
GABRIELA BARENBOIM PRESENTS
THE IνCREDIBLES
NOW PLAYING
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Standard scenario: solar $\oplus$ atmos. with 3-ν

Let us assign:

$$\Delta m_{23}^2 = m_3^2 - m_2^2 = \Delta m_{atmos}^2, \quad \Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{solar}^2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{MNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Atmos. L/E $\mu \to \tau$ Atmos. L/E $\mu \leftrightarrow e$ Solar L/E $e \to \mu, \tau$ $\beta\beta 0\nu$ decay

500km/GeV   15km/MeV

$$\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}$$

Solar and atmospheric anomalies approximately decouple as independent 2-by-2 mixing phenomena because

- Hierarchy between the two mass splittings: $|\Delta m^2_{atmos}| \gg |\Delta m^2_{solar}|$
- Small θ_{13}: $\sin\theta_{13} = V_{e3} \quad \leftrightarrow V_{ub}$

I. $E/L \sim \Delta m_{23}^2$:

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{23}^2}{4E} L\right) \\
P(\nu_e \to \nu_\tau) &= c_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{23}^2}{4E} L\right) \\
P(\nu_\mu \to \nu_\tau) &= c_{13}^4 \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m_{23}^2}{4E} L\right)
\end{aligned}
$$

Daya Bay θ_{13} miserably small !!!

$$(\Delta m_{23}^2, \theta_{23}) = (\Delta m_{atmos}^2, \theta_{\mathrm{atmos}}),$$

II. $E/L \sim \Delta m^2_{12}$:

$$P(\nu_e \to \nu_e) \simeq c^4_{13} \left(1 - \sin^2 2\theta_{12} \, \sin^2 \left(\frac{\Delta m^2_{12}}{4E} \, L\right)\right) + s^4_{13}$$

$$(\Delta m^2_{12}, \theta_{12}) = (\Delta m^2_{\rm solar}, \theta_{\rm solar})$$

When solar and atmospheric fits are done in the context of three families nothing changes too much

CP violation in neutrino oscillations

Can I have it ?

Vacuum oscillations ($W_{\alpha\beta}^{jk} \equiv [U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k}]$)

$$P(\nu_\alpha \rightarrow \nu_\beta) = -4 \sum_{k>j} \mathrm{Re}[\mathrm{W}_{\alpha\beta}^{\mathrm{jk}}] \sin^2\left(\frac{\Delta \mathrm{m}_{\mathrm{jk}}^2 \mathrm{L}}{4\mathrm{E}_\nu}\right)$$

$$\pm \quad 2 \sum_{k>j} \mathrm{Im}[\mathrm{W}_{\alpha\beta}^{\mathrm{jk}}] \sin\left(\frac{\Delta \mathrm{m}_{\mathrm{jk}}^2 \mathrm{L}}{2\mathrm{E}_\nu}\right)$$

CP violation shows up as a difference between $P(\nu_\alpha \to \nu_\beta)$ and $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$

By CPT:

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

CP and T-odd terms cancel in survival probabilities $\to$ need appearance measurements: $\alpha \neq \beta$

Observability of CP-violation $\leftrightarrow$ measurable CP-asymmetries:

$$A^{CP}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)} \quad A^{T}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)}{P(\nu_\alpha \to \nu_\beta) + P(\nu_\beta \to \nu_\alpha)}$$

$$
\begin{aligned}
P(\nu_i \to \nu_j) &= P_{CP}(\nu_i \to \nu_j) + P_{\not{CP}}(\nu_i \to \nu_j) \\
P(\bar{\nu}_i \to \bar{\nu}_j) &= P_{CP}(\nu_i \to \nu_j) - P_{\not{CP}}(\nu_i \to \nu_j)
\end{aligned}
$$

$$
P_{CP}(\nu_i \to \nu_j) = \delta_{ij} - 4\mathrm{Re}J_{12}^{ji}\sin^2\Delta_{12} - 4\mathrm{Re}J_{23}^{ji}\sin^2\Delta_{23} - 4\mathrm{Re}J_{31}^{ji}\sin^2\Delta_{31},
$$

$$
P_{\not{CP}}(\nu_i \to \nu_j) = -8\sigma_{ij} J \sin\Delta_{12}\sin\Delta_{23}\sin\Delta_{31},
$$

$s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\mathrm{CP}}$

$$
\begin{aligned}
J_{kh}^{ij} &\equiv U_{ik}U_{kj}^{\dagger}U_{jh}U_{hi}^{\dagger} \\
\Delta_{ij} &\equiv \Delta m_{ij}^2 L/4E \\
\sigma_{ij} &\equiv \sum_k \varepsilon_{ijk}
\end{aligned}
$$

CP(T)-odd terms the same for all $\alpha \neq \beta$:

$$A^{\mathrm{CP(T)-odd}}_{\nu_\alpha \nu_\beta} = \frac{2 \sin\delta\, c_{13} \sin 2\theta_{13} \overbrace{\sin 2\theta_{12} \frac{\Delta m^2_{12} L}{4E_\nu}}^{\mathrm{solar}} \overbrace{\sin 2\theta_{23} \sin^2 \frac{\Delta m^2_{13} L}{4E_\nu}}^{atmos}}{P^{\mathrm{CP-even}}_{\nu_\alpha \nu_\beta}}$$

GIM suppressed in all the Δm^2 and all the angles, because if any of them is zero, the CP-odd phase is unphysical

- Minimize GIM suppression: $E/L \sim \Delta m^2_{atmos}$
- Effects of δ are more significant in subleading transitions: $\nu_e \to \nu_\mu (\nu_\tau)$:

$$P^{\mathrm{CP-even}}_{\nu_\mu \nu_\tau} \quad = \quad \text{unsuppressed in } \theta_{13} \text{ or } \frac{\Delta m^2_{12} L}{E_\nu}$$

$$P(\nu_\mu \to \nu_\tau) \quad = \quad c^4_{13} \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{23}}{4E} L \right)$$

$$A^{\mathrm{CP(T)-odd}}_{\nu_\mu \nu_\tau} \quad \sim \quad \sin 2\,\theta_{13} \; \frac{\Delta m^2_{12} L}{E_\nu}$$

$$P^{\mathrm{CP-even}}_{\nu_e \nu_\mu(\nu_\tau)} \quad = \quad \text{suppressed in } \theta^2_{13} \text{ or } \left(\frac{\Delta m^2_{12} L}{E_\nu} \right)^2$$

$$P(\nu_e \to \nu_\mu) \quad = \quad \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m^2_{12}}{4E} L \right)$$

$$A^{\mathrm{CP(T)-odd}}_{\nu_e \nu_\mu(\nu_\tau)} \quad \sim \quad \frac{\Delta m^2_{12} L / E_\nu}{\sin 2\theta_{13}} \quad \text{or} \quad \frac{\sin 2\theta_{13}}{\Delta m^2_{12} L / E_\nu}$$

$$
\begin{aligned}
P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)} &= s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta_{23}L}{2}\right) &\equiv P^{atmos} \\
&+ c_{23}^2 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta_{12}L}{2}\right) &\equiv P^{solar} \\
+\tilde{J} \quad & \cos\left(\pm\delta - \frac{\Delta_{23}L}{2}\right) \frac{\Delta_{12}L}{2} \sin\left(\frac{\Delta_{23}L}{2}\right) &\equiv P^{inter}
\end{aligned}
$$

$$
(\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}, \qquad \Delta_{ij} \equiv \frac{\Delta m_{ij}^2}{2E_\nu})
$$

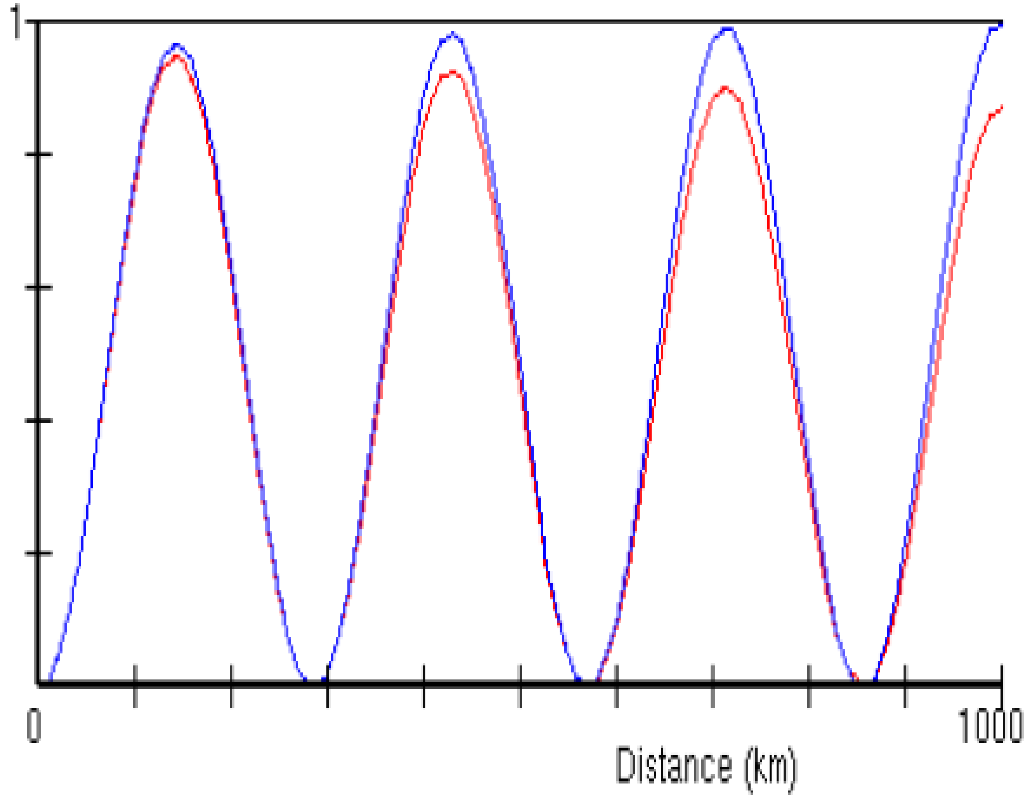
$$
P^{atmos} \gg P^{solar} \quad \rightarrow \quad A^{CP,T}_{\nu_e\nu_\mu(\nu_\tau)} \sim \frac{\Delta_{12}L}{\sin 2\theta_{13}}
$$

$$
P^{solar} \gg P^{atmos} \quad \rightarrow \quad A^{CP,T}_{\nu_e\nu_\mu(\nu_\tau)} \sim \frac{\sin 2\theta_{13}}{\Delta_{12}L}
$$

$$
P^{solar} \simeq P^{atmos} \quad \rightarrow \quad A^{CP,T}_{\nu_e\nu_\mu(\nu_\tau)} = O(1)
$$

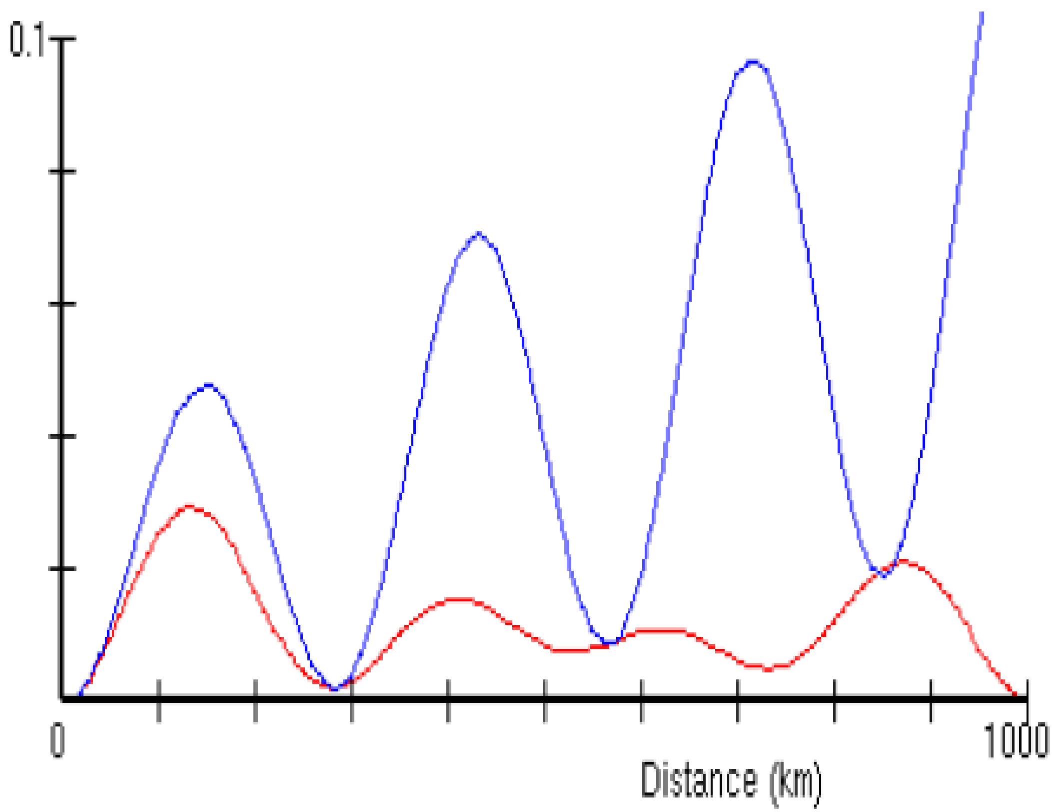
$E_\nu = 500$ MeV $\theta_{13} = 8^\circ$ $\delta = 90^\circ$

