Status and implications of fine tuning in SUSY models.

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with Hyun Min Lee and Myeonhung Park (CERN).

E-print: arxiv:1203.0569 [hep-ph].

• SUSY as a solution to hierarchy problem:

- why EW scale $v \sim M_Z \ll M_{Planck}$ is stable under quantum corrections.

• no-SUSY: $\delta v^2 \sim \delta m_h^2 \sim \, f(couplings) \, \Lambda_{UV}^2$, if $\Lambda_{UV} \sim M_{Planck}$:

$\Rightarrow$ Hierarchy Problem $\Leftrightarrow$ fine tuning (naturalness).

• SUSY: $\delta v^2 \sim \delta m_h^2 \sim m_{susy}^2 \log \Lambda_{UV}^2 / m_{susy}^2$, $m_{susy} \sim$ TeV or so;

- negative SUSY searches increase m_{susy}. If $m_{susy} \gg \mathrm{TeV} \Rightarrow$ back to SM fine-tuning.

• other (much worse) fine tunings exist: cosmological constant. Why worry?

• Problem: Measures of fine-tuning?

• Stability of EW scale in SUSY:

$$V = m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (B_0\,\mu_0\,H_1\cdot H_2 + h.c.) + \lambda_1/2\,|H_1|^4 + \lambda_2/2\,|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2$$
$$+\ \lambda_4\,|H_1\cdot H_2|^2 + \Big[\lambda_5/2\,(H_1\cdot H_2)^2 + \lambda_6\,|H_1|^2\,(H_1\cdot H_2) + \lambda_7\,|H_2|^2\,(H_1\cdot H_2) + h.c.\Big]$$

$$m^2 \equiv m_1^2\cos^2\beta + m_2^2\sin^2\beta - B_0\,\mu_0\,\sin 2\beta, \qquad \mathrm{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2$$

$$\lambda \equiv \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\,\big(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\big)$$

2 constraints: $f_1 \equiv v^2 + \dfrac{m^2}{\lambda} = 0, \quad f_2 \equiv 2\lambda\dfrac{\partial m^2}{\partial\beta} - m^2\dfrac{\partial\lambda}{\partial\beta} = 0,$

• Problem: scales vs. couplings:

$$v^2 = -m^2/\lambda, \qquad v = \mathcal{O}(100\ \mathrm{GeV}), \quad \lambda < 1, \qquad \textbf{but} \quad m_{1,2}, B_0 \text{ and } m \sim \mathcal{O}(1\,\mathrm{TeV}).$$

• Even worse: $m_h < m_Z$, LEP2: > 114.4 GeV, need large quantum corrections (QC) $\Rightarrow$ large m_{susy}

• A problem of couplings (λ_{MSSM} small). Solution: increase λ by QC or by new physics beyond MSSM

- Measures of fine tuning and their problems:

$$\Delta_{max} \equiv \max_i \left|\Delta_{\gamma_i}\right|, \qquad \Delta_{\gamma_i} \equiv \frac{\partial \ln v^2}{\partial \ln \gamma_i^2}, \quad \gamma_i = \{m_0, m_{1/2}, \mu_0, A_0, B_0\}.$$

[J. Ellis et al 1986, R. Barbieri, G. Giudice 1988]

[J. Ellis, P. Chankowski, S. Pokorski, 1997]

- based on physical grounds/intuition. Mathematical support?
- small Δ wanted; what is "acceptable" Δ ? $\leq 100?$, $\leq 500?$ ....
- different definitions for $\Delta \Rightarrow$ different results? ....

$$\Delta_q = \left[\sum_i \Delta_{\gamma_i}^2\right]^{1/2}$$

- local in $\{\gamma_i\}$ space; to compare models, something more global preferable.

- A link to Bayesian evidence $p(D)$?...

- naturalness priors: impose a penalty with priors $p \sim 1/\Delta_{max}$, B.C. Allanach et al hep-ph/0601089, 0904.2548
- change of variables/integrate over $\mu_0 \to M_z$ with $\delta(M_z - M_z^{exp}) \Rightarrow$ effective prior $1/\Delta_\mu$.

