$A_{\mathrm{FB}}^{t\bar{t}}$ and composite t' triplets from QCD-like flavor dynamics

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Seoul, Korea February 2012

Plan

- Flavor and $A_{\mathrm{FB}}^{t\bar{t}}$
  - flavor symmetric vectors B. Grinstein, A.K., M. Trott, J. Zupan
- Strong interaction realizations with J. Brod, J. Zupan, in progress
  - composite (u', c', t') weak singlet vectorlike up quarks, flavor nonet vectors,...

Flavor symmetry and $A_{\mathrm{FB}}^{t\bar{t}}$

The situation

For $M_{t\bar{t}} > 450$ GeV CDF measures:

$$A_{FB}^{t\bar{t}} = \frac{\sigma_F^{SM} + \sigma_F^{NP} - \sigma_B^{SM} - \sigma_B^{NP}}{\sigma_F^{SM} + \sigma_F^{NP} + \sigma_B^{SM} + \sigma_B^{NP}} = 0.475 \pm 0.114$$

$$\sigma^{t\bar{t}} = 1.9 \pm 0.5 \ \ \text{pb}$$

SM theory predictions for $M_{t\bar{t}} > 450$ GeV:

$$A_{\text{NLO}}^{t\bar{t}} = 0.088 \pm 0.013 \ \ 3.4\sigma \text{ discrepancy?}$$

$$\sigma_{\text{NLO+NNLL}}^{t\bar{t}} = 2.26 \pm 0.18 \ \ \text{pb} \ \ \text{Ahrens et al}$$

For $M_{t\bar{t}} < 450$ GeV CDF measures:

$$A_{FB}^{t\bar{t}} = -0.116 \pm 0.153 \ \ \text{consistent with SM}$$

D0 does not see a significant $M_{t\bar{t}}$ dependence (not unfolded)

Inclusive $A_{FB}^{\bar{t}t}$ measurements:

CDF:

$$A_{FB}^{\bar{t}t} = 0.417 \pm 0.16 \text{ (dilepton)}, \quad 0.158 \pm 0.074 \text{ (lepton + jet)}$$

D0:

$$A_{FB}^{\bar{t}t} = 0.196 \pm 0.06^{+0.018}_{-0.026} \text{ (lepton + jet)}$$

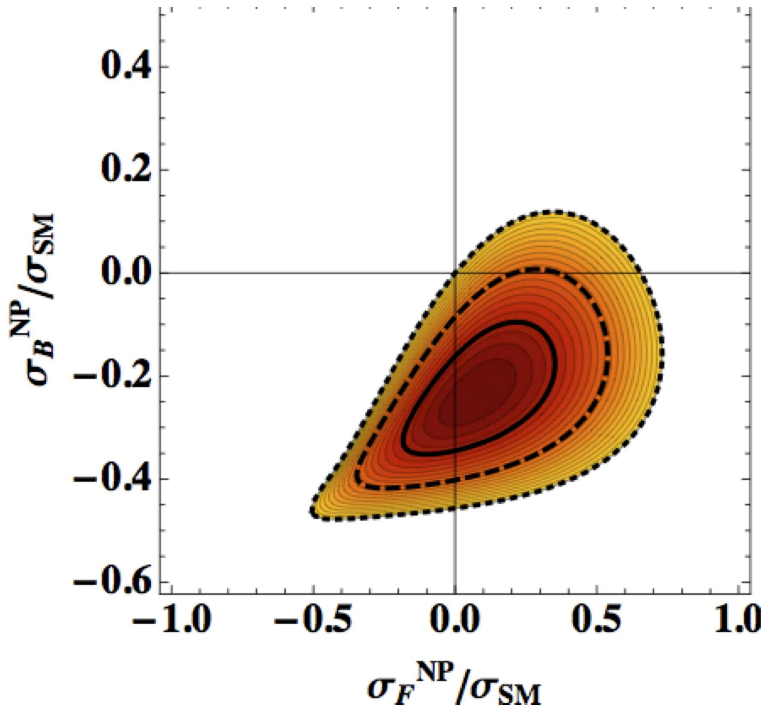
Inclusive average

$$A_{FB}^{\bar{t}t}\text{(avg)} = 0.200 \pm 0.047 \ \text{ vs. } \ A_{FB}^{\bar{t}t}\text{(SM)} = 0.09 \pm 0.01 \ \text{ Hollik, Pagani}$$

D0 inclusive leptonic asymmetry:

$$A_{FB}^{l} = 0.152 \pm 0.038^{+0.01}_{-0.013} \ \text{ vs. } \ A_{FB}^{l}\text{(SM MC@NLO)} = 0.021 \pm 0.001$$

$\sigma_B^{NP}/\sigma^{SM}$ vs. $\sigma_F^{NP}/\sigma^{SM}$ for $M_{t\bar{t}} > 450$ GeV Grinstein et al.