$P_{\nu_\mu \to \nu_\tau}$ vs. $P_{\bar{\nu}_\mu \to \bar{\tau}_\mu}$



$E_\nu = 500$ MeV $\quad \theta_{13} = 8^\circ \quad \delta = 90^\circ$

$P_{\nu_e \to \nu_\mu}$ vs. $P_{\bar{\nu}_e \to \bar{\nu}_\mu}$



The challenge

We need to measure for the first time small oscillation probabilities:
need more intense and purer ν sources

- **Superbeams** ν from K, π decay

$$\pi, K \to \quad \nu_\mu \quad O(1\%)\, \nu_e\ \bar{\nu}_\mu;$$
$$\nu_\mu \quad \to \nu_e \to e^-$$
$$\nu_e \to \nu_e \to e^-$$

- **Neutrino factory** ν from muon decay

$$\mu^- \to e^- \quad \nu_\mu \quad \bar{\nu}_e\,;$$
$$\bar{\nu}_e \to \bar{\nu}_\mu \to \mu^+$$
$$\nu_\mu \quad \to \nu_\mu \to \mu^-$$

- β**-beams** from boosted heavy ions decays

$${}^6He^{++} \to^6_3 Li^{+++}\ e^- \qquad \bar{\nu}_e$$
$$\bar{\nu}_e \to \bar{\nu}_\mu \to \mu^+$$

Matter Effects

The oscillation probabilities for three neutrino mixing in matter are not very illuminating. A particularly useful approximation is obtained for $E/L \sim \Delta m^2_{23}$ and to second order in the two small parameters: θ_{13} and Δm^2_{12} :

$$P_{\nu_e \nu_\mu (\bar\nu_e \bar\nu_\mu)} = s^2_{23}\, \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{B_\pm}\right)^2 \; \sin^2\left(\frac{B_\pm L}{2}\right)$$

$$+ c^2_{23}\, \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\left(\frac{A\,L}{2}\right)$$

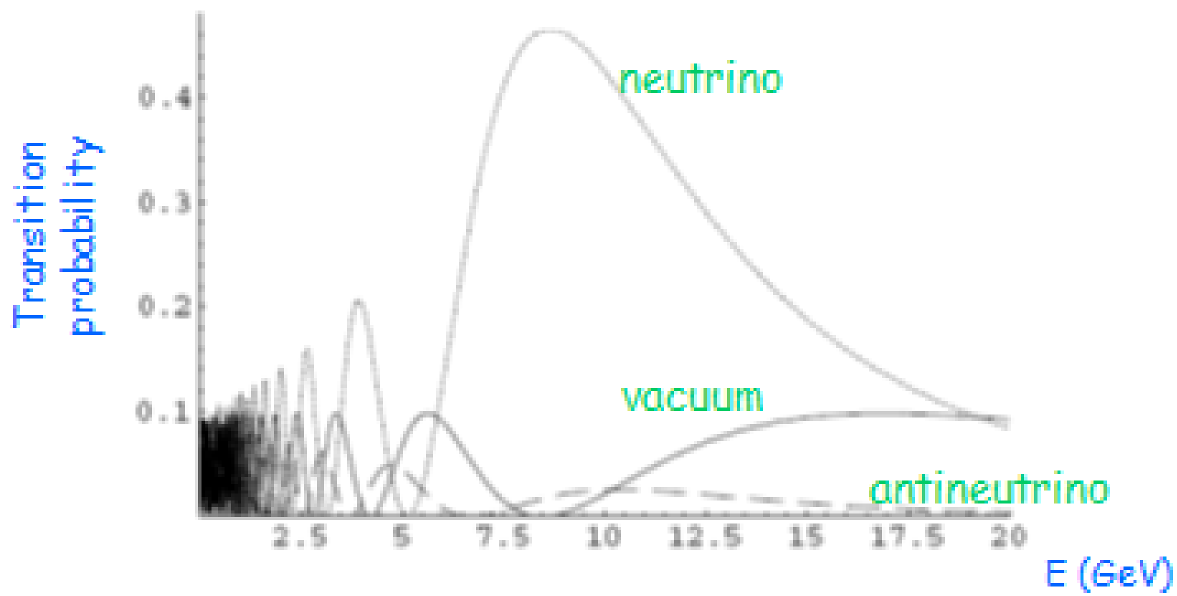
$$+ \tilde J\, \frac{\Delta_{12}}{A}\; \sin\left(\frac{AL}{2}\right) \frac{\Delta_{13}}{B_\pm}\; \sin\left(\frac{B_\pm L}{2}\right)\; \cos\left(\pm\delta - \frac{\Delta_{13}\, L}{2}\right)$$

$$B_\pm = |A \pm \Delta_{13}| \qquad \Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu}$$

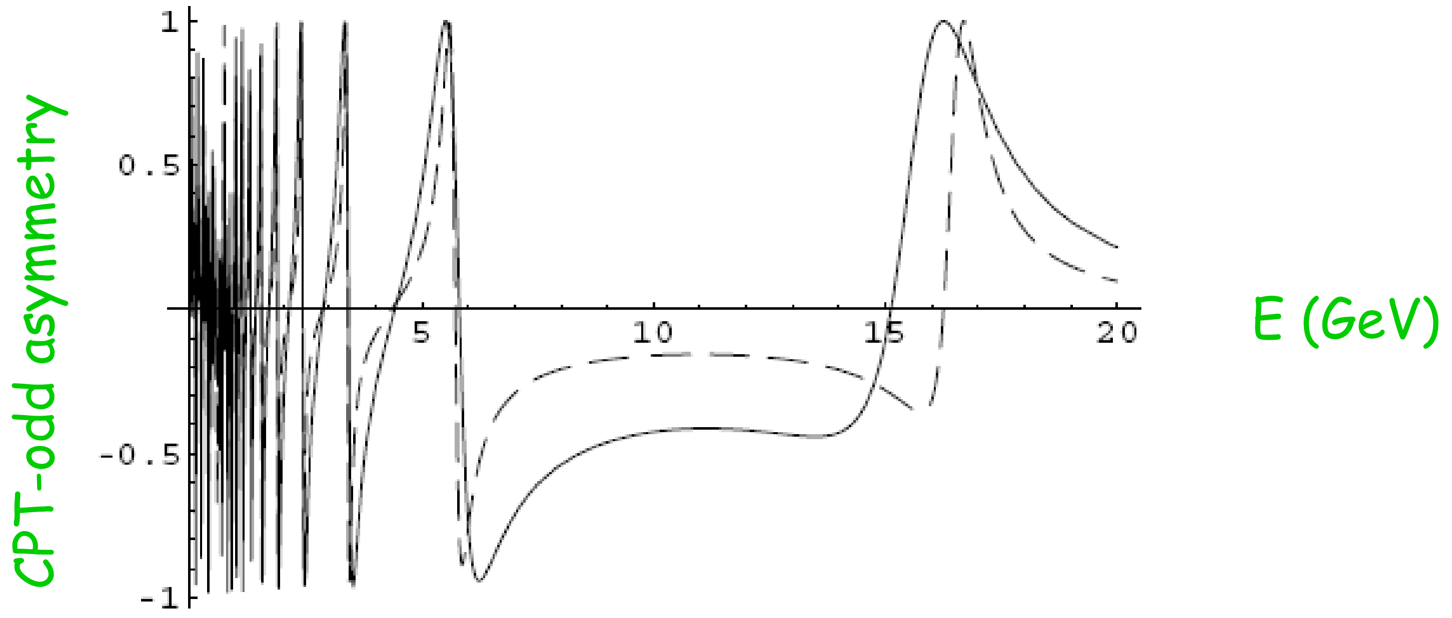
There is an MSW effect in θ_{13}! There is a huge enhacement of the ν or $\bar{\nu}$ (depending on the sign of Δm^2_{23}) channel if:

$$2E_\nu A \sim |\Delta m^2_{13}|, \quad E_\nu \sim 10 - 20 GeV$$

$$\sin^2(2\tilde{\theta}_{13}) = \frac{4s^2_{13}c^2_{13}\left(\frac{\Delta m^2_{13}}{\tilde{a}}\right)^2}{\left(E - \cos 2\theta_{13}\frac{\Delta m^2_{13}}{\tilde{a}}\right)^2 + 4s^2_{13}c^2_{13}\left(\frac{\Delta m^2_{13}}{\tilde{a}}\right)^2}$$



For very long baselines and atmospheric neutrinos...



There is an MSW effect in θ_{13}! There is a huge enhacement of the ν or $\bar{\nu}$ (depending on the sign of Δm^2_{23}) channel if:

$$2E_\nu A \sim |\Delta m^2_{13}|, \qquad E_\nu \sim 10 - 20 GeV$$

- The type of neutrino spectrum can be revealed!
- CP asymmetries exist even if $\sin\delta = 0$:

$$P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$$

E.g., MSW effect can enhance $\nu_e \leftrightarrow \nu_\mu$ and suppress $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ or vice versa.

Survival probabilities are not CP-invariant:

$$P(\nu_a \to \nu_a) \neq P(\bar{\nu}_a \to \bar{\nu}_a)$$

To disentangle fundamental $\not{CP}$ from the matter induced one in LBL experiments – need to measure energy dependence of oscillated signal or signal at two baselines

Alternatives:

- Low-E experiments ($E \sim$ 0.1 – 1 GeV) with $L \sim$ 100 – 1000 km
- Indirect measurements: CP-even terms $\sim \cos\delta_{\rm CP}$ or area of leptonic unitarity triangle

Neutrinos,

In and Beyond the Standard Model:

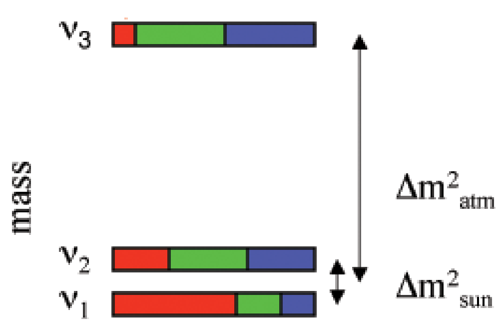
NEUTRINO MASS:

$$\delta m^2_{atm} = 2.7^{+0.4}_{-0.3} \times 10^{-3} eV^2 \qquad L/E = 500\ km/GeV$$

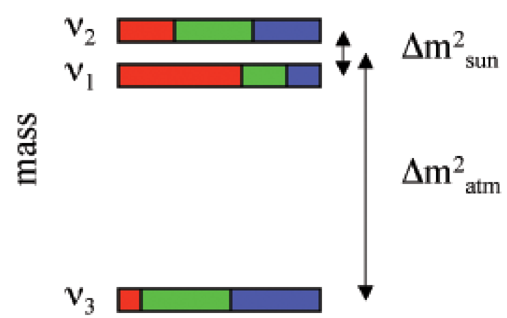
$$\delta m^2_{solar} = 8.0 \pm 0.4 \times 10^{-5} eV^2 \qquad L/E = 15\ km/MeV$$

