fourier space:

$$\langle \Phi_{\mathbf{k}} \Phi^*_{\mathbf{k}'} \rangle = 2\pi^2 k^{-3} \mathcal{P}_S(k) \delta^3 (\mathbf{k} - \mathbf{k}') \ .$$

$$\mathcal{P}_S(k) = \left[\left(\frac{H^2}{\dot{\phi}^2}\right)\left(\frac{H}{2\pi}\right)^2\right]_{k=aH} = \frac{1}{12\pi^2 m_p^6}\left(\frac{V^3}{V'^2}\right)_{k=aH}$$

Spectral index:

$$n_S(k) - 1 \equiv \frac{d \ln \mathcal{P}_S(k)}{d \ln k} = -6\epsilon + 2\eta \ .$$


Hybrid:
0 < ϵ < η
Large Field:
−ϵ < η < ϵ
Small Field:
η < −ϵ
r
n
0
0.2
0.4
0.6
0.8
1
0.8
0.85
0.9
0.95
1
1.05
1.1

$t_0 \sim 15$Gyr	E= 2.35 10^{-4}eV	z=0	now
~ Gyr	~10^{-3}	4 to 9	formation of galaxies
$t_{rec} \sim 4.10^5$ yr	0.26	1100	recombination
$t_{eq} \sim 4.10^4$ yr	0.83	3500	matter-radiation equality
3 min	6.10^4	2.10^8	nucleosynthesis
1s	10^6	3.10^9	$e^+ e^-$ annihilation
4.10^{-6} s	4.10^8	10^{12}	QCD phase transition
< 4.10^{-6} s	> 10^9		baryogenesis
			inflation
0		∞	big-bang

$kT \propto a_0/a(t) \propto (1+z)$