M.E. Cabrera et al, 0812.0536

• Fine tuning from likelihood, Bayesian evidence + theor constraints.

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$$\mathcal{P}(\gamma_i|D)=\frac{1}{p(D)}\,\mathcal{L}(D|\gamma_i)\;p(\gamma_i),\qquad p(D)=\int \mathcal{L}(D|\gamma_i)\,p(\gamma_i)d\gamma_i,\quad \gamma_i=\{m_0,m_{1/2},\mu_0,A_0,B_0\}.$$

- theor constraints: $f_1(\gamma_i;v,\beta,y_t,y_b)=f_2(\gamma_i;v,\beta,y_t,y_b)=0,\;\Rightarrow\; v(\gamma_i,...),\tan\beta_0(\gamma_i...)$.

$$\begin{aligned}\text{CMSSM:}\quad p(D)\;&=\;N\int d\gamma_1...d\gamma_5\;p(\gamma_1,...\gamma_5)\,dy_t\;dy_b\;p(y_t)\;p(y_b)\;\;dv\;\;d(\tan\beta)\\ &\times\;\delta(m_Z-m_Z^0)\;\delta(m_t-m_t^0)\;\delta(m_b-m_b^0)\\ &\times\;\delta\left(f_1(\gamma_i;v,\beta,y_t,y_b)\right)\;\delta\left(f_2(\gamma_i;v,\beta,y_t,y_b)\right)\mathcal{L}(D|\gamma_{1,2,...5};\beta,v,y_t,y_b)\end{aligned}$$

$$\Rightarrow p(D)\sim\int dS_\gamma\,\frac{\mathcal{L}(D|\gamma_i)}{\tilde{\Delta}_q}\times(priors)\Big|_{\beta_0(\gamma_i)},\quad \tilde{\Delta}_q(\gamma_i)=\Delta_q\,\frac{\partial f_2}{\partial(\tan\beta)}\Big|_{\beta_0(\gamma_i)},\quad \Delta_q=\Big[\sum_i\Delta_{\gamma_i}^2\Big]^{1/2}$$

$\Rightarrow$ points of small $\tilde{\Delta}_q\sim\Delta_q$ contribute to global "evidence" $p(D)$; so small Δ_q wanted.

$\Rightarrow$ $\Delta_q\sim\tilde{\Delta}_q$ emerges from $p(D)$ due to the 2 theoretical constraints (EW min).

$\Rightarrow$ interesting relation: $\mathcal{P}(\gamma_i|D)\sim\mathcal{L}(D|\gamma_i)/\tilde{\Delta}_q$! (no Yukawa in $\tilde{\Delta}_q$, but in $p(D),\mathcal{P}$).

- Numerical results: Models and boundary conditions:

1) CMSSM (constrained MSSM) $\gamma_i = \{m_0, \mu_0, m_{1/2}, A_0, B_0\}$
2) NUHM1 (non universal Higgs Mass) $\gamma_i = \{m_0, \mu_0, m_{H_1}^{uv} = m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
3) NUHM1 (non universal Higgs Mass) $\gamma_i = \{m_0, \mu_0, m_{H_1}^{uv}, m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
4) NUGM (non universal gaugino masses) $\gamma_i = \{m_0, \mu_0, m_{\lambda_{1,2,3}}, A_0, B_0\}$
5) NUGMd (NUGM with an GUT-like gaugino mass relation) $\gamma_i = \{m_0, \mu_0, m_{1/2}, m_{\lambda_2}, A_0, B_0\}$

