Selected Topics in the Theory of Heavy Ion Collisions

Lecture 1

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Heavy Ion Collisions - Experiments

- Alternating Gradient Synchrotron (AGS) at Brookhaven BNL
  - variety of beams, since mid 1980's

$$\left.\sqrt{s_{NN}}\right|_{Au+Au}^{AGS} \cong 2-5\,GeV$$

- CERN SPS fixed target experiments
  - variety of beams, Pb-beams since 1994

$$\left.\sqrt{s_{NN}}\right|_{Pb+Pb}^{SPS} \leq 17\,GeV$$

- Relativistic Heavy Ion Collider RHIC at BNL
  - since 2000, p+p, d+Au, Au+Au, Cu-Cu, …

$$\left.\sqrt{s_{NN}}\right|_{Au+Au}^{RHIC} \leq 200\,GeV$$

- Large Hadron Collider LHC
  - since 2000, so far p+p, Pb+Pb, …

$$\left.\sqrt{s_{NN}}\right|_{Pb+Pb}^{LHC} = 2.75\,TeV$$

- total cross section: $\sigma_{total}^{Pb+Pb} \approx 8\,barn = 8 * 10^{-24}\,cm^2$
- maximal luminosity: $L_{\max,LHC}^{Pb+Pb} \approx 10^{27}\,cm^{-2}s^{-1}$

$\Rightarrow$ 8000 collisions per second!

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What is measured when at the LHC?

$\sigma_{total}^{Pb+Pb} \approx 8\, barn = 8 * 10^{-24}\, cm^2$ $\quad L_{\max,LHC}^{Pb+Pb} \approx 10^{27}\, cm^{-2} s^{-1}$ $\quad 1\, month \approx 10^6\, s \approx 1\, LHCyr\ Pb+Pb$

When?	15 min~10^3 s (ideal)	1^{st} month, 2010	1 month, 2011	2-3 yrs
How much data?	$L_{\rm int}^{Pb+Pb} \approx 1\, \mu b^{-1}$	$L_{\rm int}^{1st\, year} \approx 7-8\, \mu b^{-1}$	$L_{\rm int}^{Pb+Pb} \approx 150\, \mu b^{-1}$	p+Pb

What?

- Event multiplicity
- Low-p_T hadron spectra
- …

- Abundant high-p_T processes such as jets

- Rare and leptonic processes

Strategy for these lectures:

- explain basic theory for data accessible at the LHC
  (and say where it is incomplete)

- explain theory in the order in which data will become accessible

- give motivation for measurement by explaining measurement
  (not before)

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…last introductory remark…

Fundamental question:

How do collective phenomena and macroscopic properties of matter emerge from the interactions of elementary particle physics?

Heavy Ion Physics: addresses this question
- for Quantum Chromodynamics (QCD)
- in the regime of the highest temperatures and densities accessible in the laboratory

How?
1. **Benchmark:** establish baseline, in which collective phenomenon is absent.
2. **Establish collectivity:** by characterizing deviations from baseline
3. **Seek dynamical explanation**, ultimately in terms of QCD.

These lectures give examples of this 'How?'

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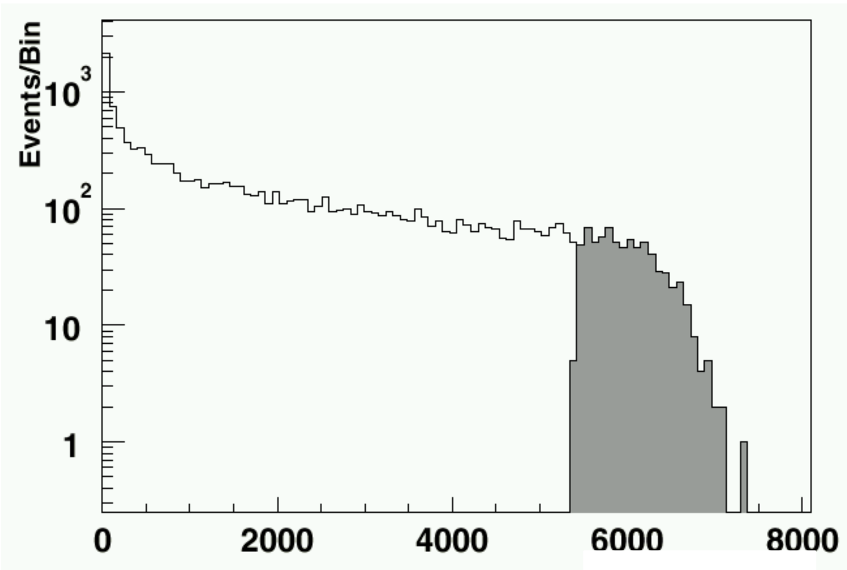
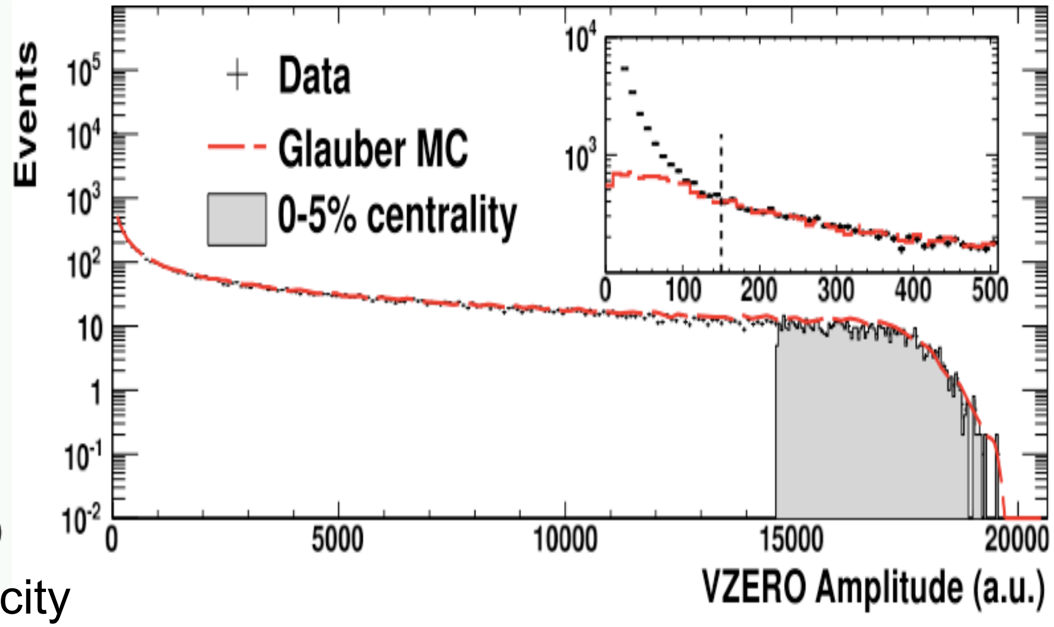
I.1. The very first measurement at a Heavy Ion Collider

PHOBOS, RHIC, 2000



Signal proportional to multiplicity

ALICE, PRL 105 (2010) 252301, arXiv:1011.3916



What is the benchmark for multiplicity distributions?

Multiplicity in inelastic A+A collisions is incoherent superposition of inelastic p+p collisions.

(i.e. extrapolate p+p -> p+A -> A+A without collective effects)

→ **Glauber theory**

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I.2. Glauber Theory

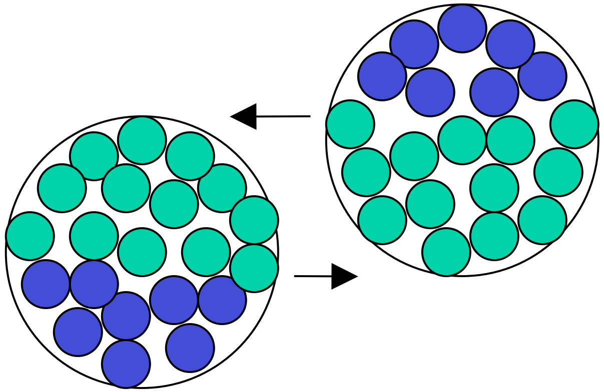
Assumption: inelastic collisions of two nuclei (A-B) can be described by incoherent superposition of the collision of "an equivalent number of nucleon-nucleon collisions".

How many?

Establish counting based on

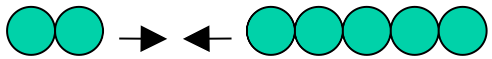
Spectator nucleons

Participating nucleons



To calculate N_{part} or N_{coll}, take

σ = inelastic n-n cross section

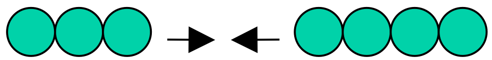


$N_{part} = 7$

$N_{coll.} = 10$

$N_{quarks+gluons} = ?$

A priori, no reason for this choice other than that it gives a useful parameterization.



$N_{inelastic} = 1$

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I.3. Glauber theory for n+A

We want to calculate:

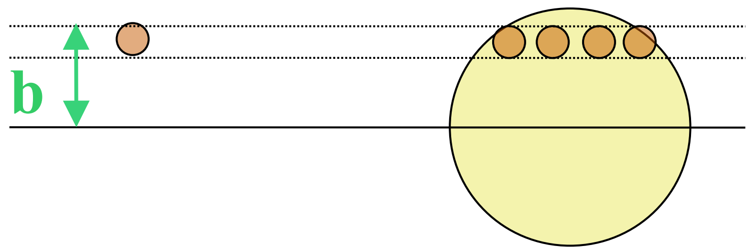
N_{part} = number of participants = number of 'wounded nucleons',
which undergo at least one collision

N_{coll} = number of n+n collisions,
taking place in an n+A or A+B collision

We know the single nucleon probability distribution within a nucleus A,
the so-called nuclear density

$$\rho(b,z)$$

(1.1)
$$\int dz\, db\, \rho(b,z) = 1$$



Normally, we are only interested in the transverse density,
the nuclear profile function

(1.2)
$$T_A(b) = \int_{-\infty}^{\infty} dz\; \rho(b,z)$$

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I.4. Glauber theory for n+A

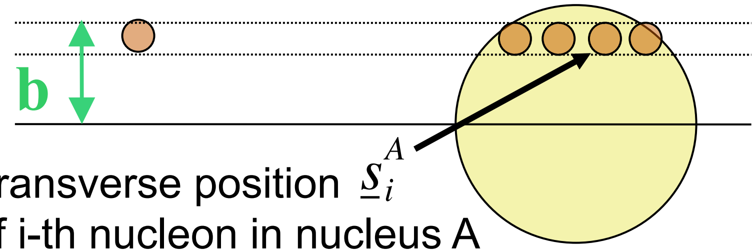
The probability that no interaction occurs at impact parameter b:

(1.3) $$P_0(\underline{b}) = \prod_{i=1}^{A} \left[1 - \int d\underline{s}_i^A T_A\left(\underline{s}_i^A\right) \sigma\left(\underline{b} - \underline{s}_i^A\right) \right] \qquad \int d\underline{s}\, \sigma(\underline{s}) = \sigma_{nn}^{inel}$$

If nucleon much smaller than nucleus

(1.4) $$\sigma(\underline{b} - \underline{s}) \approx \sigma_{nn}^{inel}\, \delta(\underline{b} - \underline{s})$$

(1.5) $$P_0(\underline{b}) = \left[1 - T_A\left(\underline{b}\right) \sigma_{nn}^{inel} \right]^A$$



Transverse position $\underline{s}_i^A$
of i-th nucleon in nucleus A

The resulting nucleon-nucleon cross section is:

(1.6) $$\sigma_{nA}^{inel} = \int d\underline{b} \left(1 - P_0(\underline{b})\right) = \int d\underline{b} \left[1 - \left[1 - T_A\left(\underline{b}\right) \sigma_{nn}^{inel} \right]^A \right]$$

$$\xrightarrow{A>>n} \boxed{\int d\underline{b} \left[1 - \exp\left[-A T_A\left(\underline{b}\right) \sigma_{nn}^{inel} \right] \right]}$$ Optical limit

(1.7) $$= \int d\underline{b} \left[A T_A\left(\underline{b}\right) \sigma_{nn}^{inel} - \frac{1}{2} \left(A T_A\left(\underline{b}\right) \sigma_{nn}^{inel} \right)^2 + \dots \right]$$

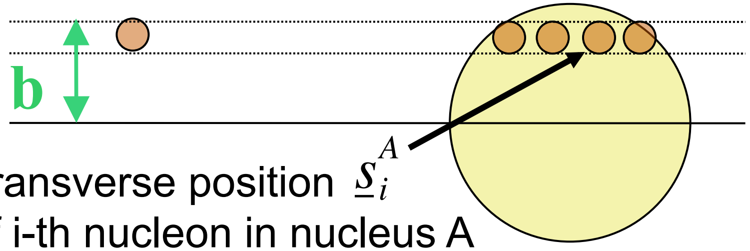
Double counting correction

I.5. Glauber theory for n+A

To calculate number of collisions: probability of interacting with i-th nucleon in A is

(1.8) $$p(\underline{b},\underline{s}_i^A) = \int d\underline{s}_i^A T_A\left(\underline{s}_i^A\right)\sigma\left(\underline{b}-\underline{s}_i^A\right) = T_A(\underline{b})\,\sigma_{nn}^{inel}$$

Probability that projectile nucleon undergoes n collisions
= prob that n nucleons collide and A-n do not



Transverse position $\underline{s}_i^A$
of i-th nucleon in nucleus A

(1.9) $$P(\underline{b},n) = \binom{A}{n}(1-p)^{A-n}\,p^n$$

Average number of nucleon-nucleon collisions in n+A

(1.10) $$\overline{N}_{coll}^{nA}(\underline{b}) = \sum_{n=0}^{A} n\,P(\underline{b},n) = \sum_{n=0}^{A} n\binom{A}{n}(1-p)^{A-n}\,p^n = A\,p$$

$$= \boxed{A\,T_A(\underline{b})\,\sigma_{nn}^{inel}}$$

Average number of nucleon-nucleon collisions in n+A

(1.11) $$\overline{N}_{part}^{nA}(\underline{b}) = 1 + \overline{N}_{coll}^{nA}(\underline{b})$$

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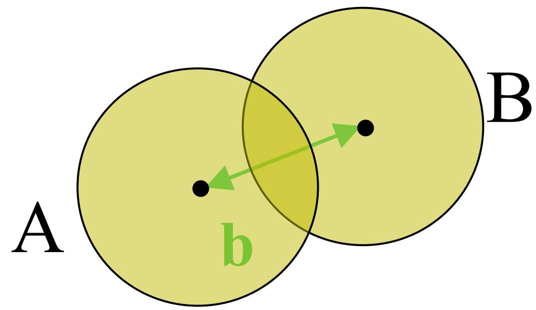
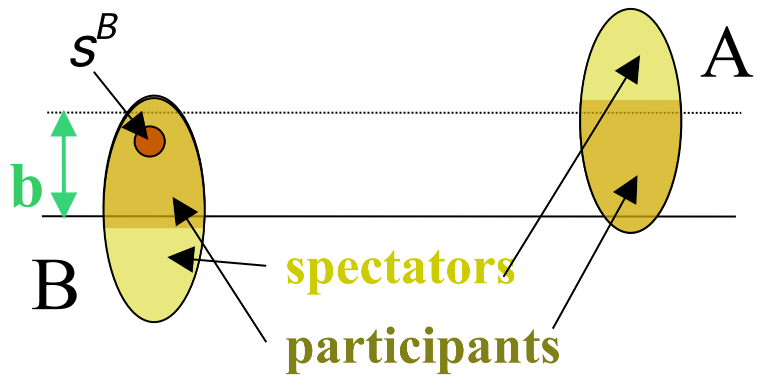
I.6. Glauber theory for A+B collisions

We define the nuclear overlap function



(1.12) $$T_{AB}(\vec{b}) = \int_{-\infty}^{\infty} d\vec{s}\; T_A(\vec{s})\, T_B(\vec{b}-\vec{s})$$

The average number of collisions of nucleon at s^B with nucleons in A is



(1.13) $$\overline{N}^{nA}_{coll}(\underline{b}-\underline{s}^B) = A\, T_A(\underline{b}-\underline{s}^B)\, \sigma^{inel}_{nn}$$

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The number of nucleon-nucleon collisions in an A-B collision at impact parameter b is

(1.14) $$\overline{N}^{AB}_{coll}(\underline{b}) = B\int d\underline{s}^B\, T_B(\underline{s}^B)\, \overline{N}^{nA}_{coll}(\underline{b}-\underline{s}^B)$$

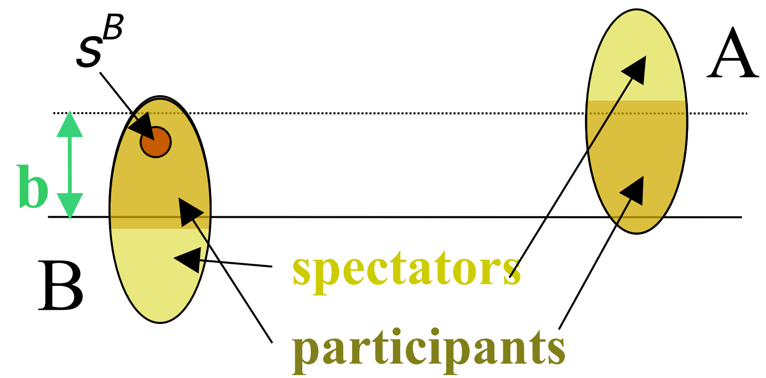
$$= AB\int d\underline{s}\, T_B(\underline{s})\, T_B(\underline{b}-\underline{s})\, \sigma^{inel}_{nn}$$

$$= A\,B\,T_{AB}(\underline{b})\, \sigma^{inel}_{nn}$$

determined in terms of nuclear overlap only

I.7. Glauber theory for A+B collisions

Probability that nucleon at s^B in B is wounded by A in configuration $\{s_i^A\}$



(1.15) $$p(\underline{s}^B,\{\underline{s}_i^A\}) = 1 - \prod_{i=1}^{A}\left[1 - \sigma(\underline{s}^B - \underline{s}_i^A)\right]$$

Probability of finding w_B wounded nucleons in nucleus B:

(1.16) $$P(w_b,\underline{b}) = \binom{B}{w_B}\left(\prod_{i=1}^{A}\prod_{j=1}^{B}\int d\underline{s}_i^A\, d\underline{s}_j^B\, T_A(\underline{s}_i^A)\, T_B(\underline{s}_j^B - \underline{b})\right) p(\underline{s}_1^B,\{\underline{s}_i^A\})\dots$$

$$\dots p(\underline{s}_{w_B}^B,\{\underline{s}_i^A\})\left[1 - p(\underline{s}_{w_B+1}^B,\{\underline{s}_i^A\})\right]\dots\left[1 - p(\underline{s}_B^B,\{\underline{s}_i^A\})\right]$$

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Nuclear overlap function defines inelastic A+B cross section.