$\sigma_B^{NP} < 0$ region $\Rightarrow$ NP interferes with SM

For NP interference with the SM

- s-channel: color octet vector
- t-channel: color singlet, or colored resonances

Low mass t-channel explanations have appealing features:

- vectors, e.g., Z' or W' with masses of a few hundred GeV yield large $A^{t\bar{t}}_{FB}$, increases with $M_{t\bar{t}}$, as observed Jung, Murayama, Pierce, Wells '10

- simultaneously, good agreement with measured spectrum at large $M_{t\bar{t}}$ Gresham, Kim, Zurek '11; Jung, Pierce, Wells '11
  - for large $M_{t\bar{t}}$, NP t-channel top production more forward
  - but CDF's acceptance decreases rapidly at large rapidity

Low mass Z', W' have some problems

- Z': same sign top production $uu \to tt$
- W': single top production
- large $Z' - u - t$ or $W' - d - t$ couplings $\Rightarrow$ FCNC's are an issue
  - why are other couplings, e.g., $Z' - u - c$ (danger for $D - \bar{D}$ mixing), much smaller?
- contribution to $\sigma_{t\bar{t}}$ at LHC via single light mediator decay, e.g. Gresham, Kim, Zurek

$$gq \to t + (Z' \to \bar{t}q)$$

Flavor Symmetric Models

- Weak scale NP models are in MFV class if invariant under

$$G_F = U(3)_Q \times U(3)_u \times U(3)_d$$

  - Yukawas and new flavor diagonal phases only source of FCNCs
  - relaxes tensions between FCNC's and weak scale NP

- NP that is invariant under the flavor subgroup

$$H_F = U(2)_Q \times U(2)_u \times U(2)_d \times U(1)_3$$

is also appealing for relaxation of FCNC constraints

To address the problems mentioned above consider models for $A^{t\bar{t}}_{FB}$ that

- do not contain additional breaking of G_F or the alternative H_F, beyond the SM Yukawas

- contain new fields in non-trivial representations of G_F or H_F

- have $O(1)$ couplings to the top and light quarks

Flavor symmetry $\Rightarrow$ no like sign top or single top production;
negligible FCNC's, e.g., $D^0 - \bar{D}^0$ mixing

- impact of single mediator decay on $\sigma_{t\bar{t}}$ (LHC) suppressed if its branching ratio to quark pairs is suppressed; also favored by dijet constraints

Flavor symmetric vector models

- Simplest possibilities are the $U(3)_{U_R}$ flavor octet color octet or color singlet vectors coupling only to RH up quarks

$$\mathcal{L} = \lambda \bar{u}_R \gamma^\mu V_\mu^{o,s} u_R + \text{MFV corrections}$$

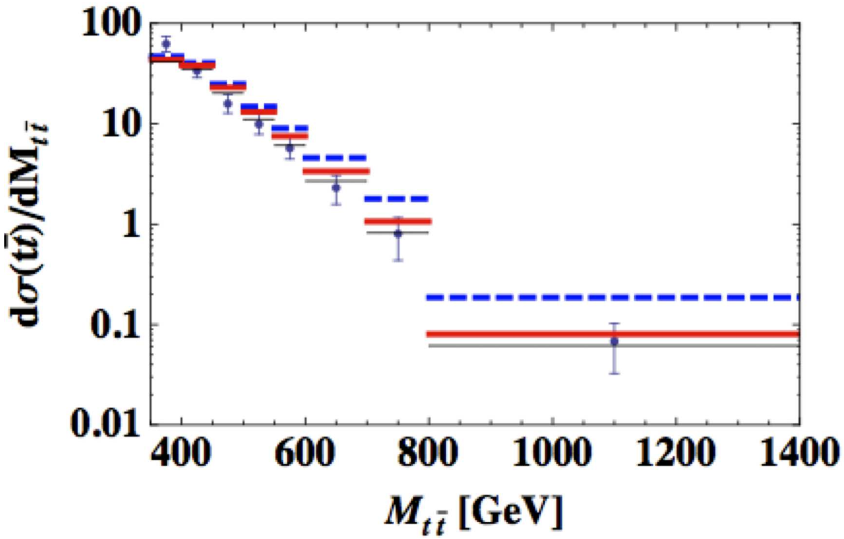
  - color octet: $V_\mu^o = V_\mu^{A,B} \mathcal{T}^A T^B$
  - color singlet: $V_\mu^s = V_\mu^A T^A$