$$\Downarrow$$

$$m_\nu^{Heavy} > \sqrt{\delta m^2_{atm}} = 50 meV$$

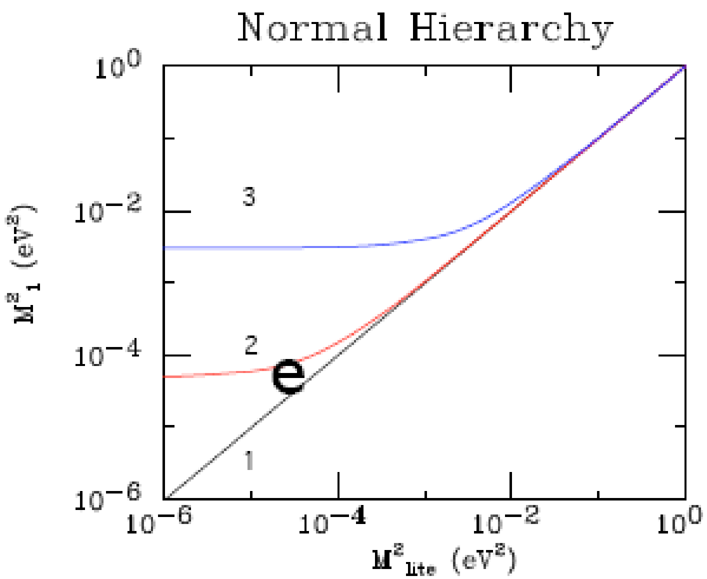


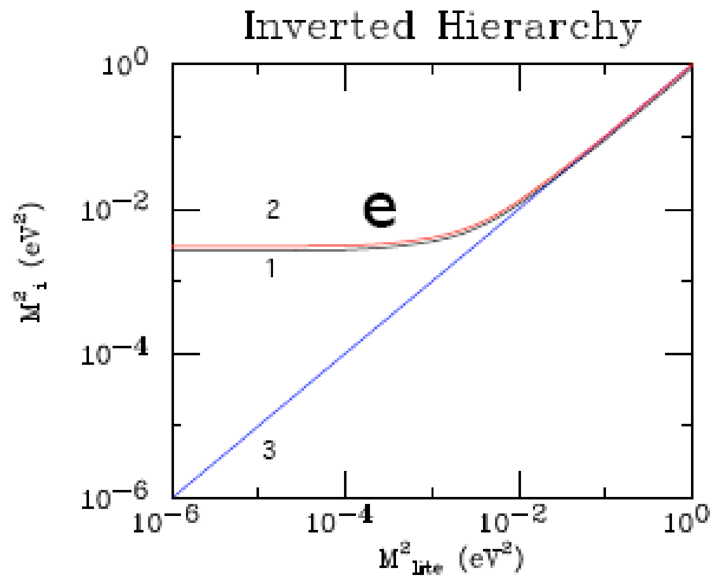
Normal mass hierarchy



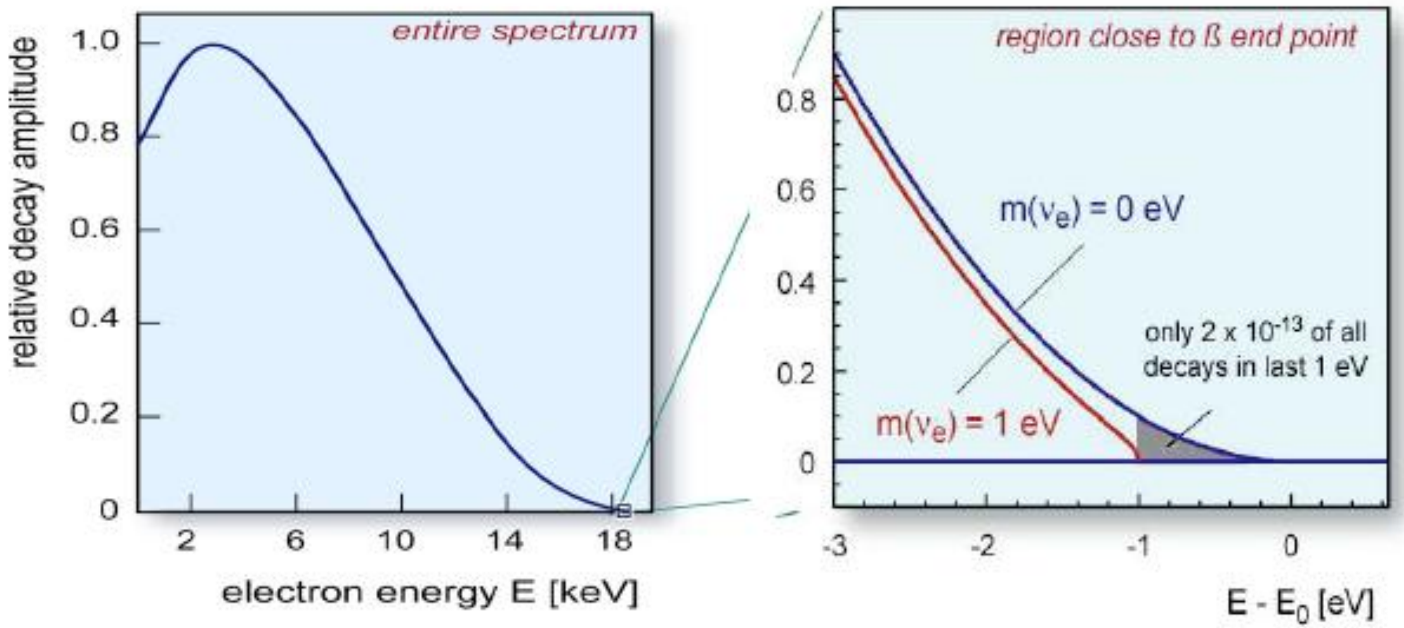
Inverted mass hierarchy

Masses:





States 1 and 2 are ν_e rich.

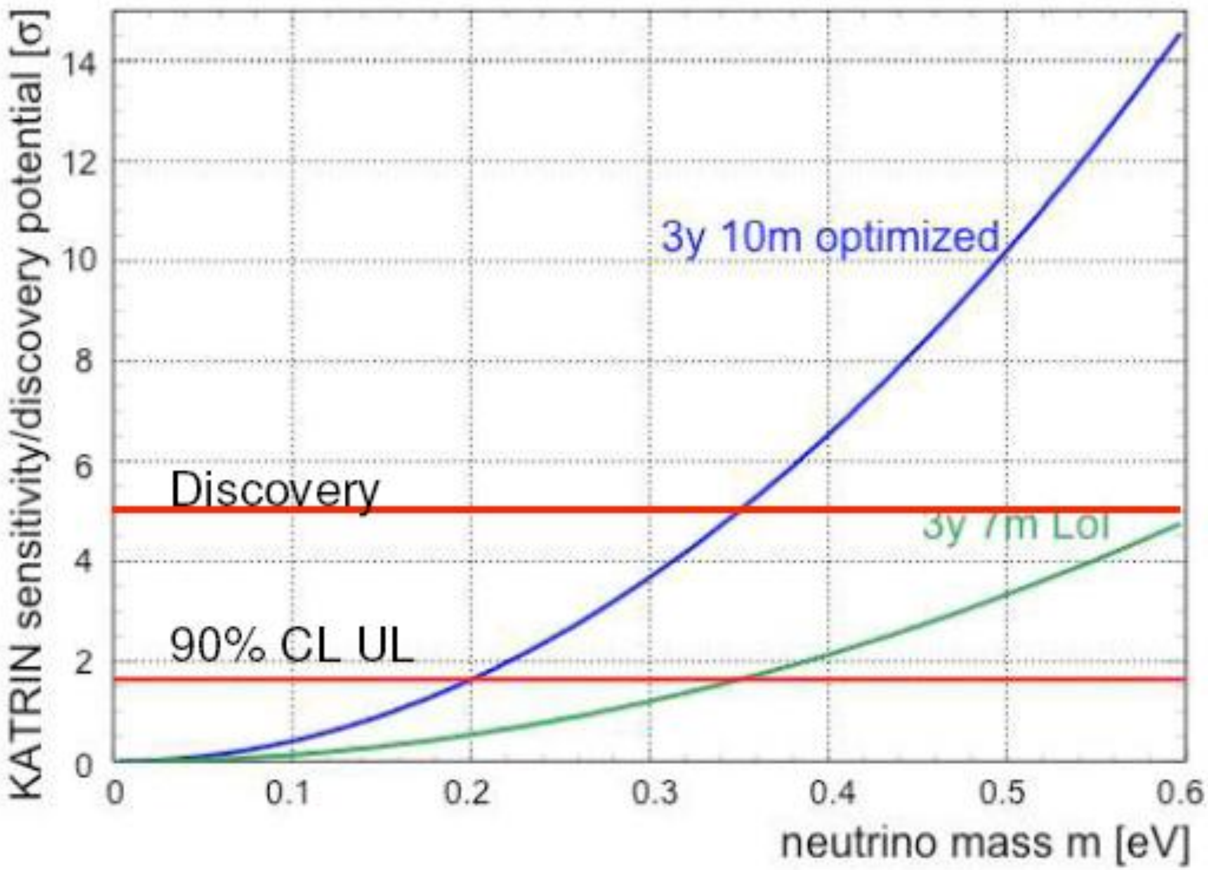


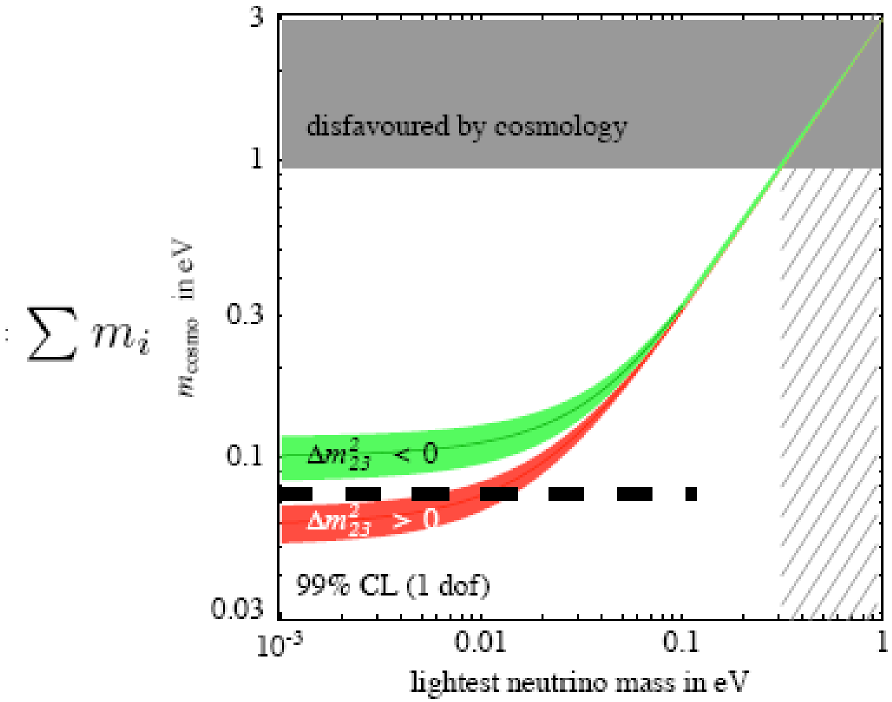
Decay Rate:

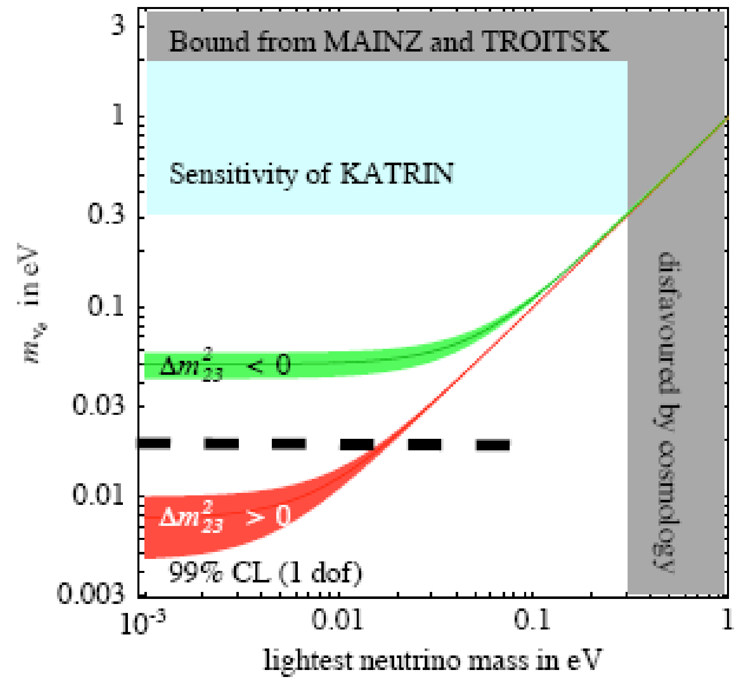
$$|\langle {}^3He + e^- + \bar{\nu}|T|{}^3H\rangle|^2 \sim pE(E_0 - E)\sum_k |U_{ek}|^2\sqrt{(E_0 - E)^2 - m_k^2}$$

if ν's quasi-degenerate: $m_1 \approx m_2 \approx m_3$

$$|\langle {}^3He + e^- + \bar{\nu}|T|{}^3H\rangle|^2 \sim pE(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}$$


3y 10m optimized
Discovery
3y 7m LoI
90% CL UL
KATRIN sensitivity/discovery potential [σ]
neutrino mass m [eV]
0
2
4
6
8
10
12
14
0
0.1
0.2
0.3
0.4
0.5
0.6

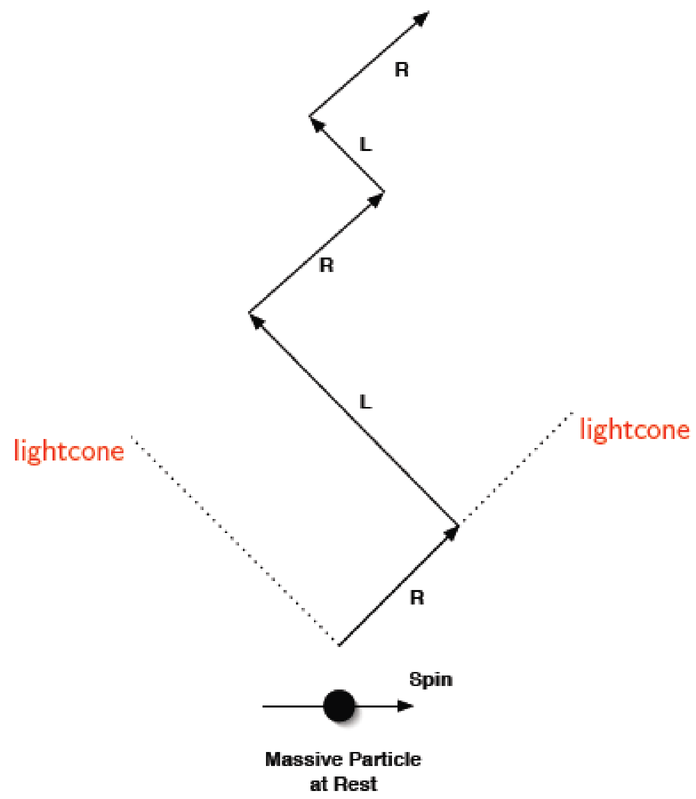




$\sum m_i \approx 60\ eV$ closes the Universe. Limit a few % of this number.