(1.17) $$\sigma_{AB}^{inel} = \int d\underline{b}\,\sigma_{AB}(\underline{b}) = \int d\underline{b}\, P(w_B = 0,\underline{b})$$

$$= \int d\underline{b}\left[1 - \left(\prod_{i=1}^{A}\prod_{j=1}^{B}\int d\underline{s}_i^A\, d\underline{s}_j^B\, T_A(\underline{s}_i^A)\, T_B(\underline{s}_j^B - \underline{b})\right)\prod_{j=1}^{B}\left[1 - p(\underline{s}_j^B,\{\underline{s}_i^A\})\right]\right]$$

$$\approx \int d\underline{b}\left[1 - \left[1 - T_{AB}(b)\,\sigma_{NN}^{inel}\right]^{AB}\right]$$

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I.8. Glauber theory for A+B collisions

It can be shown

Problem 1: derive the expressions (1.17), (1.19)
Use e.g. A. Bialas et al., Nucl. Phys. B111 (1976) 461

(1.18) Number of collisions: $$\overline{N}_{coll}^{AB}(\underline{b}) = AB\, T_{AB}(\underline{b})\, \sigma_{NN}^{inel}$$

(1.19) Number of participants: $$\overline{N}_{part}^{AB}(\underline{b}) = \frac{A\sigma_B^{inel}(\underline{b})}{\sigma_{AB}^{inel}(\underline{b})} + \frac{B\sigma_A^{inel}(\underline{b})}{\sigma_{AB}^{inel}(\underline{b})} \neq \overline{N}_{coll}^{AB}(\underline{b}) + 1$$

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1. There is a difference between 'analytical' and 'Monte Carlo' Glauber theory: For 'MC Glauber, a random probability distribution is picked from T_A.

2. The nuclear density is commonly taken to follow a Wood-Saxon parametrization (e.g. for A > 16)

(1.20) $$\rho(\vec{r}) = \rho_0 \Big/ \left(1 + \exp\left[-(r-R)/c\right]\right); \qquad R \equiv 1.07\, A^{1/3}\, fm, c = 0.545\, fm.$$

C.W. de Jager, H.DeVries, C.DeVries, Atom. Nucl. Data Table 14 (1974) 479

3. The inelastic Cross section is energy dependent, typically

(1.21) $$\sigma_{nn}^{inel} \approx 40\,(65)\, mb \qquad at \qquad \sqrt{s_{nn}} = 100\ (2700)\ GeV.$$

But σ_{nn}^{inel} is sometimes used as fit parameter.

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I.9 Event Multiplicity in wounded nucleon model

Model assumption: If $\bar{n}_{nn}$ is the average multiplicity in an n-n collision, then

(1.22) $$\bar{n}_{AB}(b) = \left(\frac{1-x}{2} \bar{N}^{AB}_{part}(b) + x\,\bar{N}^{AB}_{coll}(b) \right) \bar{n}_{NN}$$

is average multiplicity in A+B collision
(x=0 defines the wounded nucleon model).

The probability of having w_b wounded nucleons fluctuates around the mean,
so does the multiplicity n per event (the dispersion d is a fit parameter, say d~1)

(1.23) $$P(n,\underline{b}) = \frac{1}{\sqrt{2\pi\, d\, \bar{n}_{AB}(\underline{b})}} \exp\left(- \frac{\left[n - \bar{n}_{AB}(\underline{b}) \right]^2}{2d\, \bar{n}_{AB}(\underline{b})} \right)$$

How many events dN_{events} have event multiplicity dn?

(1.24) $$\boxed{\frac{dN_{events}}{dn} = \int db\, P(n,b) \underbrace{\left[1 - \left(1 - \sigma_{NN} T_{AB}(b) \right)^{AB} \right]}_{1-P_0(b)}}$$

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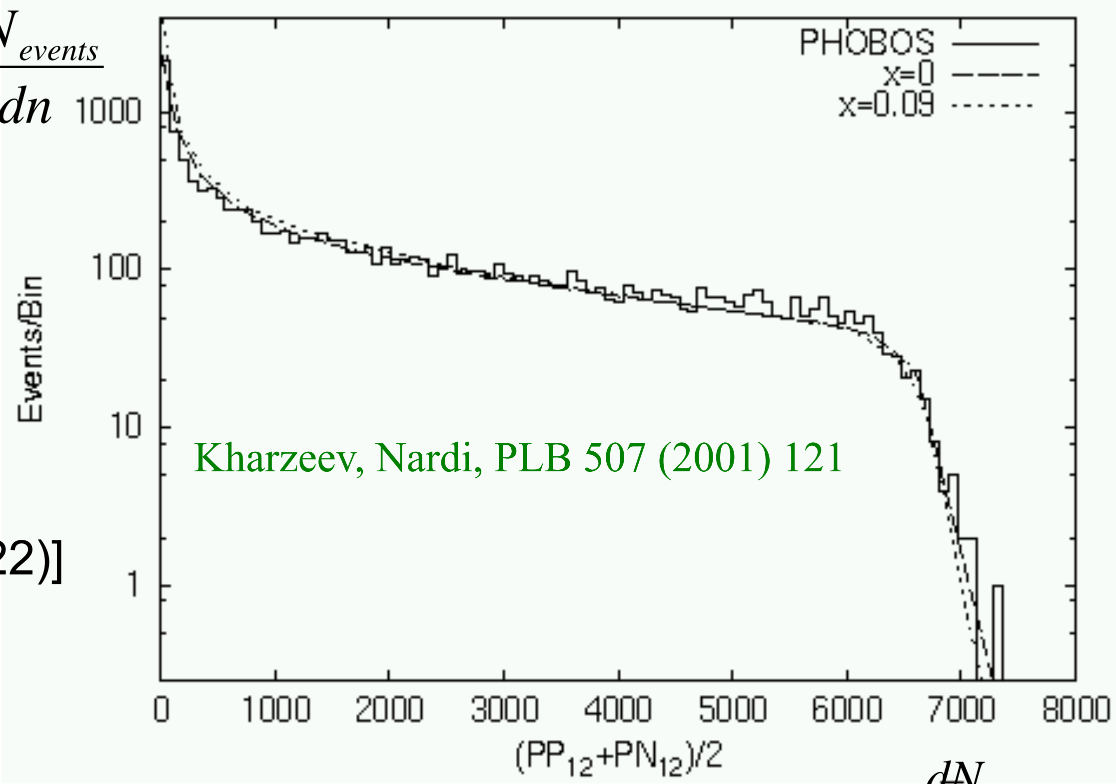
I.10 Wounded nucleon model vs. multiplicity

Compare data to multiplicity distribution (1.24): $\frac{dN_{events}}{dn} = \int d\underline{b}\, P(n,\underline{b})\left[1 - P_0(\underline{b})\right]$

$\frac{dN_{events}}{dn}$

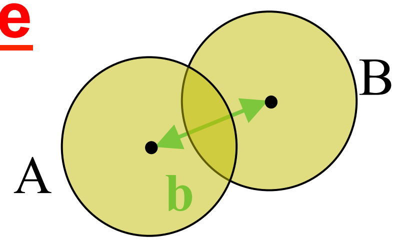


- dominated by geometry
- only weakly sensitive to details of particle production [e.g. weak dependence on x in (1.22)]
- insensitive to collective effects

Sensitivity to geometry but insensitivity to model-dependent dynamics makes $\frac{dN_{events}}{dn}$

A well-suited centrality measure

(i.e. a measure of the impact parameter b)



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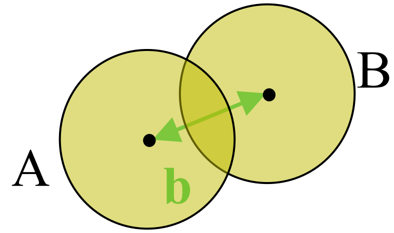
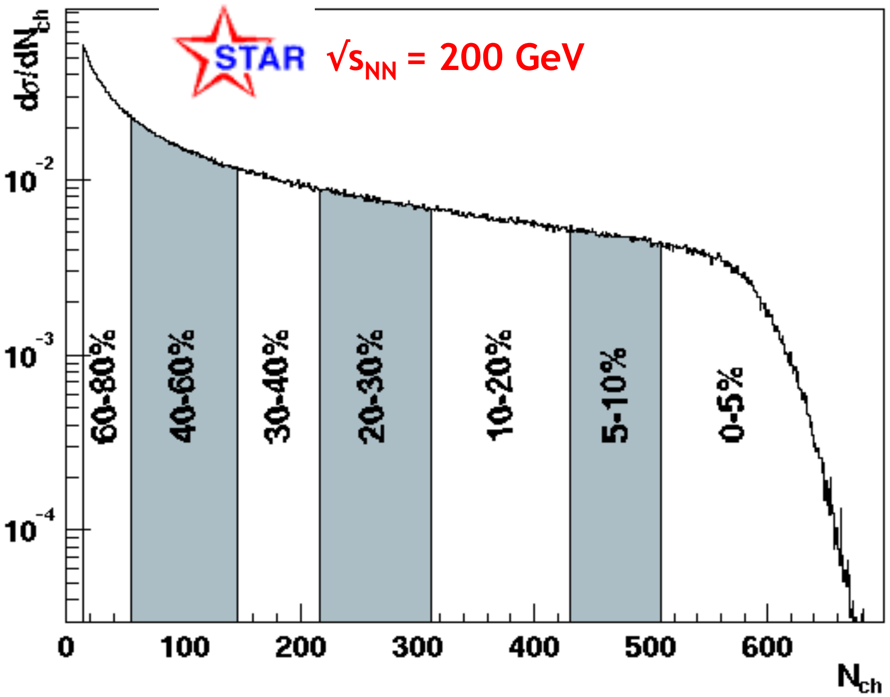
I.11. Multiplicity as a Centrality Measure

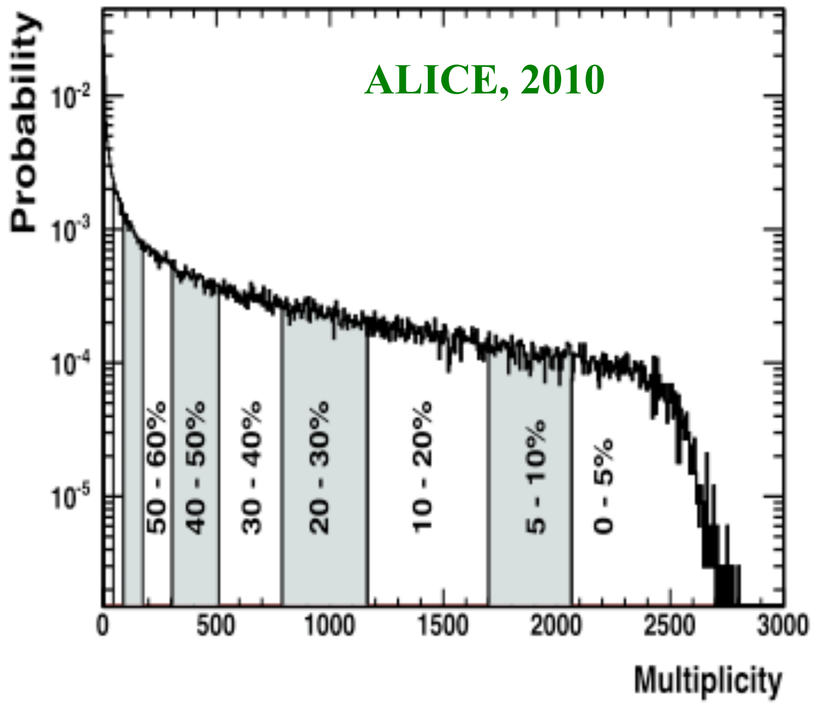
The connection between centrality and event multiplicity can be expressed in terms of

$$\text{(1.25)} \quad \left\langle N_{part}^{A+A} \right\rangle_{n>n_0} = \frac{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b}) \left[1 - P_0(\underline{b})\right] N_{part}(\underline{b})}{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b}) \left[1 - P_0(\underline{b})\right]}$$



- Centrality class = percentage of the minimum bias cross section





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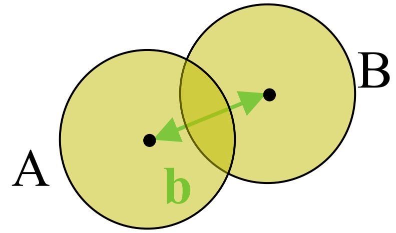
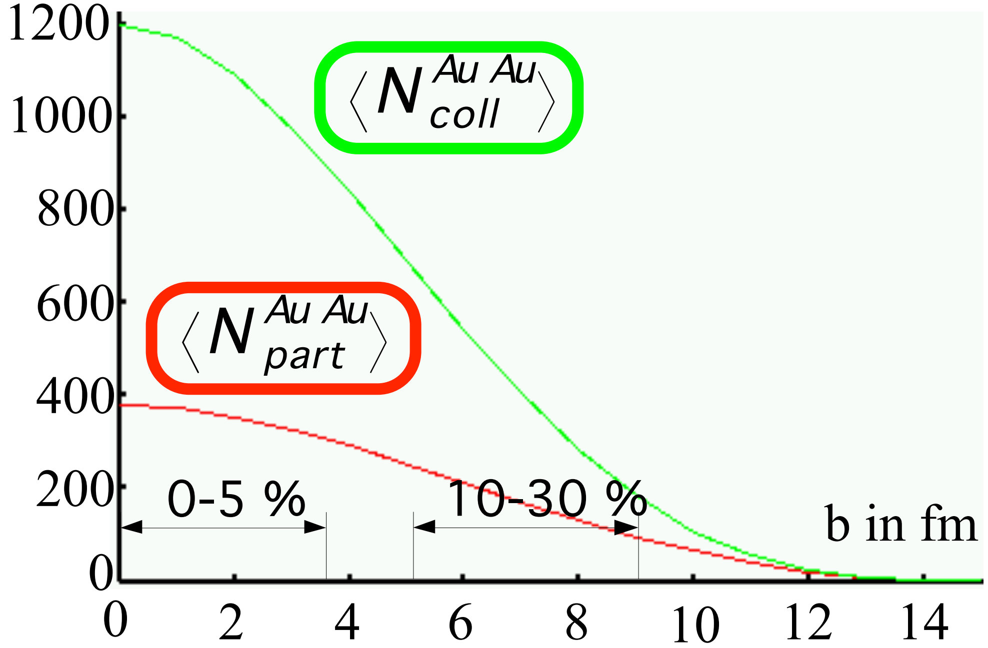
I.12. Centrality Class fixes Impact Parameter

The connection between centrality and event multiplicity can be expressed in terms of



$$\left\langle N^{A+A}_{part} \right\rangle_{n>n_0} = \frac{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b})\left[1-P_0(\underline{b})\right] N_{part}(\underline{b})}{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b})\left[1-P_0(\underline{b})\right]} \tag{1.25}$$

- Centrality class specifies range of impact parameters



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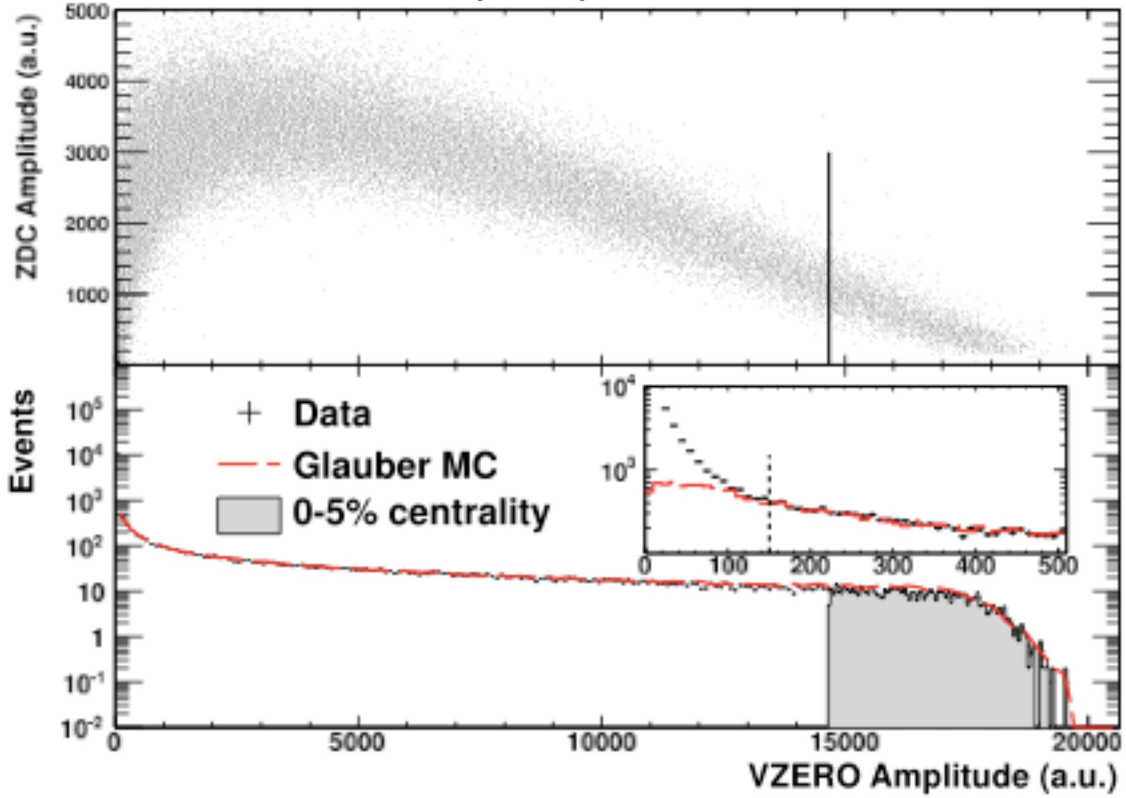
I.13. Cross-Checking Centrality Measurements

The interpretation of min. bias multiplicity distributions in terms of centrality measurements can be checked in multiple ways, e.g.

