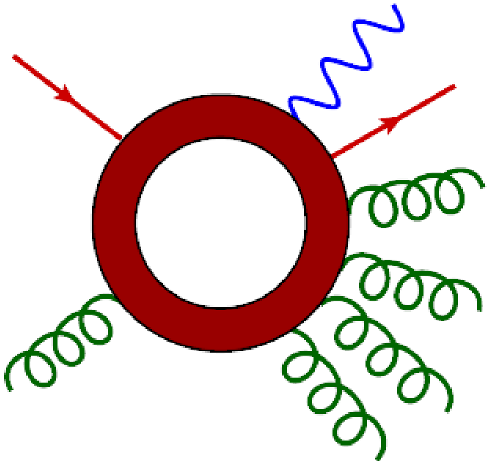
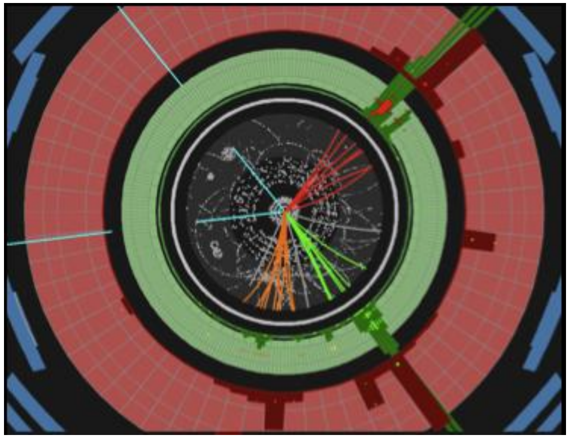
QCD at Colliders





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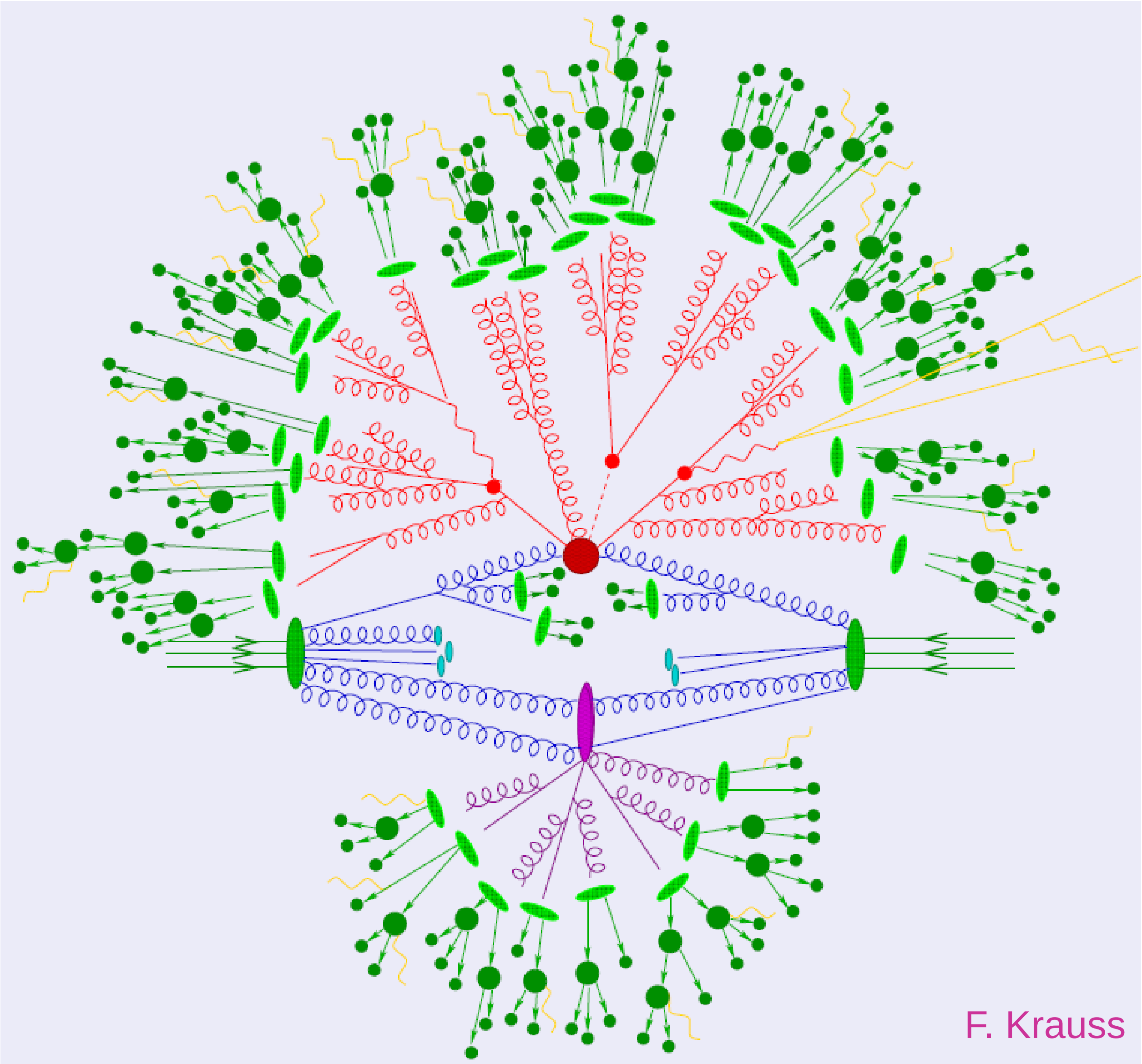
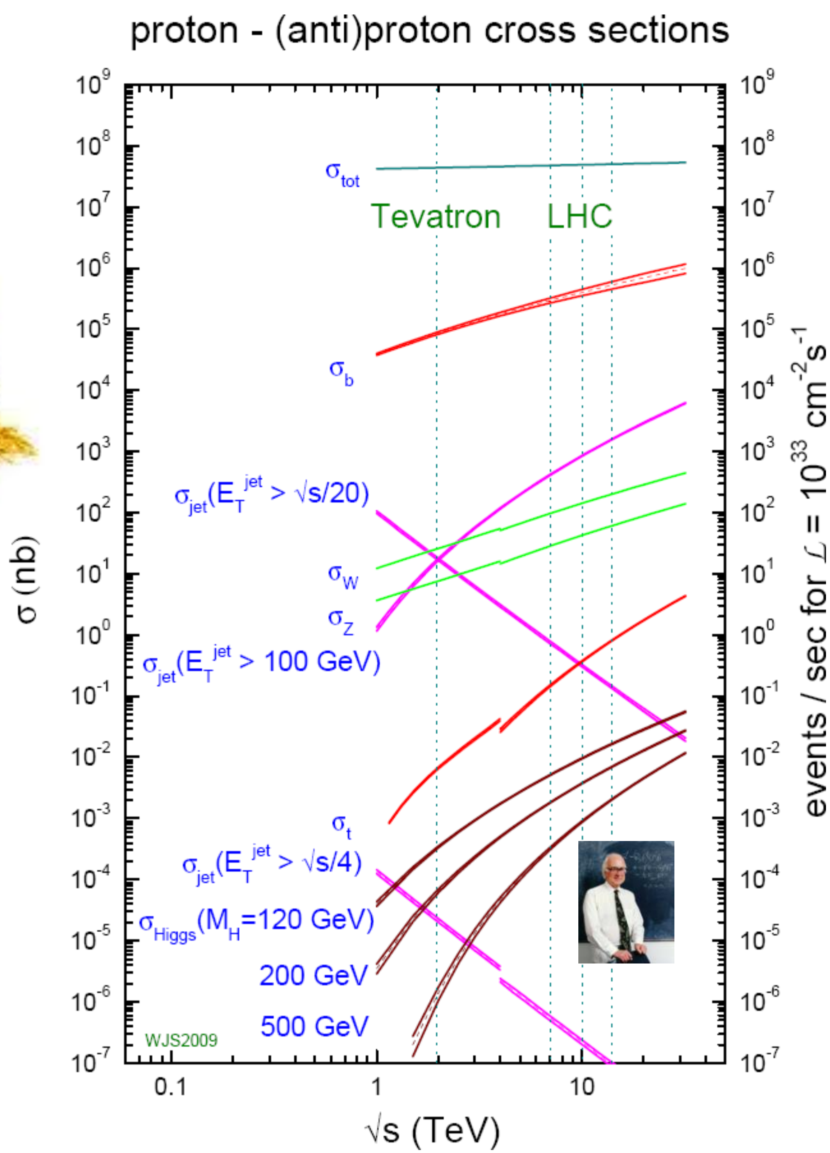
From this…

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}\gamma^{\mu}(i\partial_{\mu} - g_s A_{\mu})q - \frac{1}{4}F^{a}_{\mu\nu}F^{a\,\mu\nu}$$

… to this



LHC Data Dominated by Jets



Jets from quarks and gluons.