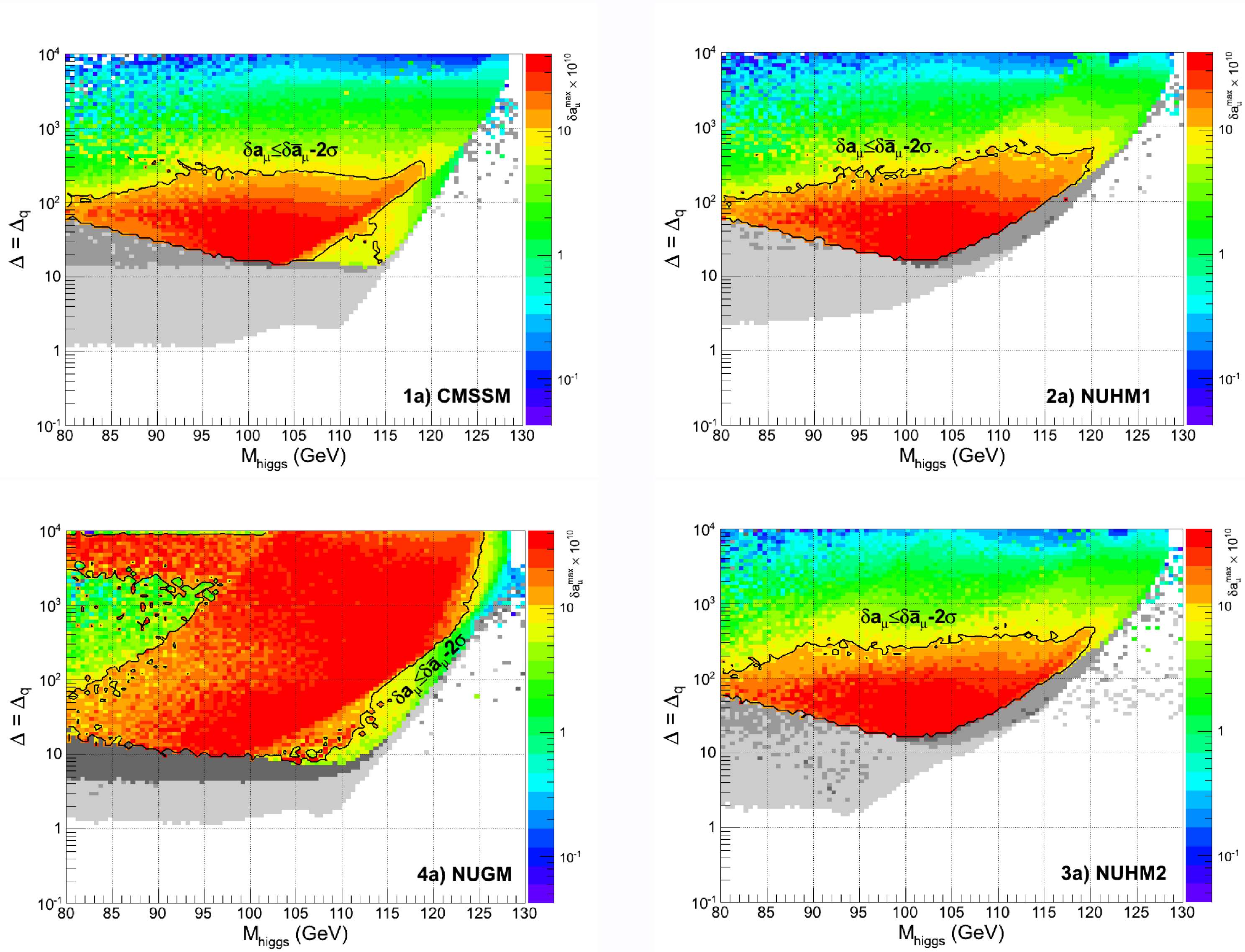
D. Horton, G. Ross arXiv:0908.0857 [hep-ph]

- Experimental Constraints:

- SUSY masses: micrOmegas 2.4.5, "MSSM/masslim.c"
- muon magnetic moment: $\delta a_\mu = (25.5 \pm 2 \times 8) \times 10^{-10}$ at 2σ
- $b \to s\,\gamma$ $3.03 < 10^4$ Br$(b \to s\gamma) < 4.07$ at 2σ
- $B \to \mu^+\mu^-$ Br$(B \to \mu^+\mu^-) < 1.08 \times 10^{-8}$ at 2σ
- ρ-parameter $-0.0007 < \delta\rho < 0.0033$ at 2σ
- dark matter $\Omega h^2 = 0.1099 \pm 3 \times 0.0062$ at 3σ
- CMS: $M_h = 125$ GeV (2.9σ), Atlas: $M_h = 126$ GeV (2.5σ), combined: $122 \leq M_h \leq 129$ GeV.
- δa_μ, M_h not imposed.