1. Energy E_F of spectators is deposited in Zero Degree Calorimeter (ZDC)

$$E_F = \left(A - N_{part}(b)/2\right)\sqrt{s}/2$$

ALICE, PRL 105 (2010) 252301, arXiv:1011.3916

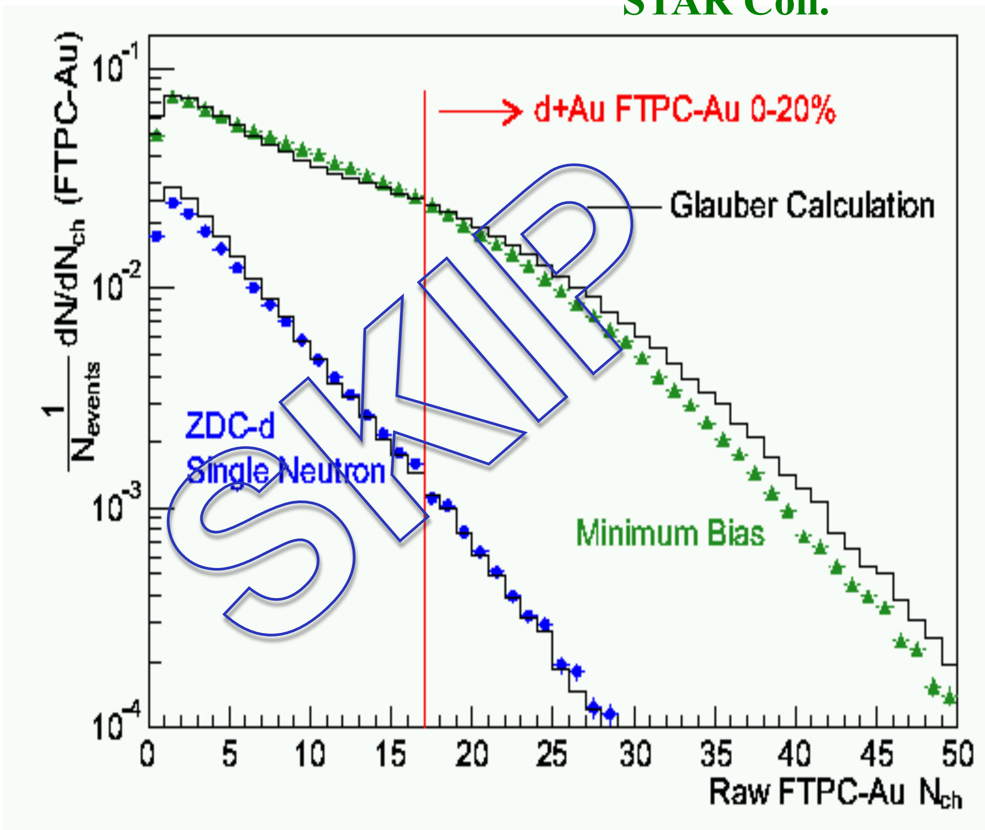


I.14. Cross-Checking Centrality Measurements

The interpretation of min. bias multiplicity distributions in terms of centrality measurements can be checked in multiple ways, e.g.

2. Testing Glauber in d+Au and in p+Au(+ n forward)

STAR Coll.



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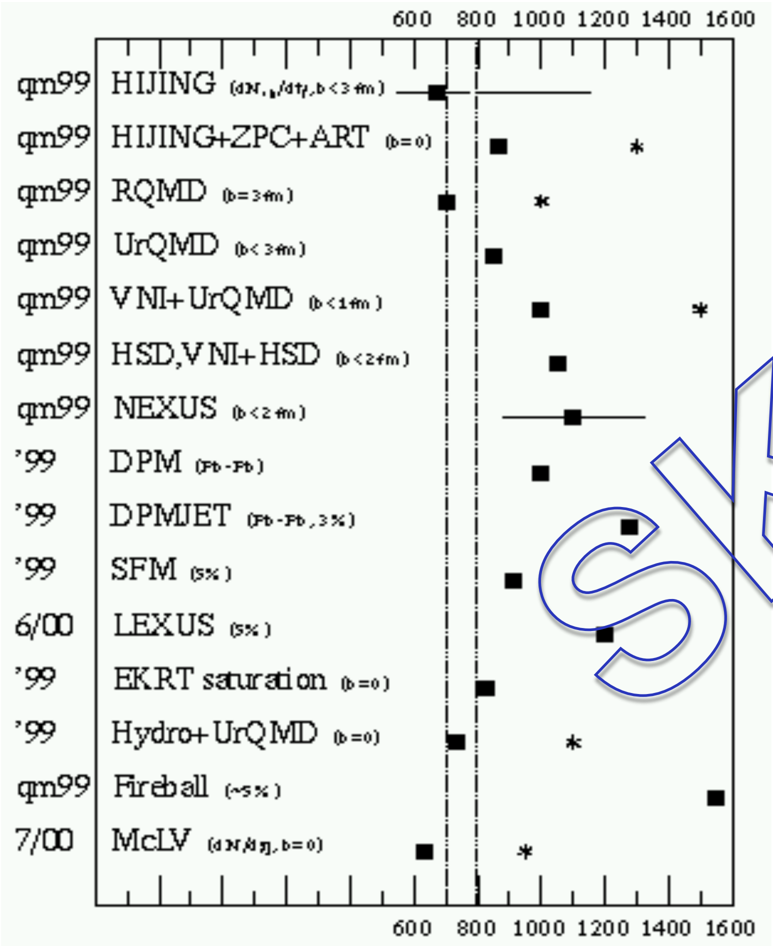
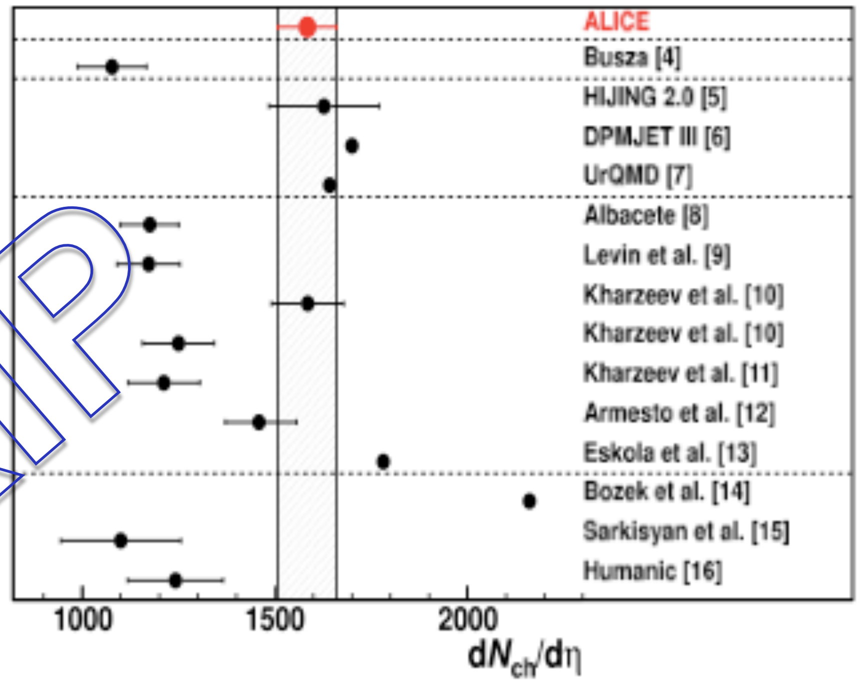
I.15. Final remarks on event multiplicity in A+B

There is **no 1st principle QCD calculation** of event multiplicity, neither in p+p nor in A+B

- Total charged event multiplicity: models failed to predict RHIC



- and failed to predict LHC



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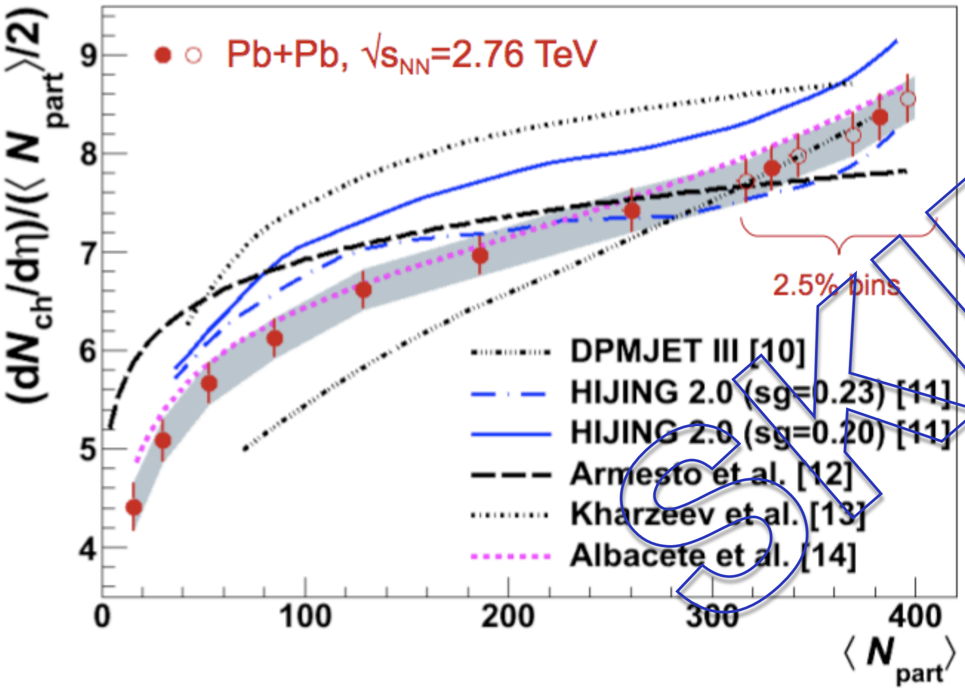
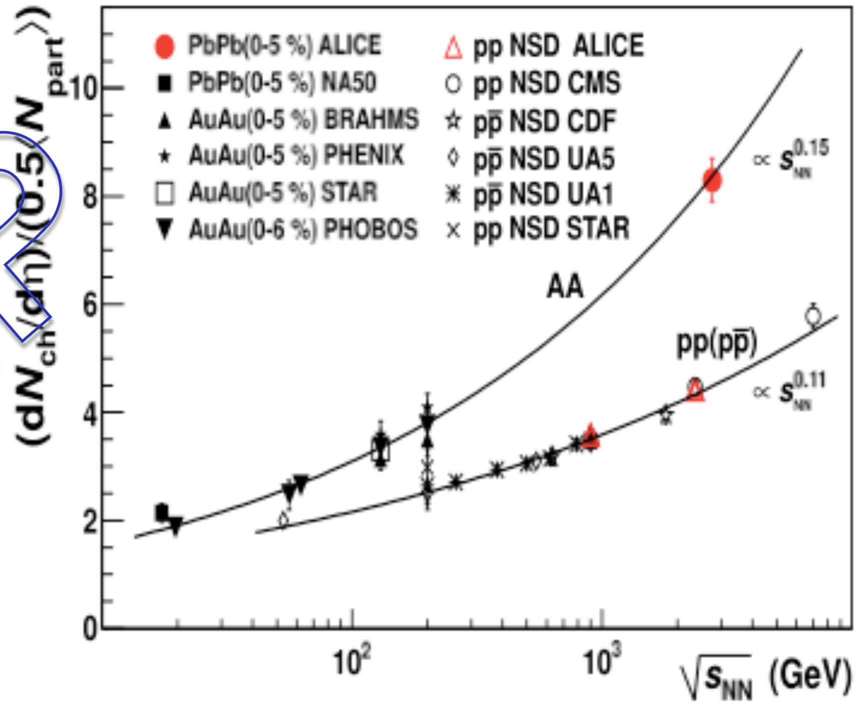
I.16. Final remarks on event multiplicity in A+B

There is **no 1st principle QCD calculation** of event multiplicity, neither in p+p nor in A+B

- Clear deviations from multiplicity of wounded nucleon model
- $\sqrt{s}$ - dependence of event multiplicity not understood in pp and AA

ALICE Coll., PRL 106, 032301 (2001) arXiv:1012.1657





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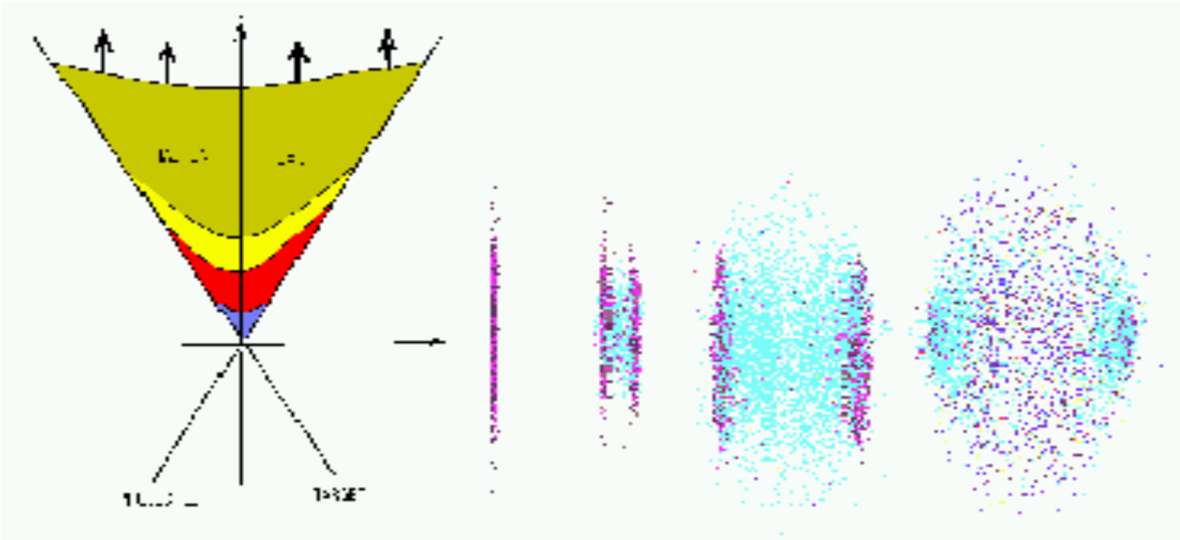
I.17. Final remarks on event multiplicity

Multiplicity distribution is not only used as centrality measure but:



Multiplicity (or transverse energy) constrains density of produced matter

Bjorken estimate

$$\boxed{\varepsilon(\tau_0) = \frac{1}{\pi R^2}\frac{1}{\tau_0}\frac{dE_T}{dy}}$$