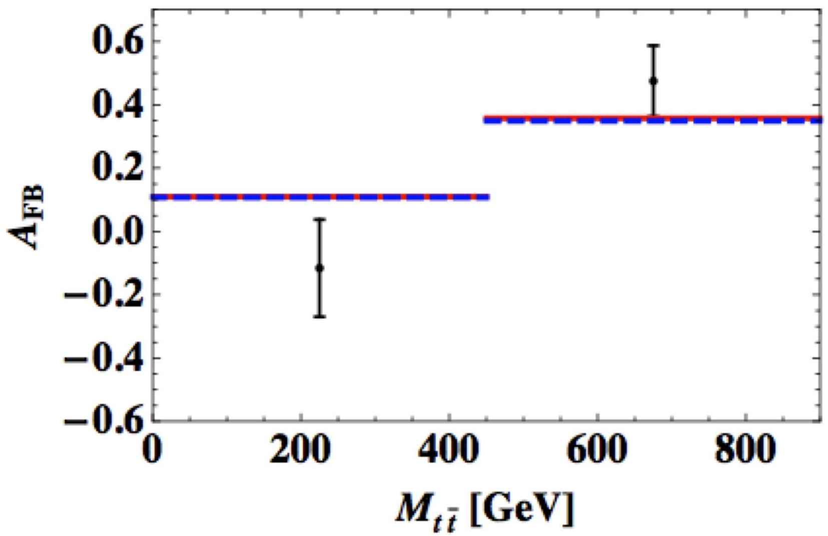
$$\text{t} - \text{channel} \qquad (V_\mu^4 - iV_\mu^5)(\bar{t}_R \gamma^\mu u_R) + \ldots.$$

$$\text{s} - \text{channel} \qquad V_\mu^8 (\bar{u}_R \gamma^\mu u_R + \bar{c}_R \gamma^\mu c_R - 2\bar{t}_R \gamma^\mu t_R)$$

- $t\bar{t}$ production t-channel dominated
- MFV corrections split $\bar{t}t$, $\bar{t}q$, and $\bar{q}q$ couplings, preserve $SU(2)_{U_R}$ symmetry
- or could have $[SU(2) \times U(1)]_{U_R}$ symmetry from the start

Ex: $A_{FB}^{t\bar{t}}$ and $d\sigma/dM_{t\bar{t}}$ for the octet of color and flavor





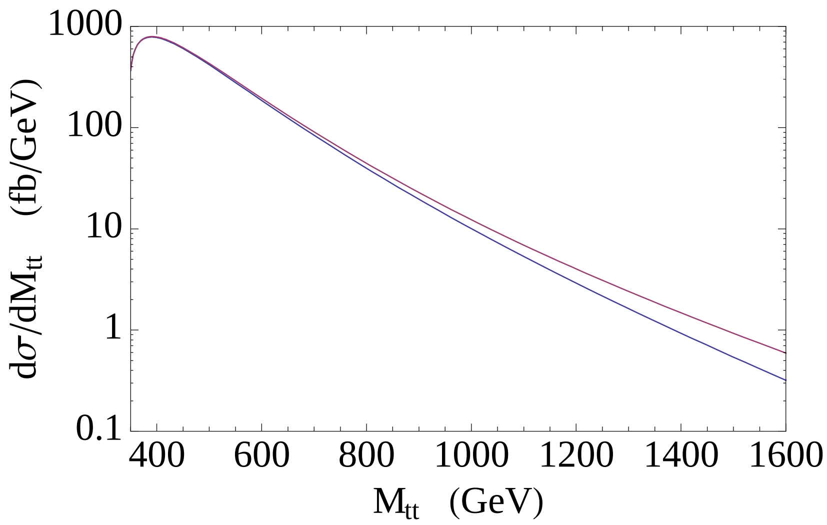
- $\mathrm{A}_{\mathrm{FB}}^{\mathrm{t\bar{t}}}$ and $d\sigma(t\,\bar{t})/dM_{t\bar{t}}$, for two different values of ($m_V\,,\sqrt{\lambda_{qq}\lambda_{33}}\,,\lambda_{q3}\,,\Gamma_V/m_V$): solid red ($300\,\mathrm{GeV}, 1, 1.33, 0.08$); dashed blue ($1200\,\mathrm{GeV}, 2.2, 4.88, 0.5$). Inclusive $\mathrm{A}_{\mathrm{FB}}^{\mathrm{t\bar{t}}} = 0.17$ in both cases

  - CDF rapidity acceptance corrections included

- For light vectors with O(10%) widths, the Tevatron and LHC dijet constraints on the smaller s-channel exchange can be satisfied with little or no flavor symmetry breaking in the quark couplings, $\lambda_{ij} \approx \lambda_{33} \approx \lambda_{i3}$

LHC $M_{t\bar{t}}$ spectrum

The $M_V = 300$ GeV color octet example:



$$\text{For } M_{t\bar{t}} \in [1400, 1600] \text{ GeV}, \quad \frac{\sigma_{NP}}{\sigma_{SM}} \approx 1.7\,, \quad \sigma_{NP} \approx 80 \text{ fb}$$

- below the $O(200)$ fb sensitivity of a recent CMS “bump hunting" search at 4.6 fb^{-1}