- Tools: micrOMEGAs 2.4.5, SoftSUSY 3.2.4. Random scan. $\Rightarrow \Delta_q$, Δ_{max} at 2-loop LL.

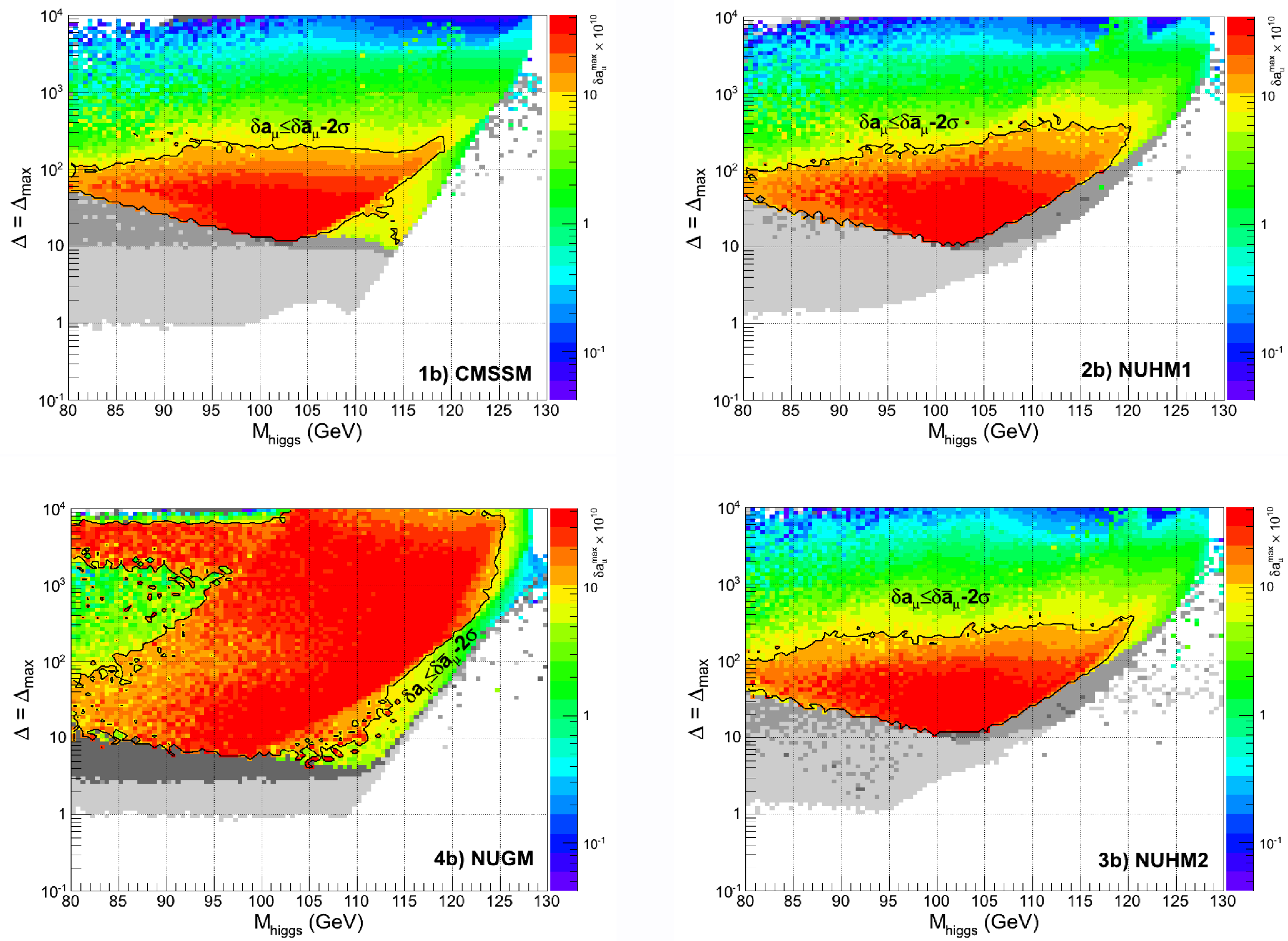
• Impact of M_{higgs} and δa_μ bounds on Δ_q. [2-loop LL, all γ_i, $\tan\beta$]