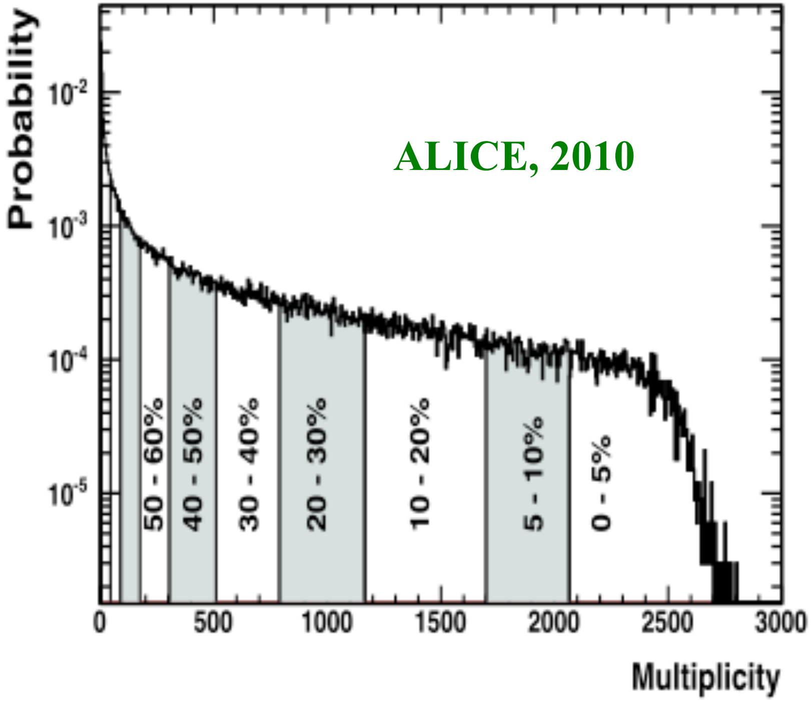
$$\frac{dE_T}{dy} \approx \frac{dN}{dy}\langle E_T \rangle$$

This estimate is based on geometry, thermalization is <u>not</u> assumed, numerically:

$$\varepsilon(\tau_0 \cong 1\, fm/c) = 3 - 4\, GeV / fm^3$$

II.1. Azimuthal Anisotropies of Particle Production

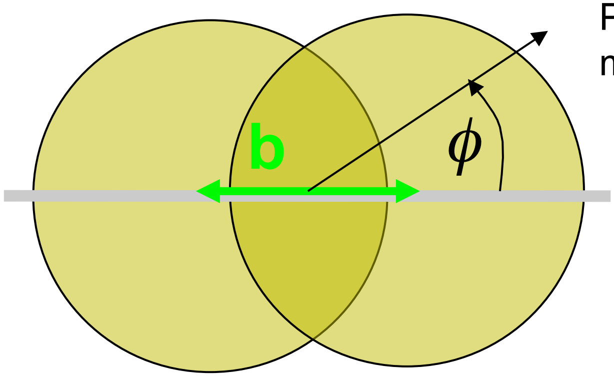
We know how to associate an impact parameter range $b \in [b_{\min}, b_{\max}]$ to an event class in A+A, namely by selecting a multiplicity class.



What can we learn by characterizing not only the modulus b, but also the orientation $\underline{b}$?

II.2. Particle production w.r.t. reaction plane

Consider single inclusive particle momentum spectrum



(2.1) $$f(\vec{p}) \equiv dN/E\,d\vec{p}$$

(2.2) $$\vec{p} = \begin{pmatrix} p_x = p_T \cos\phi \\ p_y = p_T \sin\phi \\ p_z = \sqrt{p_T^2 + m^2}\,\sinh Y \end{pmatrix}$$

To characterize azimuthal asymmetry, measure n-th harmonic moment of (2.1) in some detector acceptance *D* [phase space window in (p_T,Y)-plane].

(2.3) $$v_n \equiv \left\langle\left\langle e^{in\phi} \right\rangle\right\rangle = \left\langle \frac{\int_D d\vec{p}\, e^{in\phi} f(\vec{p})}{\int_D d\vec{p}\, f(\vec{p})} \right\rangle_{event\ average}$$

n-th order flow

<u>Problem</u>: Eq. (2.3) cannot be used for data analysis, since the orientation of the reaction plane is not known a priori.

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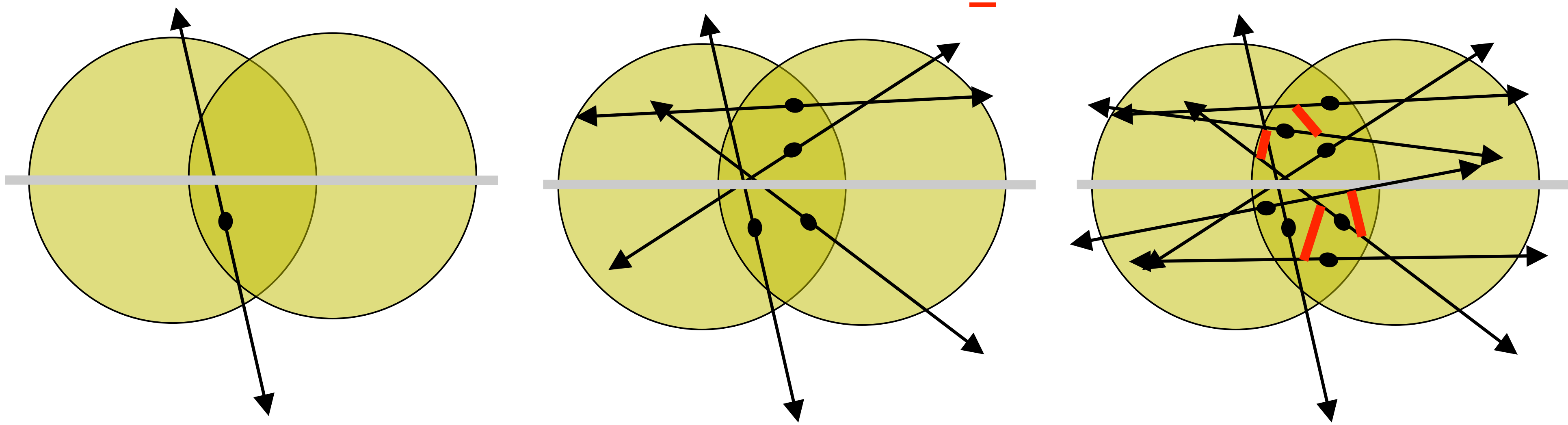
II.3. Why is the study of v_n interesting?



- Single 2->2 process
- Maximal asymmetry
- NOT correlated to the reaction plane

- Many 2->2 or 2-> n processes
- Reduced asymmetry $\sim 1/\sqrt{N}$
- NOT correlated to the reaction plane

- final state interactions
- asymmetry caused not only by multiplicity fluctuations
- collective component is correlated to the reaction plane

The azimuthal asymmetry of particle production has a collective and a random component. Disentangling the two requires a statistical analysis of finite multiplicity fluctuations.

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II.4. Cumulant Method

If reaction plane is unknown, consider particle correlations

(2.4) $$\left\langle e^{i\,n\,(\phi_1-\phi_2)}\right\rangle_{D_1 \wedge D_2} = \frac{\int_{D_1 \wedge D_2} d\vec{p}_1 d\vec{p}_2\, e^{i\,n\,(\phi_1-\phi_2)} f(\vec{p}_1,\vec{p}_2)}{\int_{D_1 \wedge D_2} d\vec{p}_1 d\vec{p}_2\, f(\vec{p}_1,\vec{p}_2)}$$

A two-particle distribution has an uncorrelated and a correlated part

(2.5) $$f(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1)f(\vec{p}_2) + \boxed{f_c(\vec{p}_1,\vec{p}_2)}$$

(2.6) Short hand $$(1,2) = (1)(2) + \boxed{(1,2)_c}$$

Correlated part

Assumption: Event multiplicity N>>1

⇒ correlated part is O(1/N)-correction to $f(\vec{p}_1)f(\vec{p}_2)$

(2.7) $$\left\langle\left\langle e^{i\,n(\phi_1-\phi_2)}\right\rangle\right\rangle = \boxed{v_n\{2\}\, v_n\{2\}} + \boxed{\underbrace{\left\langle\left\langle e^{i\,n(\phi_1-\phi_2)}\right\rangle^{corr}\right\rangle}_{O(1/N)}}$$

Flow via 2nd order cumulants

"Non-flow effects"

(2.8) If $v_n\{2\} >> \frac{1}{\sqrt{N}}$,then non-flow corrections are negligible.

What, if this is not the case?

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II.5. 4-th order Cumulants

2nd order cumulants allow to characterize v_n, if $v_n >> 1/\sqrt{N}$.
Consider now 4-th order cumulants:

(2.9)
$$(1,2,3,4) = (1)(2)(3)(4) + (1,2)_c(3)(4) + \ldots$$
$$+ (1,2)_c(3,4)_c + (1,3)_c(2,4)_c + (1,4)_c(2,3)_c$$
$$+ (1,2,3)_c(4) + \ldots$$
$$+ (1,2,3,4)_c$$

If the system is isotropic, i.e. $v_n(D)=0$, then k-particle correlations are unchanged by rotation $\phi_i \rightarrow \phi_i + \phi$ for all i, and only labeled terms survive. This defines

(2.9)
$$c_n\{4\} \equiv \left\langle\left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i\,n(\phi_1-\phi_3)}\right\rangle\right\rangle\left\langle\left\langle e^{i\,n(\phi_2-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i\,n(\phi_1-\phi_4)}\right\rangle\right\rangle\left\langle\left\langle e^{i\,n(\phi_2-\phi_3)}\right\rangle\right\rangle$$

For small, non-vanishing v_n, one finds

Borghini, Dinh, Ollitrault, PRC (2001)

(2.10)
$$c_n\{4\} = -v_n^4\{4\} + O\left(\tfrac{1}{N^3}, \tfrac{v_{2n}^2}{N^2}\right)$$

Improvement: signal can be separated from fluctuating background, if $\boxed{v_n\{4\} >> \frac{1}{N^{3/4}}}$

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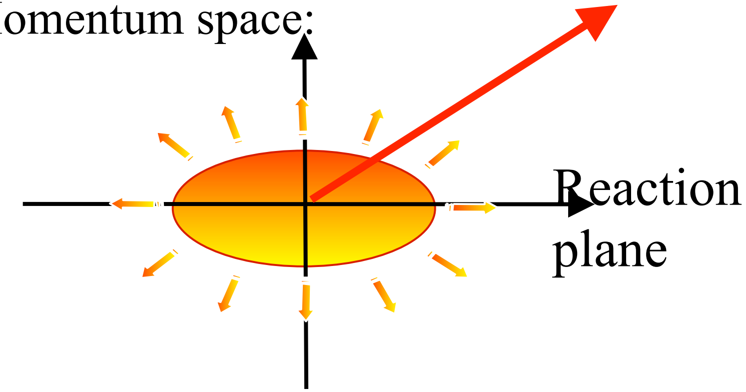
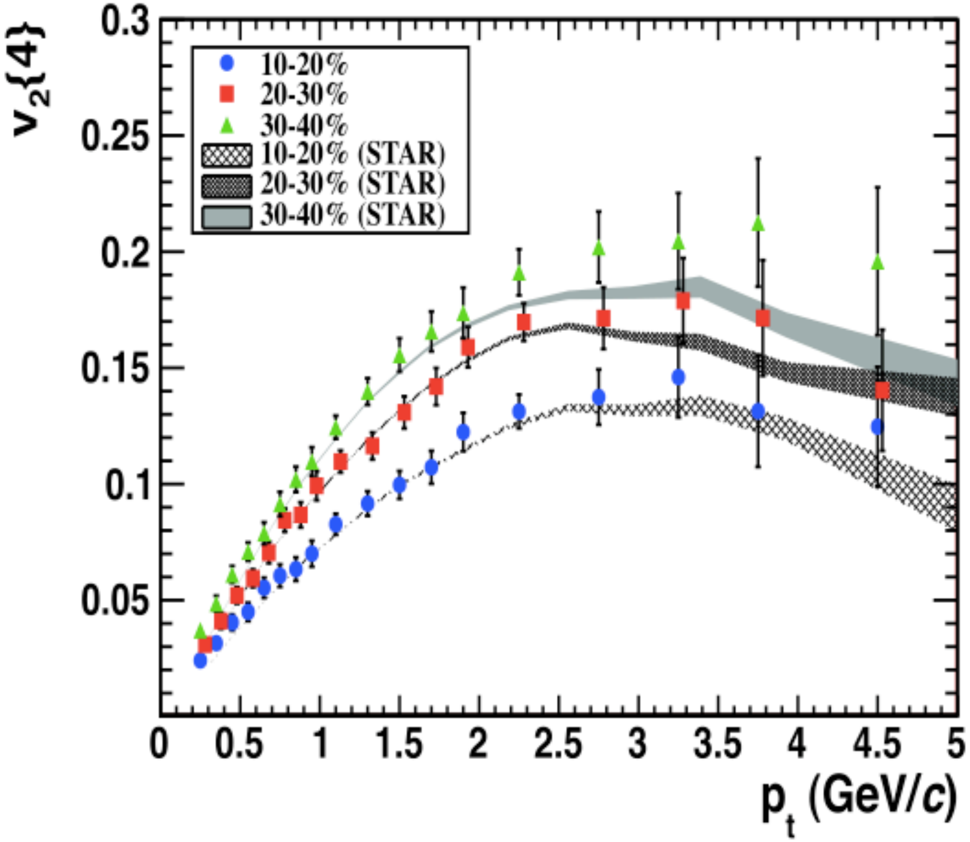
II.6. LHC and RHIC Data on Elliptic Flow: v_2

(2.11) $$E\frac{dN}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_T dp_T d\eta}\left[1 + 2v_2(p_T)\cos\left(2(\phi - \psi_{reaction\ plane})\right)\right]$$

- Momentum space:





- Signal $v_2 \approx 0.2$ implies 2-1 asymmetry of particles production w.r.t. reaction plane.

- 'Non-flow' effect for 2nd order cumulants

(2.12) $N \sim 100 \Rightarrow 1/\sqrt{N} \sim O(v_2)$

2nd order cumulants do not characterize solely collectivity.

(2.13) $1/N^{3/4} \sim 0.03 \ll v_2$

Non-flow effects should disappear if we go from 2nd to 4th order cumulants.

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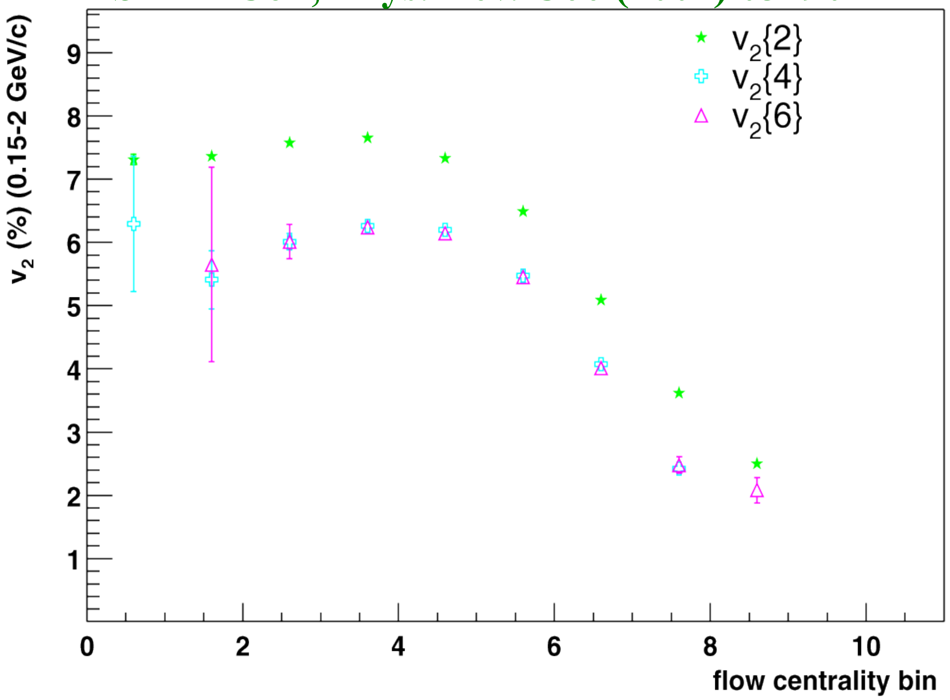
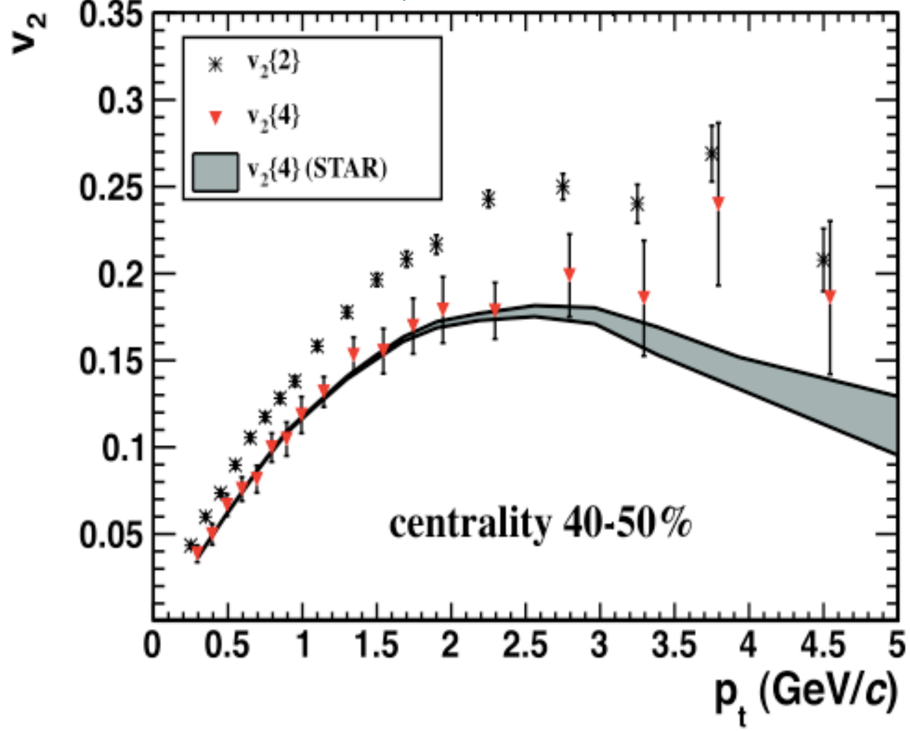
II.7. Establishing collectivity in v_2

- pt-integrated v2 stabilizes at 4th order cumulants
- pt-differential v2 from 2nd and 4th order cumulants

STAR Coll, Phys. Rev. C66 (2002) 034904



ALICE Coll., arXiv:1011.3914



Elliptic flow signal is stable if reconstructed from higher order cumulants.

