IV.1. Hard Probes

Heavy Ion Collisions produce **auto-generated probes** at high $\sqrt{s_{NN}}$

$Q \gg T \geq 150 \text{MeV}$

Q: How sensitive are such ‘hard probes’?
Bjorken 1982: consider jet in p+p collision, hard parton interacts with underlying event → collisional energy loss

\[
\frac{dE_{\text{coll}}}{dL} \approx 10 \text{ GeV/fm}
\]

(error in estimate!)

Bjorken conjectured monojet phenomenon in proton-proton

But: radiative energy loss expected to dominate

\[
\Delta E_{\text{rad}} \approx \alpha_s \hat{q} L^2
\]

(4.1)

- p+p: \( L \approx 0.5 \text{ fm}, \ \Delta E_{\text{rad}} \approx 100 \text{ MeV} \) Negligible!
- A+A: \( L \approx 5 \text{ fm}, \ \Delta E_{\text{rad}} \approx 10 \text{ GeV} \) Monojet phenomenon!

Observed at RHIC
IV.3. Parton energy loss - a simple estimate

Medium characterized by transport coefficient:

\[ \hat{q} \propto \frac{\mu^2}{\lambda} \propto n_{\text{density}} \]

- How much energy is lost?
  
  **Phase accumulated in medium:**
  \[ \left\langle \frac{k_T^2 \Delta z}{2\omega} \right\rangle \approx \frac{\hat{q} L^2}{2\omega} = \frac{\omega_c}{\omega} \]
  
  **Characteristic gluon energy**
  
  **Number of coherent scatterings:**
  \[ N_{\text{coh}} \propto \frac{t_{\text{coh}}}{\lambda}, \quad \text{where} \quad t_{\text{coh}} \propto \frac{2\omega}{k_T^2} \propto \sqrt{\omega/\hat{q}} \]

  \[ k_T^2 \propto \hat{q} t_{\text{coh}} \]

  **Gluon energy distribution:**
  \[ \omega \frac{dI_{\text{med}}}{d\omega dz} \propto \frac{1}{N_{\text{coh}}} \omega \frac{dI_1}{d\omega dz} \propto \alpha_s \sqrt{\frac{\hat{q}}{\omega}} \]

  **Average energy loss**
  \[ \Delta E = \int_0^L dz \int_0^{\omega_c} d\omega \omega \frac{dI_{\text{med}}}{d\omega dz} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2 \]
IV.4. Medium-modified Final State Parton Shower

\[
\frac{dI}{d \ln \omega dk_T} = \frac{\alpha_s C_R}{(2\pi)^2 \omega^2} 2 \text{Re} \int \int du d\bar{y} \int dy e^{-ik_Tu} e^{i\omega \bar{y} y}.
\]

(4.5)

\[
K(s = 0, y; u, y | \omega) \quad \text{hard production}
\]

Target average includes Brownian motion:

\[
K(s, y; u, \bar{y} | \omega) = \int_{s = r(\bar{y})}^{u = r(y)} Dr \exp \left[ i \int_y \widehat{d\xi} \left\{ \left( \omega r^2 / 2 \right) - n(\xi)\sigma(r) \right\} \right] \quad \omega \to \infty \quad e^{-v(s)}
\]

Two approximation schemes:

1. Harmonic oscillator approximation:

(4.6)

\[
n(\xi)\sigma(r) \triangleq \hat{q}(\xi) r^2
\]

2. Opacity expansion in powers of

(4.7)

\[
\left( \alpha_s \int_0^L d\xi n(\xi)\sigma_{el} \right)^n
\]

Radiation off produced parton

\[\omega = xE \rightarrow (1-x)E\]

BDMPS transport coefficient

\[
\langle Tr[W^A+(0)W^A(r)] \rangle = \exp \left[ -\frac{1}{4} \hat{q}L_{long} r^2 \right]
\]
IV.5. Medium-induced gluon energy distribution