CMS PAS-EXO-11-006

Strong interaction realization

The set-up

- can we build models with composite flavor octet vector mesons?
- can they naturally only couple to right-handed up quarks?
- QCD provides the prototype for flavor octet (nonet) composite vector mesons
- add asymtpotically free $SU(N)_{HC}$ "hypercolor" gauge interaction, with strong interaction scale $\Lambda_{HC} \sim 1/2 - 1$ TeV
- Minimal model: add $SU(2)_L$ singlet, vectorlike $SU(3)_{U_R}$ or $[SU(2) \times U(1)]_{U_R}$ "flavor triplet" of hypercolor quarks $(\omega_{L_i}, \omega_{R_i})$ ($i = 1, 2, 3$); and a new "flavor singlet" hypercolor scalar $\mathcal{S}$

Hypercolor matter transforms under $SU(N)_{HC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$\omega_{L_i,R_i}(N,1,1,a), \quad \mathcal{S}(\bar{N},3,1,b), \quad a+b=2/3$$

$$\mathcal{L}_{NP} = \mathbf{h}_{ij}\, \bar{u}_{Ri}\, \omega_{Lj}\, \mathcal{S} + h.c. + \mathbf{m}_{\omega\, ij}\, \bar{\omega}_i\, \omega_j + m_s^2 |\mathcal{S}|^2$$

u_R is the usual flavor triplet of RH up quarks (u_R, c_R, t_R),
the ω_i are in a flavor triplet of up quark flavors $(\omega_u, \omega_c, \omega_t)$

- imposing MFV $\Rightarrow$ $\mathbf{h}_{ij} = h\, \delta_{ij}, \quad \mathbf{m}_{\omega\, ij} = m_\omega\, \delta_{ij}$
- imposing $[SU(2) \times U(1)]_{U_R}$, or taking into account MFV corrections
  $\Rightarrow$ $\mathbf{h} = \mathrm{diag}(h_1, h_1, h_3), \quad \mathbf{m}_\omega = \mathrm{diag}(\mu_1, \mu_1, \mu_3)$
- will take $m_\omega << \Lambda$, like u, d, s in QCD
- could "supersymmetrize" in order to protect scalar mass; or could imagine that the scalar is composite

- variation on $\mathcal{L}_{NP}$:   add gauge singlet scalar, $\mathcal{N}$,

$$\mathcal{L}_{NP} = \mathbf{h}\,\bar{u}_R\,\omega_L\,\mathcal{S} + h.c. + \eta\,\mathcal{N}\,\bar{\omega}\omega + +\mu_s\,\mathcal{N}\mathcal{S}^*\mathcal{S} + m_s^2\,|\mathcal{S}|^2 + m_N^2|\mathcal{N}|^2 + ...$$

  - dynamically generate ω current masses via $SU(N)_{HC}$ condensates,

$$\langle\bar{\omega}\omega\rangle,\ \langle\mathcal{S}^*\mathcal{S}\rangle \neq 0 \ \Rightarrow\ \langle\mathcal{N}\rangle \neq 0 \ \Rightarrow\ m_\omega \neq 0$$

  - $SU(3)_c$ breaking alignment of condensates can be avoided via the new terms

$$\eta\,\mathcal{N}\,\bar{\omega}\omega + +\mu_s\,\mathcal{N}\mathcal{S}^*\mathcal{S}$$

- hypercolor sector only couples to the right-handed up quarks
  - due to choice of representations for ω, $\mathcal{S}$ (hypercharge assignments)
  - Therefore, $SU(3)_{U_R}$ or $[SU(2) \times U(1)]_{U_R}$ symmetry of $\mathcal{L}_{NP}$ could be an accidental consequence of an $SU(3)_H$ or $[SU(2) \times U(1)]_H$ horizontal gauge symmetry, under which all quarks transform
  - Spontaneous breaking of $SU(3)_H$ or $[SU(2) \times U(1)]_H$ in the UV could generate the quark mass and mixing hierarchies via a Frogatt-Nielsen type mechanism
  - At the weak scale could have the SM (or MSSM) + a new flavor symmetric hypercolor sector
- Flavor structure of the resonances could hint at a horizontal symmetry solution to the quark mass hierarchy problem

Hypercolor resonances and $A_{FB}^{t\bar{t}}$

- the lowest vector meson flavor 8+1 "nonets" (a=1,..,9):

  $\rho^a_{HC}[\bar{\omega}\omega]$ vectors; $a^a_{1\,HC}[\bar{\omega}\omega]$ axial-vectors

- $\langle\bar{\omega}\omega\rangle \neq 0$ breaks global chiral symmetry

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

$\Rightarrow$ flavor octet of pions π^a_{HC}, heavier η'_{HC}

- mass scales: scale up from QCD

$$\frac{f_\pi^{HC}}{f_\pi} \sim \frac{f_\rho^{HC}}{f_\rho} \sim \frac{m_{\rho_{HC}}}{m_\rho}\sqrt{\frac{N_{HC}}{3}}, \qquad \frac{f_\rho^{HC}}{m_\rho^{HC}} \approx 0.2\sqrt{\frac{N_{HC}}{3}}$$