We have established a **strong collective effect**, which cannot be mimicked by multiplicity fluctuations in the reaction plane.

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II.8. Alternative flow measurements: Q-cumulants

Construction of ‘standard’ cumulants involves sum over
$M(M-1)$ terms to 2nd order

(2.14) ~ M^4 terms to 4th order,
~ M^6 to 6th order, etc

$$\langle 2 \rangle \equiv \langle e^{i\,n\,(\phi_1 - \phi_2)} \rangle = \frac{1}{M(M-1)} \sum_{\substack{i,j=1 \\ (i \neq j)}}^{M} e^{i\,n\,(\phi_i - \phi_j)}$$

Problem: For typical event multiplicity M
this becomes computationally expensive

Solution: Use Q-vector of harmonic N (sum over M terms only!)

(2.15)

$$Q_n \equiv \sum_{i=1}^{M} e^{i\,n\,\phi_i}$$

to construct cumulants

SKIP

Problem: check this!
Bilandzic, Snellings, Voloshin,
arXiv:1010.0233 [nucl-ex]

(2.16)

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

(2.17)

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2\,\mathrm{Re}\left[Q_{2n}\,Q_n^*\,Q_n^*\right] - 4(M-2)|Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

II.9.Yet another method: EP

For each event, one estimates directly the orientation of the event plane (**EP**)

(2.18)

$$\boxed{\psi_n \equiv \tan^{-1} \frac{\sum_{i=1}^{M} w_i \sin(n\phi_i)}{\sum_{i=1}^{M} w_i \cos(n\phi_i)} \Big/ n}$$

(2.19) One then measures

$$\frac{dN}{d(\phi-\psi_n)} = \frac{\langle N \rangle}{2\pi}\left[1 + \sum_{k=1}^{\infty} 2\, v_{kn}^{obs} \cos\left(kn(\phi-\psi_n)\right)\right]$$

(2.20) But we want to measure w.r.t. true reaction plane orientation $\boxed{\psi_r}$

$$E\frac{dN}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_t dp_t dy}\left[1 + \sum_{n=1}^{\infty} 2\, v_n\{EP\} \cos\left(n(\phi - \boxed{\psi_r})\right)\right]$$

SKIP

(2.21) **Correction needed**

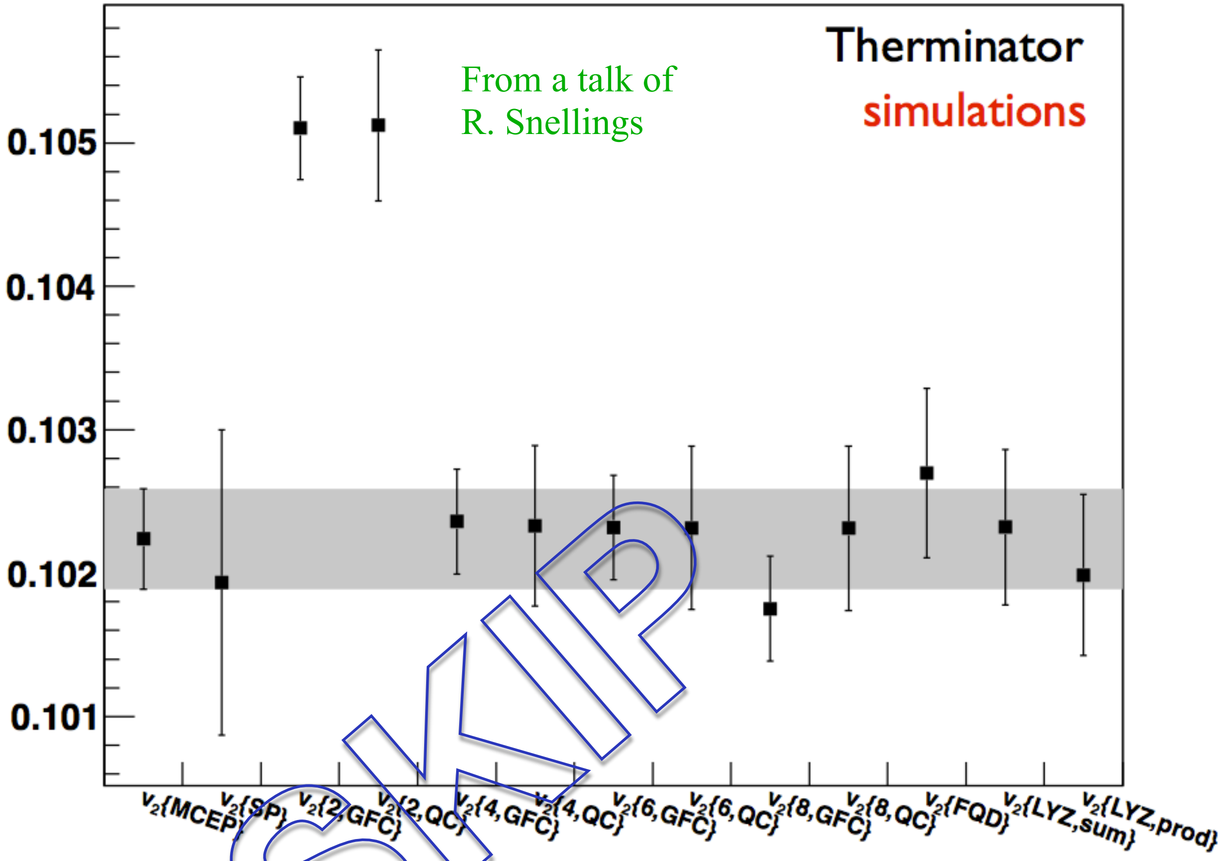
$$\boxed{v_n\{EP\} = v_n^{obs} \Big/ \left\langle \cos n(\psi_n - \psi_r) \right\rangle \equiv v_n^{obs} / R}$$

(2.22) Event-plane resolution R estimated e.g. from sub-event method (A,B,C indep. sub-events)

$$R = \sqrt{\frac{\left\langle \cos n\left(\psi_n^A - \psi_n^B\right)\right\rangle \left\langle \cos n\left(\psi_n^A - \psi_n^C\right)\right\rangle}{\left\langle \cos n\left(\psi_n^B - \psi_n^C\right)\right\rangle}}$$

Poskanzer, Voloshin, PRC58 (1998) 1671

II.10.Consistency of flow analysis methods



Many important technical issues not touched here.

Take home message:

-There are many flow analysis methods with different systematic uncertainties.

-They are “generally” consistent, deviations are “relatively well” understood.

II.11.Flow in measured two-particle correlations

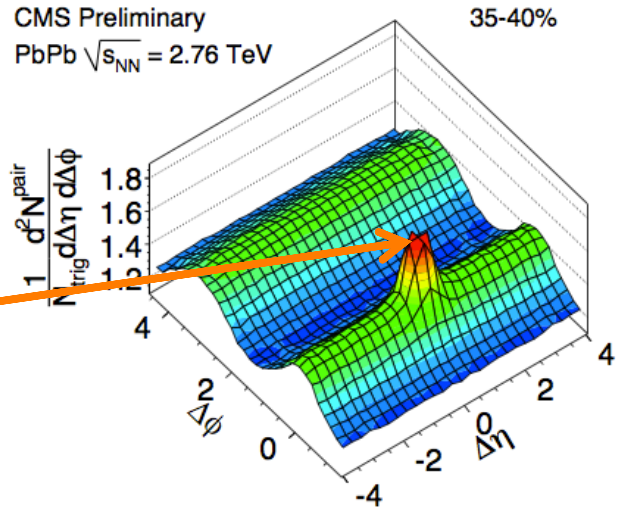
Flow harmonics measured via particle correlations.

Here: look directly at correlations of 'trigger' with 'associate' particle

(often pt-cuts on 'trig' and 'assoc')

If flow dominated, then

(2.23) $$\frac{2\pi}{N_{pairs}} \left\langle \frac{dN_{pairs}}{d\Delta\phi} \right\rangle = 1 + \sum_{n=1}^{\infty} 2 \left\langle v_n^{(trig)} v_n^{(assoc)} \right\rangle \cos\left(n\,\Delta\phi\right)$$



Characteristic features:

1.Small-angle jet-like correlations around

$\boxed{\Delta\phi \approx \Delta\eta \approx 0}$ (this is a non-flow effect)

2.Long-range rapidity correlation

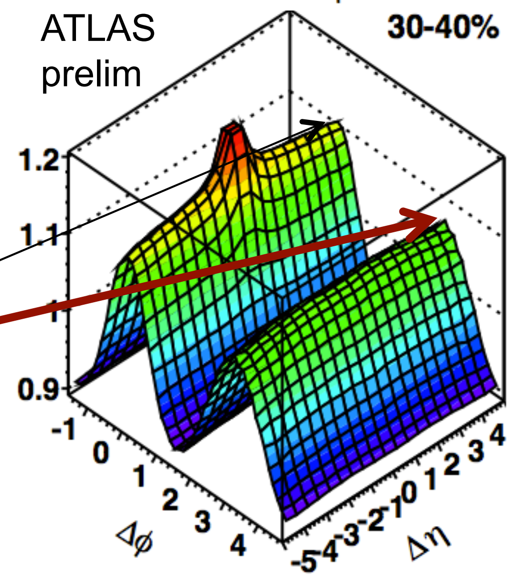
(almost rapidity-independent)

3.Elliptic flow v_2 seems to dominate

(for the semi-peripheral collisions shown here)

4.Away-side peak at $\Delta\phi \approx \pi$ is smaller

(implies ***non-vanishing odd harmonics v1, v3,*** …)



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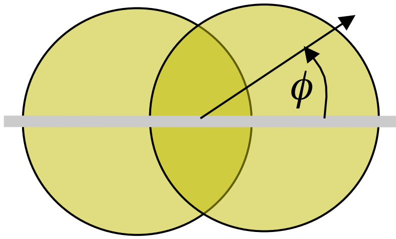
II.12.Non-vanishing odd flow harmonics



Event-averaged (non-fluctuating) **initial conditions** have nuclear overlap with $\boxed{\phi \to \phi + \pi \quad \text{symmetry}}$

Dynamics cannot break this symmetry of the initial conditions

$$\Rightarrow v_{2n+1} = 0 \quad \forall n$$

Conclusion: Non-vanishing odd harmonics are unambiguous signal for Event-by-Event fluctuations in **initial conditions.**

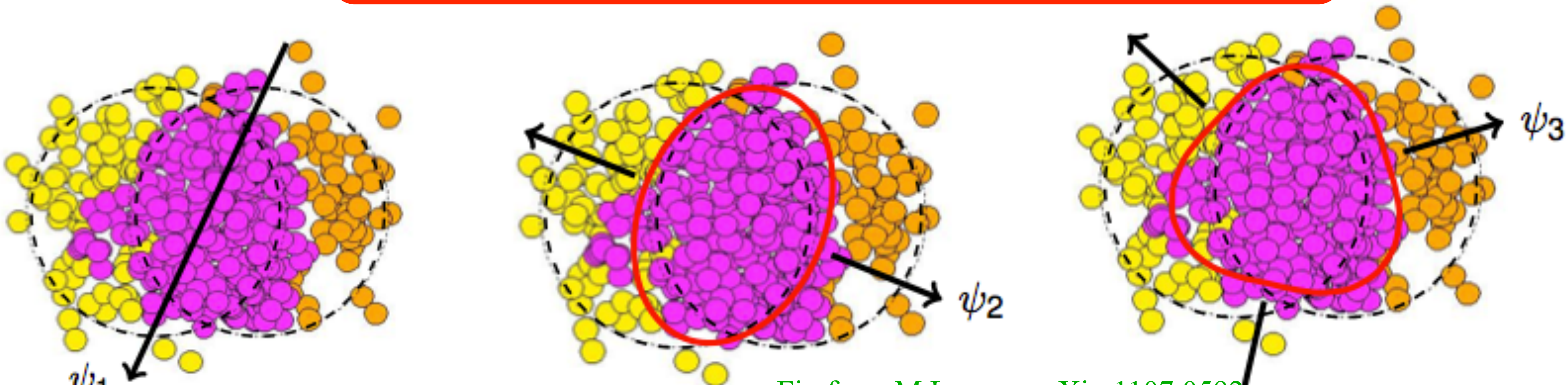


Fig from M.Luzum, arXiv:1107.0592

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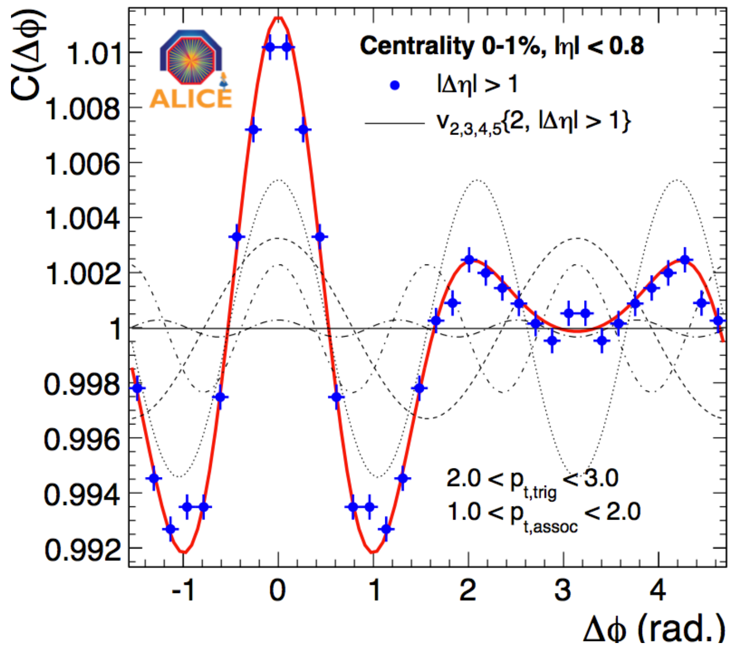
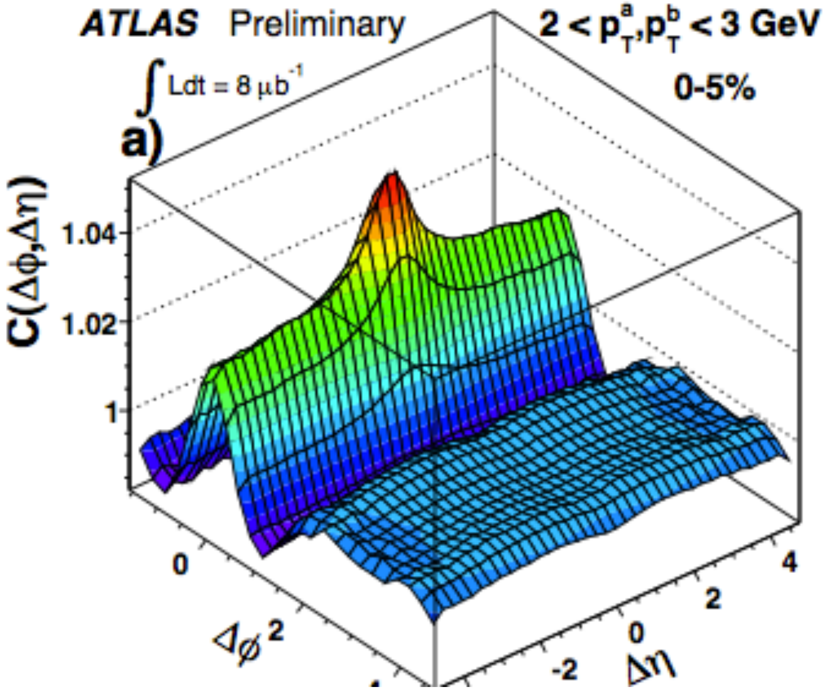
II.13. Odd harmonics dominate central collisions