Similarly, if Tritium decay exp. (Hyper-Katrin) could exclude $m_{\nu_e} > \frac{1}{30}\ eV$, then Normal Hierarchy.

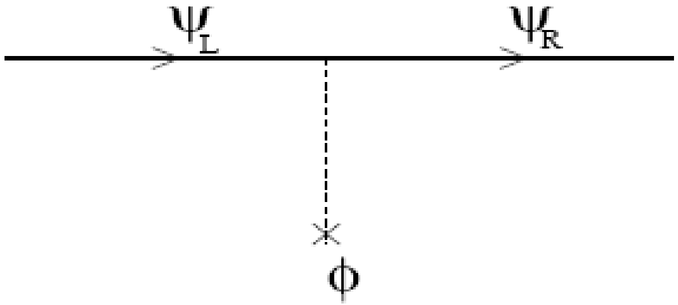
What is Fermion Mass ???



A mass can be thought of as a $L \leftrightarrow R$ transition:

$$m\ \overline{\psi_L}\psi_R + h.c.$$

In the SM fermion masses originate in the interaction with the Higgs field:



$$\lambda_f\ \overline{\psi_L}\ \Phi\ \psi_R + h.c. \rightarrow m_f = \lambda_f\ v$$

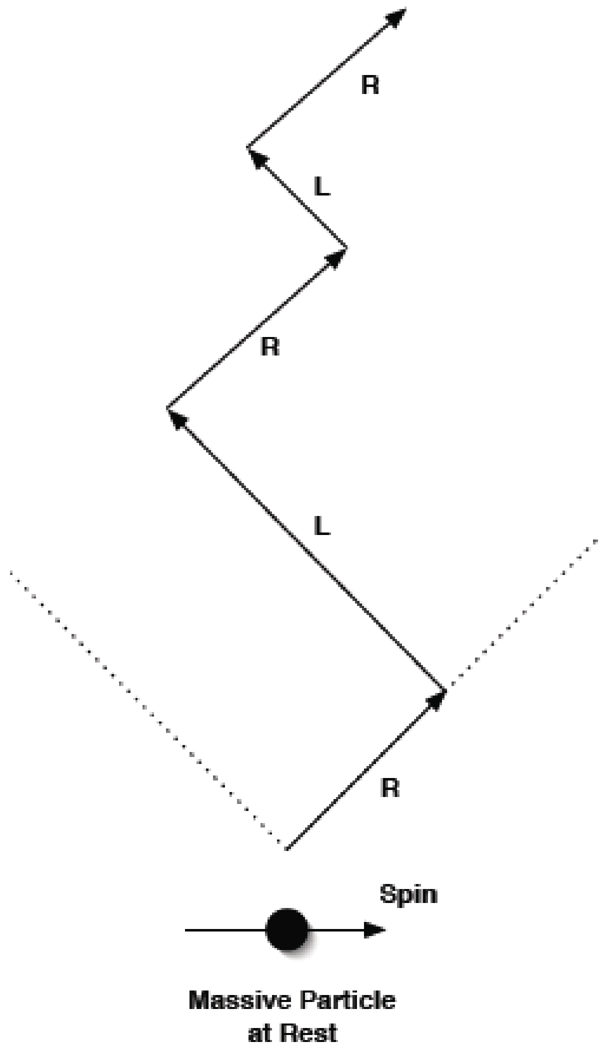
Fermion Masses:

	electron	positron	
Left Chiral	e_L	$\bar{e}_R$	SU(2)×U(1)
Right Chiral	e_R	$\bar{e}_L$	U(1)

CPT: $e_L \leftrightarrow \bar{e}_R$ and $e_R \leftrightarrow \bar{e}_L$

Mass couples L to R:

e_L to e_R AND also $\bar{e}_R$ to $\bar{e}_L$ Dirac Mass terms.



Mass couples L to R:

$$P^2 = M^2, \quad P \cdot S = 0 \quad and \quad S^2 = -1$$

$$u(P,S) = \frac{(1+\gamma_5)}{2} u(\frac{P+MS}{2}) + e^{i\phi} \frac{(1-\gamma_5)}{2} u(\frac{P-MS}{2})$$

right massless left massless

A coupling of
e_L to $\bar{e}_R$ OR e_R to $\bar{e}_L$ would be (Majorana) mass term
but this violates conservation of electric charge!

Seesaw / Dirac Neutrinos / Light Sterile Neutrinos

	Nu	CPT:	Anti-Nu	
Left Chiral	$\boldsymbol{\nu_L}$	$\Leftrightarrow$	$\boldsymbol{\bar{\nu}_R}$	
	$\Updownarrow$		$\Updownarrow$	Dirac Masses
Right Chiral	$\boldsymbol{\nu_R}$	$\Leftrightarrow$	$\boldsymbol{\bar{\nu}_L}$	
		Majorana Masses		

Coupling of

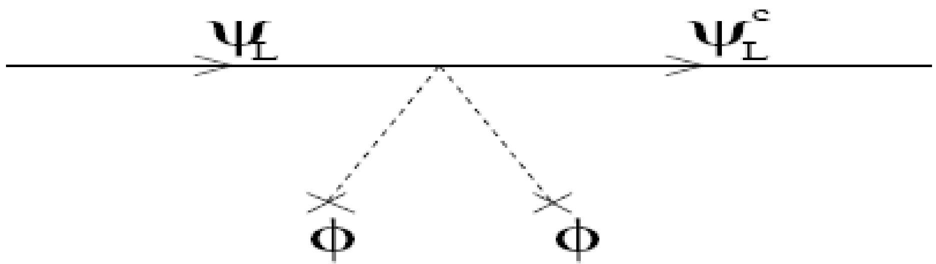
- ν_L to ν_R AND $\bar{\nu}_R$ to $\bar{\nu}_L$ are the Dirac masses.
- ν_L to $\bar{\nu}_R$ forbidden by weak isospin.
- ν_R to $\bar{\nu}_L$ allowed and coefficient is unprotected. $(\rightarrow M)$

CPT $\leftrightarrow$ Lorenzt invariance $\oplus$ unitarity:

L particle $\leftrightarrow$ R antiparticle

For neutral particles, Majorana realized that one can get rid of half of the degrees of freedom in a massive Dirac spinor by identifying:

$$\psi = \psi_L + \psi_R \qquad \psi_R \to (\psi_L)^c = C\bar{\psi}_L^T = C\gamma_0\psi_L^*$$



In the SM one can indeed write a $L \leftrightarrow R$ coupling of this form in a gauge invariant way:

$$\frac{1}{M}\nu_L^T C\ \alpha_\nu \tilde{\Phi}^T\ \tilde{\Phi}\ \nu_L + h.c. \to m_\nu = \alpha_\nu \frac{v^2}{M}$$

A new scale M is needed!

ν_L to $\bar{\nu}_R$ $\qquad$ ν_L to ν_R

$$\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

$\bar{\nu}_R$ to $\bar{\nu}_L$ $\qquad$ ν_R to $\bar{\nu}_L$

Two Majorana neutrinos
with masses m_D^2/M and M

Seesaw:
Yanagida, Gell-man-
Ramond-Slansky

- Coupling of ν_R to $\bar{\nu}_L$ allowed and coefficient is unprotected. $(\rightarrow M)$

Also applies to sterile neutrinos.

Light Sterile Neutrinos and/or Dirac Neutrinos Unexpected!!!

The consequences of this alternative are profound:

- Physics beyond the SM at a scale M!

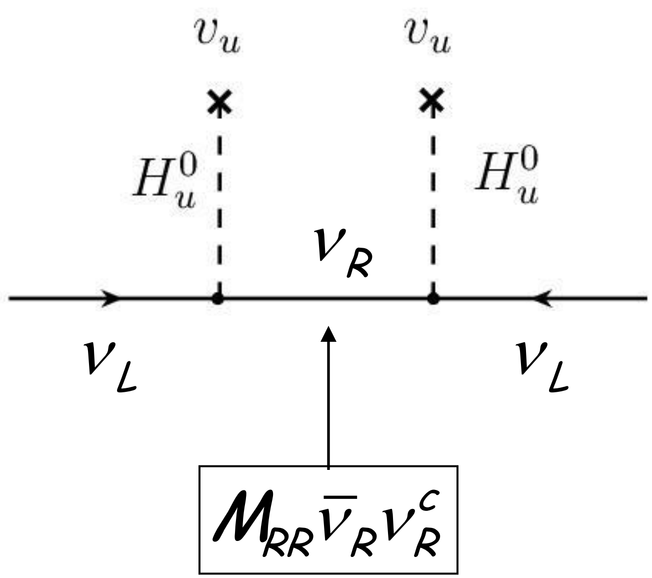
- Majorana fermions carry no conserved charge: L is violated !

$$\nu_L \to e^{i\alpha} \nu_L$$

does not leave the Majorana mass term invariant.
$\rightarrow$ Most welcome for baryogenesis: a mechanism to understand the matter-antimatter asymmetry in the Universe emerges naturally
$\rightarrow$ Most welcome by string theory: it is difficult to get global $U(1)$ charges conserved

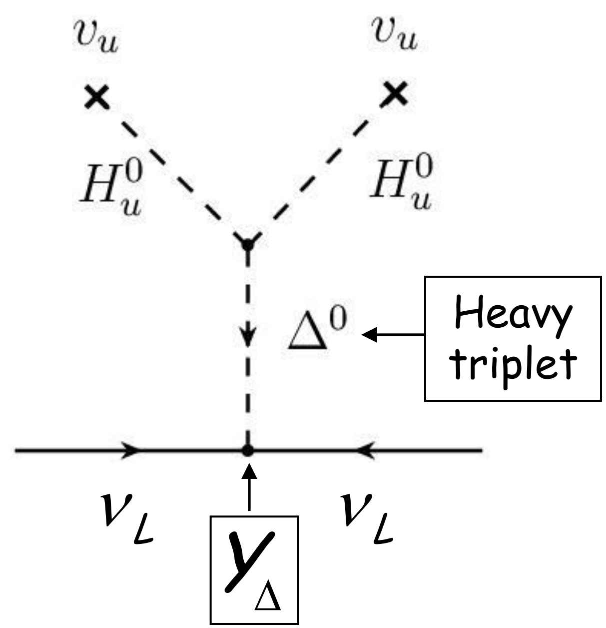
Types of see-saw mechanism

Type I see-saw mechanism



$$\boxed{m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}}$$

Type II see-saw mechanism



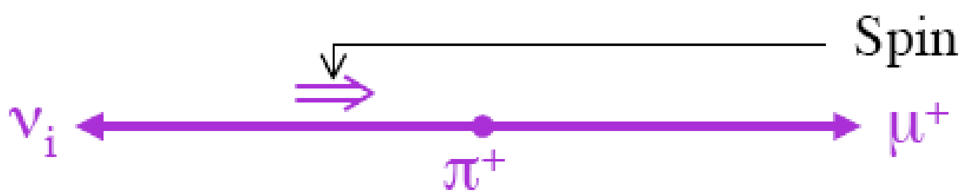
$$\boxed{m_{LL}^{II}\,\overline{\nu}_L \nu_L^c \approx Y_\Delta \frac{v_u^2}{M_\Delta}}$$

How Can We Demonstrate That $\overline{\nu}_i = \nu_i$?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve L ($\nu \to \ell^-$; $\overline{\nu} \to \ell^+$).

