

Reducing the combinatorial uncertainties using kinematic variables

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NPKI launching workshop

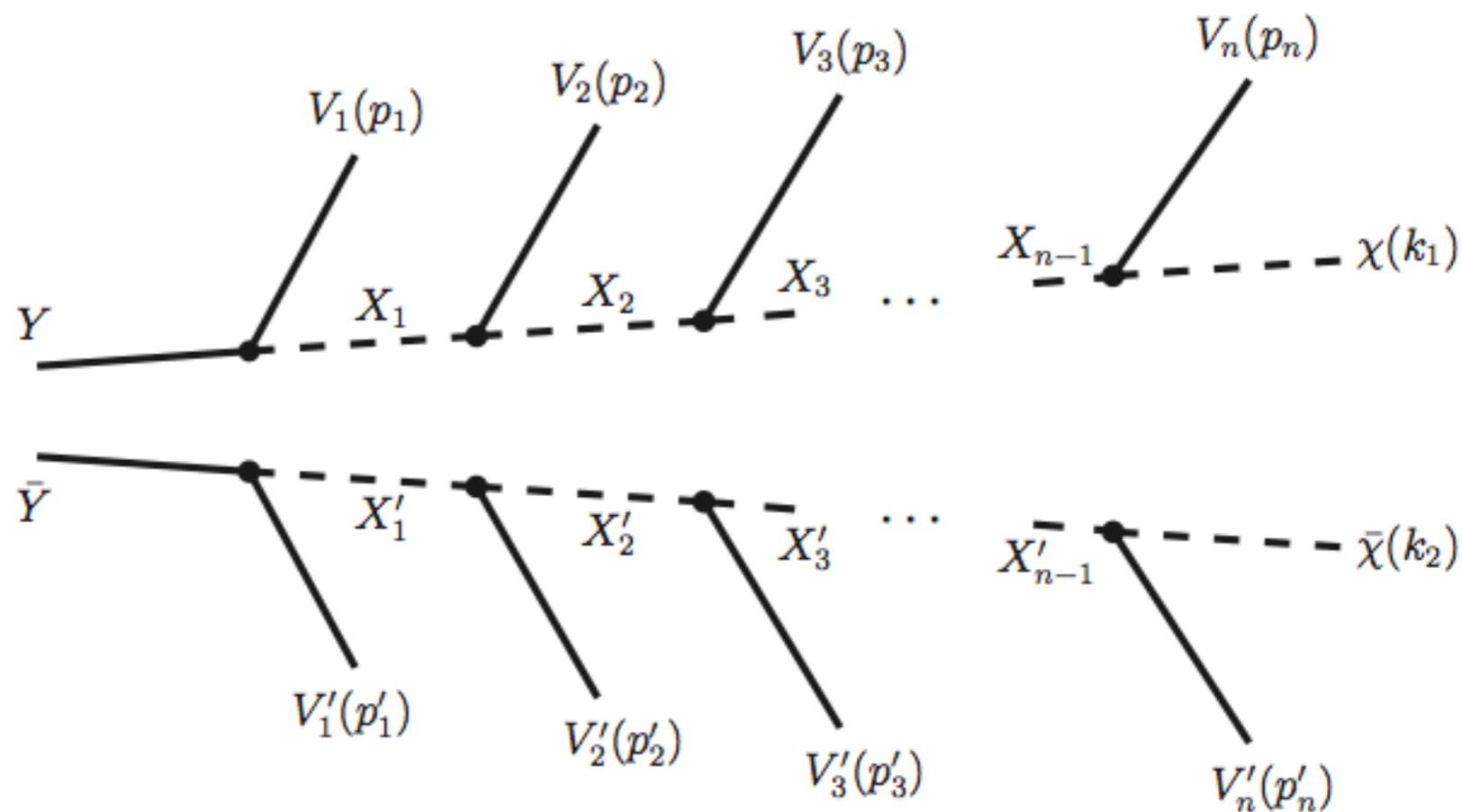
29 February 2012



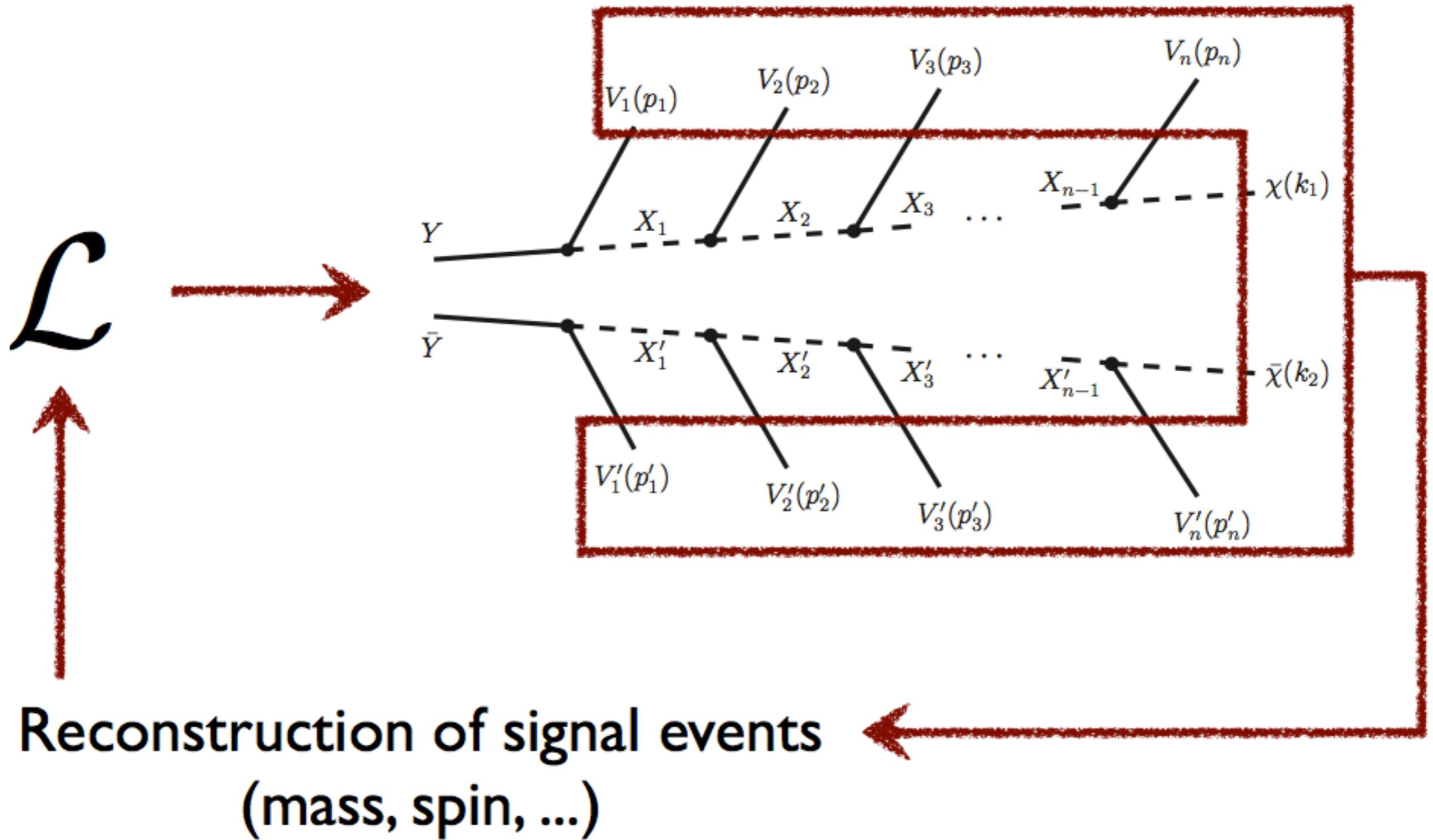
In collaboration with K. Choi and D. Guadagnoli, arXiv:1109.2201, JHEP 1111(2011)117