Consistent with estimate (4.1), spectrum is indeed determined by

$$\omega_c = \hat{q}L^2/2$$

Transverse momentum distribution is consistent with Brownian motion

### IV.6. Opacity Expansion - zeroth order

To understand in more detail the physics contained in

\[
\frac{dI}{d\ln \omega \, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2 \omega^2} 2 \text{Re} \int dy \int dy' \int du e^{-ik_T u} e^{i\pi s \left(\frac{n(x)}{\omega} - \frac{n(x')}{\omega} - \frac{n(u)}{\omega}\right)} K(s = 0, y; u, y | \omega)
\]

We expand this expression in ‘opacity’ (=density of scattering centers times dipole cross section)

\[
K(s, y; u, y) = K_0(s; u) - dr \, d\xi \, K_0(s, y; r, \xi) n(\xi) \sigma(r) K_0(r, \xi; u, y) + ....
\]

To zeroth order, there is no medium (vacuum case), and one finds:

\[
\omega \frac{dI^{(0)}}{d\omega \, dk_T} = \frac{\alpha_s C_F}{\pi^2} H(k_T) = \left| \begin{array}{c} \times \end{array} \right| 2, \quad H(k_T) = \frac{1}{k_T^2}
\]

So, in the vacuum, the gluon energy distribution displays the dominant \(1/k^2\) piece of the DGLAP parton shower.
IV.7. Opacity Expansion - up to 1st order

To first order in opacity, there is a generally complicate interference between vacuum radiation and medium-induced radiation.

\[
\omega \frac{dI^{(1)}}{d\omega dk_T} = \begin{array}{c}
\left[ \text{Figures of vacuum and medium radiation} \right] + \\
\text{Rescattering of vacuum term}
\end{array}
\]

In the parton cascade limit \( L \to \infty \), we identify three contributions:

1. **Probability conservation** of medium-independent vacuum terms.
2. **Transverse phase space** redistribution of vacuum piece.
3. **Medium-induced gluon radiation** of quark coming from minus infinity

\[
\lim_{L \to \infty} nL = \text{const} \omega \frac{dI^{(1)}}{d\omega dk_T} = -w_1 H(k_T) + nL \int_{q_T} dq_T \left[ R(q_T, k_T) + H(q_T + k_T) \right]
\]

U.A. Wiedemann
BDMPS-Z calculates in the kinematic regime

\[ E \gg \omega \gg \left| k_T \right|, \left| q_T \right| \gg \Lambda_{QCD} \]

Elastic cross section in this limit

\[ \frac{1}{\omega} |A(q)|^2 R(k,q) = \sum_{i=2}^{n} \frac{1}{\omega_i} |A(q_i)|^2 \cdot R(k, q_i) \]

Incoherent limit

\[ \propto \sum_{i=1}^{\infty} \left| A(q_i) \right|^2 R(k, q_i) \]

Inelastic cross section for multiple scattering

\[ \propto \sum_{i=1}^{\infty} \left| A(q_i) \right|^2 R(k, q_i) \]

Coherent limit
IV.9. Example: N=2 opacity

\[ \frac{dI(N = 2)}{d \ln \omega \, dk_T} = \frac{\alpha_s C_R}{\pi^2} \left( dq_1 \left( |A(q_1)|^2 - \sigma_{el} \delta(q_1) \right) dq_2 \left( |A(q_2)|^2 - \sigma_{el} \delta(q_2) \right) \right) \]

(4.15)

\[ \frac{(nL)^2}{2} R(k + q_1; q_2) - n^2 \frac{1 - \cos LQ_1}{Q_1^2} \left\{ R(k + q_1; q_2) - R(k; q_1 + q_2) \right\} \]

Incoherent

Coherent

Formation times

(4.16)

\[ \tau_{f,n} = \frac{1}{Q_n} = \frac{2\omega}{k_T + \sum_{i=1}^{n} q_i^2} \]

define interpolation scale between totally coherent and incoherent limit

(4.17)

\[ n^2 \frac{1 - \cos LQ_1}{Q_1^2} \rightarrow \begin{cases} 0 & , L > \tau_{f1} \\ n^2 L^2 / 2 & , L < \tau_{f1} \end{cases} \]