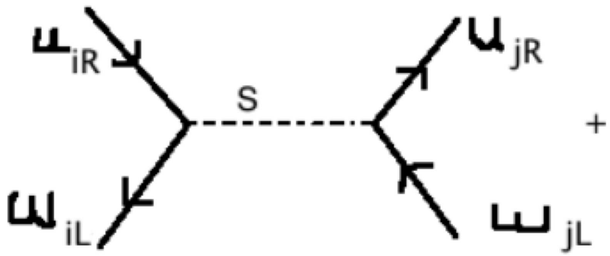
Motivated by vector flavor/color octet analysis of $A_{FB}^{t\bar{t}}$

- $m_\rho^{HC} \sim 300 - 600$ GeV $\Rightarrow$ $f_\pi^{HC} \sim 35 - 70$ GeV ($N_{HC} = 3$)
- $\Lambda_{HC}^{\chi SB} \sim 4\pi f_\pi^{HC} \sim 450 - 900$ GeV
- $m_\pi^2 \approx 8\pi\, f_\pi^{HC}\, m_\omega$

$$m_\omega \sim 10 \text{ GeV} \;\Rightarrow\; m_\pi^{HC} = O(100) \text{ GeV}$$

$$\text{VMD or scaling from QCD} \;\Rightarrow\; \frac{\Gamma(\rho_{HC} \to \pi_{HC}\,\pi_{HC})}{m_\rho^{HC}} = O(10\%)$$

vector meson - quark couplings



$+ \quad \langle \rho^a | \bar{\omega} \gamma^\mu \, T^a \, \omega | 0 \rangle \sim f_\rho m_\rho \epsilon^\mu \;\; \Rightarrow$

ρ, a_1 couplings to up quarks: $\lambda^V \, \rho^a_\mu \, \bar{u} \, T^a \, \gamma^\mu \, u \, + \, \lambda^A \, a^a_1 \, \bar{u} \, T^a \, \gamma^\mu \, \gamma_5 \, u \,,$

$$m_{\mathcal{S}} >> \Lambda \;\Rightarrow\; \lambda^V \sim h^2 \, \frac{f_\rho m_\rho}{m^2_{\mathcal{S}}}, \quad \lambda^A \sim h^2 \, \frac{f_{a_1} m_{a_1}}{m^2_{\mathcal{S}}}$$

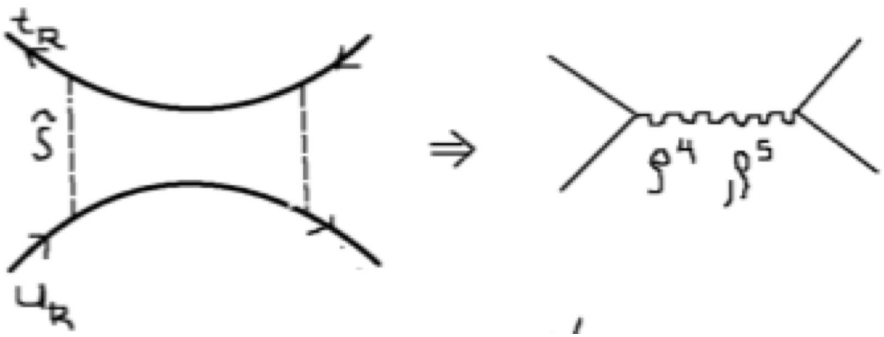
- observed $A^{t\bar{t}}_{FB} \;\Rightarrow\; \rho - u - t$ coupling $\lambda = O(1)$
  - but want perturbative $h \Rightarrow m_{\mathcal{S}} < \Lambda$
    Naive dimensional analysis (NDA) then implies

$$\lambda \sim h^2 \, \frac{f_\rho}{\Lambda} \quad \text{or} \quad h = O(\text{few})$$

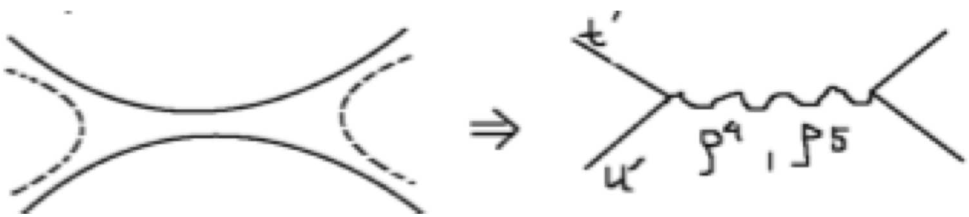
- $m_{\mathcal{S}} < \Lambda$, e.g. $O(100)$ GeV $\Rightarrow$ modified resonance picture, includes $SU(3)_{U_R}$ flavor triplet of weak singlet vectorlike up quarks, with masses of $O(m_\rho)$ or $O(1/2\ \mathrm{TeV})$

$$u'[\mathcal{S}\,\omega_u], \quad c'[\mathcal{S}\,\omega_c], \quad t'[\mathcal{S}\,\omega_t]$$

- $t\bar{t}$ production via exchange of ρ^a, a_1^a and large $u'_{R_i} - u_{R_i}$ mixing