Combinatorics in collider signature

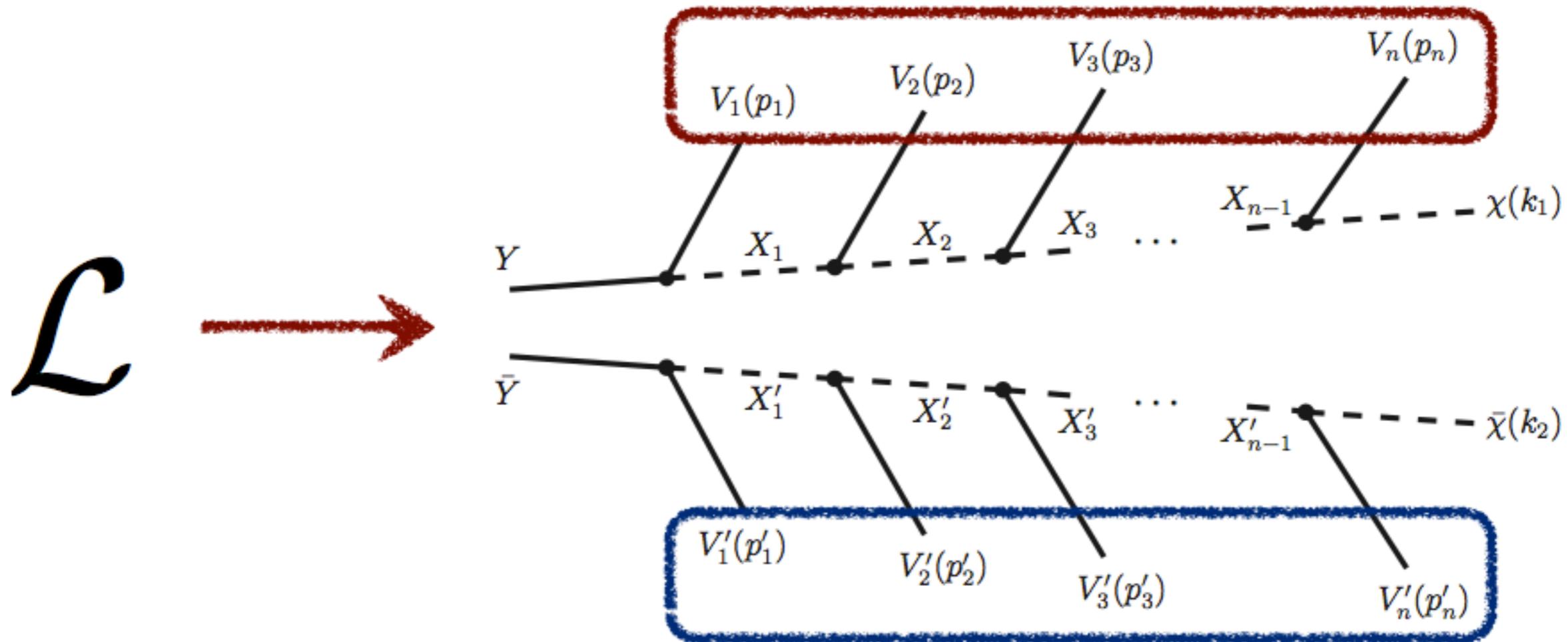
\mathcal{L}



Combinatorics in collider signature



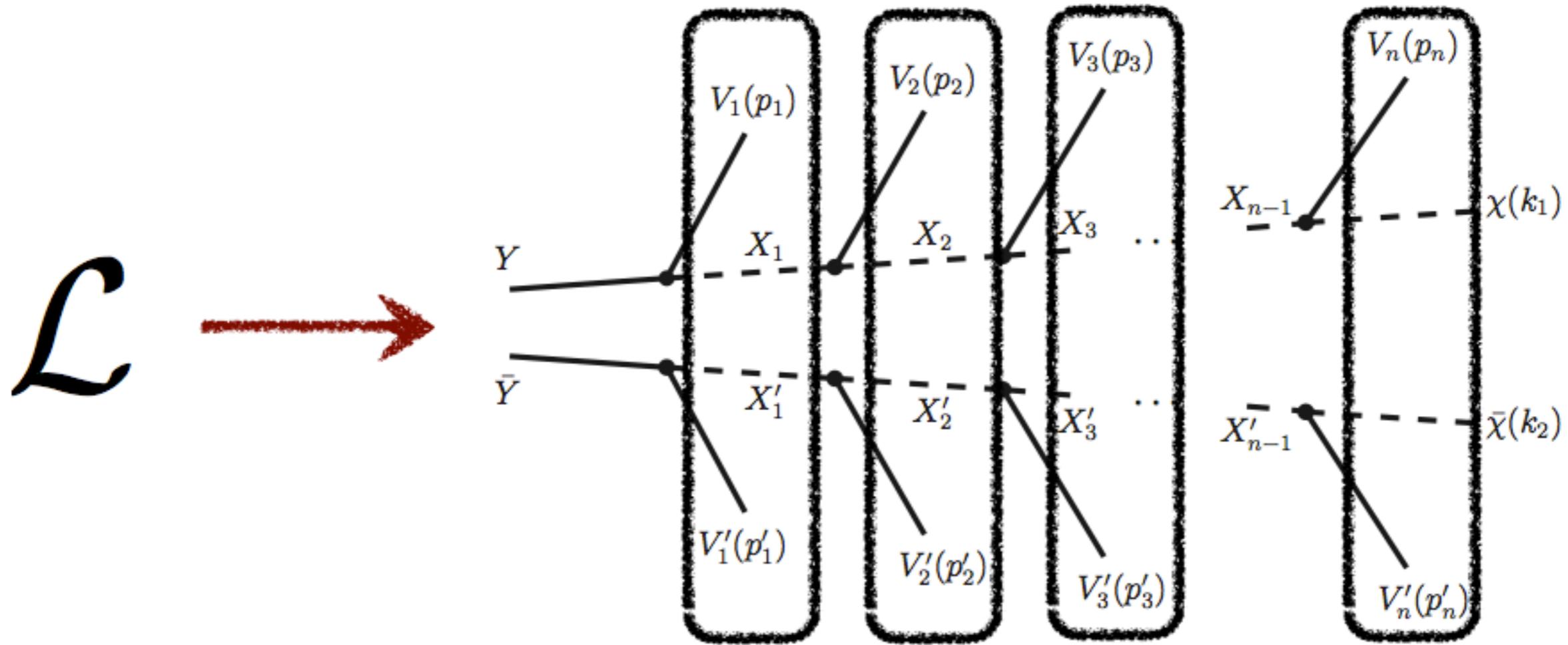
Combinatorics in collider signature



Reconstruction of signal events

* combinatorial ambiguities of **pairing**

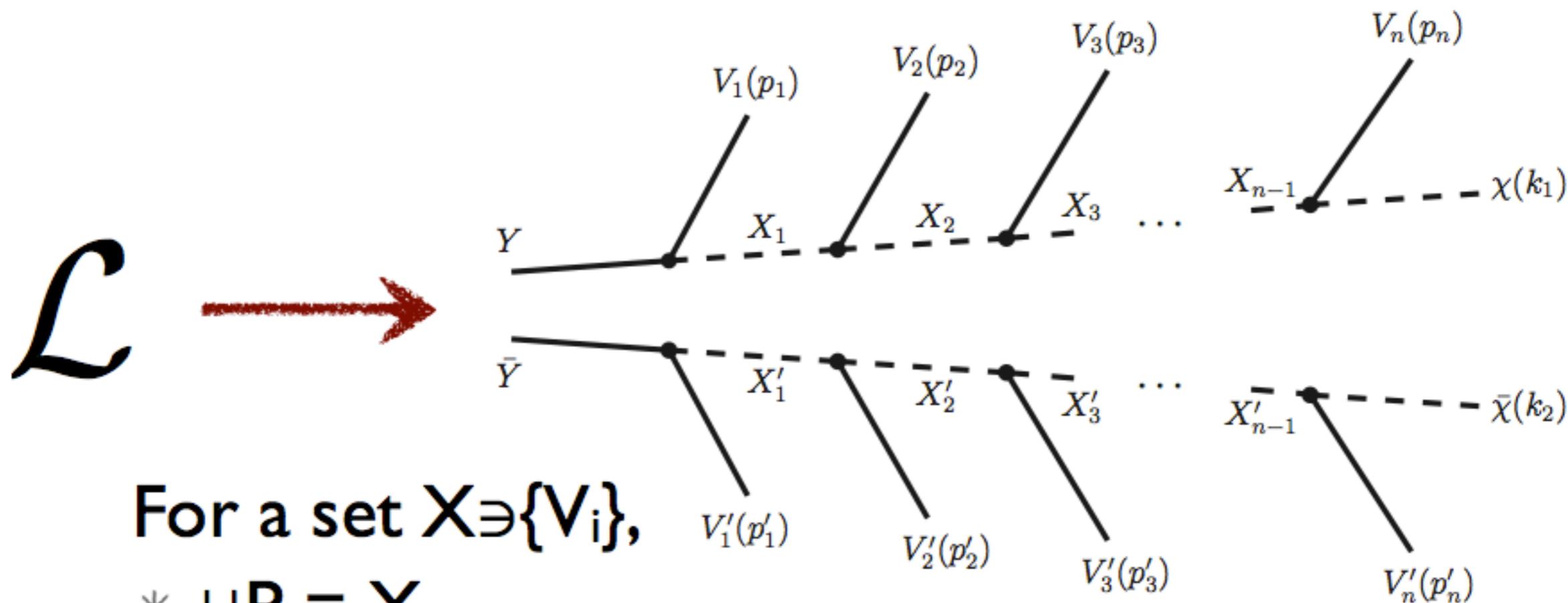
Combinatorics in collider signature



Reconstruction of signal events

* combinatorial ambiguities of **pairing** and **ordering**

Combinatorics in collider signature



* $UP = X$

* $A \cap B = \emptyset$ if $A \in P, B \in P, A \neq B$

'partition' (P)

Reconstruction of signal events

* combinatorial ambiguities of **pairing** and **ordering**

Flavor subtraction

$$\tilde{q} \rightarrow \tilde{\chi}_2^0 q \rightarrow \tilde{\ell}^\pm \ell^\mp q \rightarrow \tilde{\chi}_1^0 \ell^\pm \ell^\mp q$$

$$\tilde{q} \rightarrow \tilde{\chi}_1^0 q \quad (\ell^\pm = e^\pm, \mu^\pm)$$

$$(m_{\ell^\pm \ell^\mp})^2 = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\ell}}^2)(m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{\ell}}^2}$$

Flavor subtraction

$$\tilde{q} \rightarrow \tilde{\chi}_2^0 q \rightarrow \tilde{\ell}^\pm \ell^\mp q \rightarrow \tilde{\chi}_1^0 \ell^\pm \ell^\mp q$$

$$\tilde{q} \rightarrow \tilde{\chi}_1^0 q \quad (\ell^\pm = e^\pm, \mu^\pm)$$

same flavor leptons

main background: 2 jets + **2 leptons** + missing E_T

$$\tilde{q} \rightarrow \tilde{\chi}_1^\pm q' \rightarrow \tilde{\chi}_1^0 \ell^\pm \nu q'$$

no flavor correlation:

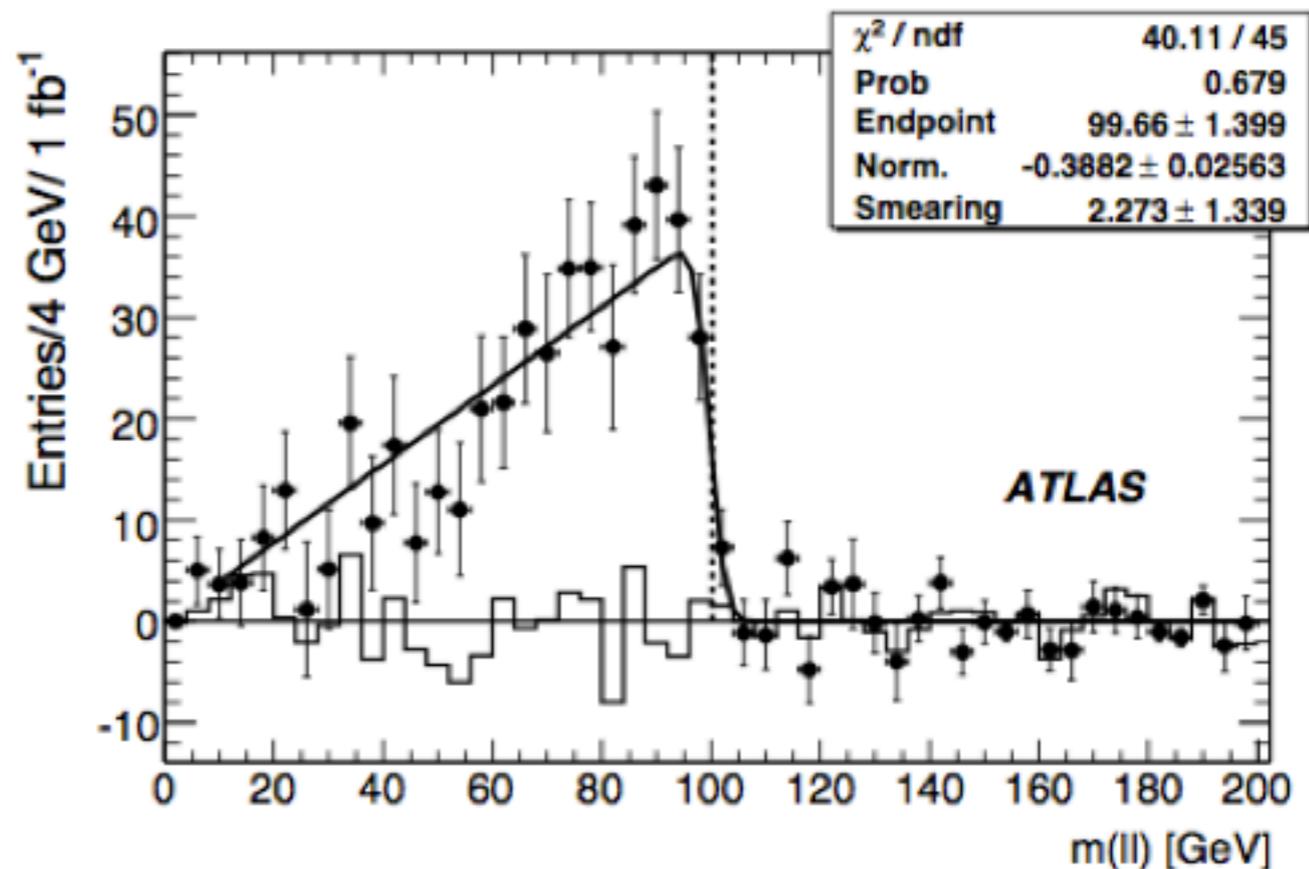
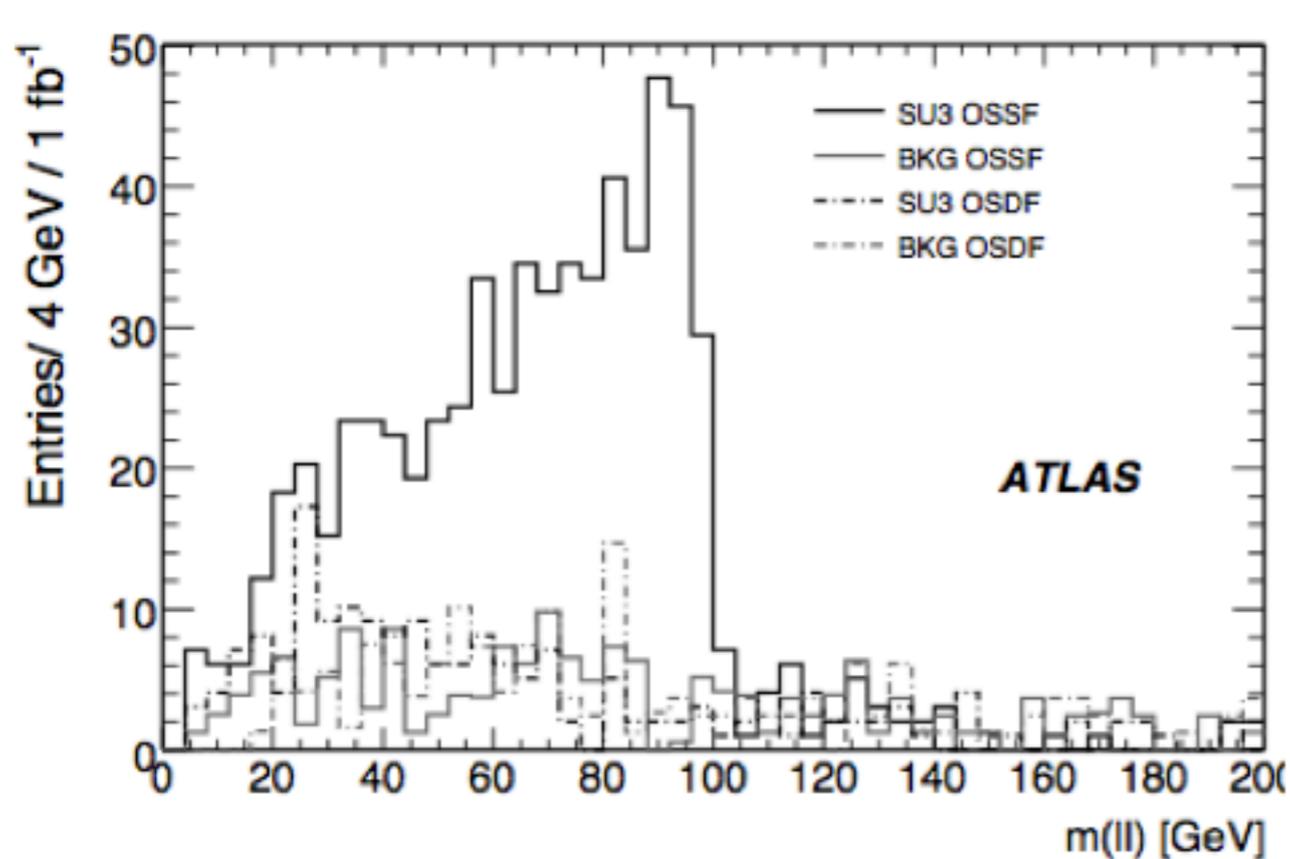
$$\tilde{q} \rightarrow \tilde{\chi}_1^\pm q' \rightarrow \tilde{\chi}_1^0 \ell^\pm \nu q'$$

(OSSF)=(OSDF)

Flavor subtraction = (OSSF) - (OSDF)

$$= e^+ e^- + \mu^+ \mu^- - e^+ \mu^- - e^- \mu^+$$

Flavor subtraction



taken from ATLAS TDR (2009)

$$\text{Flavor subtraction} = (\text{OSSF}) - (\text{OSDF})$$

$$= e^+e^- + \mu^+\mu^- - e^+\mu^- - e^-\mu^+$$

Bi-Event Subtraction

The shape of the combinatorial background can be modeled by combining observed particles from different events and subtracted away.