Formally, determine totally coherent and incoherent limiting cases by taking \( L \rightarrow 0 \) or \( L \rightarrow \infty \) for \( nL = \text{fix} \)
IV.10. Main take-home message

• In high-energy limit, the medium-induced splitting $a \rightarrow b+c$, i.e., medium-induced gluon radiation) is regarded as the **most efficient mechanism to degrade energy of partonic projectile $a$**.
It is more efficient than collisional mechanism $a+b \rightarrow a'+b'$

• Medium-induced gluon radiation has two ‘classical’ limiting cases:
  - **incoherent limit**: radiation = incoherent sum of radiation from all independent scattering centers
  - **coherent limit**: all scattering centers act coherently, as if radiation occurs from one scattering center with $q = \text{sum of the } q_i$

• The **interpolating scale** between coherent and incoherent limits is set by the **gluon formation time**

• Medium-induced quantum interference leads to characteristic **parametric dependencies** of medium-induced gluon radiation, in particular

$$\omega \frac{dI_{\text{med}}}{d\omega} \Delta \propto \alpha_s \sqrt{\frac{\omega_c}{\omega}}, \quad \omega_c = \hat{q}L^2 / 2 \quad \langle k_T^2 \rangle \propto \hat{q}L \quad \Delta E \propto \hat{q}L^2$$
IV.11. Estimating Time scales for parton E-loss

\[ L_{\text{hadr}} = \text{const} \frac{E}{Q^2} \]

\[ \Delta E \approx E \approx \alpha_s \hat{q} L_{\text{therm}}^2 \]

Dynamics of the bulk
Dynamics of hadronization
Partonic equilibration processes
Jet absorption
Jet modification

\[ L_{\text{medium}} \]

100 fm
1 fm

1 GeV
10 GeV
100 GeV

$$R_{AA}(p_T, \eta) = \frac{dN^{AA}/dp_T d\eta}{n_{coll} dN^{NN}/dp_T d\eta}$$

$R_{AA}(p_T) = 1.0$  
no suppression

$R_{AA}(p_T) = 0.2$  
factor 5 suppression

Centrality dependence:

0-5%  
L large

70-90%  
L small
IV.13. Suppression at high $p_T$ at RHIC

Centrality dependence:

- 0-5% (L large)
- 70-90% (L small)

$\pi^0$ 0-5% Central PHENIX
IV.14. Suppression persists to highest $p_T$

- Spectra in AA and pp-reference
- Nuclear modification factor shows $p_T$-dependence

ALICE, PLB 696 (2011) 30
IV.15. The Matter is Opaque at RHIC

- STAR azimuthal correlation function shows ~ complete absence of “away-side” (high-pt) particle

\[ \Delta \Phi = 0 \]

Partner in hard scatter is *completely absorbed* in the dense medium
IV.16. Dijet asymmetries at LHC

ATLAS

Trigger jet $E_T \sim 100$ GeV

Recoil GONE Or reduced

Calorimeter Towers
IV.17. Dijet asymmetry

\[ A_J = \frac{E_{T,1} - E_{T,2}}{E_{T,1} + E_{T,2}} \]

Tremendous recent progress on jet finding algorithms:

- novel class of IR and collinear safe algorithms satisfying SNOWMASS accords
  - $kt(FastJet)$
  - $anti-kt(FastJet)$
  - $SiSCon$e
- new standard for $p+p@LHC$
- fast algorithms, suitable for heavy ions!

Catchment area of a jet:

- novel tools for separating soft fluctuations from jet remnants
- interplay with MCs of jet quenching needed
IV.19. Jet Finding at high event multiplicity (exp)

- Impressive experimental checks
  - energy ‘lost’ from jet cone
    - found completely out-of-cone
    - found in soft components at very large angles
  - Angular distribution of dijets almost unchanged

Some Qualitative Considerations

**Problem 1:** How can the suppression of $R_{AA}$ and the quenching of reconstructed jets be understood in the same dynamical picture?

**Problem 2:** How can **this jet broaden** (as suggested by $A_j$-dependence) while $\Delta \Phi$ - dependence is almost unaffected?
IV.20. Jet quenching via jet collimation?