$m_s^2 >> \Lambda^2$



$m_s^2 << \Lambda^2$,

$\Rightarrow \quad m\,\bar{u}_{R_i} u'_{L_i}$ via $\langle u'_i|\bar{\omega}_i\,\mathcal{S}^*|0\rangle = \sqrt{2} f'_u \bar{u}'_i$

$\rho^a - u_i - u_j$ couplings via exchange of composite u''s

up quark mass matrix of form:

$$M_{RL} = \begin{pmatrix} m_u & \sqrt{2} h f_{u'} \\ 0 & M_{u'} \end{pmatrix}$$

m_{u_i} are ordinary up quark masses, $M_{u'_i}$ are composite up quark masses

- $\langle u'_i | \bar{\omega}_i \, \mathcal{S}^* | 0 \rangle = \sqrt{2} f_{u_i} \bar{u}'_i, \text{ with } f'_u \sim f_\rho \; \Rightarrow$

$$|u_{R_i\,(L_i)}\rangle^{\rm phys} = \cos\theta_{R_i\,(L_i)}\,|u_{R_i\,(L_i)}\rangle - \sin\theta_{R_i\,(L_i)}\,|u'_{R_i\,(L_i)}\rangle$$

$$\sin\theta_{R_i} \approx \sqrt{2}\, h_i \frac{f'_{u_i}}{M_{u'_i}}, \qquad \sin\theta_{L_i} \approx \sqrt{2}\, h_i \frac{f'_{u_i} m_{u_i}}{M^2_{u'_i}}$$

- use Vector Meson Dominance (VMD) to estimate the $\rho^a - u'_i - u'_j$ and $a_1^a - u'_i - u'_j$ couplings

$$g_V\ \rho^a_\mu\ \bar{u}'\, T^a\, \gamma^\mu\, u' \ +\ g_A\ a^a_{1\ \mu}\ \bar{u}'\, T^a\, \gamma^\mu\, \gamma_5\, u' \ \Rightarrow\ g_V \approx \frac{m_\rho}{f_\rho},\ \ g_A \approx \frac{m_{a_1}}{f_{a_1}}$$

- $\rho^a - u_i - u_j$ and $a_1^a - u_i - u_j$ couplings follow from $u' - u$ mixing:

$$\lambda_{ij}^{V_a} \approx g_V \sin^2\theta_R = \mathcal{O}\left(2h^2\,\frac{f_\rho}{m_\rho}\right) \sim\ 0.4\, h^2\ , \qquad \frac{\lambda^A}{\lambda^V} \sim \frac{m_{a_1} f_{a_1}}{m_\rho f_\rho}$$

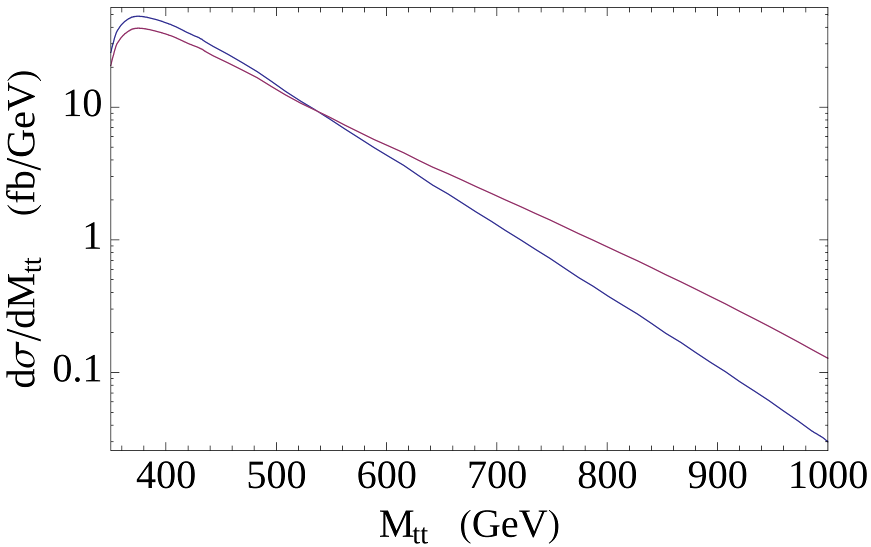
$$\lambda^V \sim 1\ \ \Rightarrow\ \ h \sim \sqrt{2}$$

- partially composite RH up quarks with $\sin\theta_{R_i} \sim 1/3$,
  and LH top with $\sin\theta_L^t \sim 1/3 \times m_t/M_{t'}$

$A_{FB}^{t\bar{t}}$ and $d\sigma/dM_{t\bar{t}}$ from the flavor nonets ρ^a, a_1^a

- for simplicity, consider $SU(3)_{U_R}$ limit (rather than $SU(2) \times U(1)$) :
- degenerate nonets $m_{\rho^a} = m_\rho, \;\; m_{a_1^a} = m_{a_1}$
- degenerate couplings for λ^V and λ^A
- in $SU(3)_{U_R}$ limit no s-channel contribution: no "$\omega - \phi$" mixing