An Idea that Does Not Work
[and illustrates why most ideas do not work]

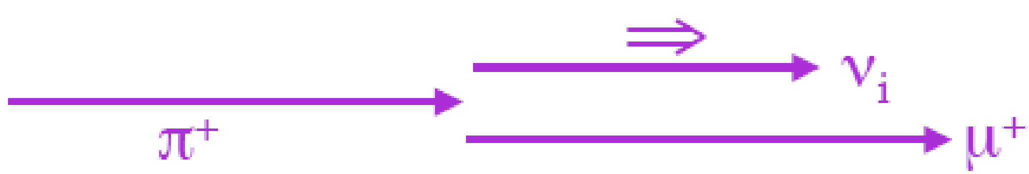
Produce a ν_i via—



Pion Rest Frame

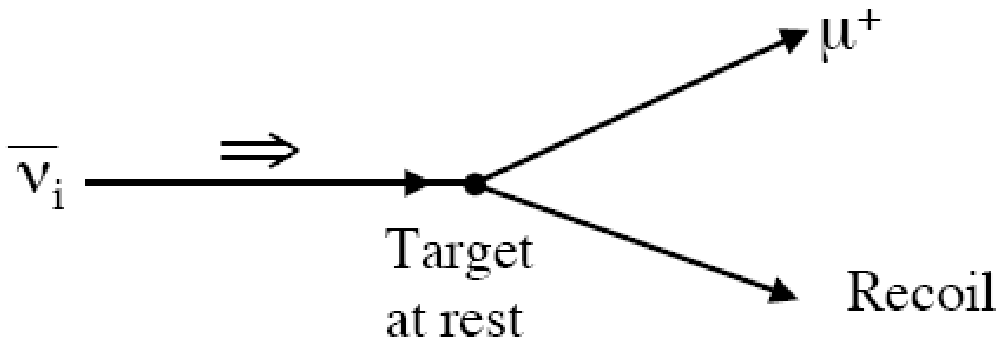
Give the neutrino a Boost:

$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$



Lab. Frame

The SM weak interaction causes—



$\nu_i = \overline{\nu}_i$ means that $\nu_i(h) = \overline{\nu}_i(h)$.

helicity

If $\nu_i \xrightarrow{\Rightarrow}$ $=$ $\overline{\nu}_i \xrightarrow{\Rightarrow}$,

our $\nu_i \xrightarrow{\Rightarrow}$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

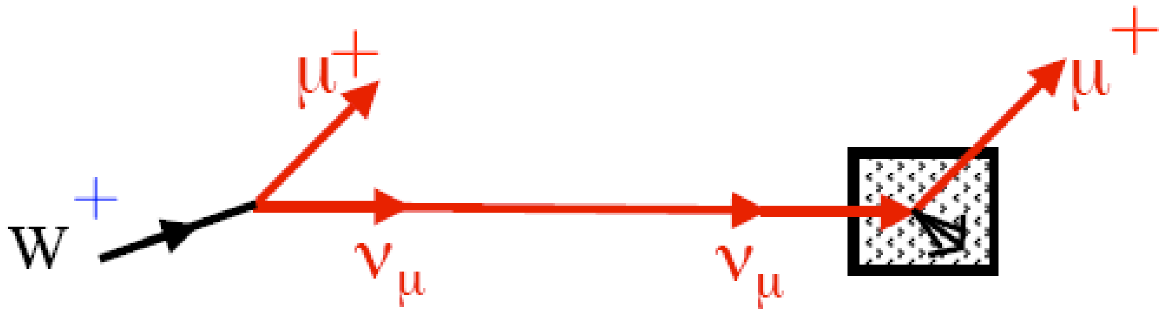
$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu^2(\pi \text{ Rest Frame})}{m_{\nu_i}^2}$$

$$\Rightarrow E_\pi \ (\text{Lab}) > 10^{12} \text{ TeV} \quad \text{if} \quad m_\nu \sim 1 \text{ eV}$$

Fraction of all π–decay that get helicity flipped

$$\approx \left(\frac{m_\nu}{E_\nu(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-16} \quad \text{if } m_\nu \sim 1 \text{ eV}$$

For Majorana Neutrinos

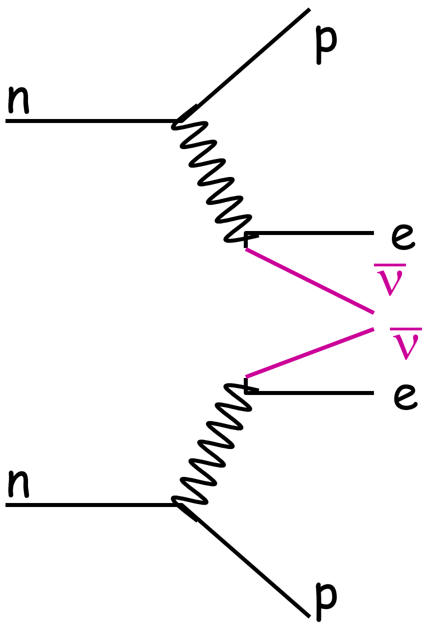


Not Observed

Allowed

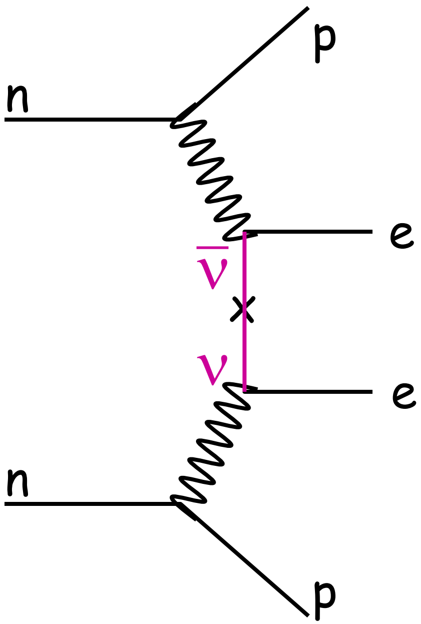
BUT Suppressed by $\frac{m_\nu^2}{E^2} \sim 10^{-20}$!!!

How we can find out ?



SM double weak process

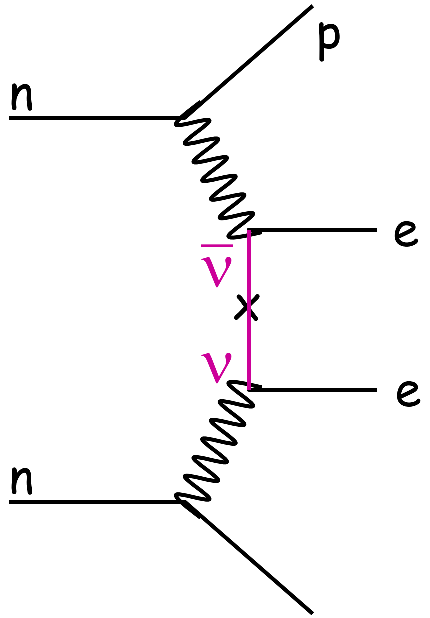
4 body decay: continuos spectrum for the e energy sum



Only allowed for Majorana ν

2 body decay: e energy sum is a delta

$\overline{\nu}_i$ is emitted (RH + $O(m_i/E)$ LH)



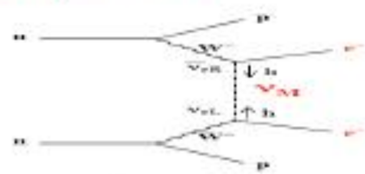
Amp[ν_i contribution] $\propto$ m_i

$$\text{Amp}[0\nu\beta\beta] \propto \left| \sum m_i U_{ei}^2 \right|$$

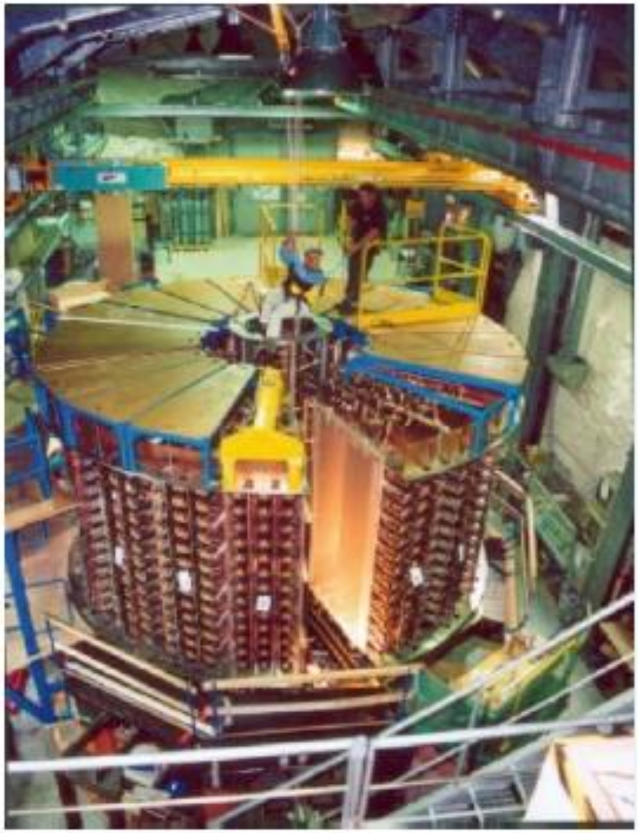
effective mass

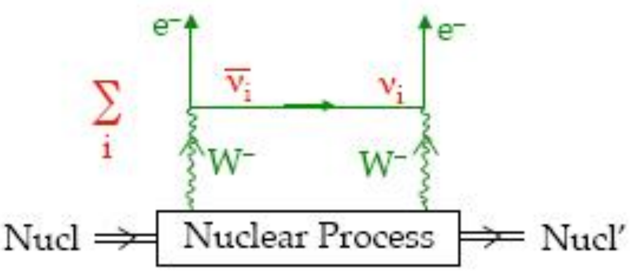
Neutrinoless double beta decay

- Most sensitive (terrestrial) probe of the absolute neutrino mass
- Unique way of proving Majorana nature of ν
- If Majorana ν is the only mechanism, ===>

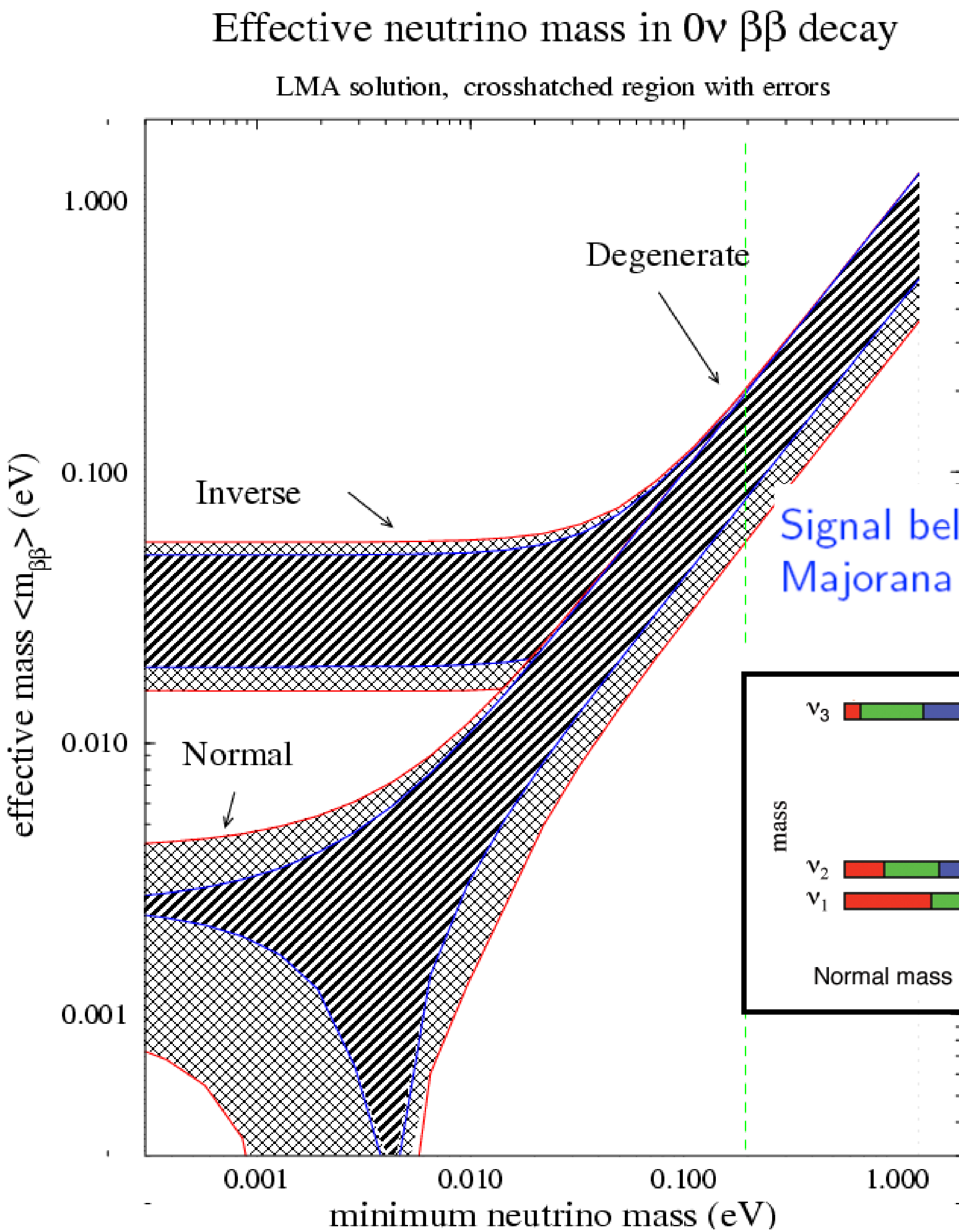


$$< m >_{\beta\beta} \equiv \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right|$$
$$= \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\beta} + m_3 s_{13}^2 e^{2i(\gamma-\delta)} \right|$$



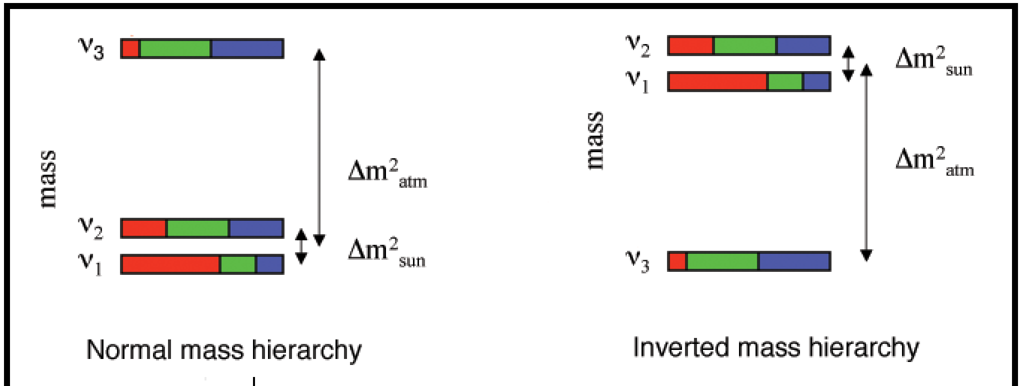


$$m_{\beta\beta} = |\sum m_i U_{ei}^2|$$



dividing point $m_{\beta\beta} \approx 10meV$

Signal below ~ 10 meV would imply Majorana and Normal Hierarchy!