(Dutta, Kamon, Kolev Krislock, 2011)

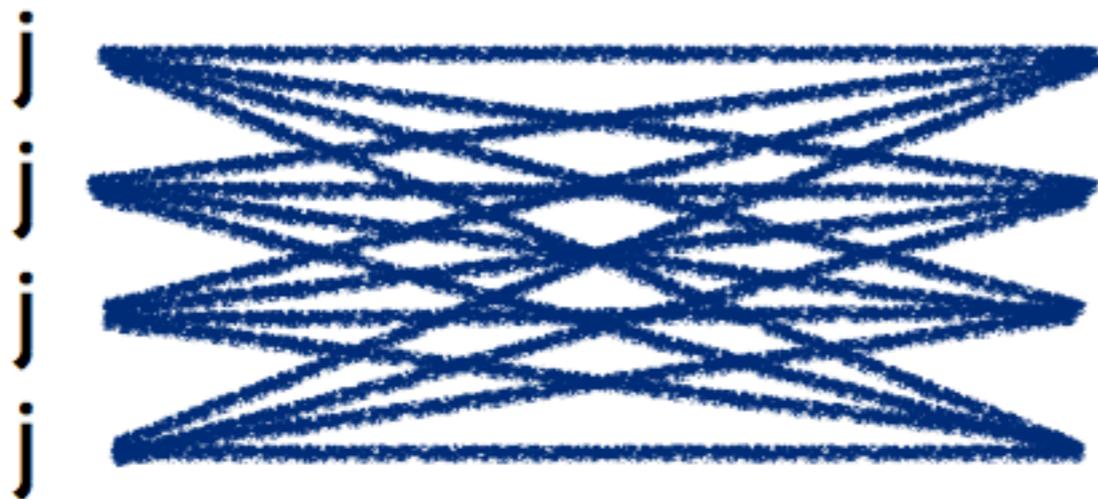
For a given event,

$$t \rightarrow bW^+ \rightarrow bj\bar{j}$$

$$\bar{t} \rightarrow \bar{b}W^- \rightarrow \bar{b}j\bar{j}$$

→ 3 pairings for m_{jj}

event 1



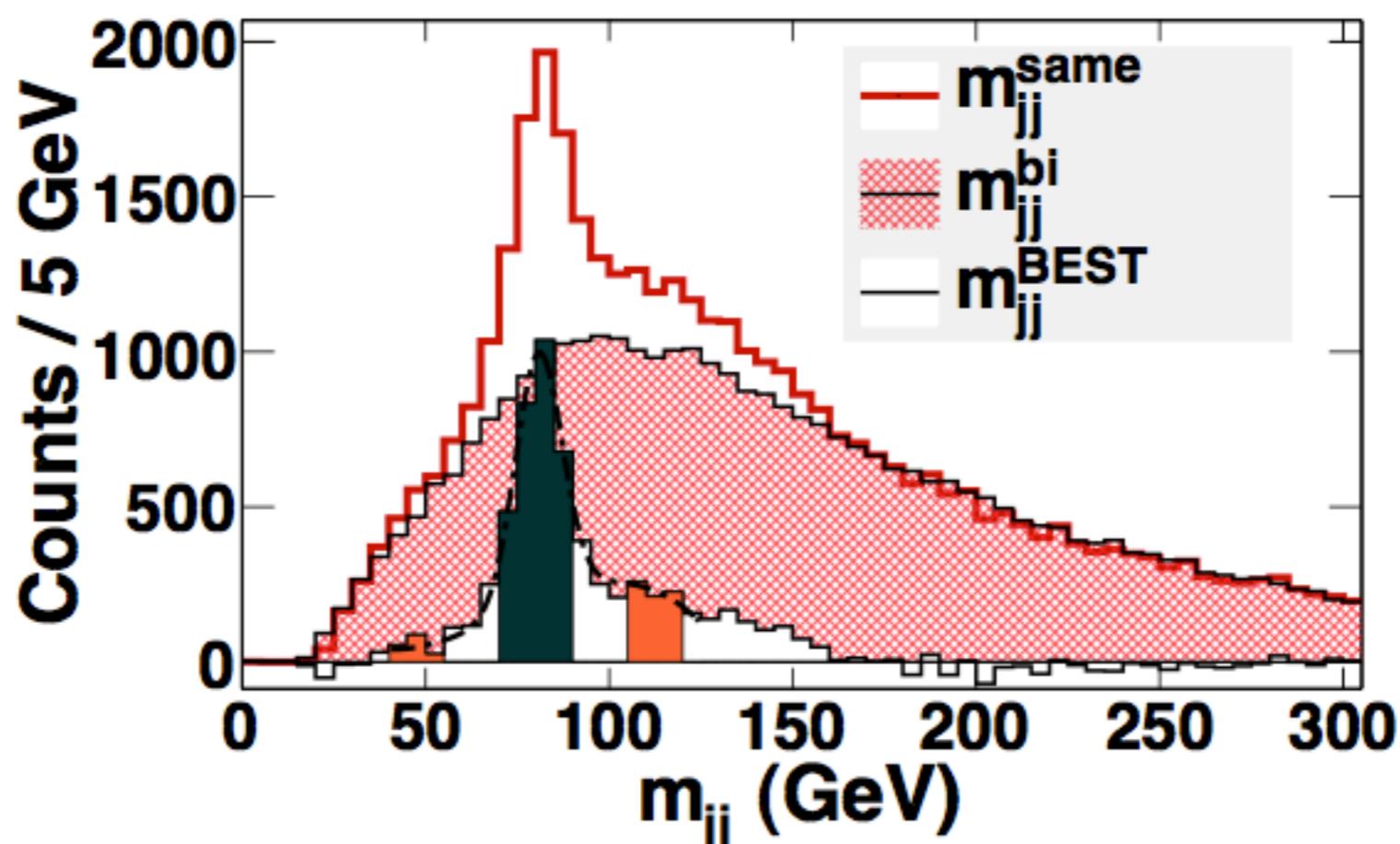
event 2

→ $m_{jj}^{\text{bi-event}}$

Bi-Event Subtraction

The shape of the combinatorial background can be modeled by combining observed particles from different events and subtracted away.

(Dutta, Kamon, Kolev Krislock, 2011)



* This method cannot resolve combinatorial ambiguities on an **event-by-event basis**.

p_T v. M method

Combinatorial ambiguities can be resolved by using selection cuts favoring high transverse momentum and low invariant mass.

(Rajaraman, Yu, 2010)

$$\begin{aligned} \tilde{g} &\rightarrow \tilde{q}^{(*)} q \rightarrow qq\tilde{\chi}_1^0 \\ \tilde{g} &\rightarrow \tilde{q}^{(*)} q \rightarrow qq\tilde{\chi}_1^0 \end{aligned} \rightarrow 3 \text{ pairings for } m_{qq}$$

take a pairing with $m_{qq} < m_{qq}^{\text{cut}}$ & $p_T > p_T^{\text{cut}}$,
discard the event if no unique choice.

The method efficiency can be improved by using M_{T2} instead of p_T (Baringer, Kong, McCaskey, Noonan, 2011).

* This method can resolve combinatorial ambiguities on an event-by-event basis, while sacrificing statistics.

Combinatorics in top-pair process

$$t \rightarrow bW^+ \rightarrow bl^+\nu$$

$$\bar{t} \rightarrow \bar{b}W^- \rightarrow \bar{b}l^-\nu$$

- * Due to the charge ambiguity on b jets,
of possible pairings = 2:
 $\{l_1, b_1\}$ & $\{l_2, b_2\}$ for right pairing (P_R),
 $\{l_1, b_2\}$ & $\{l_2, b_1\}$ for wrong pairing (P_W).

Combinatorics in top-pair process

simple, but decay topology similar as SUSY

$$t \rightarrow bW^+ \rightarrow bl^+\nu$$

$$\bar{t} \rightarrow \bar{b}W^- \rightarrow \bar{b}l^-\nu$$

- * Due to the charge ambiguity on b jets,
of possible pairings = 2:
 $\{l_1, b_1\}$ & $\{l_2, b_2\}$ for right pairing (P_R),
 $\{l_1, b_2\}$ & $\{l_2, b_1\}$ for wrong pairing (P_W).

Strategy

- * exploit kinematic variables that have predictable features: edge, threshold, peak, ... → 'test variables' and 'criterion'
- * take a partition obeying the features as much as possible (wrong partitions will generally not obey because of no correlation).

Invariant mass

For right pairing, $m_{b\ell}^2 \lesssim m_t^2 - m_W^2$

For wrong pairing,

* no definite cutoff,

If $m_{b\ell}^2 > m_t^2 - m_W^2$, wrong pairing,
else... $\forall m_{b\ell}^2(P_j) \leq m_t^2 - m_W^2$?

Invariant mass

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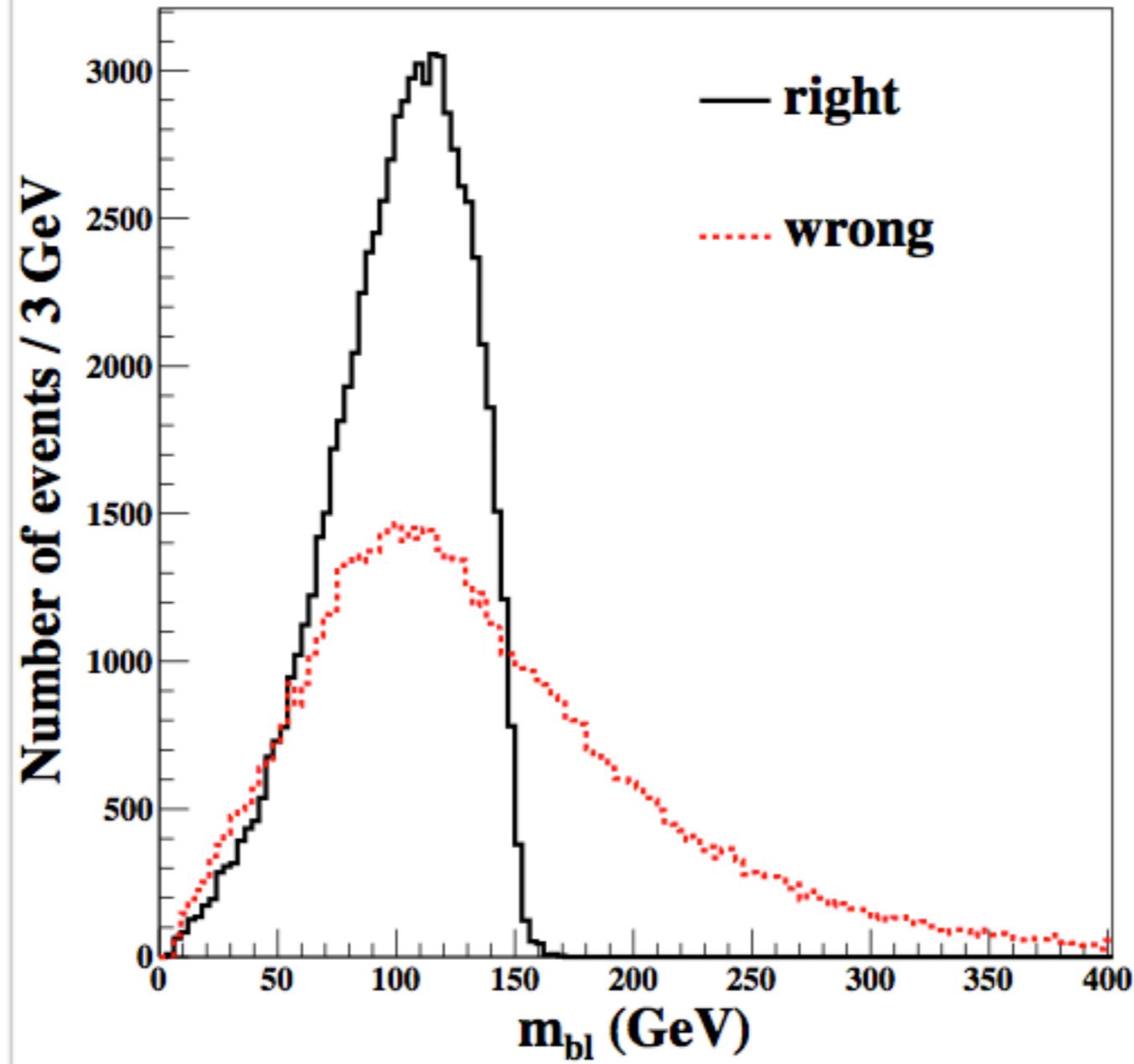
* distribution becomes broader for boosted events.

If $m_{b\ell}^2 > m_t^2 - m_W^2$, wrong pairing,

else... $\forall m_{b\ell}^2(P_j) \leq m_t^2 - m_W^2$?