In medium, formation times of soft partons are shorter

\[
\tau^\text{vac}_f \approx \frac{\omega}{k_T^2} = \frac{1}{\theta^2 \omega}, \quad \tau^\text{med}_f \approx \frac{\omega}{k_T^2} = \sqrt{\frac{\omega}{\hat{q}}}
\]

So \textbf{soft gluons} are there early in the shower and they are radiated at larger angle

\[
\langle \theta^2 \rangle = \frac{\langle k_T^2 \rangle}{\omega^2} = \frac{\hat{q} L}{\omega^2}
\]

A significant fraction of the total jet energy is soft modes

And can be radiated at angles

\[
\langle \theta \rangle \ll 1
\]

Complete decorrelation from jet axis!
IV. 21. Jet quenching via jet collimation:

Facts from first data on dijet asymmetry:

- in Pb-Pb, on average, > 10 GeV more radiated outside cone of recoil jet

- Aj-distribution is broad: some event fraction radiates >20 GeV more energy outside cone of recoil jet

- If 20 GeV were radiated in single component, this would induce significant ΔΦ-broadening, which is not observed

=> medium-induced radiation must be in multiple soft components

Estimate: $30 \leq \hat{q} L \leq 90 \text{GeV}^2$ gets $O(10 \text{GeV})$ out of jet cone.
IV. 22. Towards a MC of jet quenching

Qualitatively:
Radiative parton energy loss naturally contains key elements for understanding quenching of reconstructed jets:

- medium-induced gluon formation time shows inverted dependence on gluon energy
- size of dijet asymmetry (~ 10 GeV outside wide cone) can be accommodated naturally

How to get from qualitative considerations to quantitative analysis?
Strategy pursued here:

- start from analytically known baseline (BDMPS)
- find exact MC implementation of this baseline
- extend MC algorithm to go beyond eikonal limit
- do physics …
Recall IV.9. Example: N=2 opacity

\[
\frac{dI(N = 2)}{d \ln \omega \, dk_T} = \frac{\alpha_s C_R}{\pi^2} \left[ dq_1 \left( |A(q_1)|^2 - \sigma_{el} \delta(q_1) \right) \right] \left[ dq_2 \left( |A(q_2)|^2 - \sigma_{el} \delta(q_2) \right) \right]
\]

(4.15)

\[
\left( nL \right)^2 R(k + q_i; q_j) - n^2 \frac{1 - \cos LQ_1}{Q_1^2} \left\{ R(k + q_i; q_j) - R(k; q_i + q_j) \right\}
\]

Incoherent

\[
\text{Coherent}
\]

Formation times

(4.16)

\[
\tau_{f, n} = \frac{1}{Q_n} = \frac{2\omega}{k_T + \sum_{i=1}^{n} q_i^2}
\]

define interpolation scale between totally coherent and incoherent limit

(4.17)

\[
n^2 \frac{1 - \cos LQ_1}{Q_1^2} \rightarrow \begin{cases} 
0, & L > \tau_{f1} \\
n^2 L^2/2, & L < \tau_{f1}
\end{cases}
\]

Formally, determine totally coherent and incoherent limiting cases by taking \( L \to 0 \) or \( L \to \infty \) for \( nL = \text{fix} \).
Basic idea of jet quenching Monte Carlo:

JHEP 1107:118, 2011

Gluon fragmentation in vacuum shows interference pattern:
Implemented probabilistically e.g. via angular ordering constraint

Gluon fragmentation in medium shows interference pattern:
Implemented probabilistically via formation time constraint
IV.23. Input parameters for MC algorithm

Elastic mean free path

\[ \lambda_{el} = \frac{1}{n \sigma_{el}} = \frac{1}{n \int dq |A(q)|^2} \]

\[ |A(q)|^2 \propto \frac{1}{(q^2 + \mu^2)^2} \]

Inelastic mean free path

\[ \lambda_{inel} = \frac{1}{n \sigma_{inel}} \]

\[ \frac{d\sigma_{inel}}{d\omega dk} \propto \frac{1}{\omega} |A(q)|^2 R(k,q) \frac{\lambda_{inel}}{|A(q)|^2} \frac{\alpha_s C_R}{\omega} \delta(k - q) \]