- *q,g* from decay of new particles?
- Or from old QCD?

- Every process shown also with one more jet at ~ 1/5 the rate
- Need **accurate production rates** for *X + 1,2,3,…* jets in Standard Model

new physics →

“New physics at the LHC is a riddle, wrapped in a mystery, inside an enigma; but perhaps there is a key.” -W. Churchill

A few references

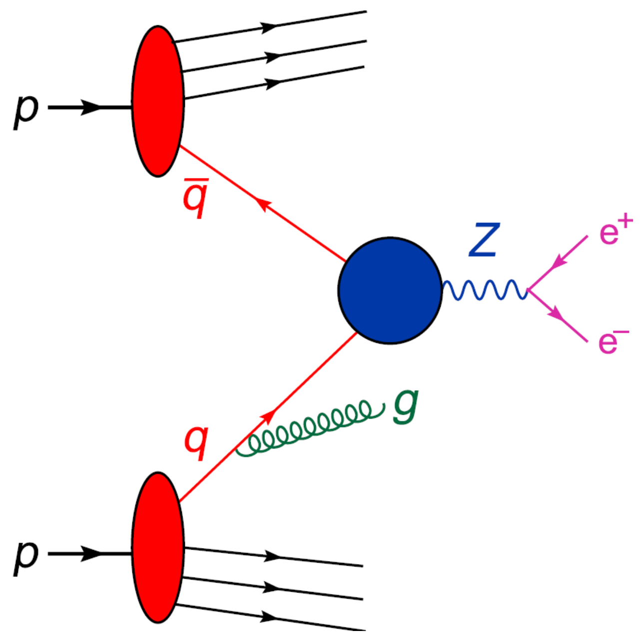
- “Elements of QCD for hadron colliders”, G. Salam, 2009 lectures at this school, arXiv:1011.5131
- "Some Basic Concepts of Perturbative QCD", G. Sterman, arXiv:0807.5118
- "QCD and Jets", G. Sterman, arXiv:hep-ph/0412013
- "Calculating scattering amplitudes efficiently", LD, hep-ph/9601359
- "Simplifying Multi-Jet QCD Computation", M. Peskin, arXiv:1101.2414
- "General-purpose event generators for LHC physics", A. Buckley et al.,

Phys. Rept. 504 (2011) 145, arXiv:1101.2599 [hep-ph].

- "On-Shell Methods in Perturbative QCD", Z. Bern, LD, D. Kosower, Annals Phys. 322 (2007) 1587, arXiv:0704.2798 [hep-ph]

Lecture 1

Parton model, QCD factorization and parton distribution functions



QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \bar{q}_i \gamma^\mu (i\partial_\mu - g_s t^a_{ij} A^a_\mu) q_j - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

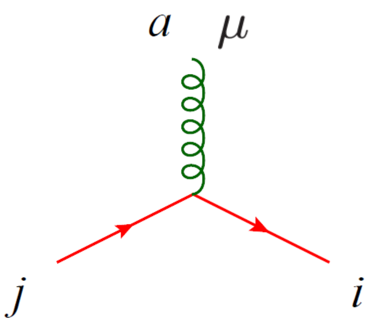
Like QED:

- couples spin ½ matter (quarks) to vector fields (gluons)
- for zero electron/quark mass, has only one dimensionless parameter at classical level ($e \rightarrow g_s$, or $\alpha \rightarrow \alpha_s$)

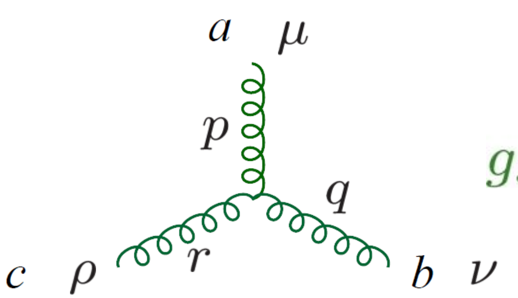
Unlike QED:

- quarks come in three colors, which can be changed by gluon exchange/emission
- gluons themselves have 8 colors, and self-interactions

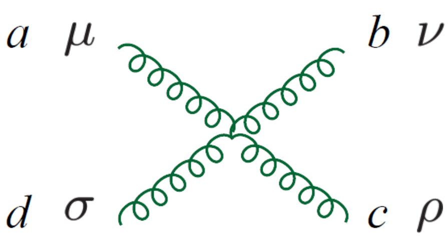
QCD Feynman rules



$$-ig_s(t^a)^i_j\gamma^\mu$$



$$g_s f^{abc}[(p-q)^\rho\eta^{\mu\nu} + (q-r)^\mu\eta^{\nu\rho} + (r-p)^\nu\eta^{\rho\mu}]$$



$$-ig_s^2 f^{ace}f^{bde}[\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\rho\nu}]$$

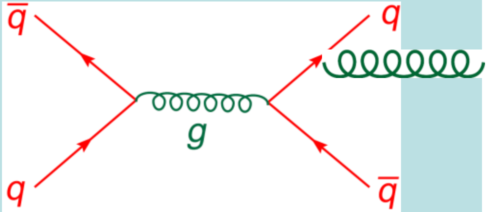
+ 2 permutations

Classical Scale Invariance

- If a theory contains a mass parameter m or other dimensionful parameter, then physics can change strongly with distance/energy scale:

$$p << m \quad vs. \quad p >> m$$

- Massless QCD (and QED) have no such parameter, and are classically scale invariant.
- From a QCD scattering process, probability of emission of a soft gluon with energy E << hard parton energies must be logarithmically distributed:

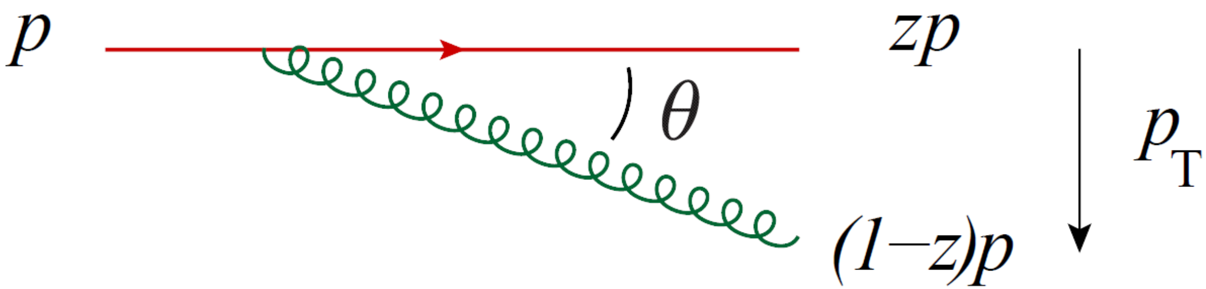


$$P(E) \propto \alpha_s \frac{dE}{E}$$

- Same is true for dependence on emission angle θ in the collinear region with θ << hard parton angles

$$P(\theta) \propto \alpha_s \frac{d\theta}{\theta}$$

Collinear splitting



In the collinear region we can trade (E,θ,ϕ) for (z,p_{T},ϕ):

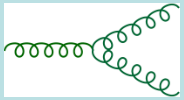
$$P(z,p_T) \propto \alpha_s P(z) \frac{dp_T}{p_T}$$

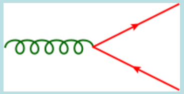
$p_T \ll$ other momentum transfers in collision

$P(z)$ = Altarelli-Parisi splitting kernel. Later we'll calculate it.