$m_{b\ell}(\text{wrong})$ is likely to be larger than $m_{b\ell}(\text{right})$

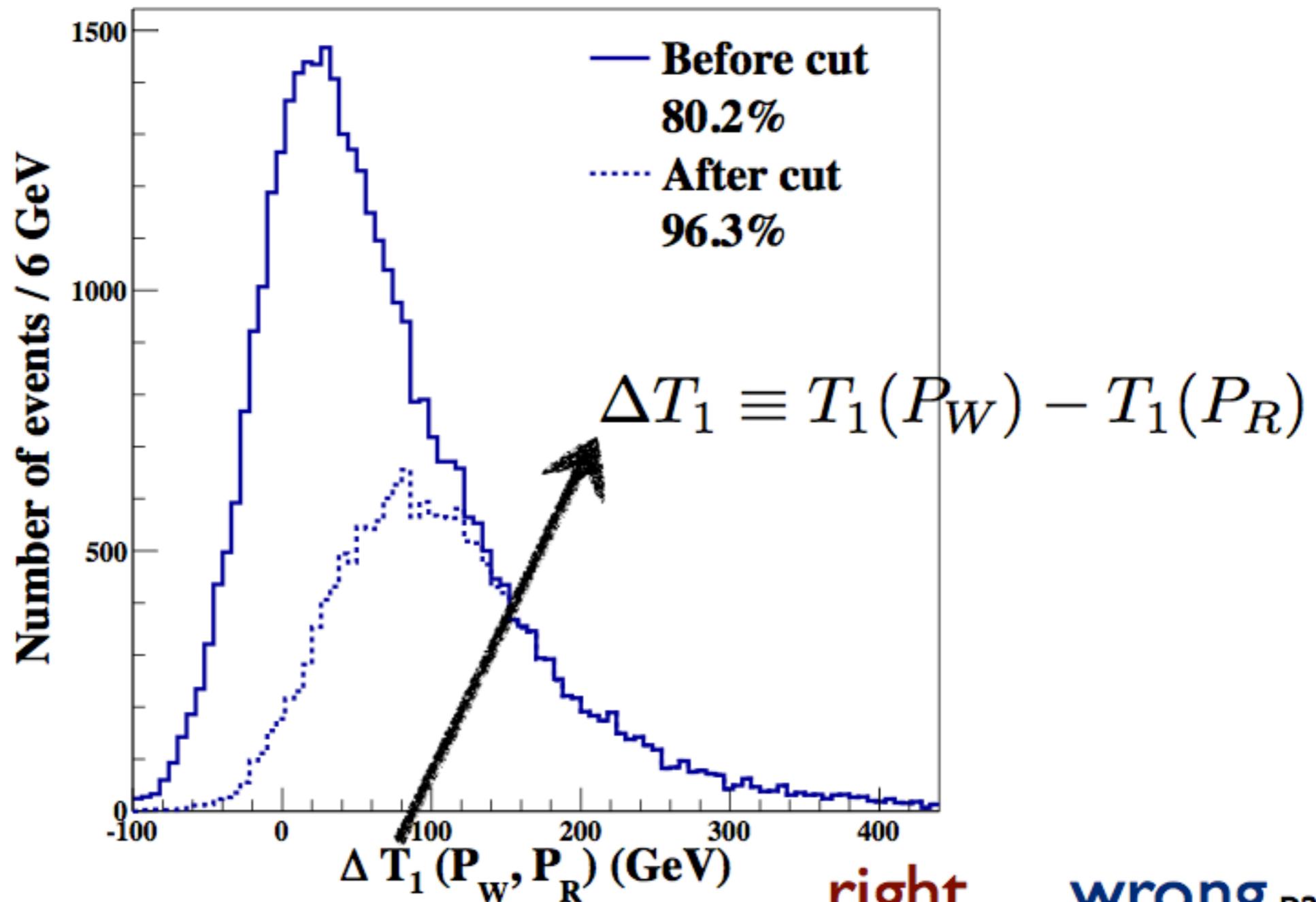
LHC, $\sqrt{s} = 7$ TeV



$m_{bl}(\text{wrong})$ is likely to be larger than $m_{bl}(\text{right})$

Combinatorics in top-pair process

LHC, $\sqrt{s} = 7$ TeV



$$T_1 \equiv \max\{m_{bl}^{(1)}, m_{bl}^{(2)}\}$$

right **wrong** pairing
 criterion: $T(P_R) < T(P_W)$.

M_{T2} (stransverse mass)

(Lester, Summers, 1999, Barr, Lester, Stephens, 2003)

$$M_{T2} \equiv \min_{\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_T^{\text{miss}}} \left[\max \left\{ M_T^{(1)}, M_T^{(2)} \right\} \right]$$

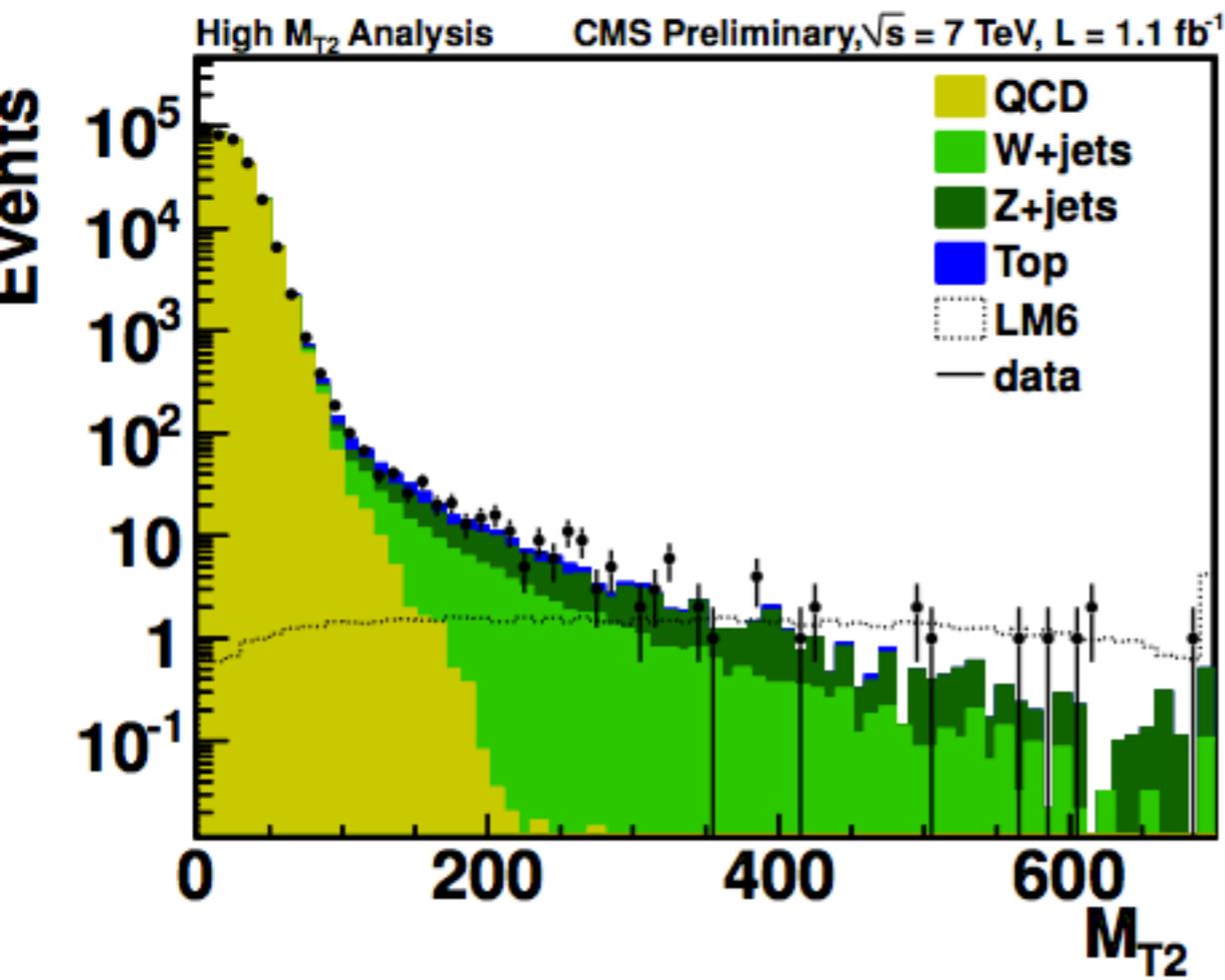
$$M_T^{(i)}(m_{bl}^{(i)}, \mathbf{p}_T^{b_i} + \mathbf{p}_T^{l_i}, \mathbf{k}_{iT})$$

transverse mass in each decay chain

* generalized transverse mass for SUSY-like event type (two invisible LSPs stabilized by R-parity).

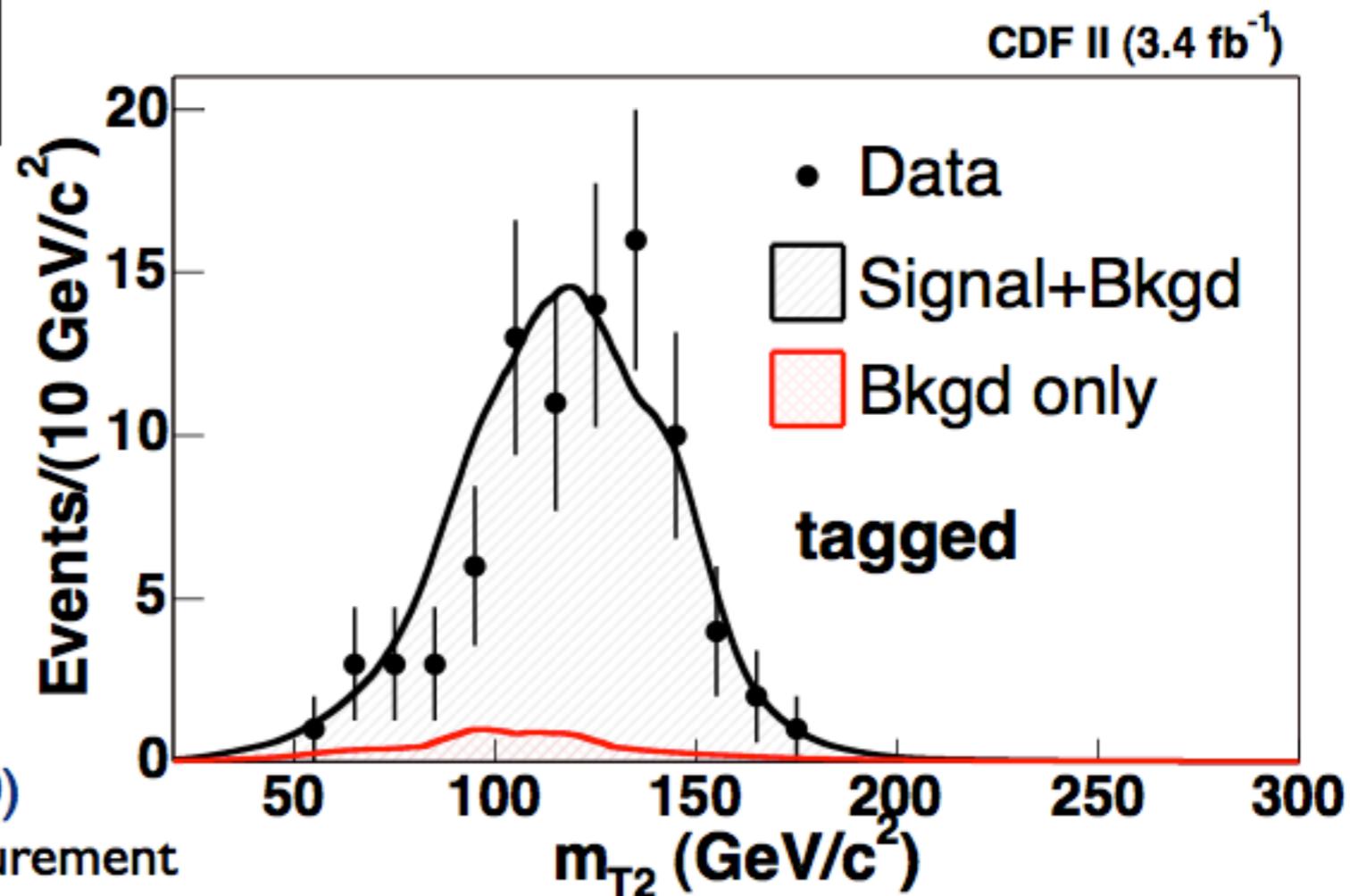
* useful for discovery as well as mass measurement
(Cho, Cho, Kim, CBP, 2007, Barr, Gripaios, Lester, 2007, Barr, Gwenlan, 2009).

M_{T2} (stransverse mass)



CMS PAS SUS-11-005
(SUSY multijet+MET search)

CDF, PRD(2010)
top mass measurement



M_{T2} (stransverse mass)

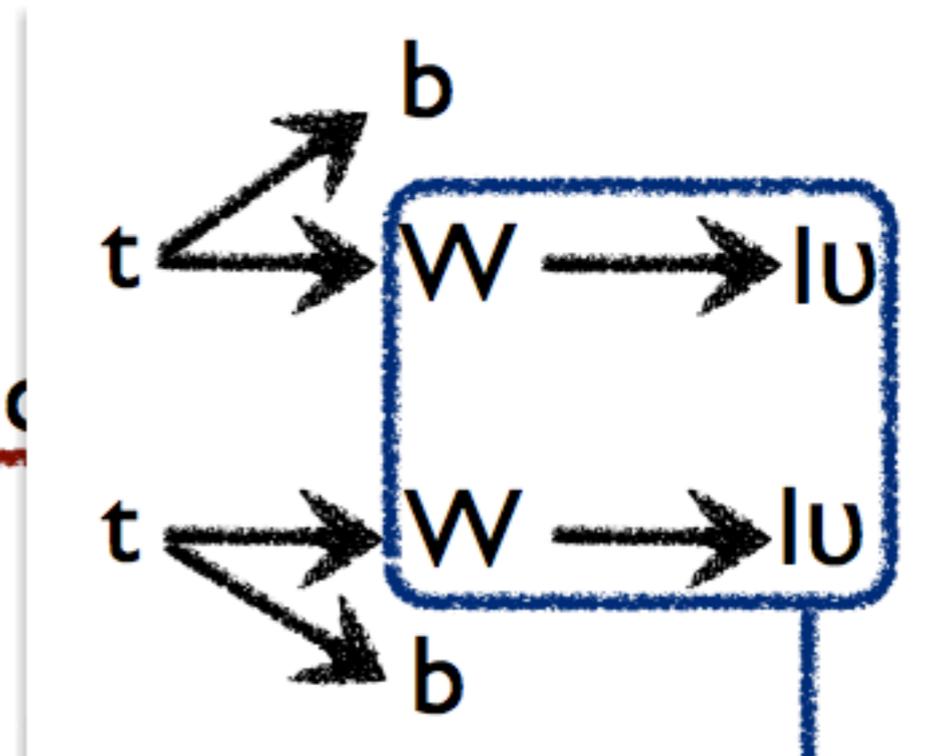
(Lester, Summers, 1999, Barr, Lester, Stephens, 2003)

$$M_{T2} \equiv \min_{\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_T^{\text{miss}}} \left[\max \left\{ M_T^{(1)}, M_T^{(2)} \right\} \right]$$

$$M_T^{(i)}(m_{bl}^{(i)}, \mathbf{p}_T^{b_i} + \mathbf{p}_T^{l_i}, \mathbf{k}_{iT})$$