• grey 0: excluded by SUSY bounds; grey 1: $b \to s\gamma$, $B \to \mu^+\mu^-$, $\delta\rho$; grey 2: excluded by $\delta a_\mu > 0$.

$\Rightarrow$ M_{higgs} strongest impact: $\Delta_q \sim e^{M_{higgs}}$. $\Delta_q \sim 1000$ (!) near 125 GeV. NUGM better behaviour.

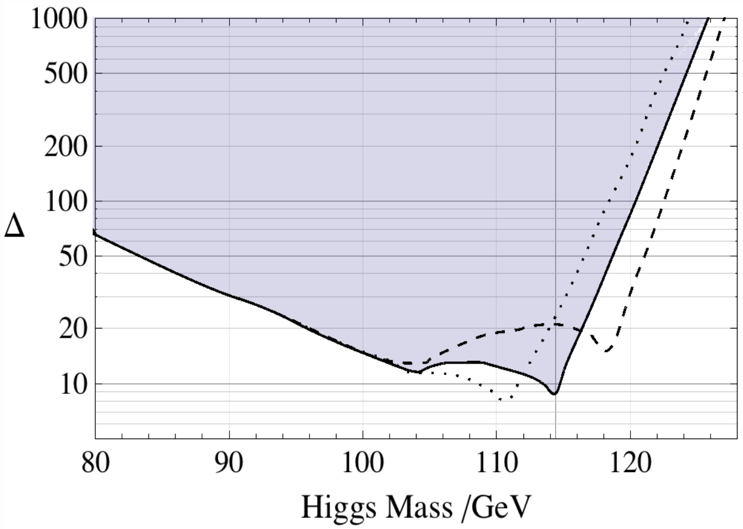
- Impact of M_{higgs} and δa_μ bounds on Δ_{max}. [2-loop LL, all γ_i, $\tan\beta$]

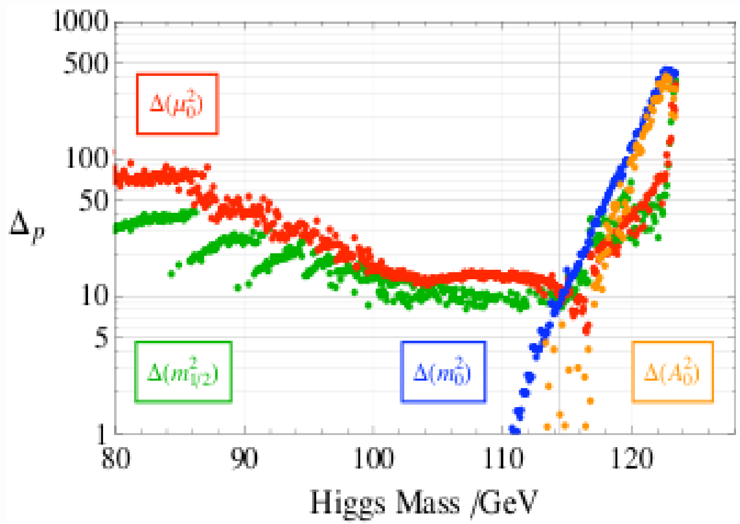


- δa_μ: 2σ contour in red. Smaller δa_μ outside it.