Evidence for $0\nu\beta\beta$ decay ?

Name	Isotope	Mass	Method	Location	Time Line
Operational & Recently completed experiments					
CUORICINO	^{130}Te	12 kg	bolometric	LNGS	2003-2008
NEMO-3	^{100}Mo/^{82}Se	6.9/0.9 kg	tracko-calo	LSM	until 2010
Construction funding					
CUORE	^{130}Te	200 kg	bolometric	LNGS	2012
EXO-200	^{136}Xe	160 kg	liquid TPC	WIPP	2009
GERDA I&II	^{76}Ge	35kg	ionization	LNGS	2009
SNO+	^{150}Nd	56 kg	scintillation	SNOlab	2011
Substantial R&D funding / prototyping					
CANDLES	^{48}Ca	0.35 kg	scintillation	Kamioka	2009
Majorana	^{76}Ge	26 kg	ionization	SUSL	2012
NEXT	^{136}Xe	80 kg	gas TPC	Canfranc	2013
SuperNEMO	^{82}Se or ^{150}Nd	100 kg	tracko-calo	LSM	2012 (first mod.)

Leptogenesis

The Universe is made of matter:

$$\eta_B \equiv \frac{N_b - N_{\bar{b}}}{N_\gamma} \sim 6 \times 10^{-10}$$

The hope is that this asymmetry might have been produced dynamically from a symmetric initial state:

- Baryon number violation
- Deviation from thermal equilibrium
- C and CP violation

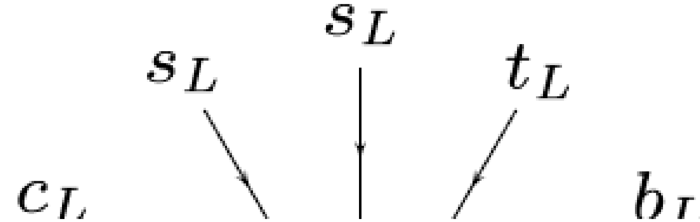
Sakharov

All these conditions are present in the SM !

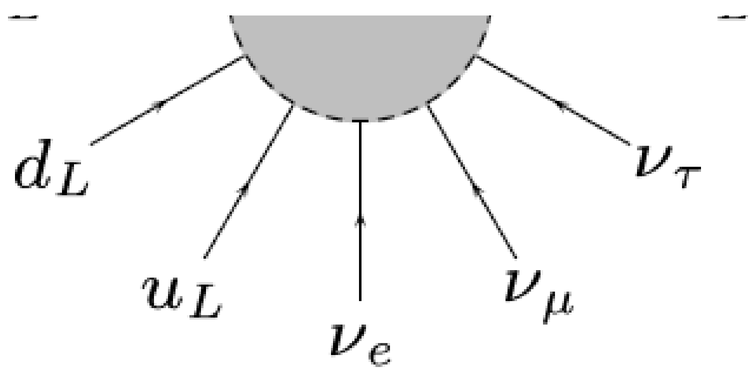
Baryon number violation:

$B + L$ is anomalous in the SM both with and without massive neutrinos, while $B - L$ is preserved

At high T in the early Universe, $B + L$ violating transitions could be in thermal equilibrium due to the thermal excitation of configurations with topological charge called sphalerons :



If there are heavy Majorana singlets, as in the sea-saw mechanism, there is a additional source of L violation (and $B - L$):



$\Delta B = \Delta L$

Deviation from equilibrium:

Sphalerons are in equilibrium for $T \geq 100$ GeV: to get these processes out-of-equilibrium we need to go to the EW phase transition.

EW baryogenesis is disfavoured because:

1. The sources of CP violation involved are too small in the SM
2. The out-of-equilibrium condition is not well met: the EW phase transition is not strongly first order

Majorana singlets are in equilibrium until they decouple: $T \sim M_R$

This happens much earlier then EW transition since $M_R \gg v$ and sphalerons are still in equilibrium:

$$Y_B = aY_{B-L} = \frac{a}{a-1} Y_L \quad a = \frac{28}{79} \quad \text{in SM}$$

Leptogenesis

Baryon Asymmetry is created by a Lepton Asymmetry produced by the decays of super heavy Majorana Neutrinos.

$$\frac{\Gamma(N\to l^+\phi^-)-\Gamma(N\to l^-\phi^+)}{\Gamma(N\to l^+\phi^-)+\Gamma(N\to l^-\phi^+)}$$

$\Gamma(N \to l^{\pm}\phi^{\mp})$ depends on the Majorana Phases in the MNS mixing matrix.

$$B_{now} = \frac{1}{2}(B-L) + \frac{1}{2}(B+L) = \frac{1}{2}(B-L)_{ini} = -\frac{1}{2}L_{ini}$$

$$0$$

Final asymmetry:

$$Y_B = 10^{-2} \overbrace{\epsilon_1}^{\mathrm{CP-asym}} \overbrace{\kappa}^{\mathrm{eff.\ factor}}$$

$$\epsilon_1 = \frac{\Gamma(N \to \Phi l) - \Gamma(N \to \Phi \bar{l})}{\Gamma(N \to \Phi l) + \Gamma(N \to \Phi \bar{l})}$$

κ efficiency factor which depends on the non-equilibrium dynamics.

A relation between the baryon number of the Universe and the neutrino flavour parameters!

Some exotic (and not so exotic) scenarios

Non standard neutrino interactions

CPT violation

Violations of Lorentz invariance

Non standard neutrino interactions

They can be described by effective four-fermion operators of the form

$$2\sqrt{2}G_F\varepsilon_{\alpha\beta}\left(\overline{\nu}_\beta\gamma^\mu P_L l_\alpha\right)\left(\bar{f}\gamma_\mu P_{L,R} f'\right)$$

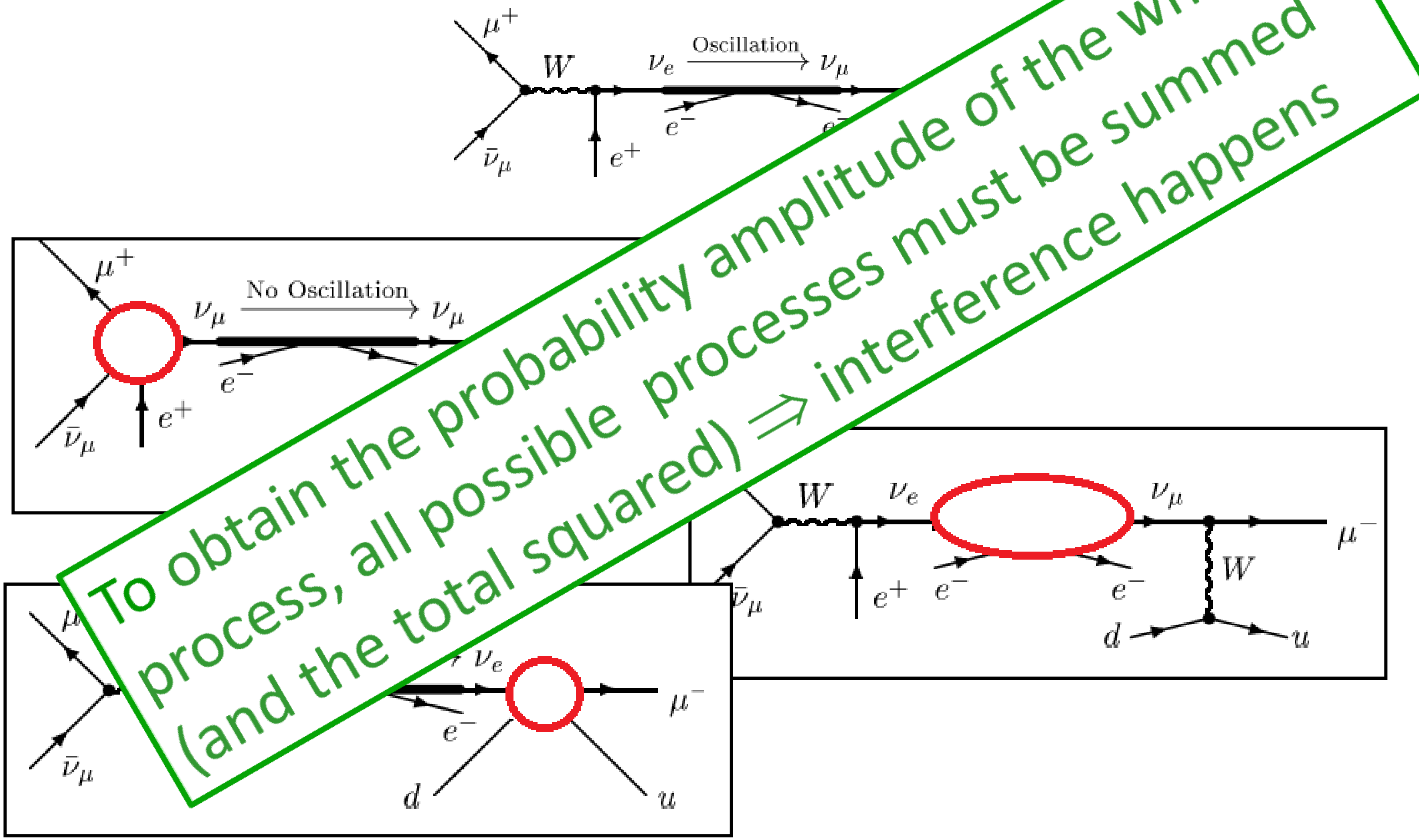
normalizing the operator with the Fermi constant

$$\varepsilon_{\alpha\beta} = \frac{M_W^2}{M^2_{NSNI}}$$

NSNI can appear at every step. It is therefore necessary to break down the analysis in three stages

- the production process
- the time evolution
- the detection process

Lets have a look at what is called the “golden ” channel in a neutrino factory



To obtain the probability amplitude of the whole process, all possible processes must be summed (and the total squared) $\Rightarrow$ interference happens

The system consists of an initial state

state of the target

state of the parent particle

$$A + T$$

intermediate state B

and a final state

unobserved particles

state of the particle producing and identifiable event

$$C + U$$

$$P\,(\,A + T \;\rightarrow C + U) = |\;\textstyle\sum_B\; \Phi(A, T; B; C, U)\;|^2$$

$$\boxed{P\,(\,A \;\rightarrow C) = \sum_{T,U} P(A + T \rightarrow C + U)}$$

Assuming that the amplitude $\Phi\,(A, T_o\,; B_o; C\,, U_o\,)$ is dominant

$$P\,(A \rightarrow C) \approx P(A + T_o \;\; \rightarrow \;\; C + U_o)$$

$$= |\; \Phi\,(A, T_o\,; B_o; C\,, U_o)|^2$$

$$+\ 2\ \mathrm{Re}\Big[\; \Phi\,(A, T_o\,; B_o; C\,, U_o)^* \;\textstyle\sum_{B \neq B_o} \Phi\,(A, T_o\,; B; C\,, U_o\,)\Big]$$

$$+\ \cancel{|\textstyle\sum_{B \neq B_o} \Phi\,(A, T_o\,, B; C\,, U_o\,)\,|^2}$$

For a neutrino factory : $A \rightarrow \mu^-$ $\quad C \rightarrow \mu^+$

Production and detection involve charged current NSNI

$$\pi \longrightarrow \mu + \nu_\alpha \qquad\qquad \mu^- \rightarrow e^- \ \nu_\mu \ \overline{\nu_\alpha}$$

$$n + \nu_\alpha \longrightarrow p + l_\beta$$

$$2\sqrt{2}G_F \varepsilon^{ud}_{\alpha\beta}\left(\bar{l}_\beta \gamma^\mu P_L \nu_\alpha\right)\left(\bar{u}\gamma_\mu P_{L,R} d\right) \qquad 2\sqrt{2}G_F \varepsilon^{\mu e}_{\alpha\beta}\left(\bar{\mu}\gamma^\mu P_L \nu_\beta\right)\left(\bar{\nu}_\alpha \gamma_\mu P_L e\right)$$

$$\left|\varepsilon^{ud}\right| < \begin{pmatrix} 0.041 & 0.025 & 0.041 \\ 2.6\cdot 10^{-5} & 0.078 & 0.013 \\ 0.011 & 0.016 & 0.13 \end{pmatrix} \qquad \left|\varepsilon^{\mu e}\right| < \begin{pmatrix} 0.025 & 0.03 & 0.03 \\ 0.025 & 0.03 & 0.03 \\ 0.025 & 0.03 & 0.03 \end{pmatrix}$$

bounds are ~ 10^{-2}

We are left "only" with neutral current NSNI

$$2\sqrt{2}G_F\varepsilon_{\alpha\beta}\left(\bar{\nu}_\beta\gamma^\mu P_L\nu_\alpha\right)\left(\bar{f}\gamma\right.$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{2E}\left[U\begin{pmatrix}0 & 0 & 0\\ 0 & \Delta m^2_{21} & 0\\ 0 & 0 & \Delta\end{pmatrix} \ldots \begin{pmatrix} \ldots & \varepsilon_{e\mu} & \varepsilon_{e\tau}\\ \ldots & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau}\\ \varepsilon^*_{e\tau} & \varepsilon^*_{\mu\tau} & \varepsilon_{\tau\tau}\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