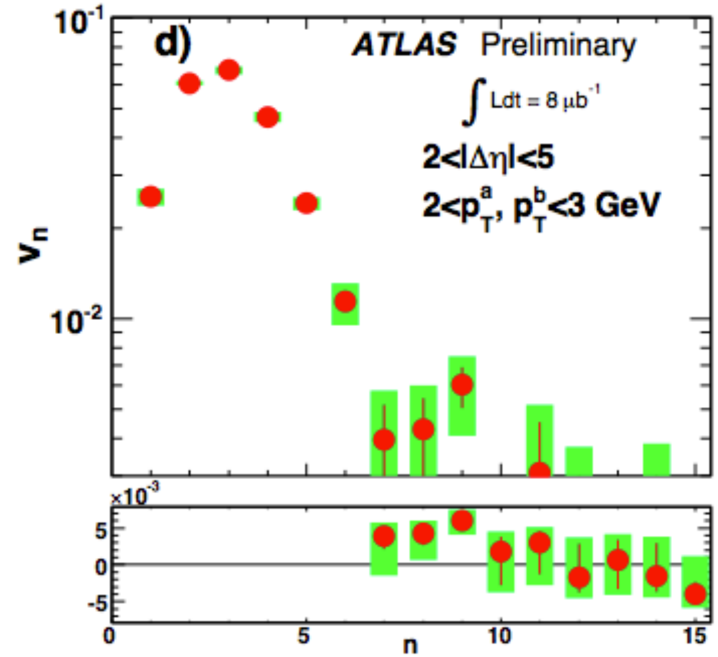
In the most central 0-5% events,

(2.24) $$v_3 \geq v_2$$

Fluctuations in initial conditions dominate flow measurements





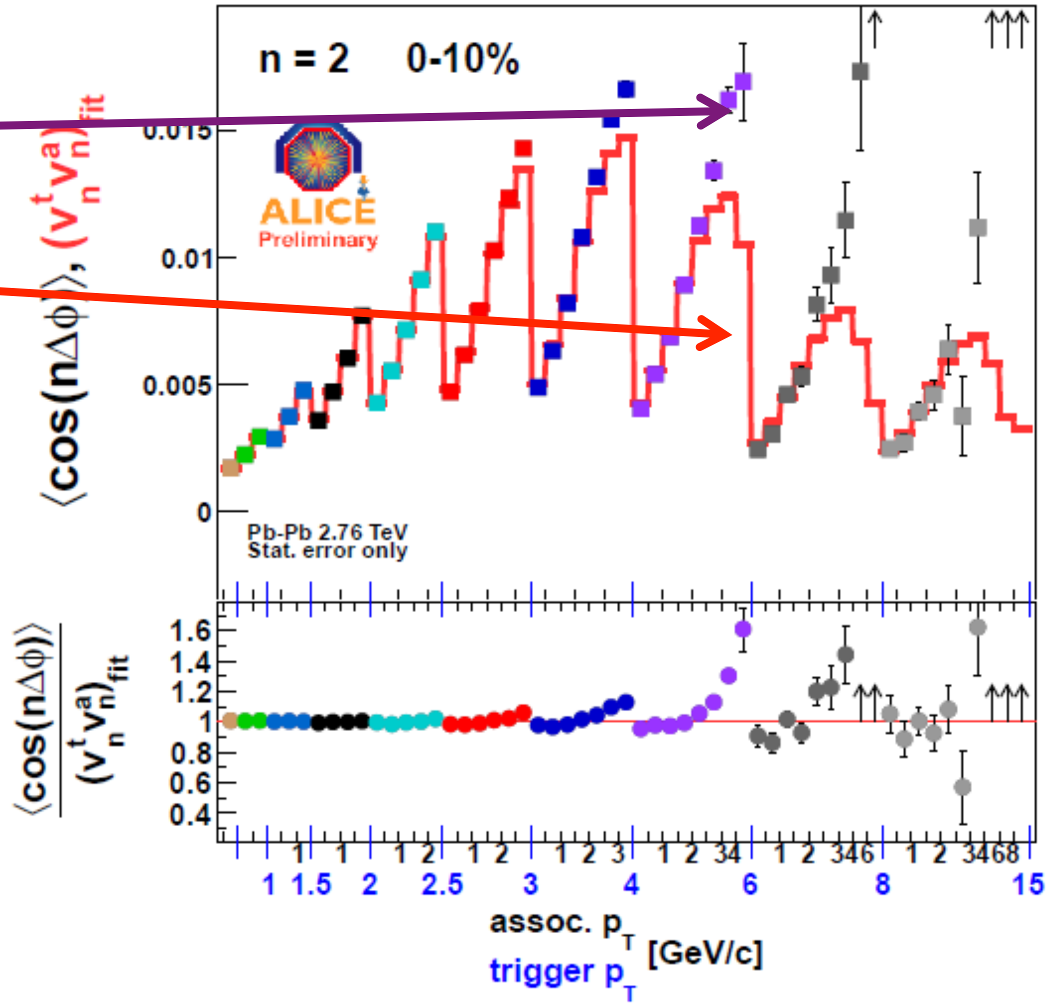


II.14. Factorization of 2-particle correlations

If these fluctuations in the initial conditions *propagate collectively* to the measured flow harmonics,
then 2-particle-correlations must factorize.

Do they? Check (2.23)

$$\boxed{\frac{2\pi}{N_{pair}}\frac{dN_{pair}}{d\Delta\phi}} = 1 + 2\sum_{n=1}^{\infty} \boxed{\left\langle v_n^{(t)} v_n^{(a)} \right\rangle} \cos(n\,\Delta\phi)$$



At sufficiently low p_T, data consistent with assumption of collective propagation.

II.15.Characterizing spatial asymmetries

To discuss propagation of fluctuations in initial conditions, need to quantify them.
Characterize **spatial eccentricities**, e.g., via moments of transverse density

(2.24) $$\varepsilon_{m,n} e^{in\phi_{m,n}} \equiv -\frac{\left\{ r^m e^{in\phi} \right\}}{\left\{ r^m \right\}}, \quad \varepsilon_n \equiv \varepsilon_{n,n} \qquad \left\{ \ldots \right\} \equiv \frac{\int d^2x\, \rho(x) \ldots}{\int d^2x\, \rho(x)}$$

Simplifying working hypothesis (commonly used)

- EbyE asymmetry of initial condition is a **purely spatial eccentricity**
- **spatial eccentricity** is related to (**momentum**) **flow** by ***linear response***

(2.25) $$\boxed{v_n \exp\left[i n \psi_n \right] = k\, \varepsilon_n \exp\left[i n \phi_n \right] + corr}$$

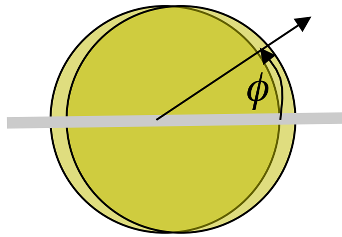
For tests, see e.g.
F. Gardim et al, arXiv:1111.6538

Final aim: to understand the dynamical mechanism that maps
fluctuating initial conditions onto flow harmonics

Aside: In most central collision, event-averaged (non-fluctuating) initial conditions would lead to

$$\varepsilon_n \approx 0 \Rightarrow v_n \approx 0$$

Thus, no geometric reason for 2nd harmonics to dominate fluctuating initial conditions (see II.13).



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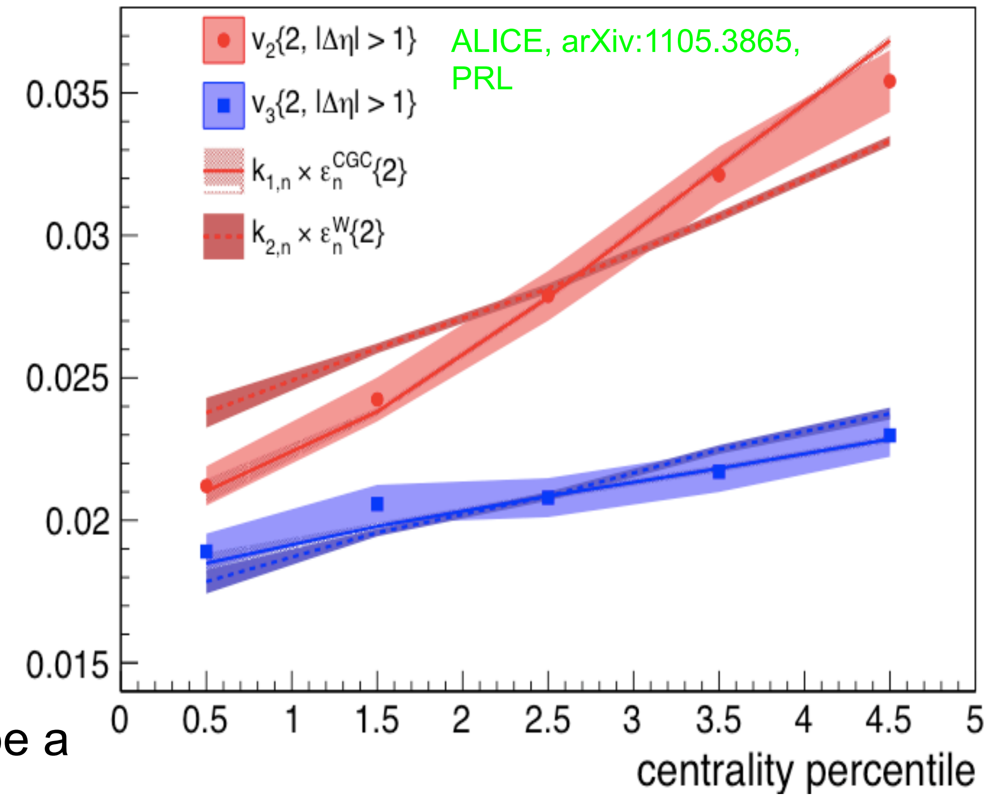
II.16. Comparing spatial eccentricities with flow

Simple models for initial spatial eccentricities and their centrality dependence can be based on supplementing e.g. Glauber model with notion of energy density:

(2.26)

$$\varepsilon\left(\underline{x}\right) \equiv \sum_{i=1}^{N_{part}} \varepsilon_{NN}\left(\underline{x}-\underline{x}_i\right) = \frac{K}{2\pi\sigma^2}\sum_{i=1}^{N_{part}} \exp\left[-\frac{\left(\underline{x}-\underline{x}_i\right)^2}{2\sigma^2}\right]$$



<u>Spatial eccentricities</u>

- details model-dependent
- for some models

(2.27) $$\boxed{v_n \propto \varepsilon_n}$$

- Linear response (2.27) seems to be a fair first approximation
- But <u>deviations from linear response (2.27)</u> do not disprove a model of eccentricity in initial conditions. They could be accounted for by <u>non-linear dynamics.</u> (**to which we turn now**).

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III. Dynamical framework for collective flow

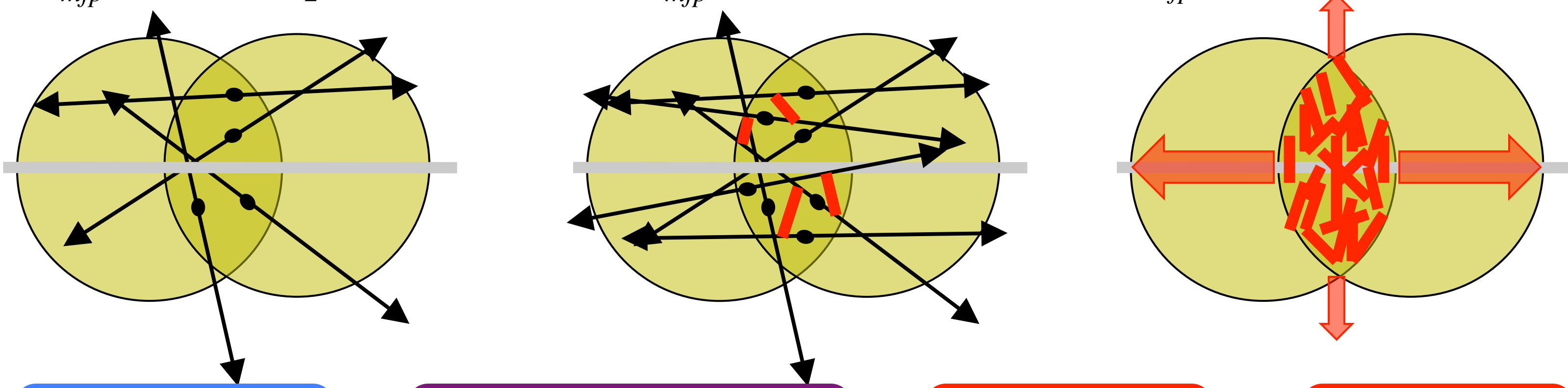
We seek a dynamical framework that maps

initial conditions
- their average eccentricities
- their EbyE fluctuations

→

particle spectra
- their p_T - and η - dependence
- their flow harmonics

Mean free path vs. collectivity: $\lambda_{mfp} \approx \infty \Rightarrow v_2 = 0$ | $\lambda_{mfp} \approx \mathit{finite}$ | $\lambda_{mfp} \approx 0 \Rightarrow v_2 = \max$



Theory tools: Free streaming → Particle cascade (QCD transport theory) → Dissipative fluid dynamics → Perfect fluid dynamics

System: p+p ?? … A+A … ??

Study **fluid dynamics** as relevant theoretical baseline for discussing collective effects …

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III.1. Fluid dynamics - the basics

Consider matter in local equilibrium, characterized locally by its energy momentum tensor, the density of n charges, and a flow field:

- energy momentum tensor $T^{\mu\nu}$ ……. 10 indep. components
- conserved charges N_i^μ …………… 4n indep. components

Tensor decomposition w.r.t. flow field $u_\mu(x)$ projector $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

(3.1) $N_i^\mu = n_i\, u^\mu + \bar{n}_i$

(3.2) $T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$

(3.3) (1 comp.) $\varepsilon \equiv u_\mu T^{\mu\nu} u_\nu$ energy density

(3.4) (1 comp.) $p \equiv -T^{\mu\nu}\Delta_{\mu\nu}/3$ isotropic pressure

(3.5) (3 comp.) $q^\mu \equiv \Delta^{\mu\alpha} T_{\alpha\beta} u^\beta$ heat flow

(3.6) (5 comp.) $\Pi^{\mu\nu} \equiv \left[\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right] T^{\alpha\beta}$ shear viscosity

In Local Rest Frame (LRF) $u_\mu = (1,0,0,0)$

Convenient choice of frame: Landau frame: $u = u_L \Rightarrow q^\mu = 0$

Eckard frame: …

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III.2. Equations of motion for a perfect fluid

A fluid is perfect if it is locally isotropic at all space-time points. This implies

(3.7) $$N_i^\mu = n_i\, u^\mu + \cancel{\overline{\nu}_i}$$ (n comp.)

(3.8) $$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + \cancel{q^\mu u^\nu} + \cancel{q^\nu u^\mu} + \cancel{\Pi^{\mu\nu}}$$ (5 comp.)

The equations of motion are then determined by conservation laws

(3.9) $$\partial_\mu N_i^\mu \equiv 0$$ (n constraints)

(3.10) $$\partial_\mu T^{\mu\nu} \equiv 0$$ (4 constraints)

and the equation of state

(3.11) $$p = p(\varepsilon, n)$$ (1 constraint)

Here, information from ab initio calculations (lattice) or models enters.

Hydrodynamic simulations are numerical solutions of (3.7),(3.8).
'Systematic' model uncertainties arise from

- specifying initial conditions
- specifying the decoupling of particles ('freeze-out')
- assuming that non-perfect terms in (3.7),(3.8) can be dropped
- specifying (3.11)

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III.3. Two-dimensional Bjorken fluid dynamics

Main assumption: initial conditions for thermodynamic fields do not depend on space-time rapidity

(3.12)
$$\eta = \frac{1}{2}\ln\left[\frac{t+z}{t-z}\right]$$

Longitudinal flow has 'Hubble form':

(3.13)
$$v_z = z/t$$

Bjorken scaling means that hydrodynamic equations preserve Hubble form

(3.14) $u^\mu = \cosh y_T \left(\cosh\eta, v_x, v_y, \sinh\eta\right)$ **Longitudinally boost-invariant flow profile**

(3.15) at mid-rapidity
$$v_r(\tau, r, \eta = 0) \equiv \tanh y_T(\tau, r)$$

(3.16) at forward rapidity
$$v_r(\tau, r, \eta) \equiv \frac{v_r(\tau, r, \eta = 0)}{\cosh\eta}$$

Problem: show that e.o.m. (3.10) preserve longitudinal boost-invariance of initial conditions.
solution see e.g. Kolb+Heinz, PRC62 (2000) 054909

III.4. 2-dim "perfect" Hydro Simulations: Input…

Initialization: thermo-dynamic fields $\varepsilon(\tau, r, \eta = 0)$ have to be initialized, e.g. by

(3.17) $$\varepsilon_{init}(\underline{r}) = \varepsilon(\tau_0, \underline{r}, \eta = 0) \propto \left(\frac{1-x}{2} \overline{N}^{AB}_{part}(\underline{b}, \underline{r}) + x \overline{N}^{AB}_{coll}(\underline{b}, \underline{r}) \right)$$

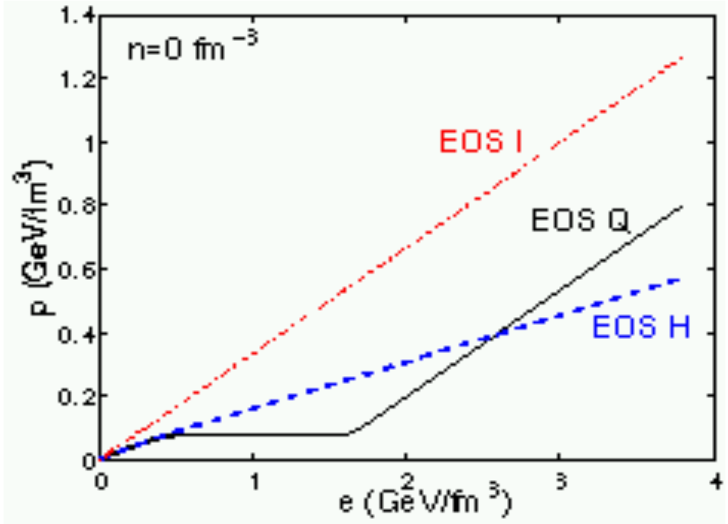
Equation of state: $p(\varepsilon, n)$

(3.18) Velocity of sound: $c_s^2 = \frac{\partial p}{\partial \varepsilon}$

(3.19) Expectations: $c_s^2 \approx 0.15$ **Soft EOS**

$c_s^2 = 1/3$ **Hard EOS**

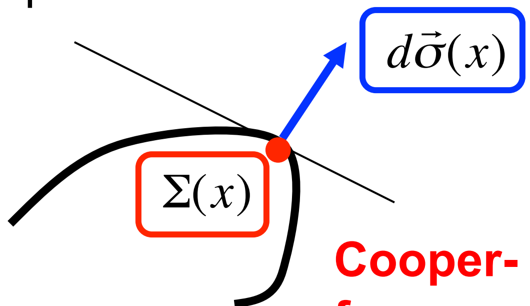
Input from (many) models and from lattice QCD.