- example: exchange of ρ^a flavor nonet vector mesons ($m_\rho = 300$ GeV; $m_{a_1} = 450$ GeV; $\lambda^V = 0.8$; $\lambda^A = \lambda^V m_{a_1}/m_\rho$, $\Gamma_\rho/m_\rho = \Gamma_{a_1}/m_{a_1} = 10\%$):
  - $M_{\bar{t}t}$ spectrum:



blue $= SM$

  - $A^{\bar{t}t}_{FB}(M > 450\text{ GeV}) = 0.37, \quad A^{\bar{t}t}_{FB}(M < 450\text{ GeV}) = 0.02$
    $A^{\bar{t}t}_{FB}(\text{inclusive}) = 0.11$
- CDF rapidity acceptance correction not included - should bring the true excess at large $M_{\bar{t}t}$ into better agreement with data
- preliminary: ρ, a_1 exchange satisfies Tevatron dijet contsraints for $\frac{\Gamma_{\rho\to\pi\pi}}{m_\rho} = O(10\%)$

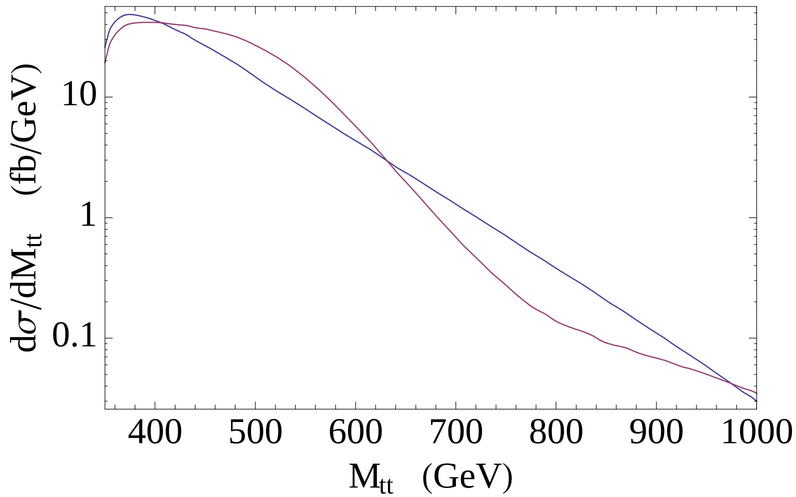
- could also have s-channel exchange of a P-wave vector meson bound states of the scalars, $V^{\mu}\,[S^*S]$,
  - a flavor singlet color octet $V^{[8]}$, and flavor singlet color singlet $V^{[1]}$,

$$\langle V^a|\mathcal{S}^* T^a \partial_\mu \mathcal{S} - (\partial_\mu \mathcal{S}^*) T^a \mathcal{S}|0\rangle \sim f_V m_V \epsilon_\mu$$

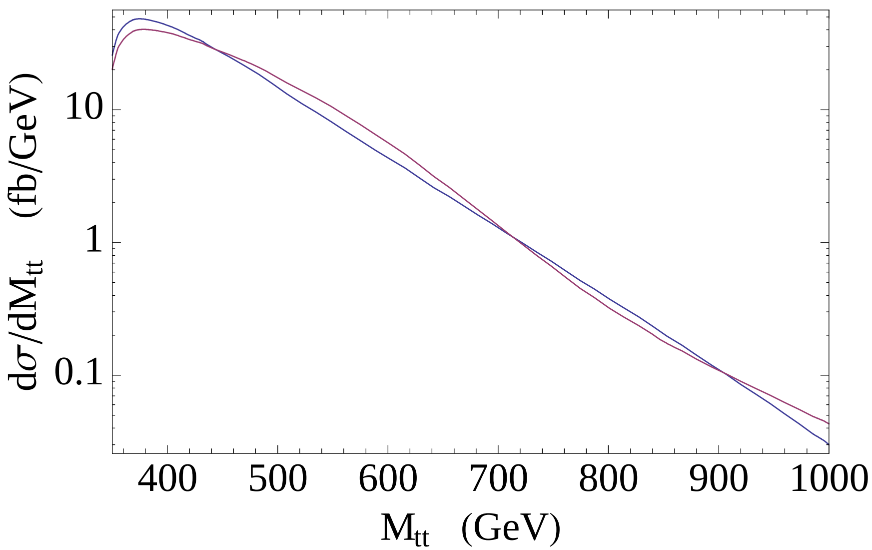
  - can gain insight on masses, decay constants from QCD tensor mesons $f_2(1270)$, $f_2'(1525)$
  - however, V expected to be very broad, e.g. $\Gamma/M = O(1)$, follows from NDA estimates of $V \to \bar{u}_i' u_i$
  - P-wave resonance picture likely to break down
  - nevertheless proceed, to gauge possible impact on the $M_{t\bar{t}}$ spectrum and A_{FB} of "s-channel exchanges"