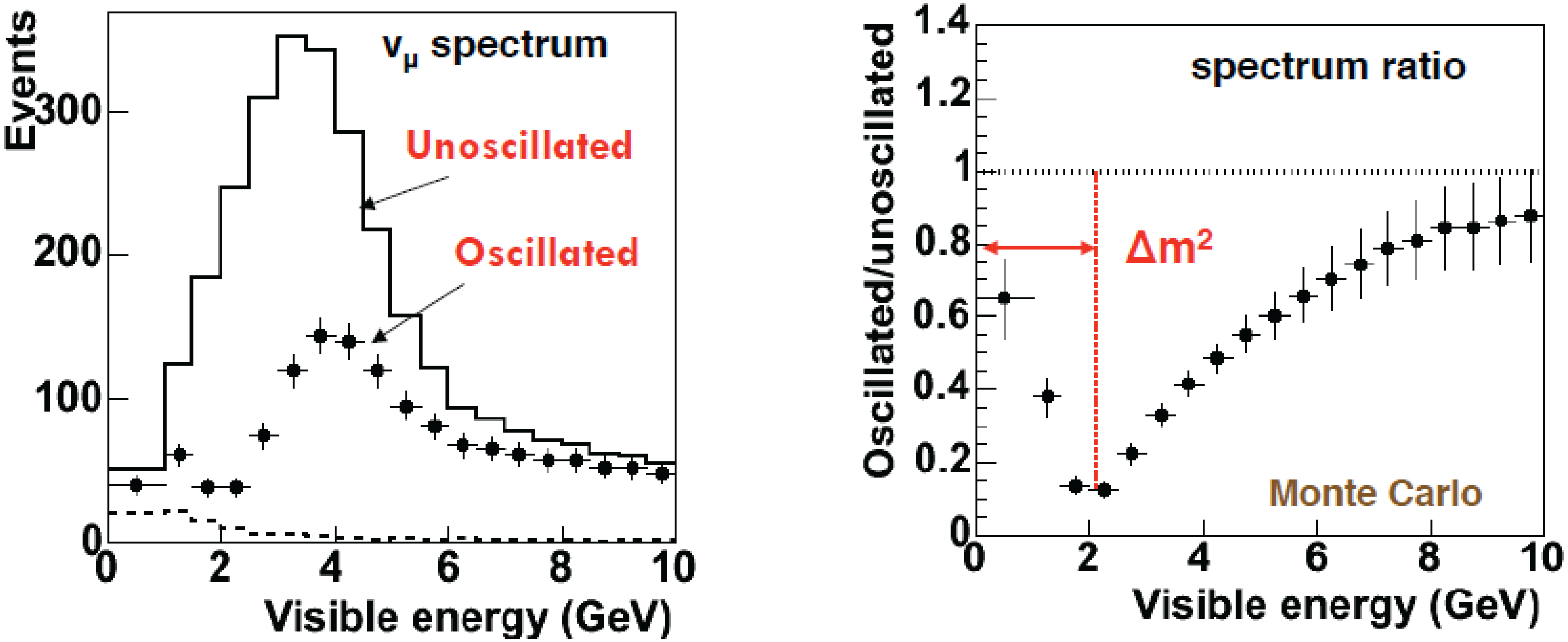
$$a \equiv 2\sqrt{2}G_F n_e E$$

$$H \ldots m^2_{32}\Big)U^\dagger + a\begin{pmatrix}\epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon^*_{\mu\tau} & \epsilon_{\tau\tau}\end{pmatrix}\Big]$$

Anomalies

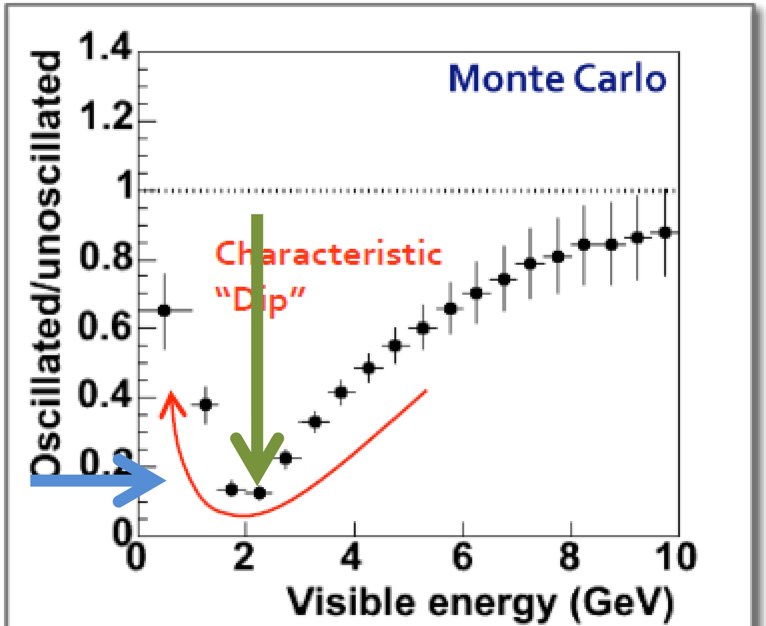
the driving force in neutrino physics for 30+ years !!!!!

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2(1.27\,\Delta m^2 L / E)$$



$\varepsilon_{\mu\tau}$ changes the disspearence probability at large energies shifts the position of the minimum in energy

$$\Delta m^2$$

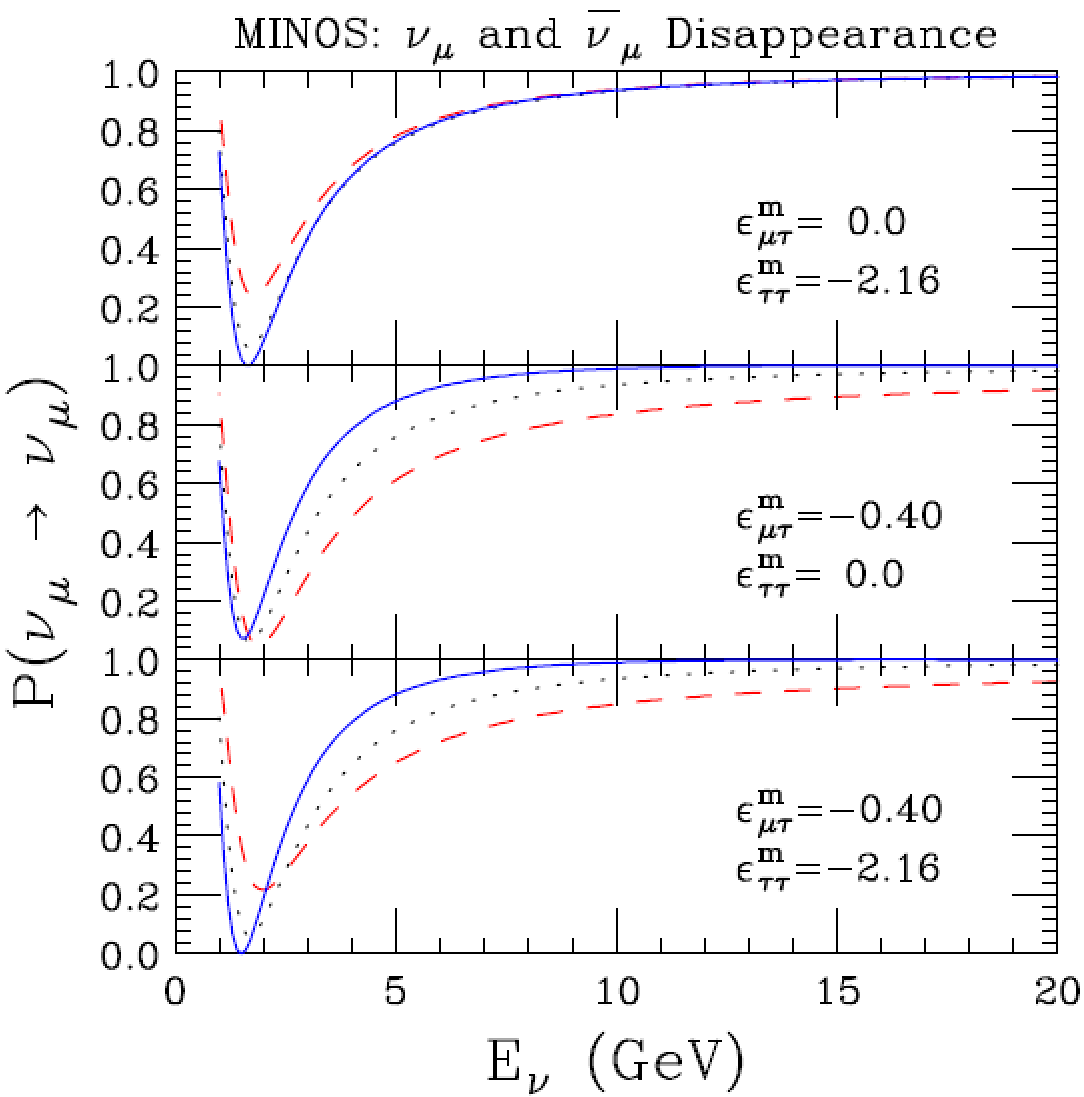


$\varepsilon_{\tau\tau}$ modifies the dissapearence probability near the first oscillation minimum, especially the depth of the minimum

$$sin^2(2\,\theta_{23})$$

neutrino

antineutrino



Kopp et al. 1009.0014

Summary

- With 7x10^{20} POT of neutrino beam, MINOS finds
  - muon-neutrinos disappear

$$\left|\Delta m^2\right| = 2.35^{+0.11}_{-0.08} \times 10^{-3}\,\mathrm{eV}^2,$$
$$\sin^2(2\theta) > 0.91\ (90\%\ \mathrm{C.L.})$$

  - NC event rate is not diminished

$$f_s < 0.22\,(0.40)\ \text{at}\ 90\%\ \mathrm{C.L.}$$

  - electron-neutrino appearance is limited

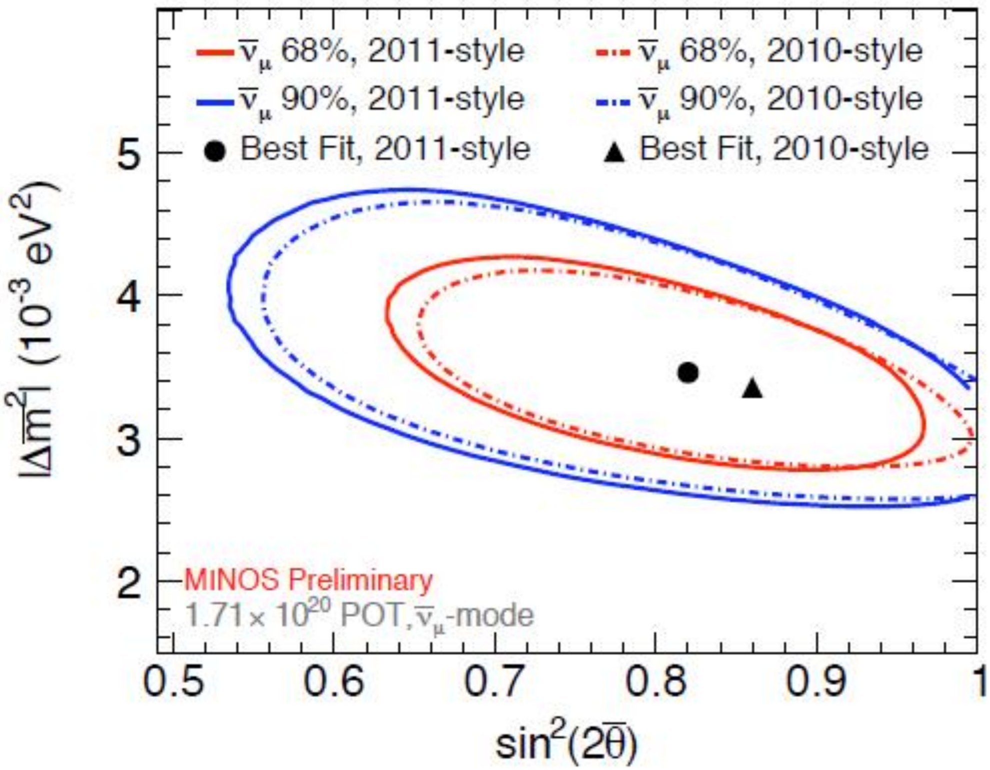
$$\sin^2(2\theta_{13}) < 0.12\ (0.20)\ \text{at}\ 90\%\ \mathrm{C.L.}$$

- With 1.71x10^{20} POT of anti-neutrino beam
  - muon anti-neutrinos also disappear with

$$\left|\overline{\Delta m^2}\right| = 3.36^{+0.45}_{-0.40} \times 10^{-3}\,\mathrm{eV}^2,$$
$$\sin^2(2\bar{\theta}) = 0.86 \pm 0.11$$

  - we look forward to more anti-neutrino beam!