$\Rightarrow$ Δ_{max}: identical behaviour to Δ_q, only marginally smaller.

- CMSSM - impact of 1σ shifts of $\alpha_3(M_z)$, m_{top} on Δ:





$(\alpha_3, m_t) = (0.118, 173.1)$.

$\Delta \equiv \max |\Delta_p|, \quad p = \mu_0^2, m_0^2, A_0^2, B_0^2, m_{1/2}^2,$

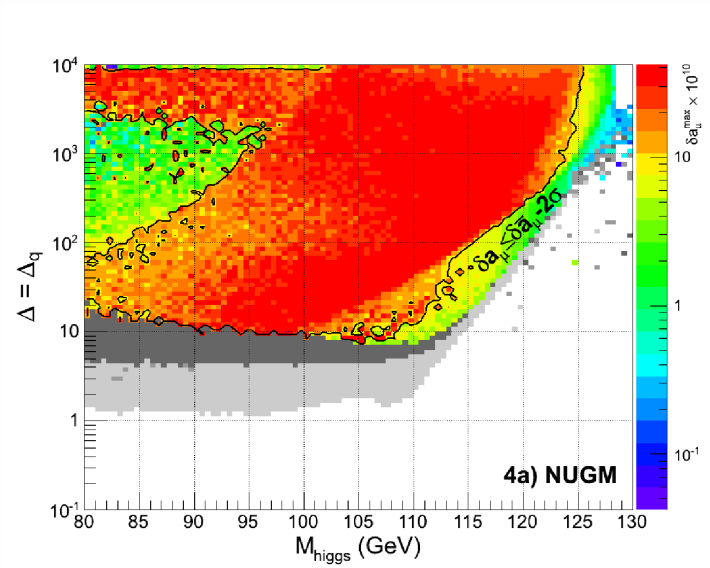
$\Delta_{\mu_0^2}$, $\Delta_{m_0^2}$ dominant (EW vs. QCD)

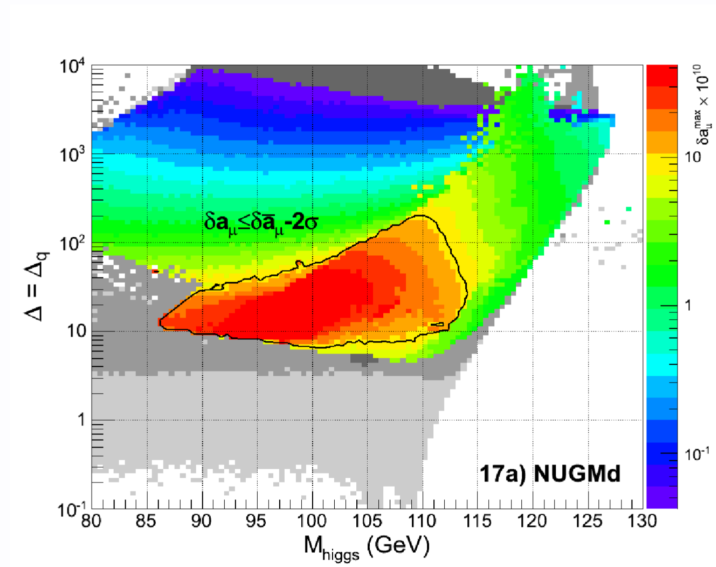
$\Rightarrow$ if $\alpha_3 - 1\sigma$, $m_t + 1\sigma$ $\Rightarrow$ dashed line: reduce Δ by a factor of ≈ 2 (same higgs mass)

$\Rightarrow$ if $\alpha_3 + 1\sigma$, $m_t - 1\sigma$ $\Rightarrow$ dotted line: QCD does not like large m_h (fine-tuning cost)

- Similar impact if using Δ_q. Also expected for NUHM1, NUHM2, NUGM.