transverse mass in each decay of

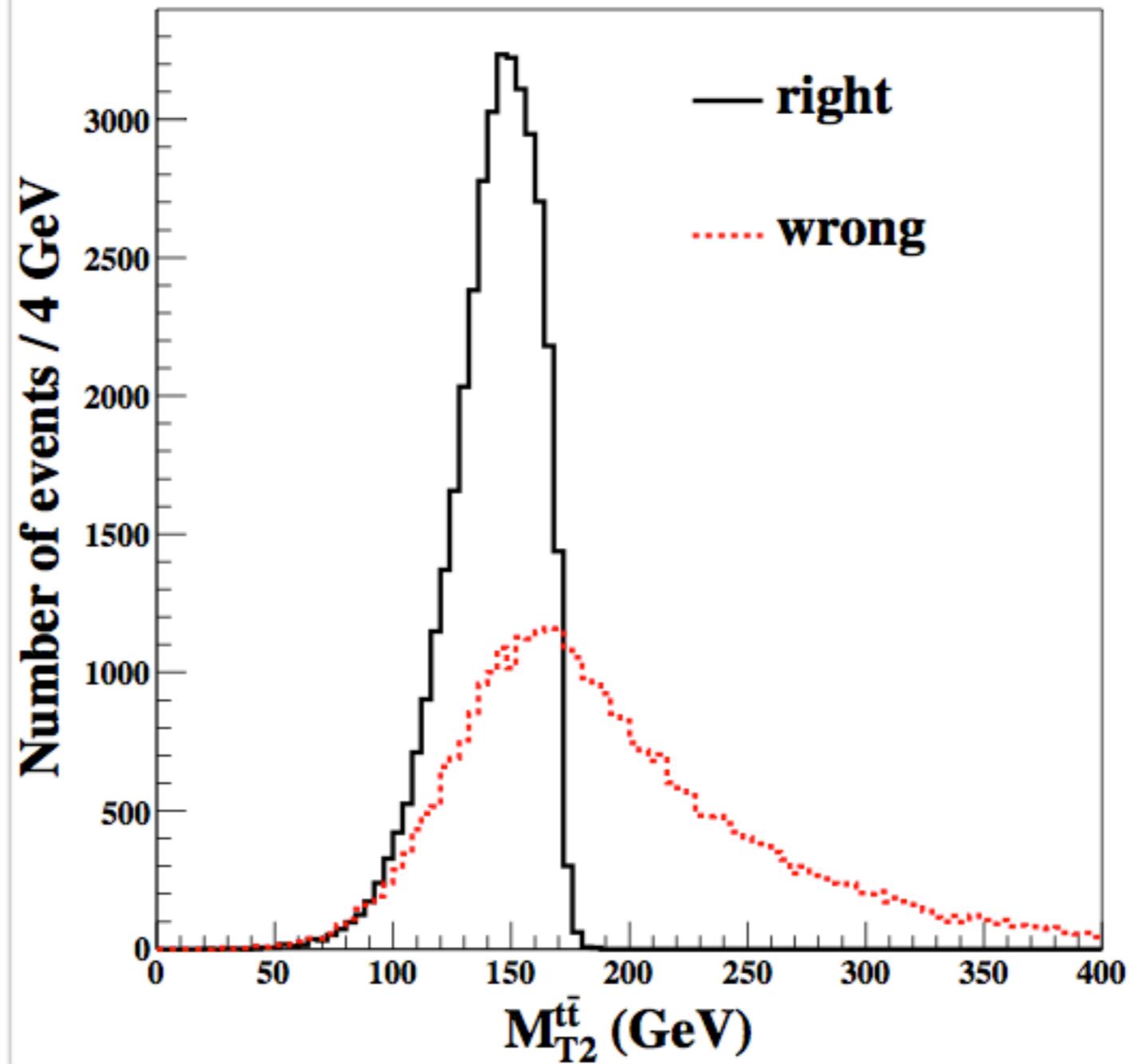
$$M_{T2} \leq m_t \text{ for right pairing.}$$



top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

LHC, $\sqrt{s} = 7$ TeV

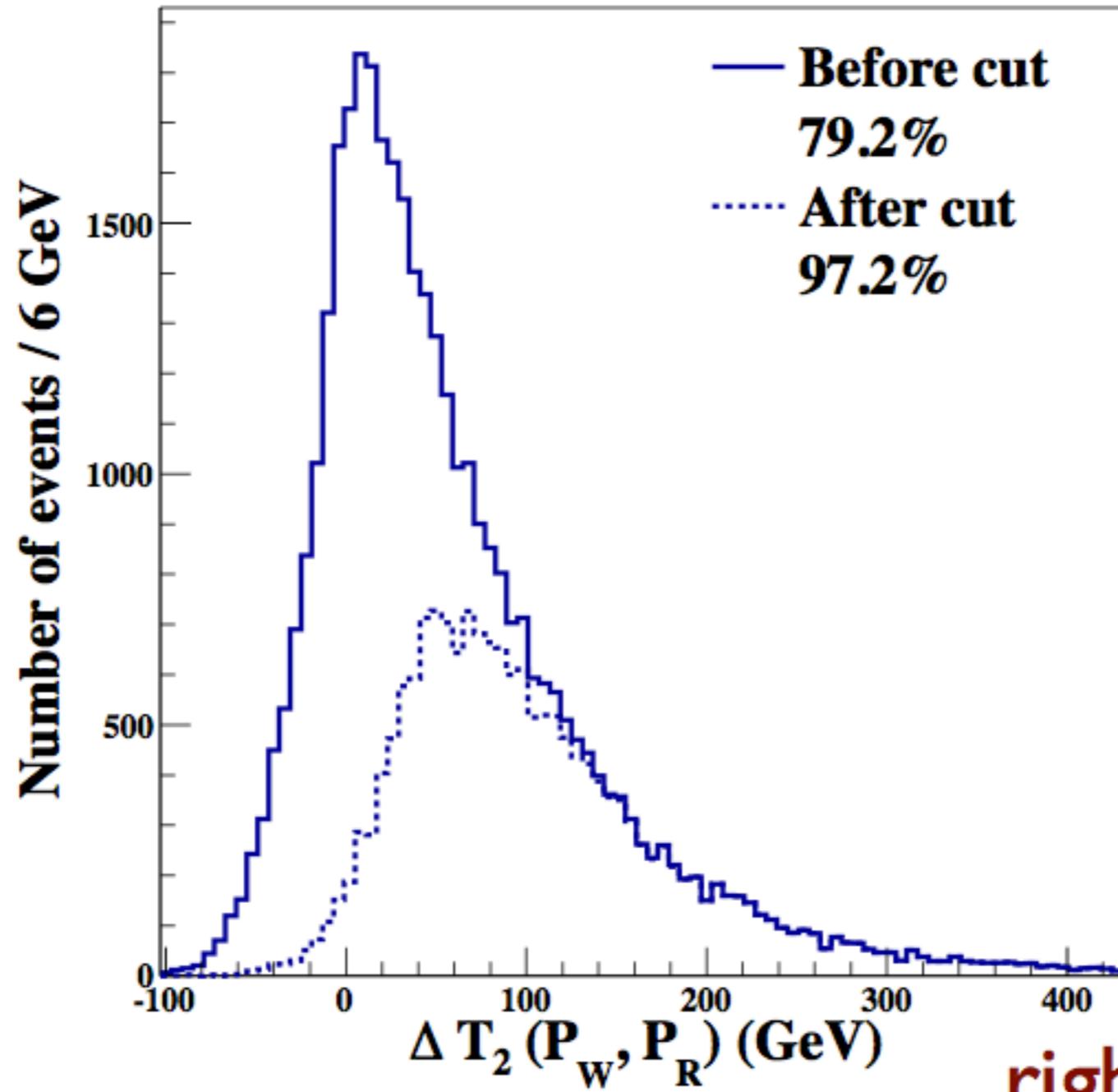


$M_{T2} \leq m_t$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

Combinatorics in top-pair process

LHC, $\sqrt{s} = 7$ TeV



right wrong pairing

$$T_2 \equiv M_{T_2}^{tt}$$

criterion: $T(P_R) < T(P_W)$.

Approximation of invisible momenta

$$M_{T2} \equiv \min_{\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_T^{\text{miss}}} \left[\max \left\{ M_T^{(1)}, M_T^{(2)} \right\} \right]$$

gives the invisible (neutrino) transverse momenta (k_T) as a result of minimization.

For given k_T values, k_L can be obtained by solving on-shell eqs $(p^b + p^l + k)^2 = m_t^2$,

$$(p^l + k)^2 = m_W^2.$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

Approximation of invisible momenta

$$M_{T2} \equiv \min_{\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_T^{\text{miss}}} \left[\max \left\{ M_T^{(1)}, M_T^{(2)} \right\} \right]$$

gives the invisible (neutrino) momenta (\mathbf{k}_T) as a function of the visible momenta (\mathbf{p}_T) .

M_{T2} -assisted on-shell (MAOS) reconstruction

MAOS can be obtained by solving on-shell eqs

$$(p^b + p^l + k)^2 = m_t^2,$$

$$(p^l + k)^2 = m_W^2.$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

MAOS momenta

$$k^{\text{maos}-WW} \quad \text{vs} \quad k^{\text{maos}-t\bar{t}}$$

$$(p^b + p^l + k)^2 = m_t^2,$$

$$(p^l + k)^2 = m_W^2.$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

MAOS momenta

$$k^{\text{maos-}WW} \quad \text{vs} \quad k^{\text{maos-}t\bar{t}}$$

$$\text{If } k^{\text{maos-}WW} = k^{\text{true}}$$

$$\text{then } (p^b + p^l + k^{\text{maos-}WW})^2 = m_t^2$$

$$\text{If } k^{\text{maos-}t\bar{t}} = k^{\text{true}}$$

$$\text{then } (p^l + k^{\text{maos-}t\bar{t}})^2 = m_W^2$$

$$(p^b + p^l + k)^2 = m_t^2,$$

$$(p^l + k)^2 = m_W^2.$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

MAOS invariant mass (I)


$$k^{\text{maos-WW}}$$

If $k^{\text{maos-WW}} = k^{\text{true}}$

then $(p^b + p^l + k^{\text{maos-WW}})^2 = m_t^2$
 $\equiv (m_t^{\text{maos}})^2$


$$(p^l + k)^2 = m_W^2.$$



W-pair subsystem (2l+MET): M_{T2}^{WW}

MAOS invariant mass (I)

$k^{\text{maos-}WW}$ always real

If $k^{\text{maos-}WW} = k^{\text{true}}$

then $(p^b + p^l + k^{\text{maos-}WW})^2 = m_t^2$

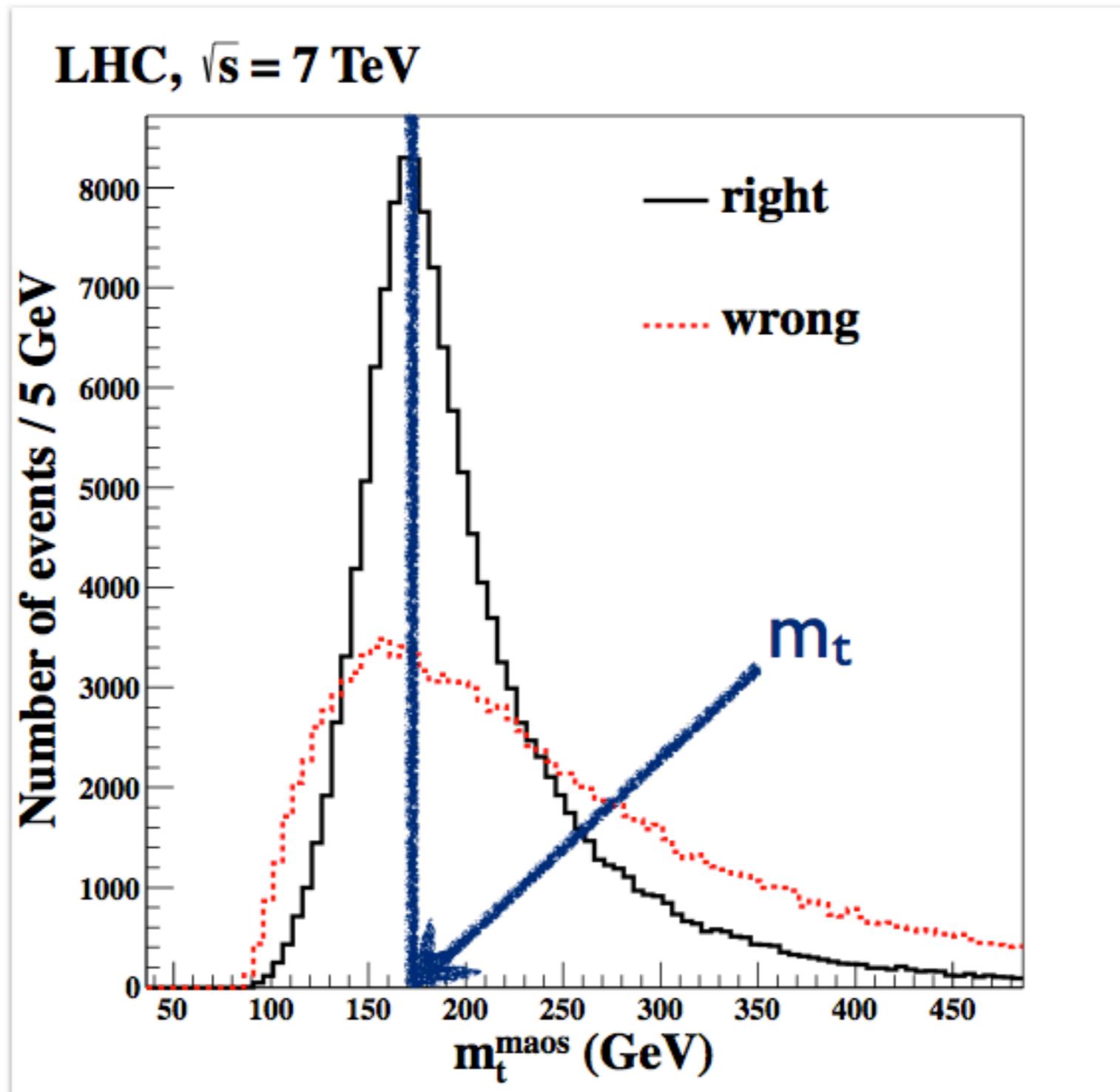
$\equiv (m_t^{\text{maos}})^2$

combinatorial ambiguity here!