Soft behavior dE/E, $E \sim (1\text{-}z)$ implies $P(z) \sim \frac{1}{1-z}$ as $z \rightarrow 1$

Two big qualitative differences from QED:





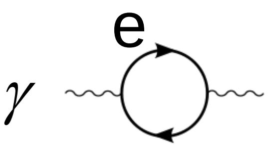
- Gluons can also split into pairs of gluons (as well as quarks)
- Quantum effects (running of coupling) have opposite sign

Asymptotic Freedom

Gross, Wilczek, Politzer (1973)

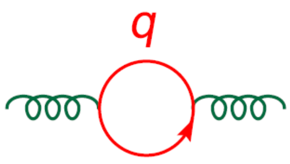
Quantum fluctuations of massless virtual particles polarize vacuum

QED: electrons screen charge (e larger at short distances)



> 0 → $$e^2(r) = \frac{e^2(r_0)}{1+\frac{2e^2(r_0)}{3\pi}\, ln\frac{r}{r_0}}$$

QCD: gluons anti-screen charge (g_s smaller at short distances)



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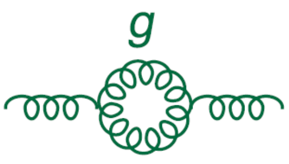

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Gluon self-interactions make quarks almost free, and make QCD calculable at short distances (high energies)

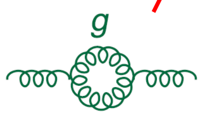
Asymptotic Freedom (cont.)

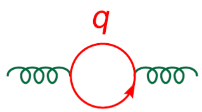
- Write QCD coupling α_s as function not of distance but of momentum transfer = renormalization scale μ
- QCD does not predict coupling at any given point, e.g. $\mu = M_Z$
- But it does determine its evolution with μ:

$$\frac{d\alpha_s(\mu^2)}{d\ln\mu^2} = \beta(\alpha_s(\mu^2))$$

$$\beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \cdots)$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}, \qquad b_1 = \frac{17C_A^2 - (5C_A + 3C_F)n_f}{24\pi^2}$$





$$C_A = N_c = 3 \qquad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

Leading order solution

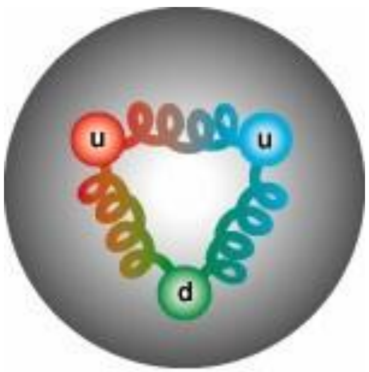
$$\frac{d\alpha_s}{d \ln \mu^2} = -b_0 \alpha_s^2 \quad \Rightarrow \quad d(1/\alpha_s) = b_0 \, d \ln \mu^2$$

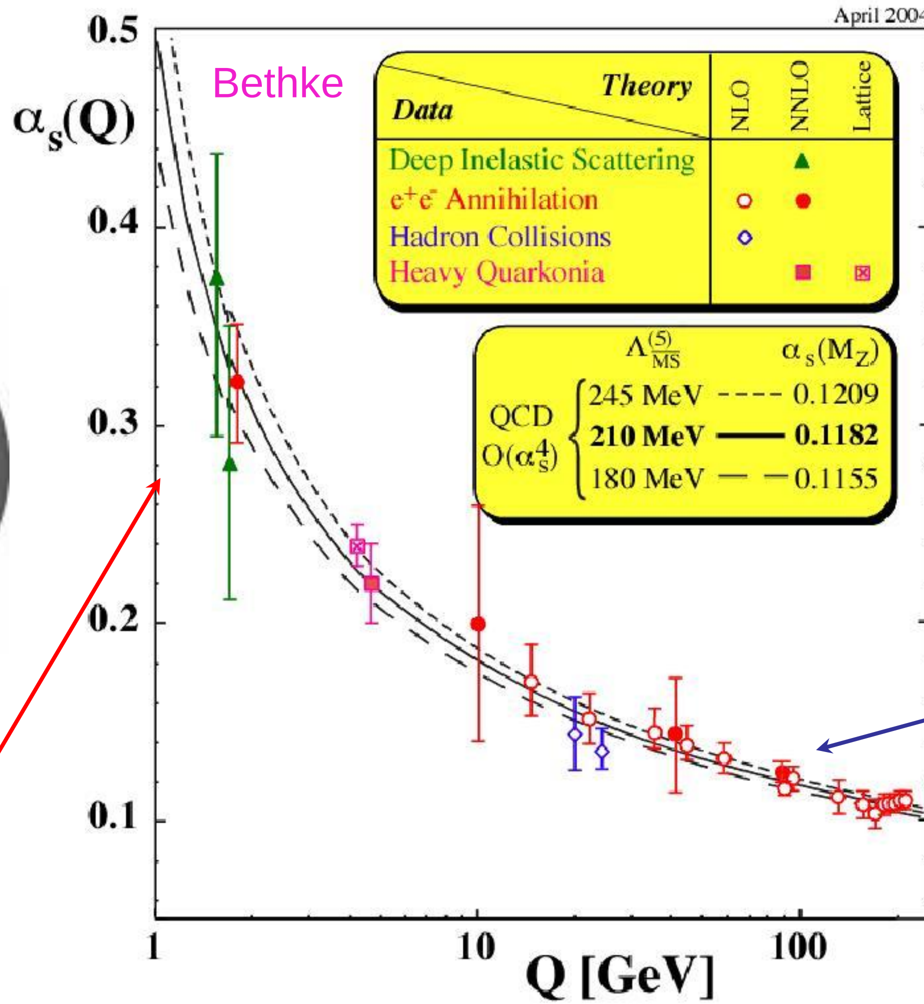
$$\Rightarrow \quad \frac{1}{\alpha_s} = b_0 \ln(\mu^2/\Lambda^2)$$

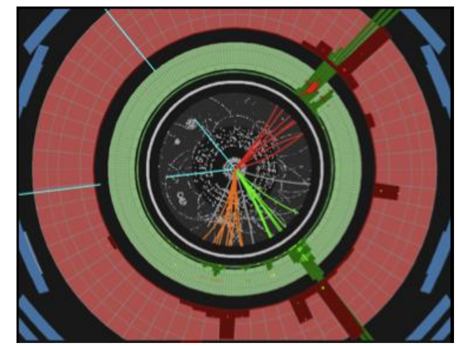
$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln(\mu^2/\Lambda^2)} = \frac{\alpha_s(\mu_0^2)}{1 + b_0 \alpha_s(\mu_0^2) \ln(\mu^2/\mu_0^2)}$$