Freeze-out: local temperature $T(x) = T_{fo}$ defines space-time hypersurface $\Sigma(x)$,
from which particles decouple with spectrum

(3.20) $$E \frac{dN_i}{d\vec{p}} = \frac{g_i}{(2\pi)^3} \int_\Sigma \vec{p}.d\vec{\sigma}(x)\, f_i(p.u(x), x)$$

(3.21) $$f_i(E, x) = \frac{1}{\exp[(E - \mu_i(x))/T(x)] \pm 1}$$



Cooper- Frye freeze-out

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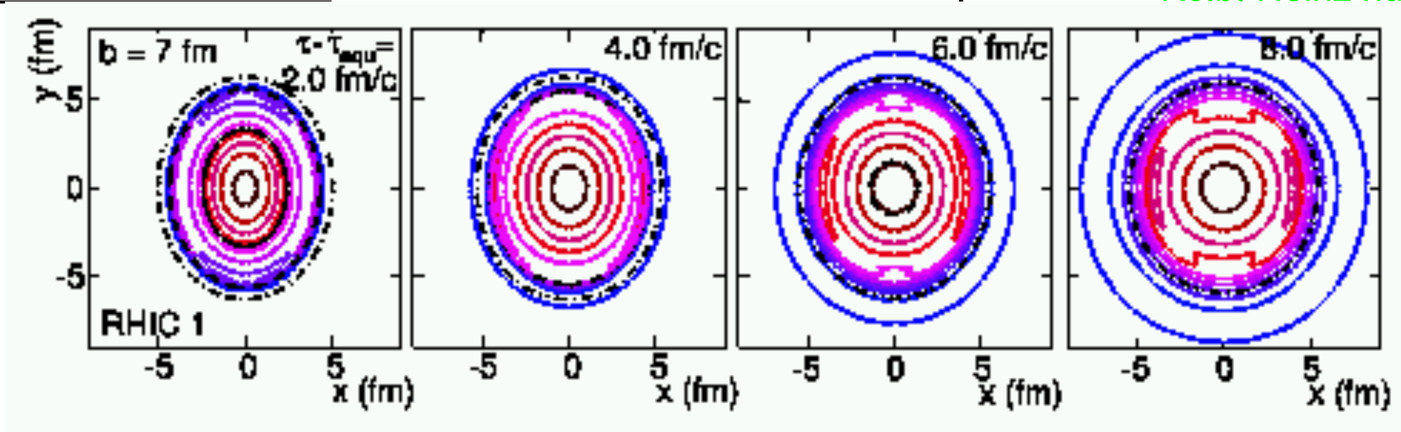
III.5. 2D-simulations with event-averaged IC

Results of simulations: time evolution in transverse plane

Kolb. Heinz nucl-th/0305084



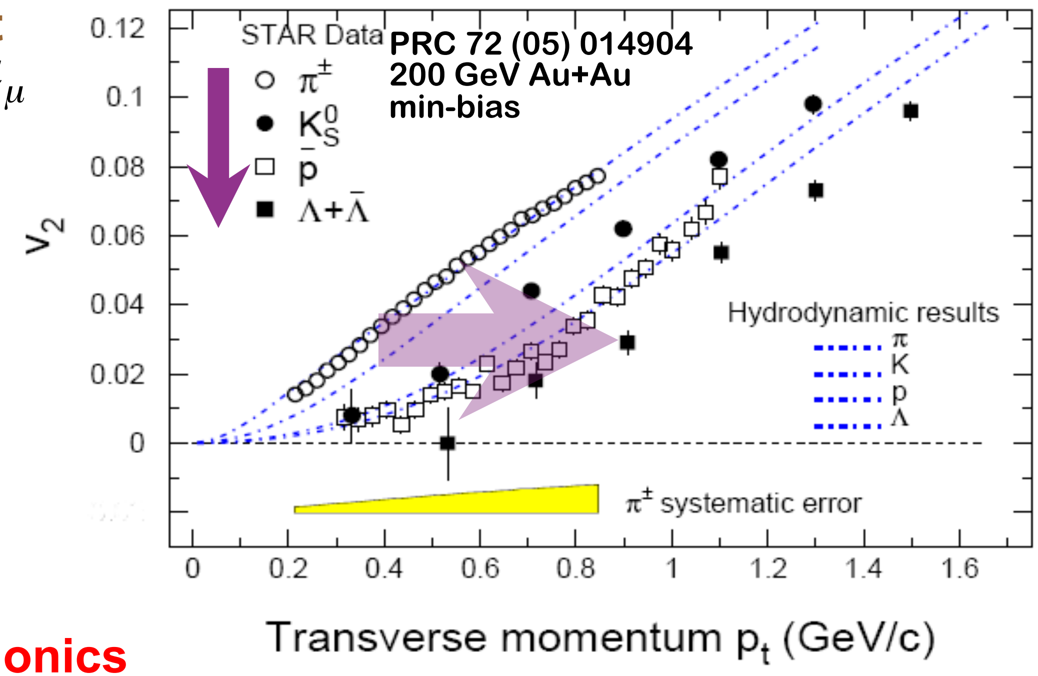
Conclusions from such studies:

- initial **transverse pressure gradient**
  ⇒ ϕ - dependence of flow field u_μ
  ⇒ elliptic flow $v_2(p_T)$

- size and pt-dependence of v_2 data accounted for by hydro ('maximal')

- characteristic **mass dependence**, since all particle species emerge from common flow field u_μ

- **BUT: no fluctuations, no odd harmonics**



III.6. Dissipative corrections to a perfect fluid

Small deviations from a locally isotropic fluid can be accounted for by restoring

(3.7) $$\boxed{N_i^\mu = n_i\, u^\mu + \bar{n}_i}$$ (4n comp.)

(3.8) $$\boxed{T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}}$$ (10 comp.)

When does perfect fluid assumption fail? Consider conserved current:

(3.22) $$\partial_\mu j^\mu = \partial_\mu\left(\rho\, u^\mu\right) = \rho \underbrace{\partial_\mu u^\mu}_{\text{expansion scalar}} + \underbrace{u^\mu \partial_\mu}_{\text{comoving } t-\text{derivative}} \rho = 0$$

Spatio-temporal variations of macroscopic fluid should be small if compared to microscopic reaction rates

(3.23) $$\Gamma \cong n\sigma >> \theta = \partial_\mu u^\mu$$

Dissipative corrections characterized by gradient expansion!

Now, the conservation laws and equation of state

$\partial_\mu N_i^\mu \equiv 0$ (n constraints)

$\partial_\mu T^{\mu\nu} \equiv 0$ (4 constraints)

$p = p(\varepsilon, n)$ (1 constraint)

are not sufficient to constrain all independent thermo-dynamic fields in (3.7),(3.8).

How do we obtain additional constraints?

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III.7. 1st order dissipative fluid dynamics

Since conservation laws + eos do not close equations of motion, one seeks additional constraints from expanding 2nd law of thermodynamics to 1st order

(3.24) $S^\mu = s\,u^\mu + \beta\, q^\mu$ Entropy to first order

Use $\varepsilon + p = \mu n + Ts$ and $u_\nu \partial_\mu T^{\mu\nu} \equiv 0$ to write:

(3.25) $$T\partial_\mu S^\mu = \left(T\beta - 1\right)\partial q + q\left(\dot{u} + T\partial\beta\right) + \Pi^{\mu\nu}\partial_\nu u_\mu + \Pi\theta \geq 0$$

To warrant that entropy increases, require:

$$\beta \equiv 1/T$$

Navier-Stokes
1st order hydro

(3.26) bulk viscosity $$\Pi \equiv \varsigma\,\theta$$

(3.27) heat conductivity $$q^\mu \equiv \kappa T \Delta^{\mu\nu}\left(\partial_\nu \ln T - \dot{u}_\nu\right)$$

(3.28) shear viscosity $$\Pi^{\mu\nu} \equiv 2\eta\left[\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right]\partial^\alpha u^\beta$$

Determines $\Pi, q^\mu, \Pi^{\mu\nu}$ in terms of flow, energy density and dissipative coeff.

(3.29) $$\partial_\mu S^\mu = \frac{\Pi^2}{\varsigma T} - \frac{q\,q}{\kappa T^2} + \frac{\Pi^{\mu\nu}\Pi_{\mu\nu}}{2\eta T} \geq 0$$

Problem: instantaneous acausal propagation.

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III.8. 2nd order viscous hydro – entropy derivation

Expand entropy to 2nd order in dissipative gradients

(3.30) $$S^\mu = s u^\mu + \beta q^\mu + \alpha_0 \Pi q^\mu + \alpha_1 \Pi^{\mu\nu} q_\nu + u^\mu \left(\beta_0 \Pi^2 + \beta_1 q q + \beta_2 \Pi^{\mu\nu} \Pi_{\mu\nu} \right)$$

Now, need 9 eqs. to determine $\Pi, q^\mu, \Pi^{\mu\nu}$

$\partial_\mu S^\mu \geq 0$ leads to differential equations for $\Pi, q^\mu, \Pi^{\mu\nu}$

which involve $\alpha_0, \alpha_1, \beta, \beta_0, \beta_1, \beta_2, \varsigma, \kappa, \eta$

Entropy increase determined by **shear viscosity** (if vorticity neglected)

(3.31) $$\boxed{T \partial_\mu S^\mu = \Pi_{\mu\nu} \left[-\beta_2 D \Pi^{\mu\nu} + \tfrac{1}{2} \left\langle \nabla^\mu u^\nu \right\rangle \right] \equiv \frac{1}{2\eta} \Pi_{\mu\nu} \Pi^{\mu\nu}} \qquad \beta_2 = \tau_\Pi / 2\eta$$

Equations of motion involve **relaxation time** and **viscosity**.

Notations: covariant derivative $d_\mu u^\nu \equiv \partial_\mu u^\nu + \Gamma^\nu_{\alpha\mu} u^\alpha$

Convective derivative $D \equiv u^\mu d_\mu$

Nabla operator $\nabla^\mu \equiv \Delta^{\mu\nu} d_\nu = d^\mu - u^\mu D$

Angular bracket $\left\langle A^{\mu\nu} \right\rangle \equiv \left[\tfrac{1}{2} \left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha \right) - \tfrac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] A^{\alpha\beta}$

III.9. Fluid dynamics from transport theory

Dissipative fluid dynamics can also be derived as the long wave-length limit of transport theory.

Consider Boltzmann equation with relaxation time approximation

(3.32) $$p^\mu d_\mu f(x,p) = C \approx -\left(u^\mu p_\mu\right)\frac{f - f_{eq}}{\tau_\pi}$$

Consider small departures from local thermal equilibrium, quadratic ansatz

(3.33) $$f = f_{eq}\left[1 + \varepsilon_{\mu\nu}(x,p)p^\mu p^\nu\right] \qquad \varepsilon_{\mu\nu} = \frac{1}{2T^2\left(\varepsilon + p\right)}\Pi_{\mu\nu}$$

With this ansatz, we write momentum moments from the Boltzmann eq.

… long journey …

(3.34)
$$\left(\varepsilon + p\right)Du^\mu = \nabla^\mu p - \Delta^\mu_\nu \nabla^\sigma \Pi^{\nu\sigma} + \Pi^{\mu\nu} Du_\nu$$
$$D\varepsilon = -(\varepsilon + p)\nabla_\mu u^\mu + \tfrac{1}{2}\Pi^{\mu\nu}\left\langle \nabla_\nu u_\mu \right\rangle$$
$$\tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^{\alpha\beta} + \Pi^{\mu\nu} = \eta\left\langle \nabla^\mu u^\nu \right\rangle - 2\tau_\pi \Pi^{\alpha(\mu}\omega^{\nu)}_\alpha$$

2nd order Israel-Stewart fluid dynamic equations of motion.

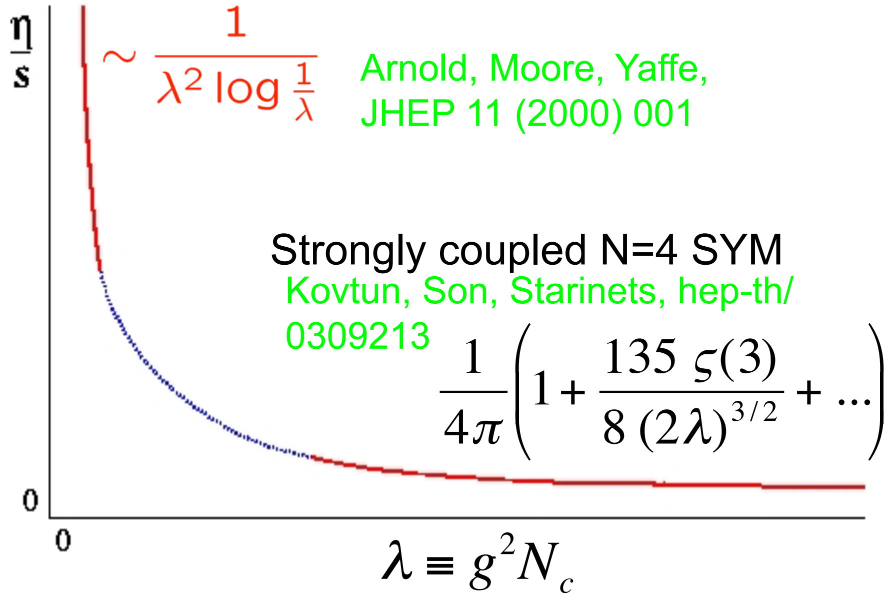
III.10. Input: transport coefficients are fundamental properties of hot QCD matter

The Green-Kubo formula defines transport coefficient as long wave-length limit of retarded Green's function of energy-momentum tensor

(3.35)
$$G^R_{xy,xy}(\omega,0) \equiv \int dt\, dx\, e^{i\omega t}\, \Theta(t) \left\langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \right\rangle_{eq}$$

$$\eta \equiv -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G^R_{xy,xy}(\omega,0)$$

Calculable from first principles in quantum field theory (QCD)



First attempts in finite temperature lattice QCD: H. Meyer, PRD76 (2007) 101701

Motivates the scanning of

η / s

in units of $1/4\pi$

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III.11. Input: relaxation times

Also relaxation times are calculable from first principles in QFT …
In some theories with gravity dual, e.g. N=4 SYM, ***all*** relaxation times and transport coefficients are known

Bhattacharyya, Hubeny, Minwalla,Rangamani 2008
Kanitschneider, Skenderis (2009)
Buchel, Myers (2009)
Romatschke (2009)

in the **weak coupling limit**,

(3.36)
$$\tau_\pi\big|_{\lambda<<1} \sim 5.9 \frac{\eta}{\varepsilon + p}$$

and in the **strong coupling limit**

(3.37)
$$\tau_\pi\big|_{\lambda>>1} \sim \left(4 - 2\ln 2 + \frac{375}{8}\varsigma(3)\lambda^{-3/2}\right)\frac{\eta}{\varepsilon + p} \approx \boxed{\frac{0.2}{T}}$$

Relaxation time is very short

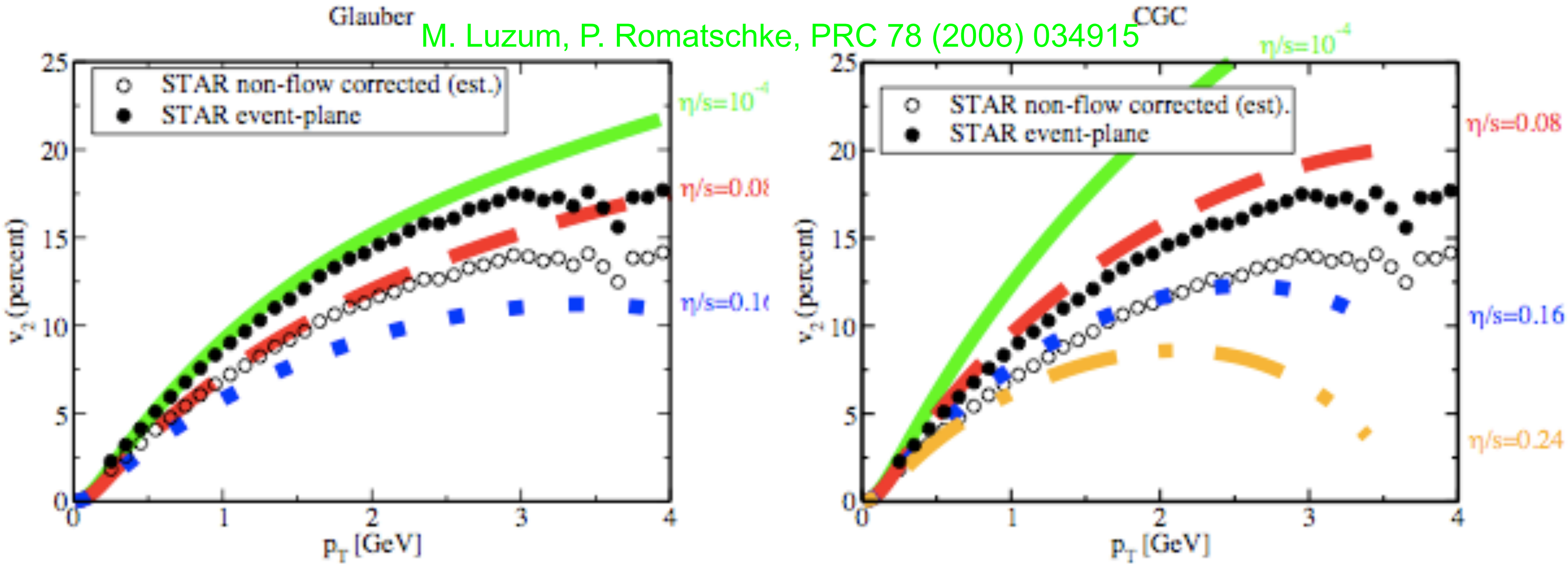
Remarkable curiosity: all modes propagate causal
(need not be the case since hydro holds in long wavelength limit only)

Numerical simulations show very weak dependence on value of relaxation time (see following slides).