$$(p^l + k)^2 = m_W^2.$$

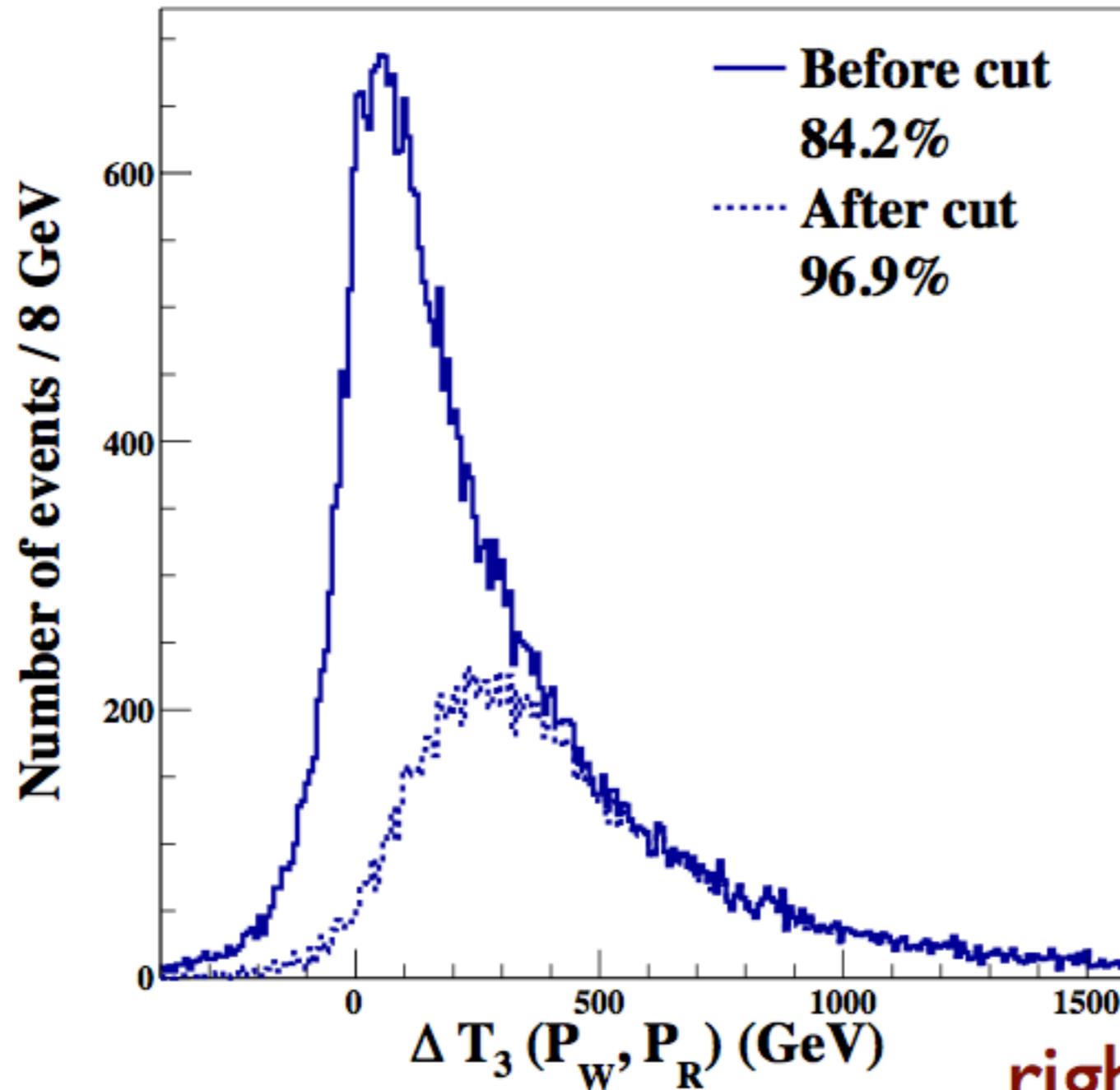
W-pair subsystem (2l+MET): M_{T2}^{WW}

MAOS invariant mass (I)



Combinatorics in top-pair process

LHC, $\sqrt{s} = 7$ TeV



$$T_3 \equiv |m_t^{\text{maos}} - m_t|$$

right wrong pairing
criterion: $T(P_R) < T(P_W)$.

MAOS invariant mass (II)

$$k^{\text{maos}-t\bar{t}}$$

If $k^{\text{maos}-t\bar{t}} = k^{\text{true}}$
then $(p^l + k^{\text{maos}-t\bar{t}})^2 \equiv (m_W^{\text{maos}})^2 = m_W^2$

$$(p^b + p^l + k)^2 = m_t^2,$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

MAOS invariant mass (II)

$$k^{\text{maos}-t\bar{t}}$$

complex solutions may occur in wrong pairing
(~38 %)

If $k^{\text{maos}-t\bar{t}} = k^{\text{true}}$

$$\text{then } (p^l + k^{\text{maos}-t\bar{t}})^2 \equiv (m_W^{\text{maos}})^2 = m_W^2$$

$$(p^b + p^l + k)^2 = m_t^2,$$

combinatorial ambiguity here!

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

MAOS invariant mass (II)

complex solutions may occur in wrong pairing
(~38 %)

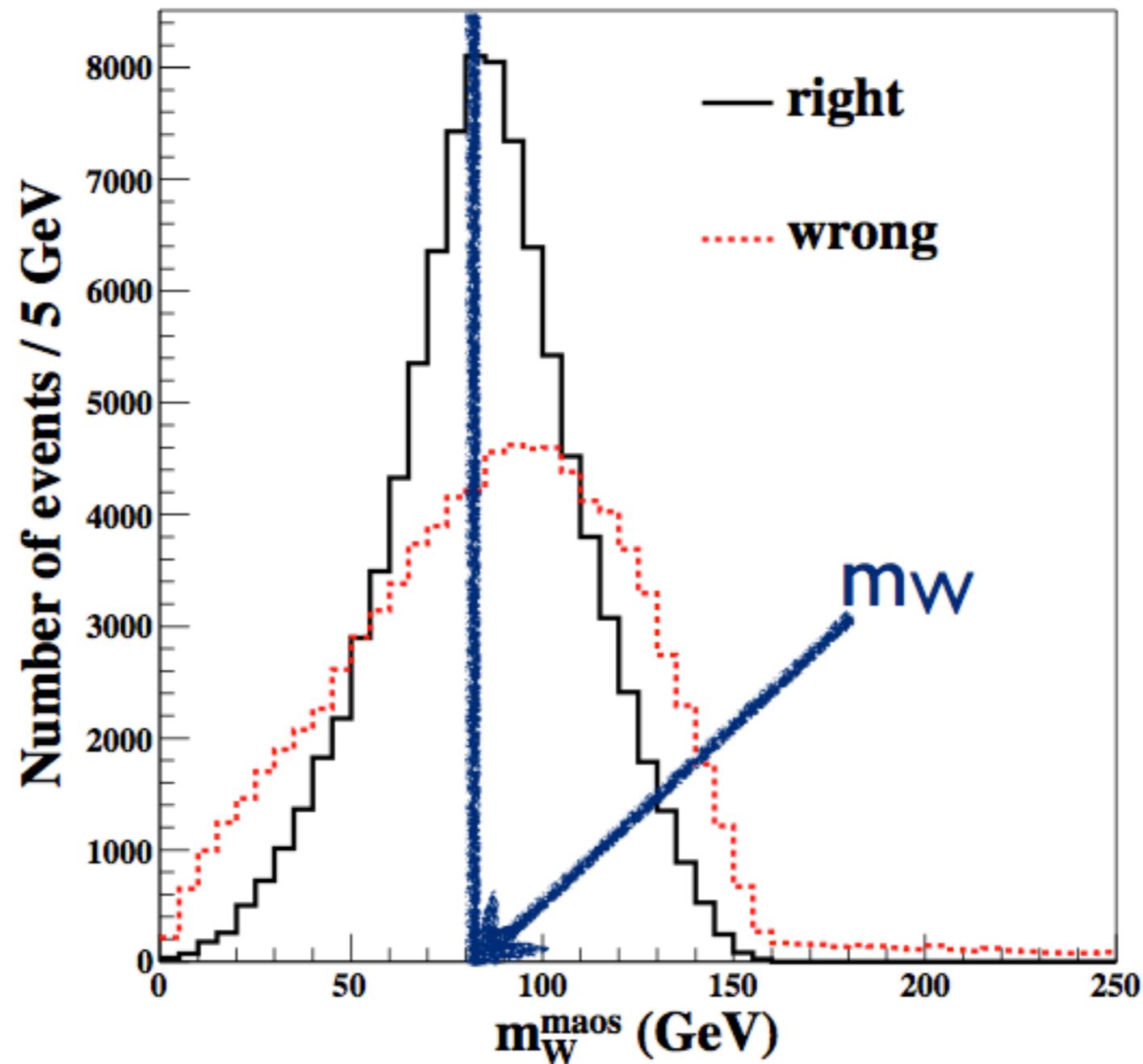


if the MAOS calculation returns at least a complex solution, but only for one of the partitions, then this partition will be regarded as the wrong one.

→ *'complex-MAOS-solution criterion'* ~99%

MAOS invariant mass (II)

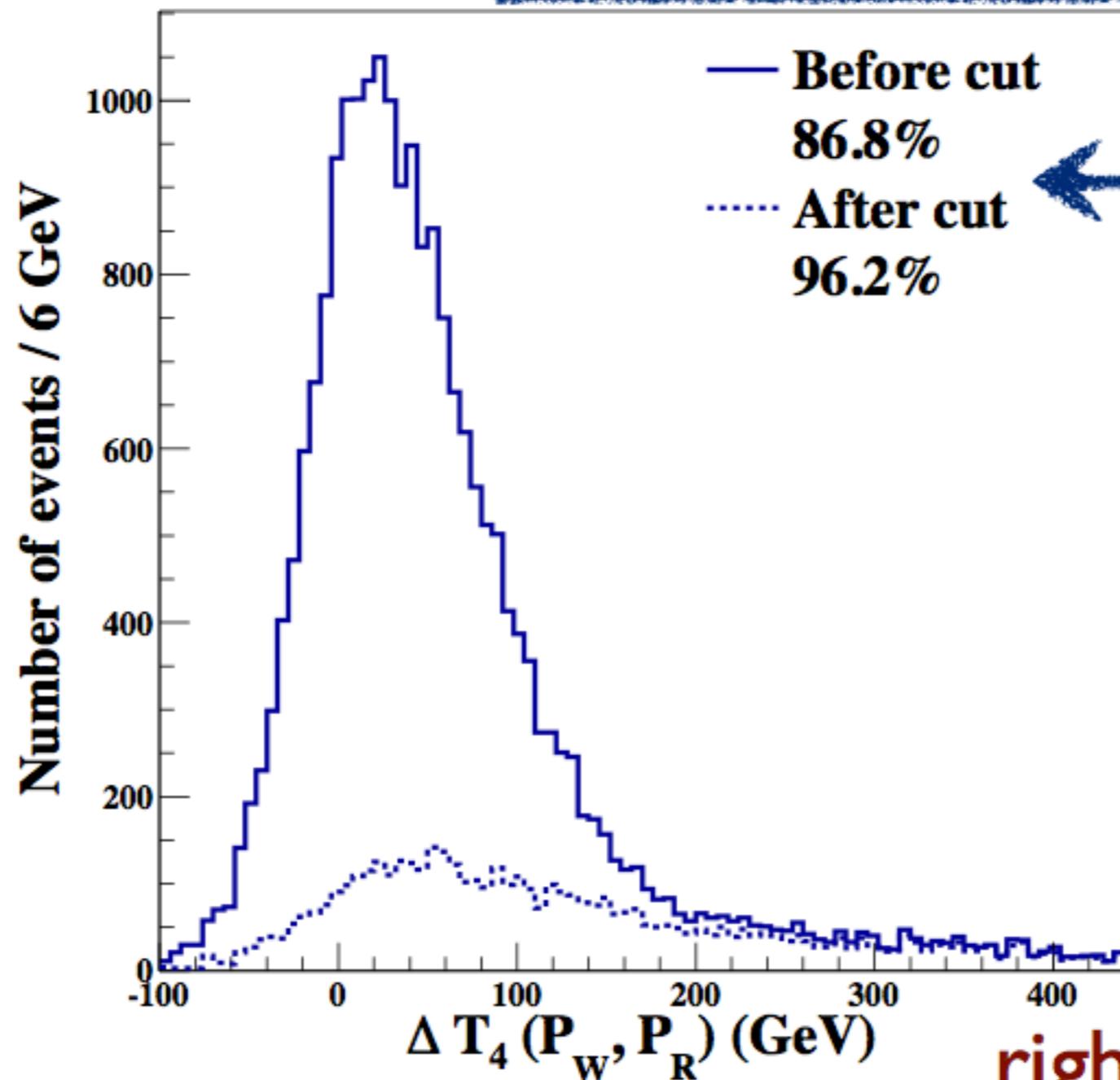
LHC, $\sqrt{s} = 7$ TeV



for events with only real solutions ~62%

Combinatorics in top-pair process

LHC, $\sqrt{s} = 7$ TeV including 'complex-solution criterion'



right

wrong pairing

$$T_4 \equiv |m_W^{\text{maos}} - m_W|$$

criterion: $T(P_R) < T(P_W)$.