Higher order solutions qualitatively similar

α_s Measurements

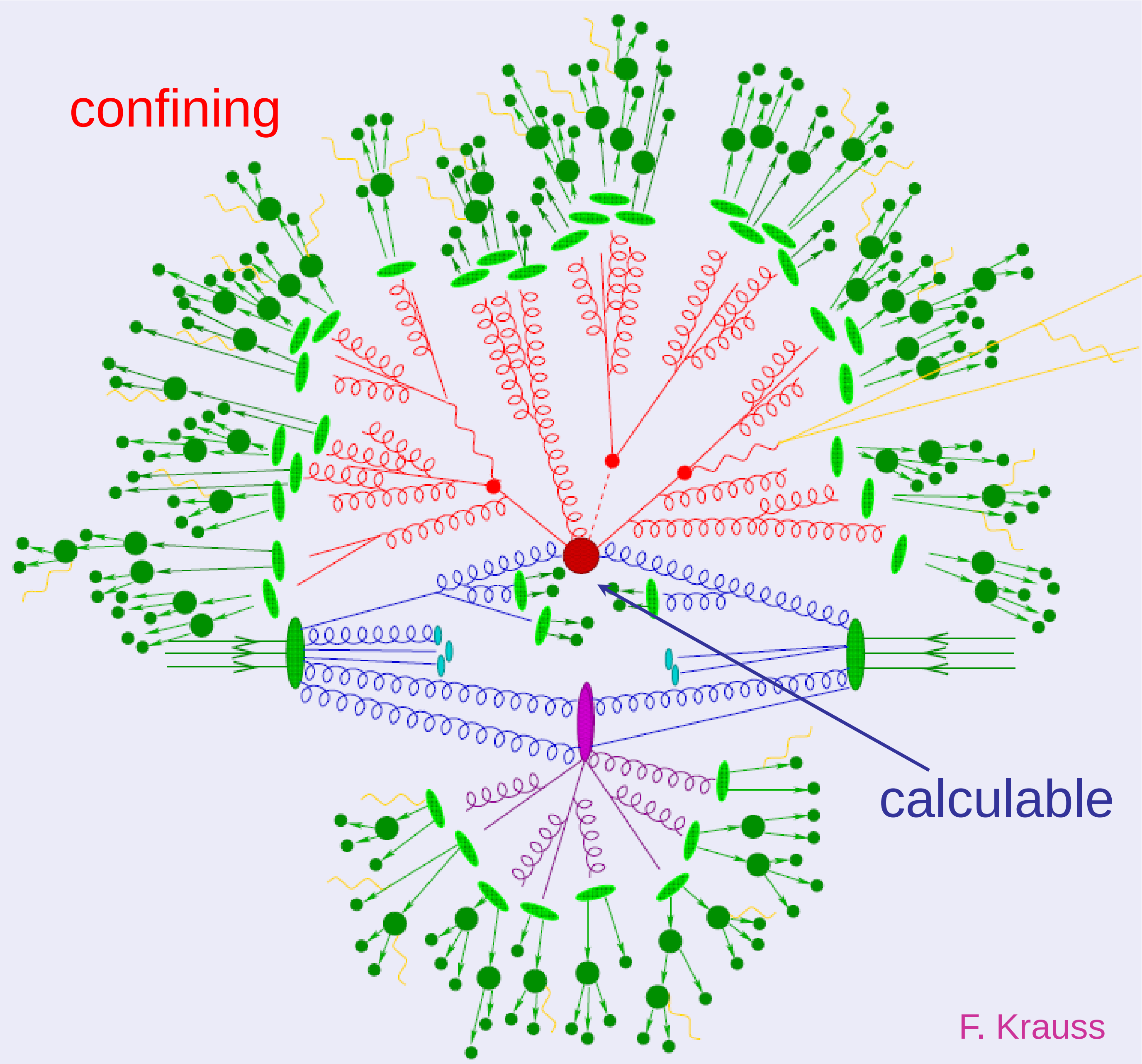






calculable

confining

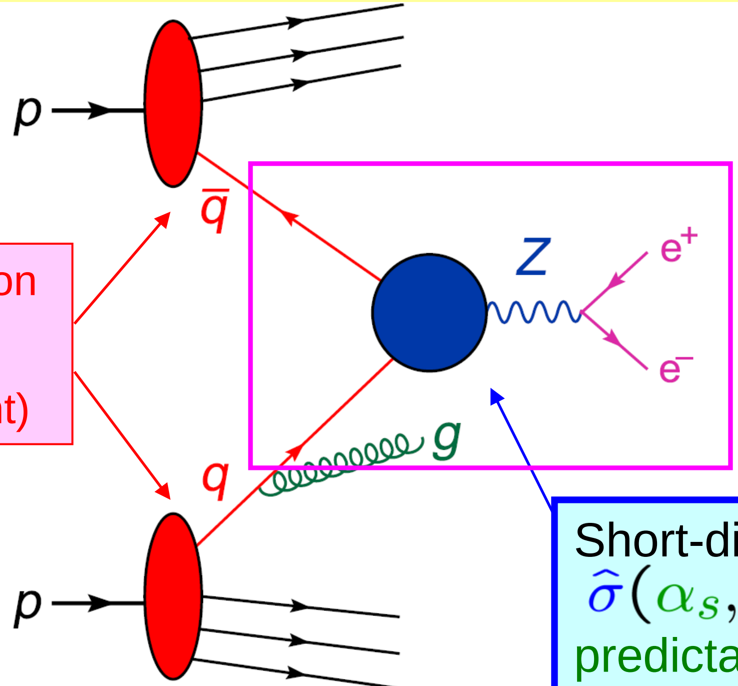


confining
calculable
F. Krauss

QCD Factorization & Parton Model

Quarks and gluons (partons) in proton almost free, sampled one at a time in hard collisions

Parton distribution functions (from experiment)



Box separates "femto-universe" from long-distance effects like parton distributions.

size = factorization scale μ_F ("arbitrary")

Short-distance cross section $\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$ predictable using perturbative QCD

Renorm. scale ("arbitrary")

Basic factorization formula

Any "suitable" Inclusive final state

Parton distribution function: prob. of finding parton a in proton 1, carrying fraction x_1 of its momentum

factorization scale ("arbitrary")

$$\sigma^{pp\to Z+X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to Z+X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

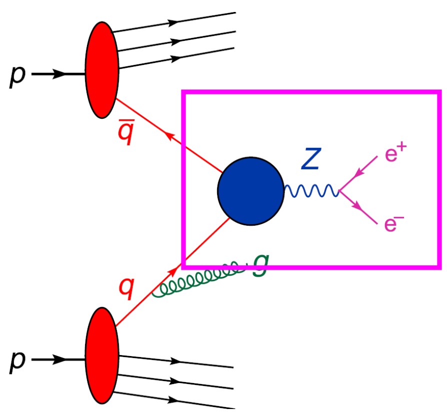
Partonic cross section, computable in perturbative QCD

partonic CM energy^2

renormalization scale ("arbitrary")

$$s = (p_1+p_2)^2 \approx 2p_1\cdot p_2$$
$$\hat{s} = (x_1p_1+x_2p_2)^2 \approx 2x_1x_2p_1\cdot p_2 \approx sx_1x_2$$

" $+ X$ " (inclusive) very important: proton is allowed to do anything, as long as it provides a quark or gluon with the right longitudinal momentum fraction x

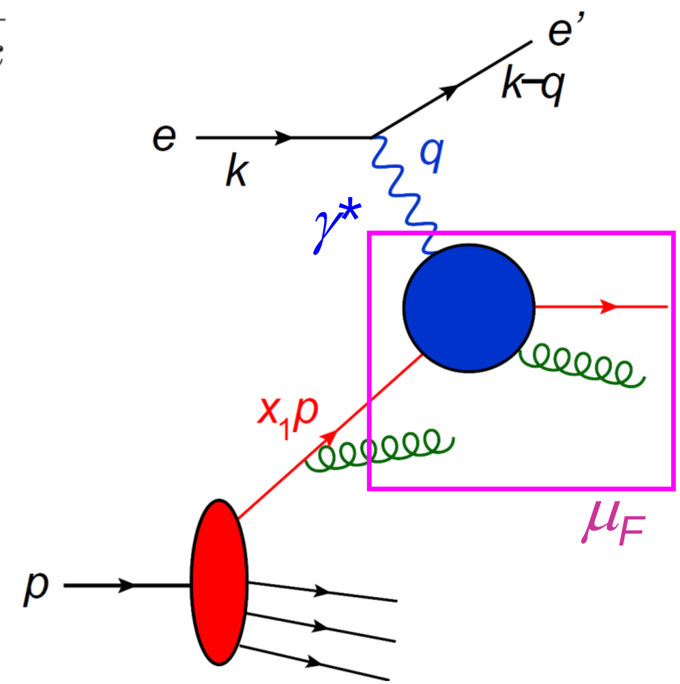


Each piece on RHS varies logarithmically with μ_F while LHS is (or should be) independent of μ_F, μ_R

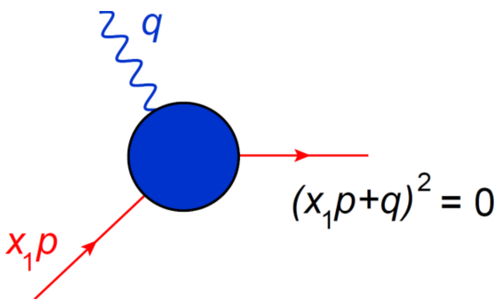
Factorization in deep inelastic scattering

$$\sigma^{ep\to e'+X}(q^2,p\cdot q,p\cdot k;\alpha_s,\mu_R,\mu_F) \ = \ \sum_a \int_0^1 dx_1 f_a(x_1,\alpha_s,\mu_F) \times \ C^a(x_1,q^2,p\cdot q,p\cdot k;\alpha_s,\mu_R,\mu_F)$$

Define: $Q^2 \equiv -q^2 \qquad x \equiv \dfrac{Q^2}{2p\cdot q} \qquad y \equiv \dfrac{p\cdot q}{p\cdot k}$



LO kinematics:



$$0 = (x_1 p+q)^2 = x_1^2 p^2 + 2x_1 p\cdot q + q^2 = 2x_1 p\cdot q - Q^2$$

$\Rightarrow$ $\boxed{x_1 = x \text{ at LO}}$ $\qquad C^a|_{\mathrm{LO}} \propto \delta(x-x_1)$

x = momentum fraction of struck quark

DIS (cont.)