Run IV-Only Contours 2011 vs. 2010



2010 Analysis, 1.71×10^{20} POT

$$|\Delta\overline{m}^2_{atm}| = 3.36^{+0.46}_{-0.40} \times 10^{-3}\mathrm{eV}^2$$
$$\sin^2(2\bar{\theta}_{23}) = 0.86^{+0.11}_{-0.12}$$

2011 Analysis, 1.71×10^{20} POT

$$|\Delta\overline{m}^2_{atm}| = 3.46^{+0.47}_{-0.43} \times 10^{-3}\mathrm{eV}^2$$
$$\sin^2(2\bar{\theta}_{23}) = 0.82^{+0.10}_{-0.11}$$

CPT violation

$$\frac{|\, m(K_0) - m(\overline{K_0}\,)\,|}{m_{K-av}} < 10^{-18}$$

$$m_{K-av} \approx \frac{1}{2}\; 10^{-9}\; \mathrm{eV}$$

$$\left(m(K_0) - m\left(\overline{K_0}\right)\;\right)\left(m(K_0) + m\left(\overline{K_0}\,\right)\right) < 2\;\; 10^{-18}\, {m_{K-av}}^2$$

$$\left|m^2(K_0)\; - m^2\left(\overline{K_0}\,\right)\right| \qquad \approx \frac{1}{2}\; \mathrm{eV}^{\,2}$$

$$\boxed{|\; \Delta m^2 \text{ - } \overline{\Delta m^2}\; | \approx\; 10^{-6} \;-\; 10^{-3}\, \mathrm{eV}^{\,2}}$$

Distinguishing CPT violation from NSNI

The muon neutrino survival probability in matter can be written as

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_\nu \sin^2\left(\frac{\Delta m_\nu^2 L}{4E}\right) .$$

in matter

$\Delta m_\nu^2 \cos 2\theta$ … $- \epsilon_{\tau\tau} A ,$

$\Delta m_\nu^2 \sin 2\theta$ … $- 2\epsilon_{\mu\tau} A .$

$$4\Delta m^4 = \Delta m_\nu^4 + \Delta m_{\bar\nu}^4 + 2\Delta m_\nu^2 \Delta m_{\bar\nu}^2 \cos(2\theta_\nu - 2\theta_{\bar\nu})$$

$$\sin^2(2\theta) = \frac{\left(\Delta m_\nu^2 \sin(2\theta_\nu) + \Delta m_{\bar\nu}^2 \sin(2\theta_{\bar\nu})\right)^2}{\Delta m_\nu^4 + \Delta m_{\bar\nu}^4 + 2\Delta m_\nu^2 \Delta m_{\bar\nu}^2 \cos(2\theta_\nu - 2\theta_{\bar\nu})}$$

$$\begin{aligned} 2\epsilon_{\tau\tau}^m A &= \Delta m_\nu^2 \cos(2\theta_\nu) - \Delta m_{\bar\nu}^2 \cos(2\theta_{\bar\nu}) \\ 4\epsilon_{\mu\tau}^m A &= \Delta m_\nu^2 \sin(2\theta_\nu) - \Delta m_{\bar\nu}^2 \sin(2\theta_{\bar\nu}) \end{aligned}$$

Violations of Lorentz invariance

Lorentz violation

$$(h_{\text{eff}})_{ab} = \frac{m_{ab}^2}{2E} + \frac{1}{E}\left[(a_L)^{\alpha} p_{\alpha} - (c_L)^{\alpha\beta} p_{\alpha} p_{\beta}\right]_{ab}$$

standard Lorentz covariant term

violates both CPT and Lorentz invariance

As usual, the oscillation probability is governed by the difference of the eigenvalues of the effective hamiltonian.

$$\sin^2(\Delta_{ab}\ L/2)$$

$${m_{ab}}^2 L/E$$

$$(a^{\alpha})_{ab}\ L$$

$$(c^{\alpha\beta})_{ab}\ L\,E$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2(1.27\Delta m^2 L/E)$$

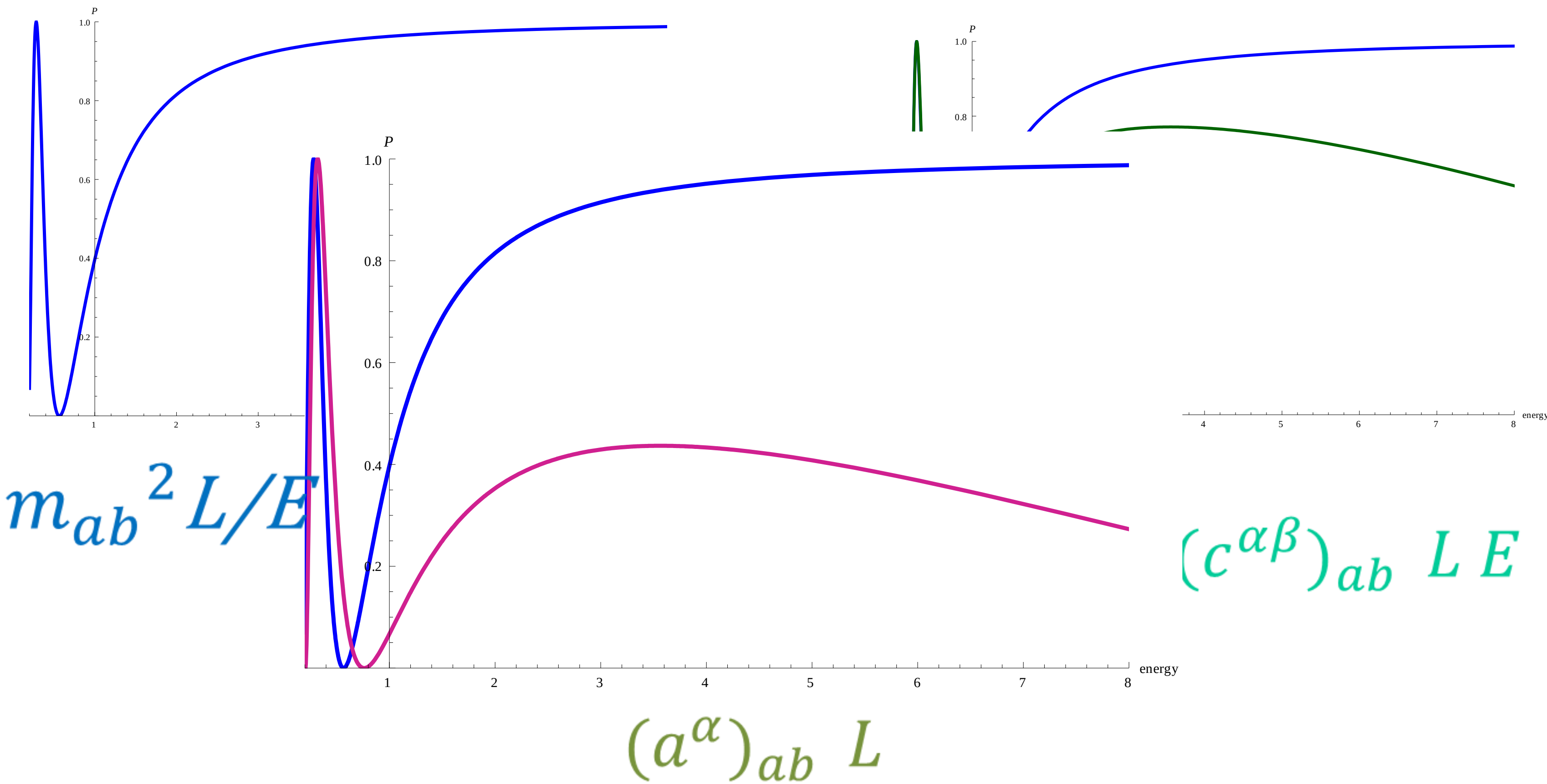


TABLE X: Neutrino sector

Combination	Result	System	Ref.
$(a_L)^T_{e\mu}$	$(-3.1 \pm 0.9) \times 10^{-20}$ GeV	MiniBooNE	[93]
$(a_L)^X_{e\mu}$	$(0.6 \pm 1.9) \times 10^{-20}$ GeV	"	[93]
$(a_L)^Y_{e\mu}$	$(-0.9 \pm 1.8) \times 10^{-20}$ GeV	"	[93]
$(a_L)^Z_{e\mu}$	$(-4.2 \pm 1.2) \times 10^{-20}$ GeV	"	[93]
$(c_L)^{TT}_{e\mu}$	$(7.2 \pm 2.1) \times 10^{-20}$	"	[93]
$(c_L)^{TX}_{e\mu}$	$(-0.9 \pm 2.8) \times 10^{-20}$	"	[93]
$(c_L)^{TY}_{e\mu}$	$(1.3 \pm 2.6) \times 10^{-20}$	"	[93]
$(c_L)^{TZ}_{e\mu}$	$(5.9 \pm 1.7) \times 10^{-20}$	"	[93]
$(c_L)^{XZ}_{e\mu}$	$(-1.1 \pm 3.7) \times 10^{-20}$	"	[93]
$(c_L)^{YZ}_{e\mu}$	$(1.7 \pm 3.4) \times 10^{-20}$	"	[93]
$(c_L)^{ZZ}_{e\mu}$	$(2.6 \pm 0.8) \times 10^{-19}$	"	[93]
$a_L^X,\ a_L^Y$	$< 1.8 \times 10^{-23}$ GeV	IceCube	[94]
$\lvert(a_L)^X_{\mu\tau}\rvert$	$< 5.9 \times 10^{-23}$ GeV	MINOS FD	[95]
$\lvert(a_L)^Y_{\mu\tau}\rvert$	$< 6.1 \times 10^{-23}$ GeV	"	[95]
$\lvert a_L^X\rvert,\ \lvert a_L^Y\rvert$	$< 3.0 \times 10^{-20}$ GeV	MINOS ND	[96]
$c_L^{TX},\ c_L^{TY}$	$< 3.7 \times 10^{-27}$	IceCube	[94]
$\lvert(c_L)^{TX}_{\mu\tau}\rvert,\ \lvert(c_L)^{TY}_{\mu\tau}\rvert$	$< 0.5 \times 10^{-23}$	MINOS FD	[95]
$\lvert(c_L)^{XX}_{\mu\tau}\rvert$	$< 2.5 \times 10^{-23}$	"	[95]
$\lvert(c_L)^{YY}_{\mu\tau}\rvert$	$< 2.4 \times 10^{-23}$	"	[95]
$\lvert(c_L)^{XY}_{\mu\tau}\rvert$	$< 1.2 \times 10^{-23}$	"	[95]
$\lvert(c_L)^{YZ}_{\mu\tau}\rvert,\ \lvert(c_L)^{XZ}_{\mu\tau}\rvert$	$< 0.7 \times 10^{-23}$	"	[95]
$\lvert c_L^{TX}\rvert,\ \lvert c_L^{TY}\rvert$	$< 9 \times 10^{-23}$	MINOS ND	[96]
$\lvert c_L^{XX}\rvert$	$< 5.6 \times 10^{-21}$	"	[96]
$\lvert c_L^{YY}\rvert$	$< 5.5 \times 10^{-21}$	"	[96]
$\lvert c_L^{XY}\rvert$	$< 2.7 \times 10^{-21}$	"	[96]
$\lvert c_L^{YZ}\rvert$	$< 1.2 \times 10^{-21}$	"	[96]
$\lvert c_L^{XZ}\rvert$	$< 1.3 \times 10^{-21}$	"	[96]

V.Kostelecky and N. Russell
Rev.Mod.Phys. 83 (2011) 11
e-Print: arXiv:0801.0287

SUMMARY I

$$|\nu_e,\ \nu_\mu,\ \nu_\tau\rangle^T_{flavor} = U_{\alpha i}\ |\nu_1,\ \nu_2,\ \nu_3\rangle^T_{mass}$$

$$U_{\alpha i} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}$$

Atmos. L/E $\mu \to \tau$ — Atmos. L/E $\mu \leftrightarrow e$ — Solar L/E $e \to \mu, \tau$ — $0\nu\beta\beta$ decay

500km/GeV — 15km/MeV

$$\delta m^2_{atm} = 2.7^{+0.4}_{-0.3} \times 10^{-3} eV^2 \qquad \delta m^2_{solar} = 8.0 \pm 0.4 \times 10^{-5} eV^2$$

$$\sin^2\theta_{12} = 0.31 \pm 0.03$$

$$\sin^2\theta_{23} = 0.50 \pm 0.14$$

$$\sin^2(2\theta_{13}) = 0.094\ \pm 0.017$$

SUMMARY

neutrino mass $\Leftrightarrow$ flavor change

Unknowns:

- Majorana v Dirac
- Light Steriles ???
- Mass Hierarchy $m_3 > m_2 > m_1$ OR $m_2 > m_1 > m_3$

  using $|U_{e3}|^2 < |U_{e2}|^2 < |U_{e1}|^2$
- Is CP violated ? $\sin\delta \neq 0$
- Mass of Heaviest Neutrino
- Mass of Lightest Neutrino
- New Interactions, Surprises !!!