Combinatorics in top-pair process

* Alternative definition of the MAOS-based variables, thanks to the identical decay chains.

$$T_3 \equiv |m_t^{\text{maos}} - m_t|$$

$$T_4 \equiv |m_W^{\text{maos}} - m_W|$$

$$\tilde{T}_3 \equiv |m_t^{\text{maos}}(1) - m_t^{\text{maos}}(2)|$$

$$\tilde{T}_4 \equiv |m_W^{\text{maos}}(1) - m_W^{\text{maos}}(2)|$$

$$\tilde{T}(P_R) = 0, \quad \tilde{T}(P_W) > 0$$

$$\Rightarrow \tilde{T}(P_R) < \tilde{T}(P_W)$$

with comparable efficiency.

Combinatorics in top-pair process

$$T_1 \equiv \max\{m_{bl}^{(1)}, m_{bl}^{(2)}\}$$

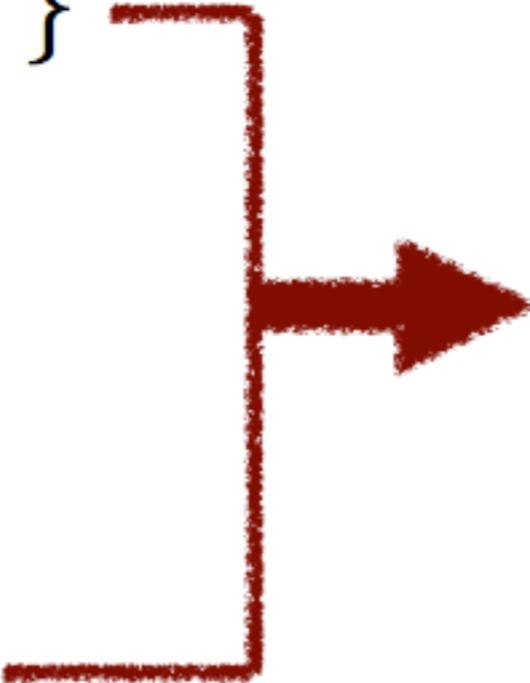
$$T_2 \equiv M_{T_2}^{t\bar{t}}$$

$$T_3 \equiv |m_t^{\text{maos}} - m_t|$$

$$T_4 \equiv |m_W^{\text{maos}} - m_W|$$

with complex-MAOS-solution criterion

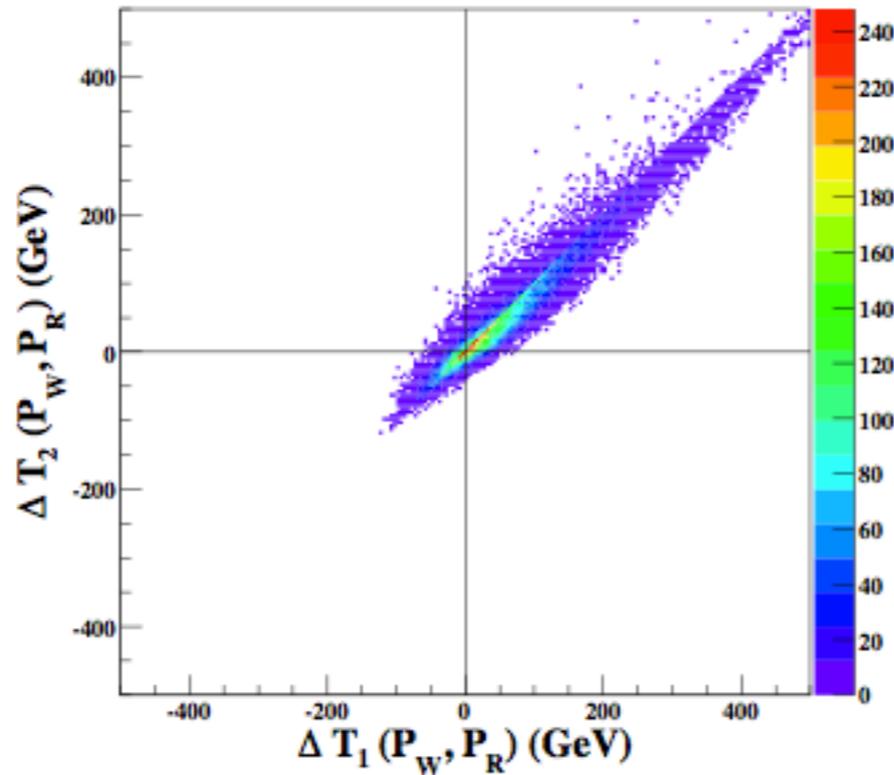
* One should find a set of independent test variables (as weakly correlated as possible).



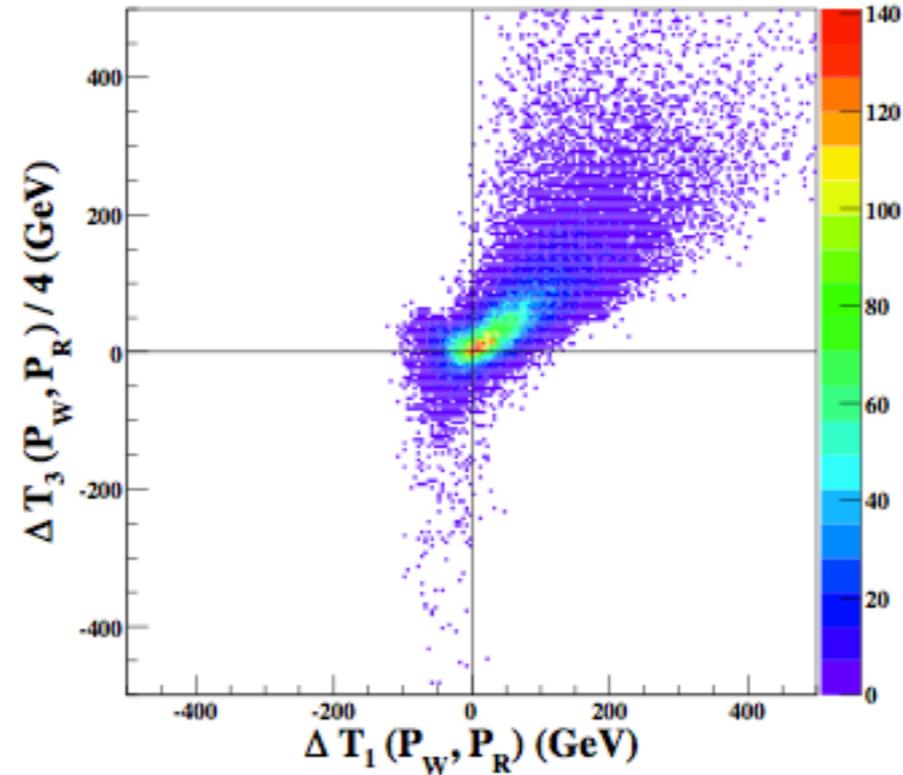
combined method?