More precisely:

$$\frac{d^2\sigma^{ep\to e'+X}}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\,\frac{1+(1-y)^2}{2} F_2(x,Q^2)$$

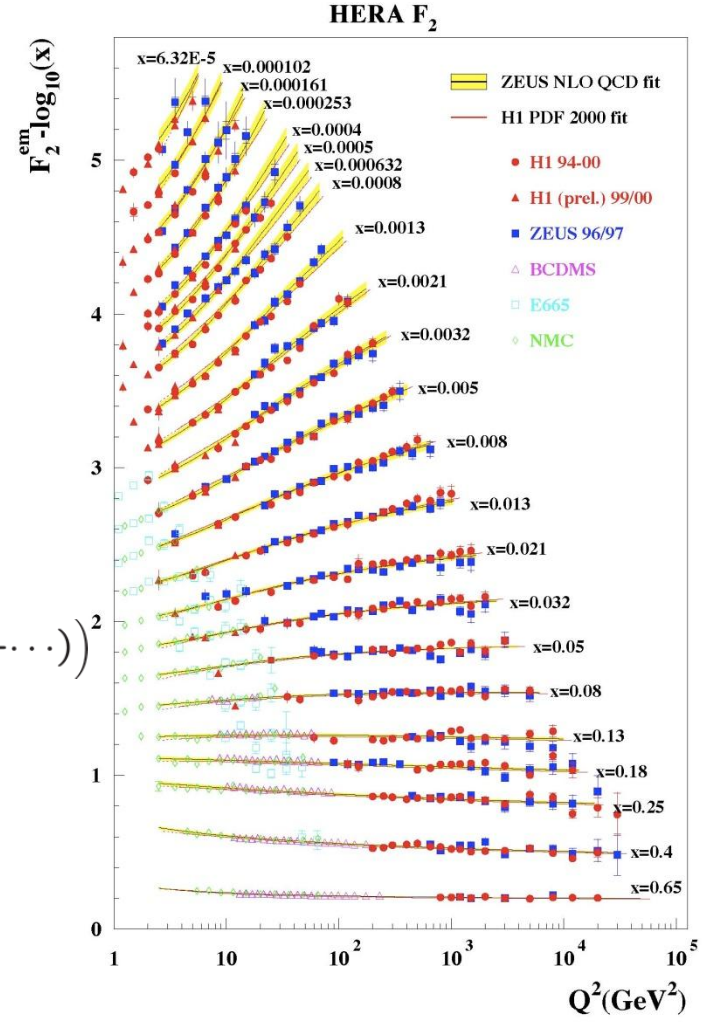
$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_a^2 f_a(x,\mu_F) \;+\; \mathcal{O}(\alpha_s)$$

Usually take $\mu_F = Q$

Proton:

$$F_2^p = x\left(\tfrac{4}{9}(u(x)+\bar{u}(x)+\cdots)+\tfrac{1}{9}(d(x)+\bar{d}(x)+s(x)+\bar{s}(x)+\cdots)\right)$$

High precision DIS data over large range of (x,Q^2) from ep collider HERA



Separate *u* from *d*?

Proton: $F_2^p = x\left(\frac{4}{9}(u(x)+\bar{u}(x)+\cdots)+\frac{1}{9}(d(x)+\bar{d}(x)+s(x)+\bar{s}(x)+\cdots)\right)$

Neutron: Isospin symmetry exchanges
p ←→ *n* at hadron level *u* ←→ *d* at quark level
→ Second linear combination:

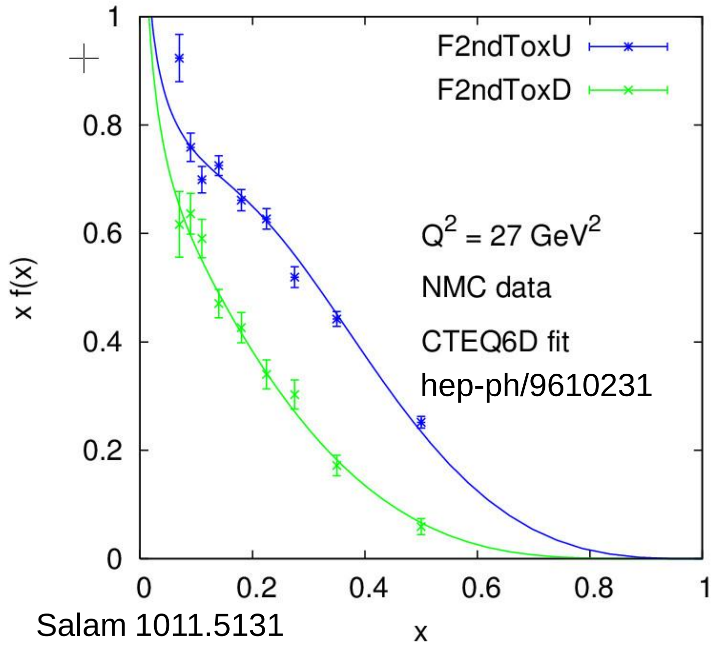
$$u_n(x) = d_p(x) \equiv d(x)$$
$$d_n(x) = u_p(x) \equiv u(x)$$

$$F_2^n = x\left(\frac{4}{9}(d(x)+\bar{d}(x)+\cdots)+\frac{1}{9}(u(x)+\bar{u}(x)+s(x)+\bar{s}(x)+\cdots)\right)$$

In practice deuteron or heavier nuclear targets used instead of neutron

$$3F_2^p - \frac{6}{5}F_2^d = \text{“}xu(x)\text{”}$$
$$-3F_2^p + \frac{24}{5}F_2^d = \text{“}xd(x)\text{”}$$

Reasonable picture for large x where “valence quarks” dominate.



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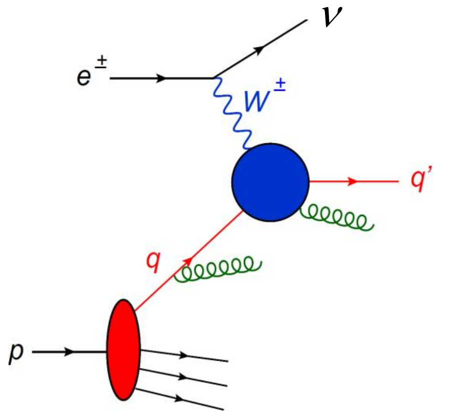
Quark vs. antiquark?