The QCD coupling is then defined by

\[ \alpha_s C_R = \frac{\lambda_{el}}{\lambda_{inel} \log[\omega_{\text{max}} / \omega_{\text{min}}]} \]
MC algorithm for gluon number distributions

Aim: first illustration without kinematic complications

Algorithm in the totally incoherent limit:

1. Decide whether and where projectile parton scatters via no scattering probability and density distribution of scatterer

\[ S_{no}^{proj}(0;L) = \exp \left( -\frac{L}{\lambda_{inel}} \right) \]

\[ \Sigma(0;\xi) = -\frac{dS_{no}^{proj}(0;\xi)}{d\xi} \]

1. After scattering, continue propagating projectile parton inelastically
2. After inelastic scattering, continue propagating produced gluon by counting the number of elastic scatterers between \( \xi \) and \( L \)

Analytical result from BDMPS:
average number of gluons produced with exactly \( j \) momentum in incoherent limit

\[ \langle N_{gluons}^{incoh} \rangle_j = \exp \left( \frac{L}{\lambda_{el}} \right) \times \frac{1}{N!} \left( \frac{L}{\lambda_{el}} \right)^{N-1} \frac{L}{\lambda_{inel}} = \frac{L}{\lambda_{inel}} \frac{\Gamma(j) - \Gamma\left(j;\frac{L}{\lambda_{el}}\right)}{\lambda_{el} \Gamma(j)} \]
MC algorithm in the totally coherent limit

- **Basic problem 1:**
  must start with the same no-scattering probability as in incoherent case
  But: $S^{\text{proj}}_{\text{no}}(0;L)$ overestimates scattering probability in the case of coherence
  $\Rightarrow$ Accept produced gluon with **coherent weight**

- **Basic problem 2:**
  algorithm selects initially sharp position $\xi$ for production of gluon
  But: non-zero formation time $\Rightarrow$ **production point localized around** $\xi$.
  $\Rightarrow$ in MC, gluon can undergo additional elastic scattering in a region, starting as early as $\xi - \tau_f$
Comparing MC algorithm and analytic results

The average number of gluons produced with exactly $j$ momentum transfers from the medium:

$$\lambda_{\text{inel}} = 1.0 \text{ fm}, \quad \lambda_{\text{el}} = 0.1 \text{ fm}, \quad L = 1.3 \text{ fm}$$

$$\left\langle N_{\text{gluons}}^{\text{coh}} \right\rangle_j = \left\langle N_{\text{gluons}}^{\text{coh}} \right\rangle(N = j) = \frac{1}{N!} \frac{L}{\lambda_{\text{el}}} \frac{N^{-1}}{\lambda_{\text{inel}}} \exp \left( -\frac{L}{\lambda_{\text{el}}} \right)$$
Basic idea of jet quenching Monte Carlo:
Gluon fragmentation in vacuum shows interference pattern:
Implemented probabilistically e.g. via angular ordering constraint
Gluon fragmentation in medium shows interference pattern:
Implemented probabilistically via formation time constraint

JHEP 1107:118, 2011

The resulting local and probabilistic Monte Carlo algorithm provides a quantitatively exact implementation of all features of the BDMPS-Z formalism.
IV.24. JEWEL: jet evolution with energy loss


Anchor modeling on theoretically well-controlled limits:

Note: there are many complementary works to implement jet quenching in MC event generators
IV.25. JEWEL: baseline and $R_{AA}$

K. Zapp, et al., arXiv:1111.6838v2

The graph shows the comparison between PHENIX $p+p$ data and JEWEL+PYTHIA for $\pi^0$ production. The plots are on a log scale for the data density and a linear scale for the transverse momentum ($p_\perp$) range from 2 to 18 GeV. The MC/data ratio is also displayed, indicating the agreement between the model predictions and experimental data.
Nuclear Modification Factor @ RHIC & LHC

K. Zapp, et al., arXiv:1111.6838v2