Combinatorics in top-pair process

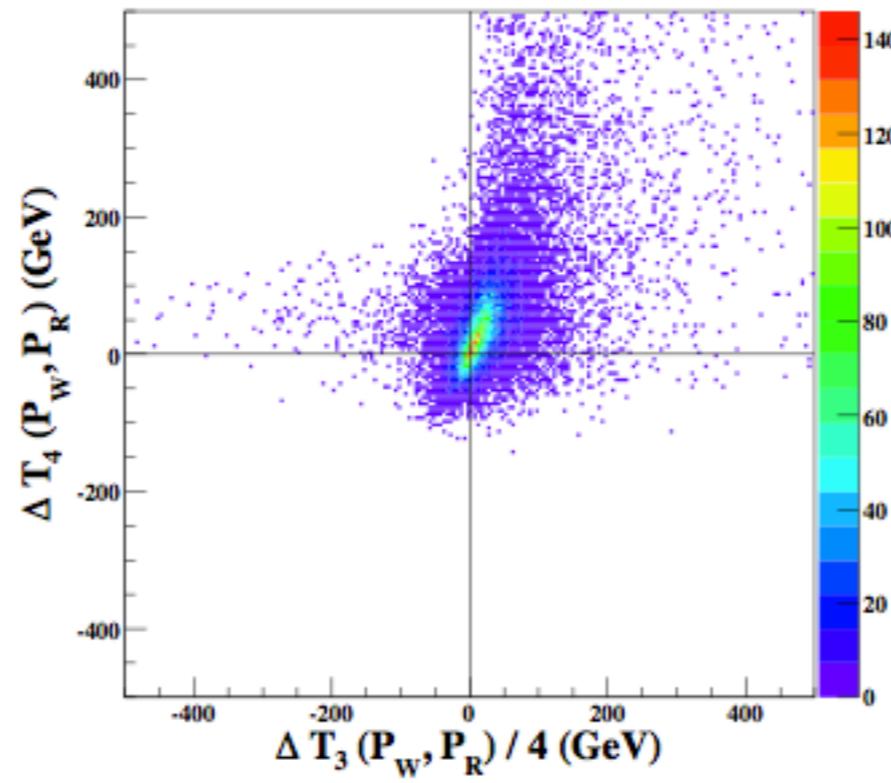
LHC, $\sqrt{s} = 7$ TeV



LHC, $\sqrt{s} = 7$ TeV

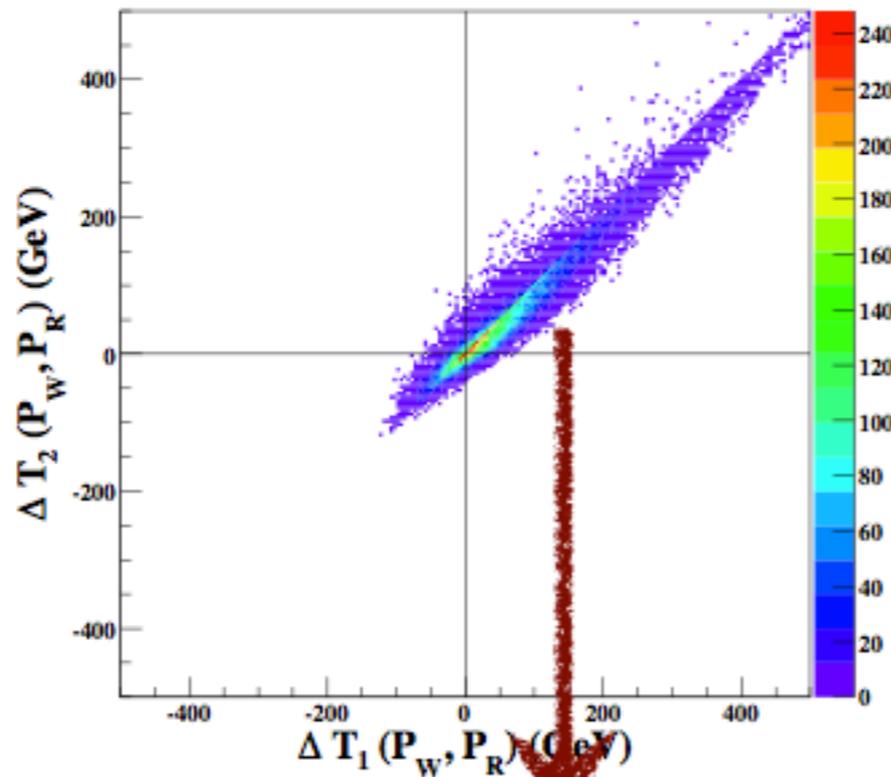


LHC, $\sqrt{s} = 7$ TeV

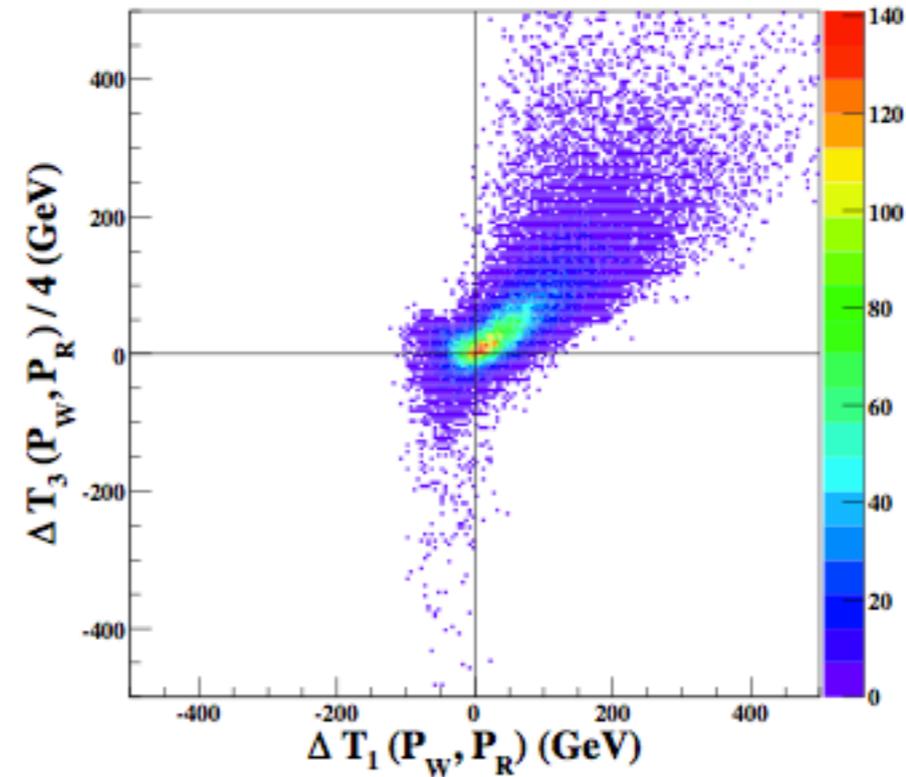


Combinatorics in top-pair process

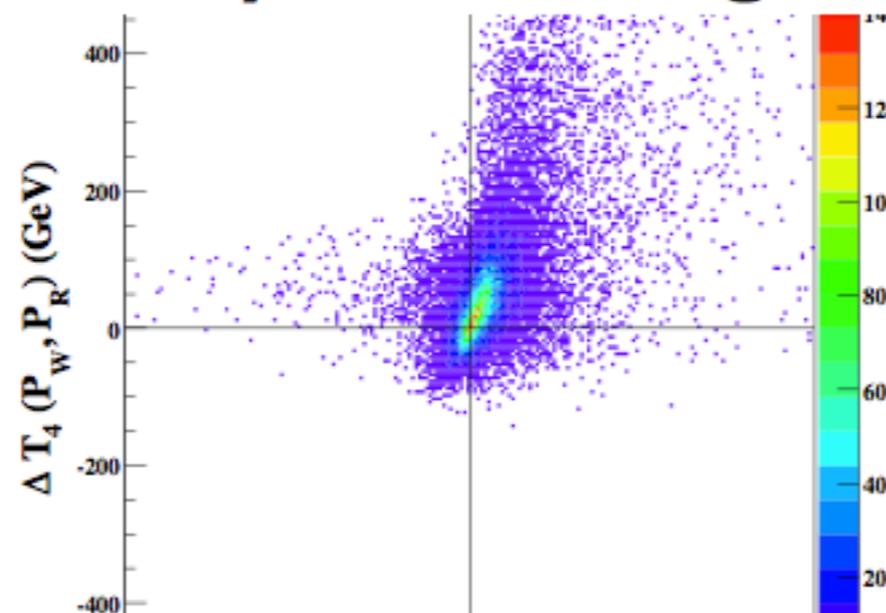
LHC, $\sqrt{s} = 7$ TeV



LHC, $\sqrt{s} = 7$ TeV



M_{T2} is a monotonically increasing function of m_{bl} .



* We choose $\{T_2, T_3, T_4\}$ as a set of indep. test variables.

Combinatorics in top-pair process

$$T_2 \equiv M_{T_2}^{t\bar{t}} \rightarrow \text{combined method?}$$

$$T_3 \equiv |m_t^{\text{maos}} - m_t|$$

$$T_4 \equiv |m_W^{\text{maos}} - m_W|$$

with complex-MAOS-solution criterion

* take a partition obeying the criteria as much as possible.

→ select a pairing P_i
if majority of $T(P_i) < T(P_j)$ ($i \neq j$).

(~ **89%** efficiency **without loss of statistics!**)

Combinatorics in top-pair process

ex)

$$T_2 \equiv M_{T_2}^{t\bar{t}}$$

$$T_2(P_1) < T_2(P_2)$$

$$T_3 \equiv |m_t^{\text{maos}} - m_t|$$

$$T_3(P_1) < T_3(P_2)$$

$$T_4 \equiv |m_W^{\text{maos}} - m_W|$$

$$T_4(P_1) > T_4(P_2)$$

without complex MAOS solutions

Take P_1 as
the 'right'
pairing

* take a partition obeying the criterions as much as possible.

→ select a pairing P_i

if majority of $T(P_i) < T(P_j)$ ($i \neq j$).

(~ **89%** efficiency **without loss of statistics!**)

Combinatorics in top-pair process

combination of test variables + event selection cuts

* wrong pairing: no definite cutoff,
distribution becomes broader for boosted events.

Combinatorics in top-pair process

combination of test variables + event selection cuts

* wrong pairing: no definite cutoff,
distribution becomes broader for boosted events

Partition-insensitive kinematic variables:

$M_T^{t\bar{t}}$ (transverse mass of top-pair full system)

m_V (invariant mass of $2b+2l$ system)

M_{eff} (scalar sum of all P_T + missing E_T)

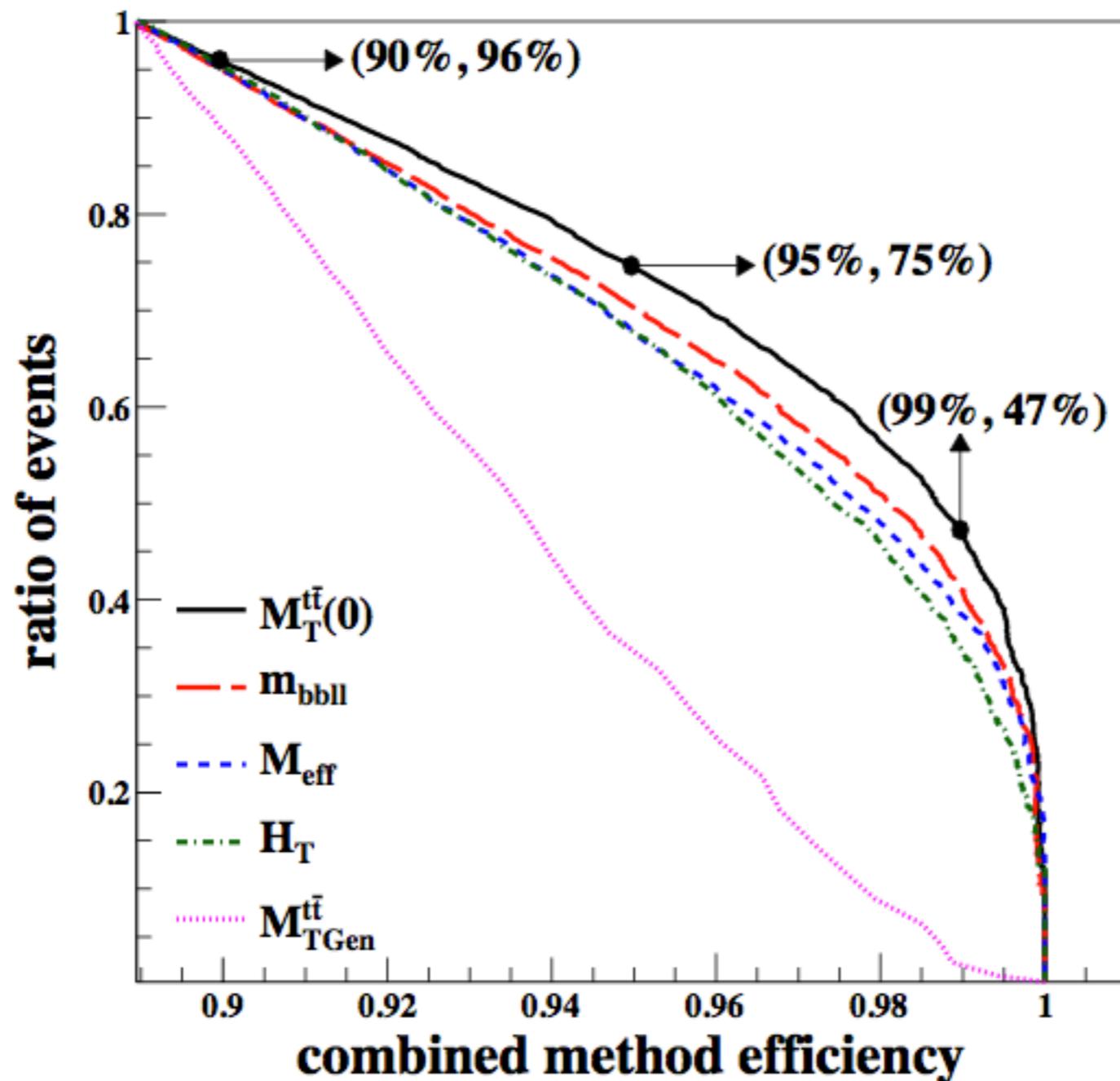
H_T (scalar sum of all P_T)

$M_{T\text{Gen}}^{t\bar{t}}$ (smallest M_{T2} of all possible in full system)

Combinatorics in top-pair process

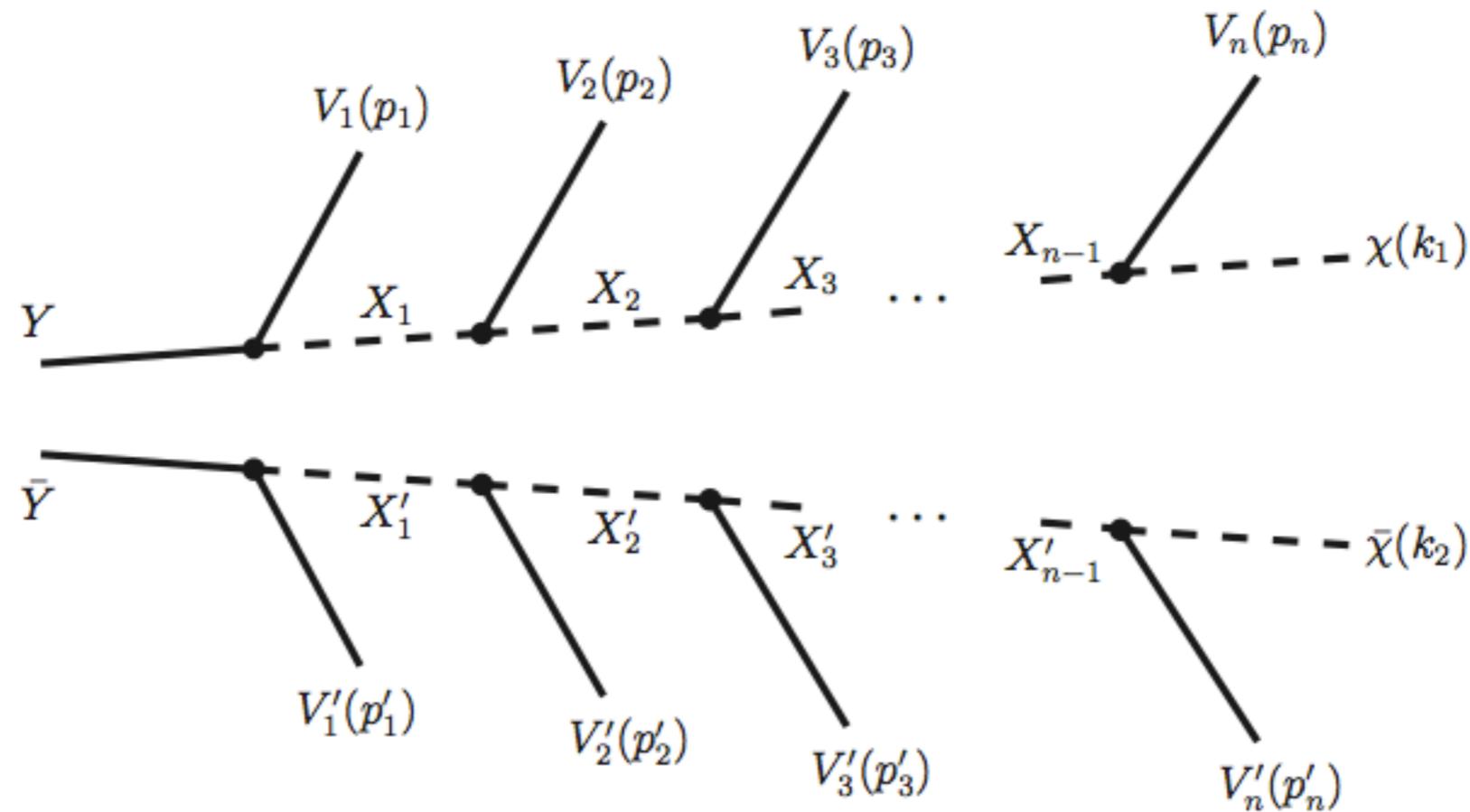
combination of test variables + event selection cuts

LHC, $\sqrt{s} = 7$ TeV



Application to new physics signature

(work in progress)



$$T_2^{Y\bar{Y}} \leq m_Y, T_2^{X_1 X'_1} \leq m_{X_1}, \dots, T_2^{X_{n-1} X'_{n-1}} \leq m_{X_{n-1}}$$

$$T_a^{\text{maos}}(P_j, k^{\text{maos}}) \equiv \sum_{c=1,2} |\Delta_{X_a} m_{X_a}^{\text{maos}}(p_c, k_c^{\text{maos}})|(P_j)$$

$$\Delta_{X_a} m_{X_a}^{\text{maos}}(p_c, k_c^{\text{maos}}) = m_{X_a}^{\text{maos}}(p_c, k_c^{\text{maos}}) - m_{X_a}$$

Application to new physics signature

(work in progress)

* select a right partition from the requirement that it gives the smallest value of T_i with respect to the other partitions for the largest number of test variables.

$$\tilde{q} \rightarrow \tilde{\chi}_2^0 q \rightarrow \tilde{\ell}^\pm \ell^\mp q \rightarrow \tilde{\chi}_1^0 \ell^\pm \ell^\mp q$$

→ **ordering** as well as pairing ($e^+e^-e^+e^-$ vs $e^+e^- \mu^+\mu^- + \text{MET}$)

$$|m_{\ell\tilde{\chi}_1^0}^{\text{maos}} - m_{\tilde{\ell}}|(\text{right}) < |m_{\ell\tilde{\chi}_1^0}^{\text{maos}} - m_{\tilde{\ell}}|(\text{wrong})$$

Summary and outlook

- * We proposed a novel method to resolve combinatorial ambiguities in hadron collider events involving two invisible particles in the final state.
- * The method is based on the kinematic variables with the assumption of decay topology.
- * It can be utilized for new physics processes with pair production of heavy particles and a WIMP pair - in case of pairing and/or ordering ambiguities - (e.g. R-parity conserving SUSY), as well as the dileptonic top-pair process.
- * For practical use, we will also study the effects of momentum smearing, ISR, and poor mass information.