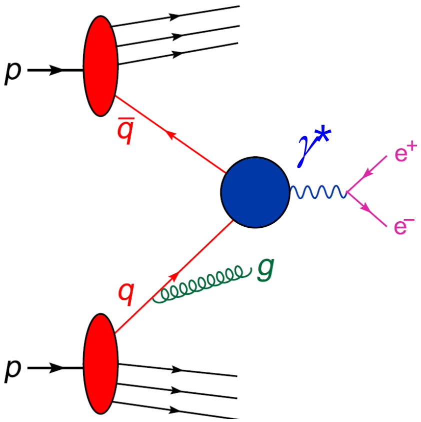
1) Use charged current scattering via *W* exchange:
- Neutrino beams back in fixed target era
- At HERA:

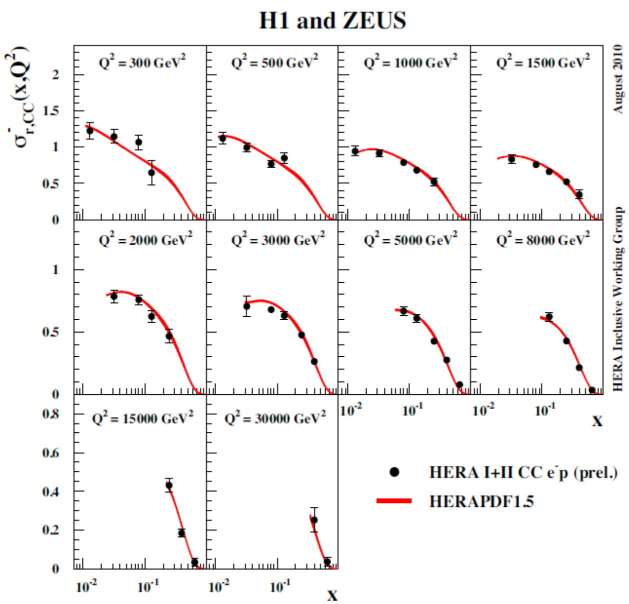
$$\sigma^{e^+p\to\bar\nu+X} \propto x(\bar u+\bar c)+(1-y)^2x(d+s)$$
$$\sigma^{e^-p\to\nu+X} \propto x(u+c)+(1-y)^2x(\bar d+\bar s)$$



2) Drell-Yan production, pp → γ^* + X, etc.

$$\sigma^{pp\to l^+l^-+X} \propto \tfrac{4}{9}u\bar u+\tfrac{1}{9}d\bar d+\cdots$$





Sum rules

- As factorization scale changes, parton distributions change.
- For example $g \to q + \bar{q}$ and $q \to q + g$
- However, certain quantities are conserved:
- Net number of quarks of each flavor:

$$\int_0^1 dx\ (u(x) - \bar{u}(x)) \ = \ 2$$

$$\int_0^1 dx\ (d(x) - \bar{d}(x)) \ = \ 1$$

$$\int_0^1 dx\ (s(x) - \bar{s}(x)) \ = \ 0$$

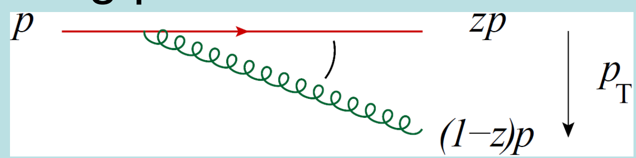
- Momentum of proton:

$$\int_0^1 dx\ x\Big(\sum_q (q(x) + \bar{q}(x)) + g(x)\Big) \ = \ 1$$

Initial state splitting and PDF evolution

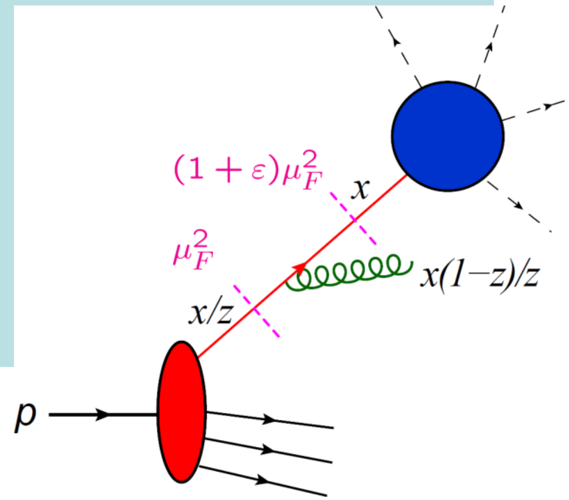
- Recall the general collinear splitting process

$$P(z,p_T) = \frac{\alpha_s}{2\pi} p_{qq}(z) \frac{dp_T^2}{p_T^2}$$



$$p_{qq}(z) = C_F \frac{1+z^2}{1-z}$$ ← (calculate later)

Let's specify to case of initial state radiation before a hard collision, and vary the factorization scale $\mu_F = p_T^{\text{cut}}$ infinitesimally:



$$\frac{dq(x,\mu_F^2)}{d\ln\mu_F^2} = \frac{1}{\varepsilon}\frac{\alpha_s}{2\pi}\int_{\mu_F^2}^{(1+\varepsilon)\mu_F^2}\frac{dp_T^2}{p_T^2}\int_0^1 dz\ p_{qq}(z)\frac{q(x/z,\mu_F^2)}{z}$$

$$= \frac{\alpha_s}{2\pi}\int_x^1 dz\ p_{qq}(z)\frac{q(x/z,\mu_F^2)}{z}$$

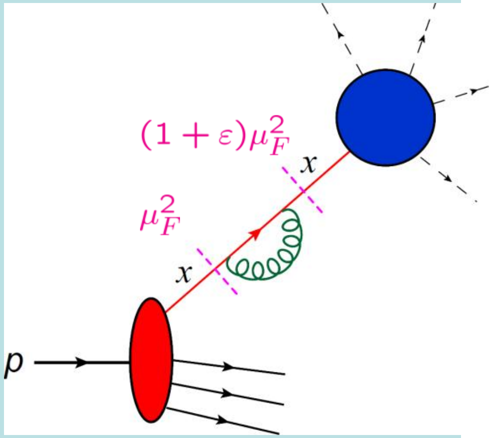
almost the DGLAP equation

The $z \to 1$ limit

• A soft-gluon singularity in
$$\frac{dq(x,\mu_F^2)}{d\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\int_x^1 dz\, \frac{1+z^2}{1-z}\frac{q(x/z,\mu_F^2)}{z}$$
is cancelled by a virtual correction at $z = 1$:
$$\sim -\frac{\alpha_s C_F}{2\pi}\, q(x,\mu_F^2)$$

• It's conventional to regularize the $z \to 1$ limit using a "plus" prescription, defining

$$\int_0^1 dz\, \frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz\, \frac{f(z)-f(1)}{1-z}$$



• The full DGLAP kernel for $q \to qg$ is

$$P_{qq}(z) = C_F\Big[\frac{1+z^2}{(1-z)_+} + c\,\delta(1-z)\Big]$$

where c can be fixed by quark-number conservation:

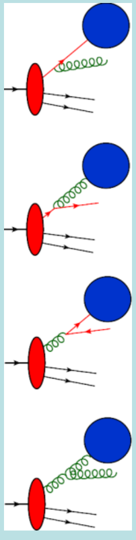
$$0 = \int_0^1 dy\, q(y) \int_0^1 dz \Big[\frac{1+z^2}{(1-z)_+} + c\,\delta(1-z)\Big]$$

$$\Rightarrow \quad 0 = c + \int_0^1 dz \frac{1+z^2-2}{1-z} = c - \frac{3}{2} \qquad \Rightarrow \quad c = \frac{3}{2}$$

DGLAP Equations

- Write $$\int_x^1 dz\, P(z) \frac{f(x/z)}{z} \equiv P \otimes f$$
- Full DGLAP equations couple q and g together:

$$\frac{d}{d\ln\mu_F^2}\begin{pmatrix} q \\ g \end{pmatrix} = \frac{\alpha_s(\mu_F^2)}{2\pi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$



$$\begin{aligned}
P_{qq}(z) &= C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \\
P_{gq}(z) &= C_F\,\frac{1+(1-z)^2}{z} \\
P_{qg}(z) &= T_R\left[z^2+(1-z)^2\right] \\
P_{gg}(z) &= 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] \\
&\quad + \frac{11C_A - 4n_f T_R}{6}\,\delta(1-z)
\end{aligned}$$

$$T_R = \frac{1}{2}$$

Sample evolutions

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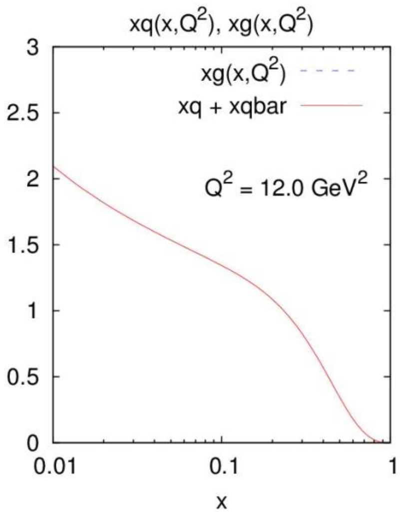
No initial gluons