Add exchange of color octet V^a ($m_V = 500$ GeV, $\lambda^V = 1.0$): Tevatron $M_{t\bar{t}}$ spectrum

- $\Gamma_V/m_V = 50\%$,



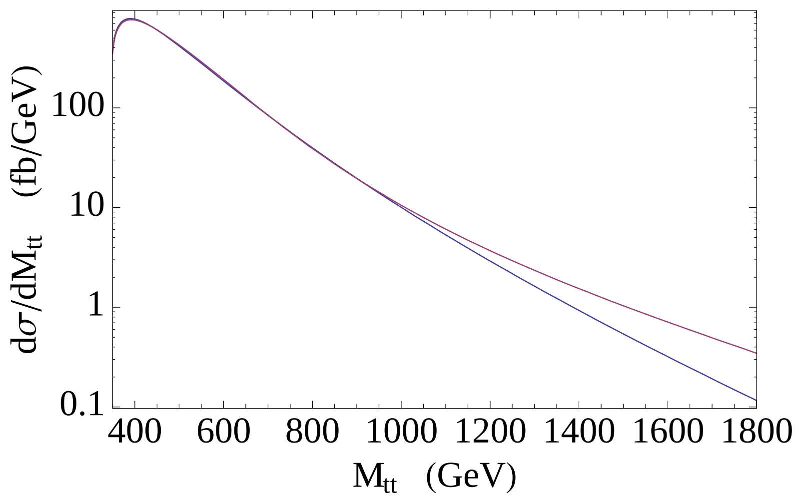
- $\Gamma_V/m_V = 100\%$,



$A_{FB}^{\bar{t}t}(M > 450 \text{ GeV}) = 0.36, \quad A_{FB}^{\bar{t}t}(M < 450 \text{ GeV}) = 0.13,$
$A_{FB}^{\bar{t}t}(\text{inclusive}) = 0.18$

LHC $M_{t\bar{t}}$ spectrum

- $\Gamma_V/m_V = 100\%,$



$$\frac{\sigma_{NP}}{\sigma_{SM}} \approx 1.9 \;\; (M_{t\bar{t}} \in [1400, 1600] \text{ GeV}), \quad \frac{\sigma_{NP}}{\sigma_{SM}} \approx 2.5 \;\; (M_{t\bar{t}} \in [1600, 1800] \text{ GeV})$$

On the composite u''s

- They can be quite broad! Due to decays to ordinary RH up quarks and HC pions, π^a, e.g.,

$$u'_i \to \pi^a + t$$

- the couplings of π^a to the u''s (similar to $\pi - N$ coupling)

$$\sim \frac{2}{f_\pi} \bar{u}' \gamma_5 T^b \, \gamma^\mu \partial_\mu \, \pi^b \, u' \;\Rightarrow\; \Gamma_{u'}/M_{u'} = \mathcal{O}(10\%),$$

compared with $\approx 1\%$ for the top quark

- composite quark masses $\sim 1/2$ TeV, depending on scalar mass $m_{\mathcal{S}}$

- detection at LHC via $u'_i \to t + \pi^a$
- π^a are color singlets, decay via $\pi^a \to \bar{u}u,\ \bar{c}c$, or $\pi^a \to \bar{t}^{(*)}u,\ \bar{t}^{(*)}c$
- final states with two tops: $\bar{u}'_i u'_i \to \bar{t}\, t\, \bar{q}\, q\, \bar{q}\, q$,
  $\bar{u}'_i u_i \to \bar{t}\, t\, \bar{u}_i\, u_i$ (suppressed)
- final states with 4 tops: $\bar{u}' u' \to \bar{t}\, t\, \bar{t}\, t\, \bar{q}\, q$
- Production mechanism:
  - $\bar{u}'_i\ u'_i$: via QCD and $\rho^a,\ a_1^a$, $V^{[8],[1]}$ exchange
  - $\bar{u}'_i\, u_i$ (single u' production): via $\rho^a,\ a_1^a,\ \ V^{[8],[1]}$ exchange

In progress

- Hypercolor
  - coding everything in madgraph: detailed study of phenomenology, including LHC $t\bar{t}$ charge asymmetry A_C
  - non-relativistic quark model for dependence of vector and axial vector nonet mass spectra on m_{ω_i}
  - include resulting "$\omega - \phi$ mixing" contributions to s-channel $t\bar{t}$ and dijet production
  - phenomenology of flavorful HC $[\omega\omega\omega]$ baryons, DM implications
- exploring connection to low scale bosonic technicolor with a light Higgs
  - $\Lambda_{\rm HC} \sim 1/2$ TeV suggests a low scale for $\Lambda_{\rm TC}$
  - identification of HC with TC disfavored: sufficient asymptotic freedom for $\mathrm{N}_{\rm TC} = 2$ is problematic
  - studying an $SU(3)_{\rm HC} \times SU(2)_{\rm TC}$ model in which $\Lambda_{\rm TC} > \Lambda_{HC}$, and the two scales are linked