III.12. Sensitivity of flow on shear viscosity

Elliptic flow decreases strongly even for close to minimal values of η / s

M. Luzum, P. Romatschke, PRC 78 (2008) 034915



(3.38) To understand order of magnitude, consider 1st order Navier-Stokes dissipative hydrodynamics

$$\frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T}$$

'Perfect liquid' description applicable, if change of entropy small compared to s

$$\frac{\eta}{\tau T}\frac{1}{s} << 1$$

(3.39) Put in numbers $\tau \sim 1\, fm/c, \quad T \sim 200\, MeV$ $\Rightarrow$ $\frac{\eta}{s} << 1$

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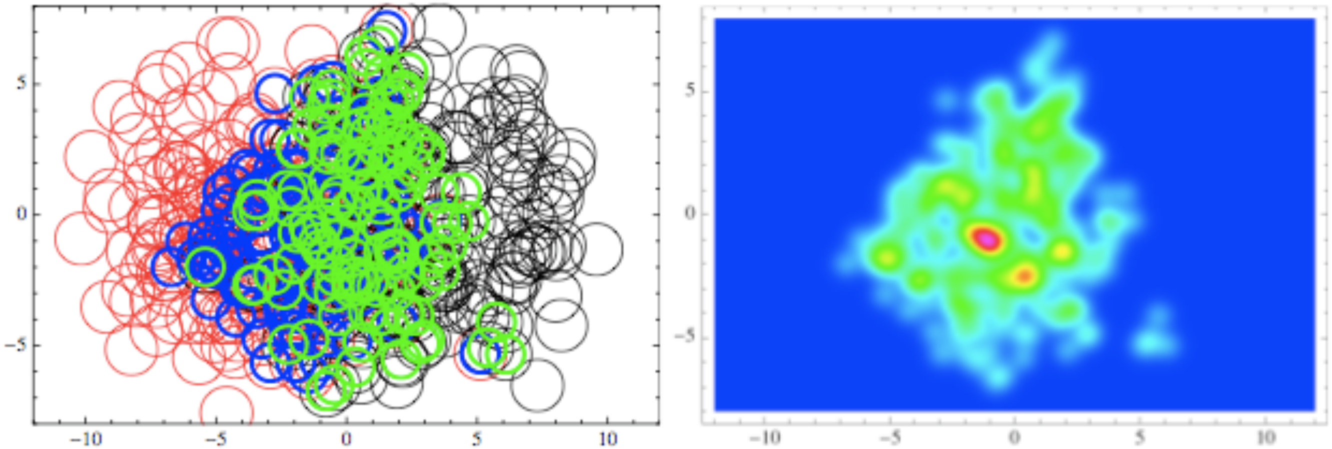
III.13. Input with EbyE fluctuations

EbyE fluctuations needed to account for odd harmonic flow coefficients.

- Typical transverse energy density distribution from Glauber model

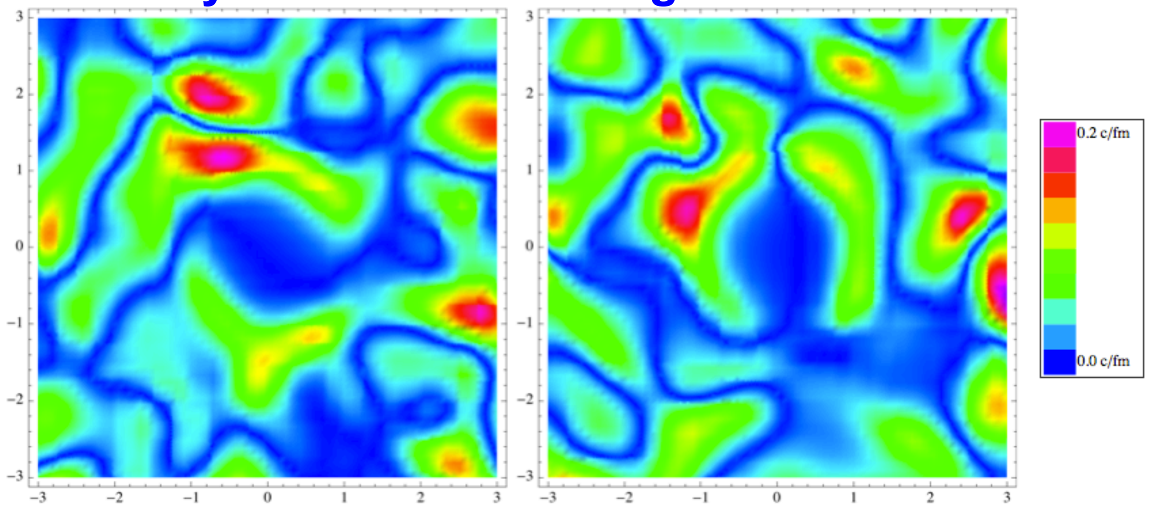
Relevance for v3 first pointed out by B. Alver and G. Roland, PRC81 (2010) 054905



S. Flörchinger, UAW, arXiv:1108.5535, JHEP in press

- Fluctuations in initial velocity fields (normally not included)

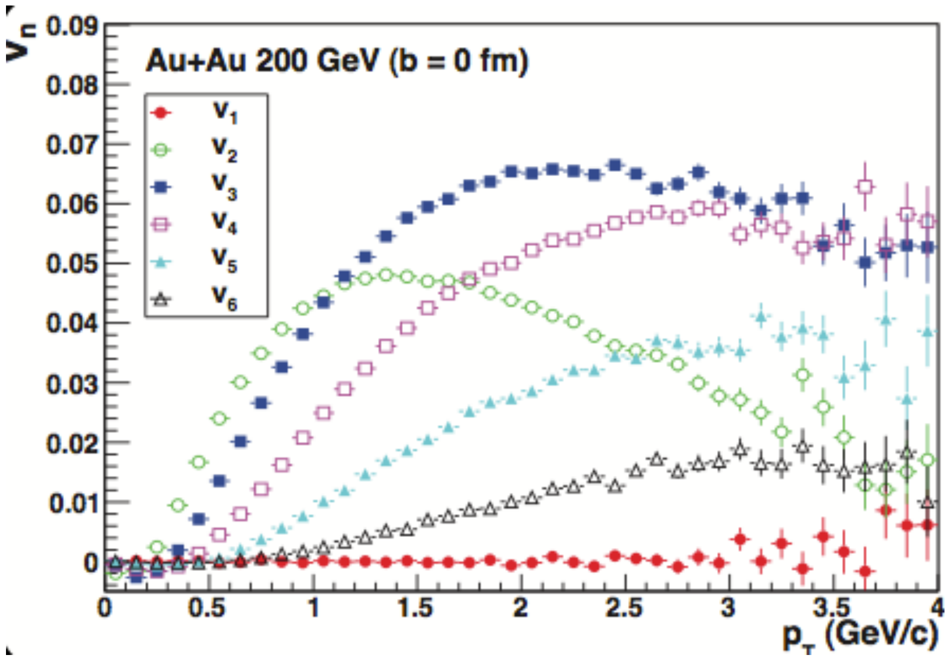
Vorticity of flow field **Divergence of flow field**

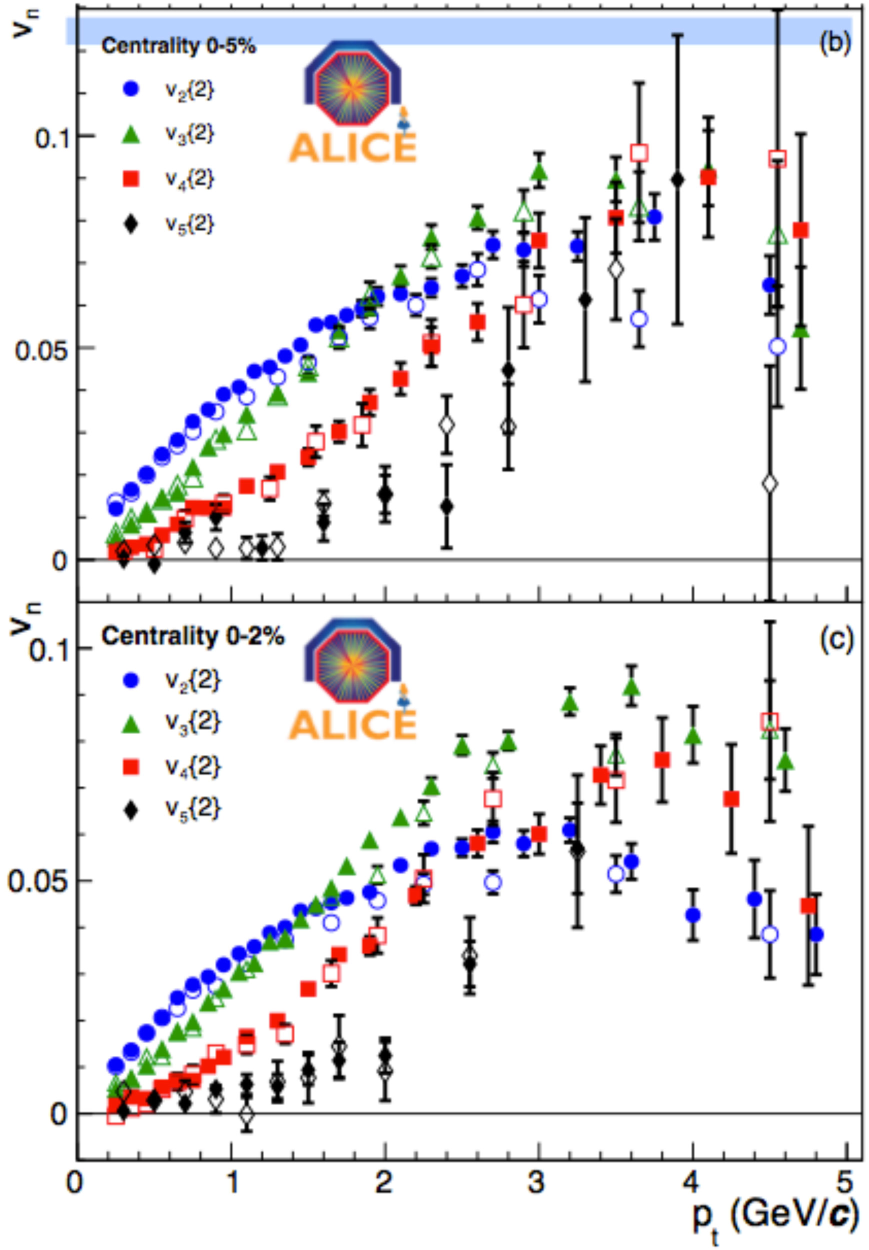


III.14. Odd harmonics in transport models…

• AMPT: includes fluctuations in the initial state …

G-L Ma & X.N. Wang, arXiv:1011.5249v2



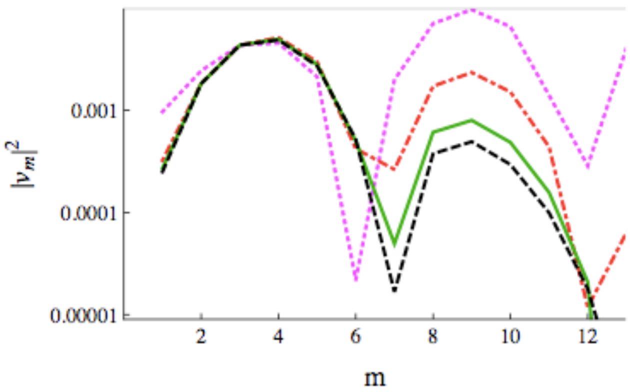


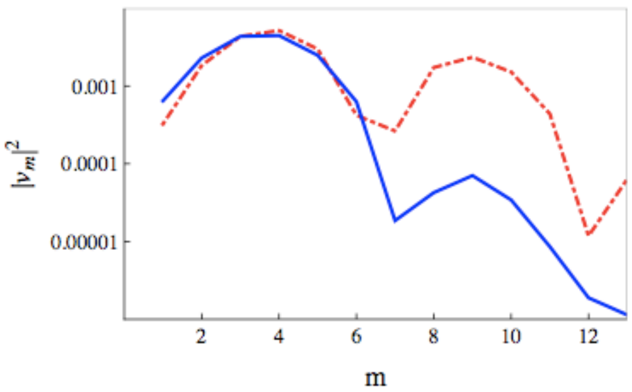
• This is not a fluid dynamic simulation but the AMPT transport model has very small m.f.p's

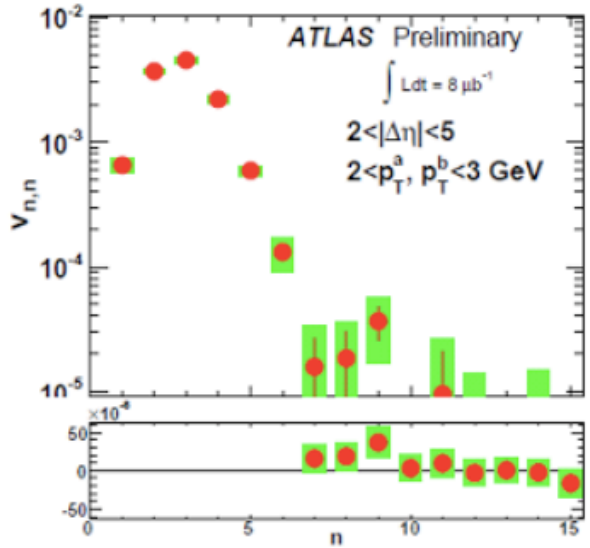
III.15. How does fluid dynamics propagate fluctuations in heavy ion collisions?

P. Staig and E. Shuryak, arXiv:1109.6633

- Consider linear fluid dynamic perturbations on top of analytically known event-averaged fluid dynamic solution (Gubser's model)
- Find that higher Fourier modes of fluid dynamic perturbations dissipate faster
- Emphasize analogy with CMB radiation spectrum







III.16. How does fluid dynamics propagate fluctuations in heavy ion collisions?

S. Flörchinger, UAW, arXiv:1108.5535, JHEP in press

- If fluid dynamic description holds, Reynold's number is

$$\mathrm{Re} \propto 1/(\eta/s) \cong 1-10$$

- consider linear and non-linear propagation of fluid dynamic perturbations on top of analytically known Bjorken model:

  late time dynamics governed (after coord. trafo) by 2-dim Navier-Stokes equation

Heavy Ions

- Bjorken expansion (1-dim)
- time-scale sufficient for fluid dynamic description? (exp support but no deep th understanding)
- expansion delays onset of non-linearities only in longitudinal dimension
- **dynamics of fluctuations gives access to material properties** (viscosities, relaxation times, calculable from 1^{st} principles of QFT)

CMB

- Hubbel expansion (3-dim)
- time scale clearly sufficient for fluid dynamic description
- expansion delays onset of non-linearities
- dynamics of fluctuations gives access to matter content of Universe

Much more to come …