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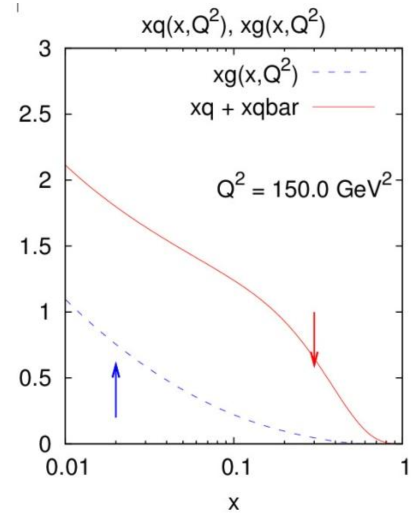
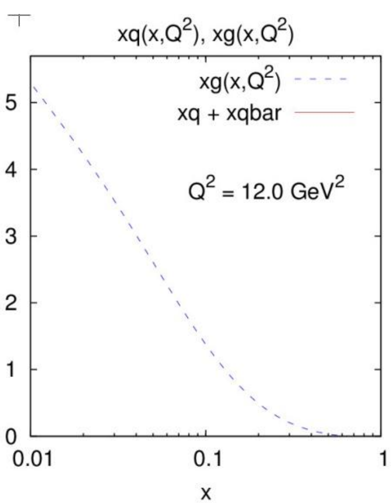


No initial quarks



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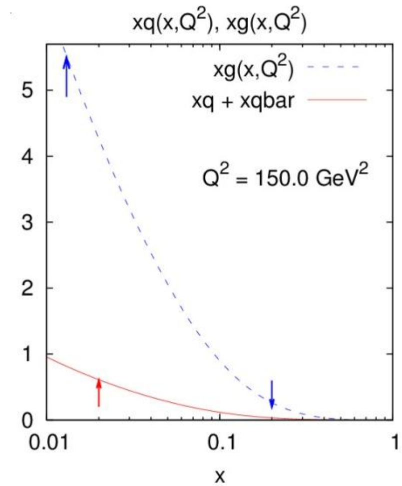
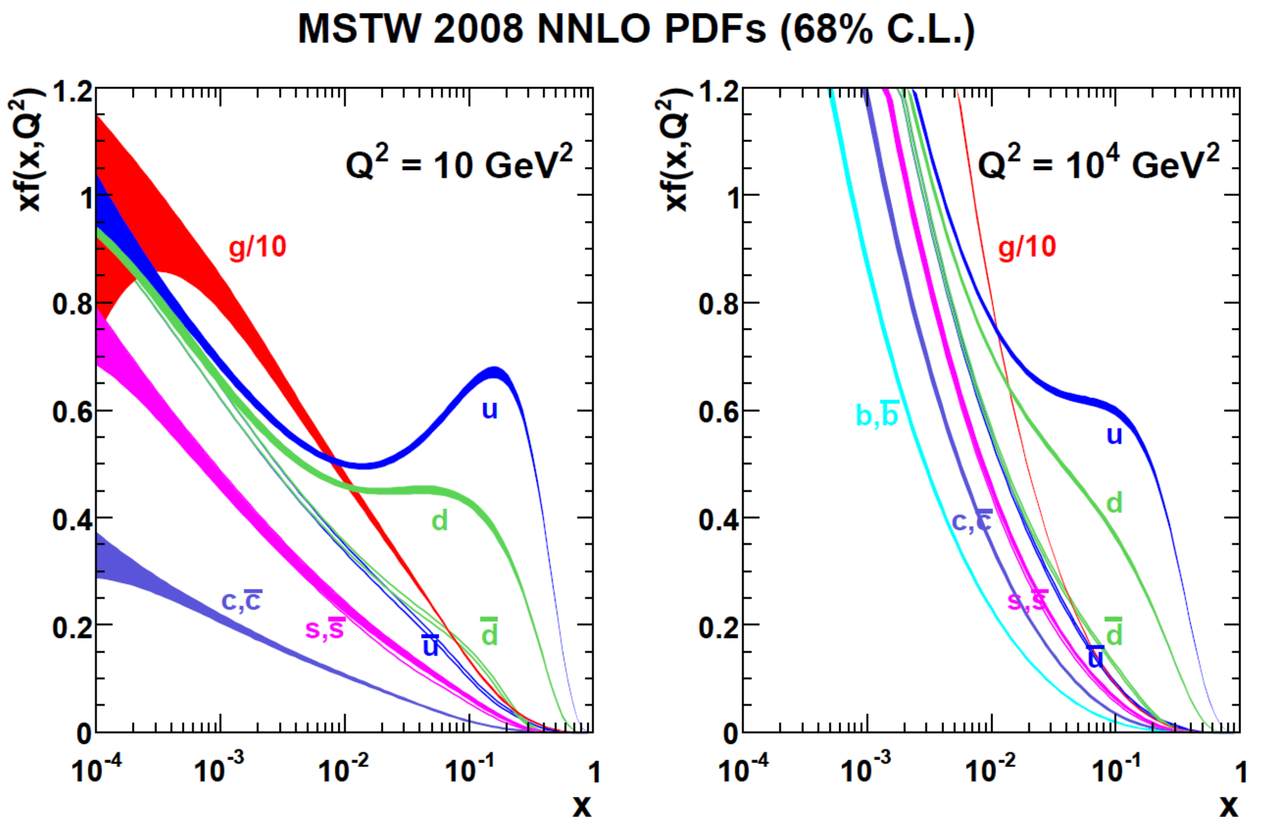


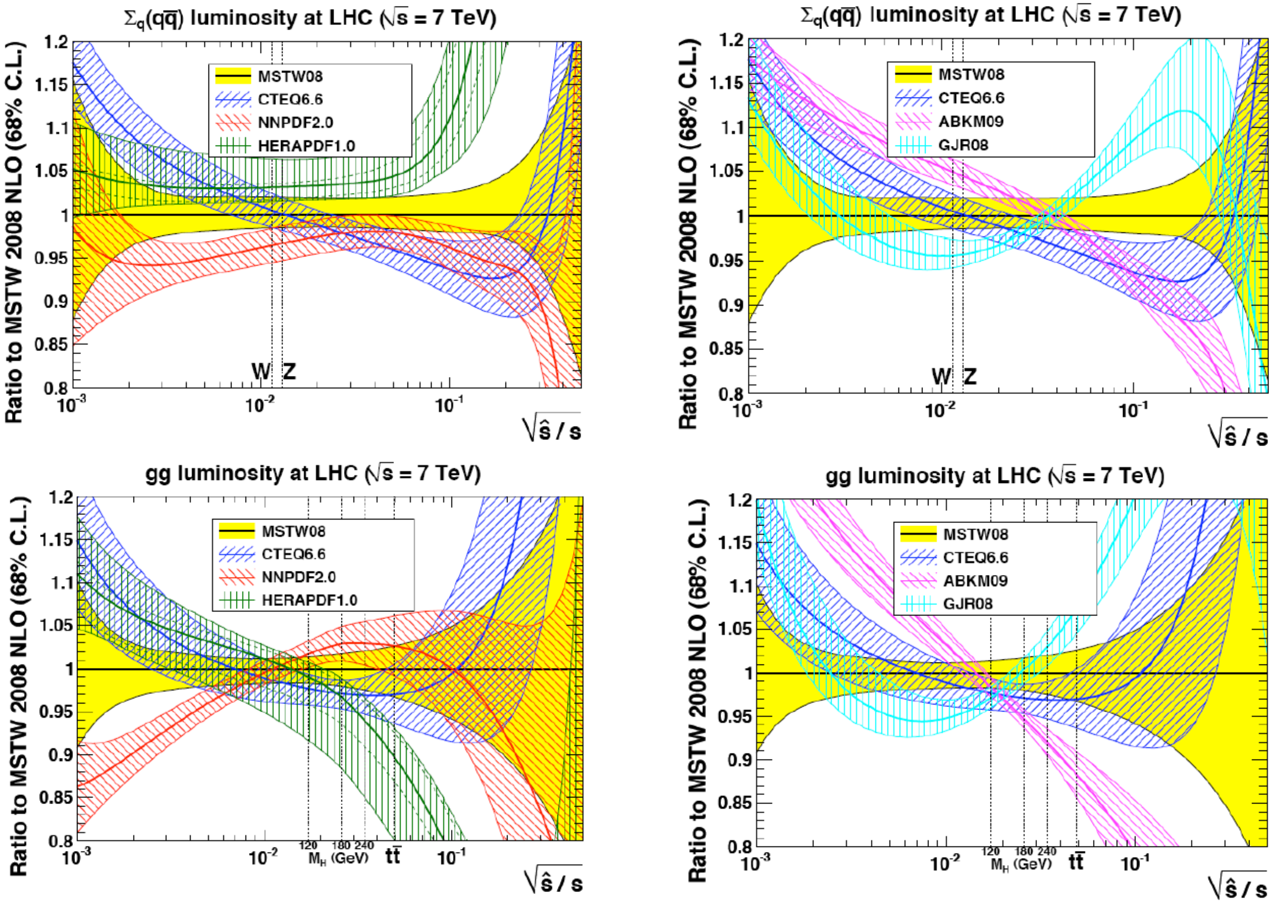
Modern Global PDF Fits

NNPDF, MSTW, CTEQ, ABKM, HERAPDF, GJR,…
Performed at LO, NLO, NNLO in QCD, with "error sets"
Different analyses do not always get identical results



Compare "Luminosity Functions"

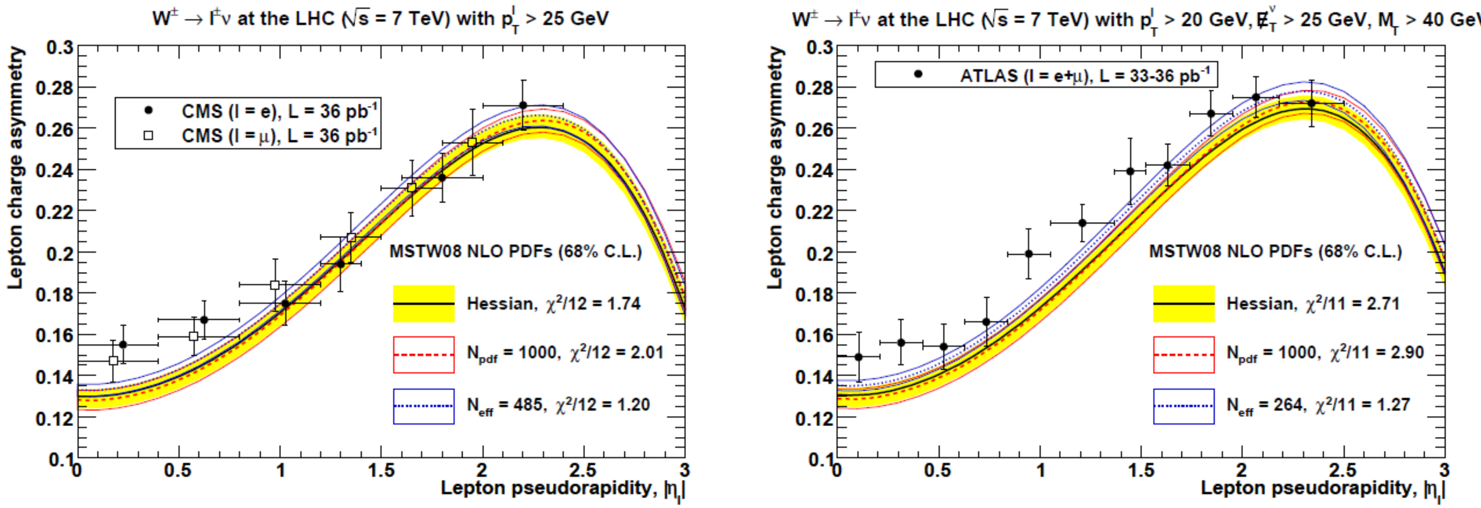
$$\sigma^{ab\to X}(\hat{s}) \propto \int_0^1 dx_1 dx_2 f_a(x_1) f_b(x_2)\delta(x_1 x_2 - \hat{s}/s) \equiv \mathcal{L}_{ab}(\hat{s}/s)$$



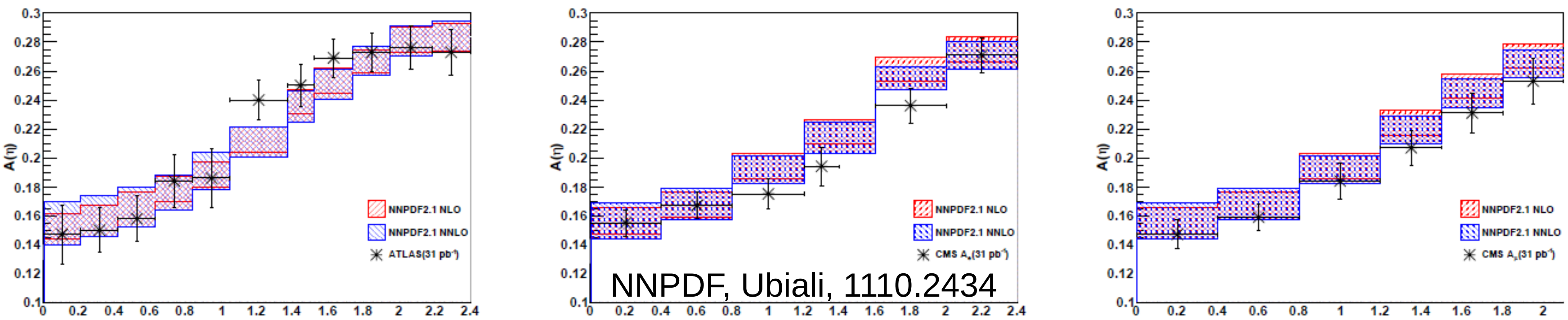
G. Watt

LHC lepton charge asymmetry now discriminating between pdf sets

$$A_l(\eta_l) = \frac{d\sigma(pp \to W^+ \to l^+\nu) - d\sigma(pp \to W^- \to l^-\bar{\nu})}{d\sigma(pp \to W^+ \to l^+\nu) + d\sigma(pp \to W^- \to l^-\bar{\nu})} \quad \sim u_v - d_v$$

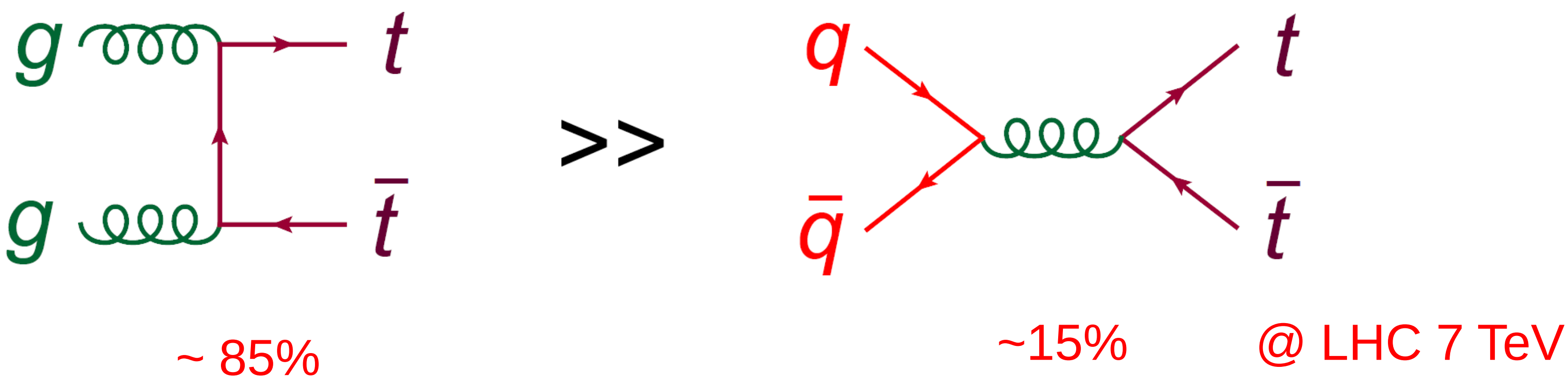


MSTW, Thorne, Watt, 1205.4024



NNPDF, Ubiali, 1110.2434

Top quark cross section as gluon luminosimeter



Need higher order QCD corrections to σ^{top}
to fully utilize the experimental measurements