• NUGM versus NUGM with GUT-like gaugino mass relation (NUGMd):





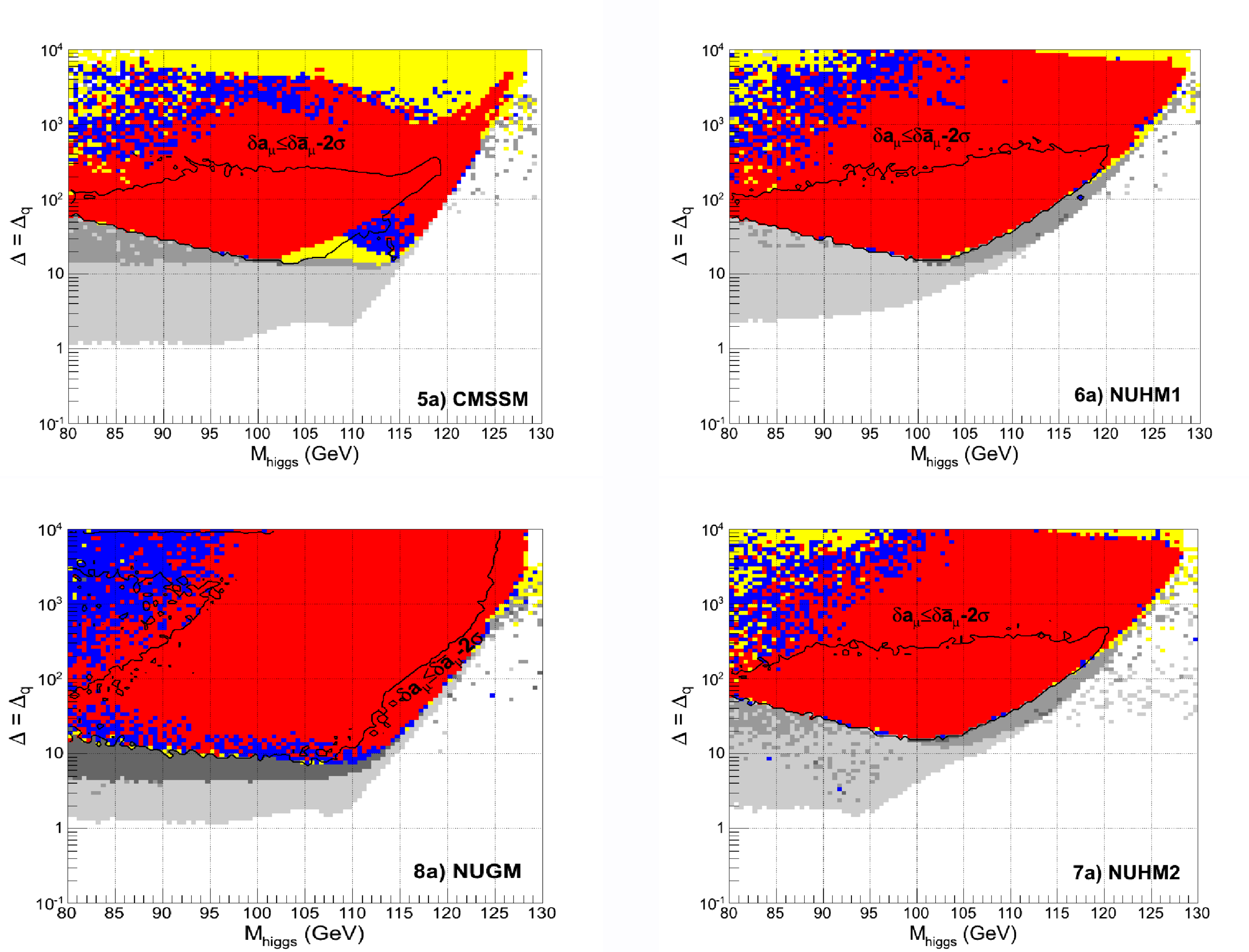
non-univ gaugino mass with a GUT relation:

$$m_{\lambda_1} = 5/3 m_{1/2}, m_{\lambda_2} = m_{1/2}, m_{\lambda_3} = (1/3)\, m_{1/2}$$

Horton, Ross, arXiv:0908.0857 [hep-ph]

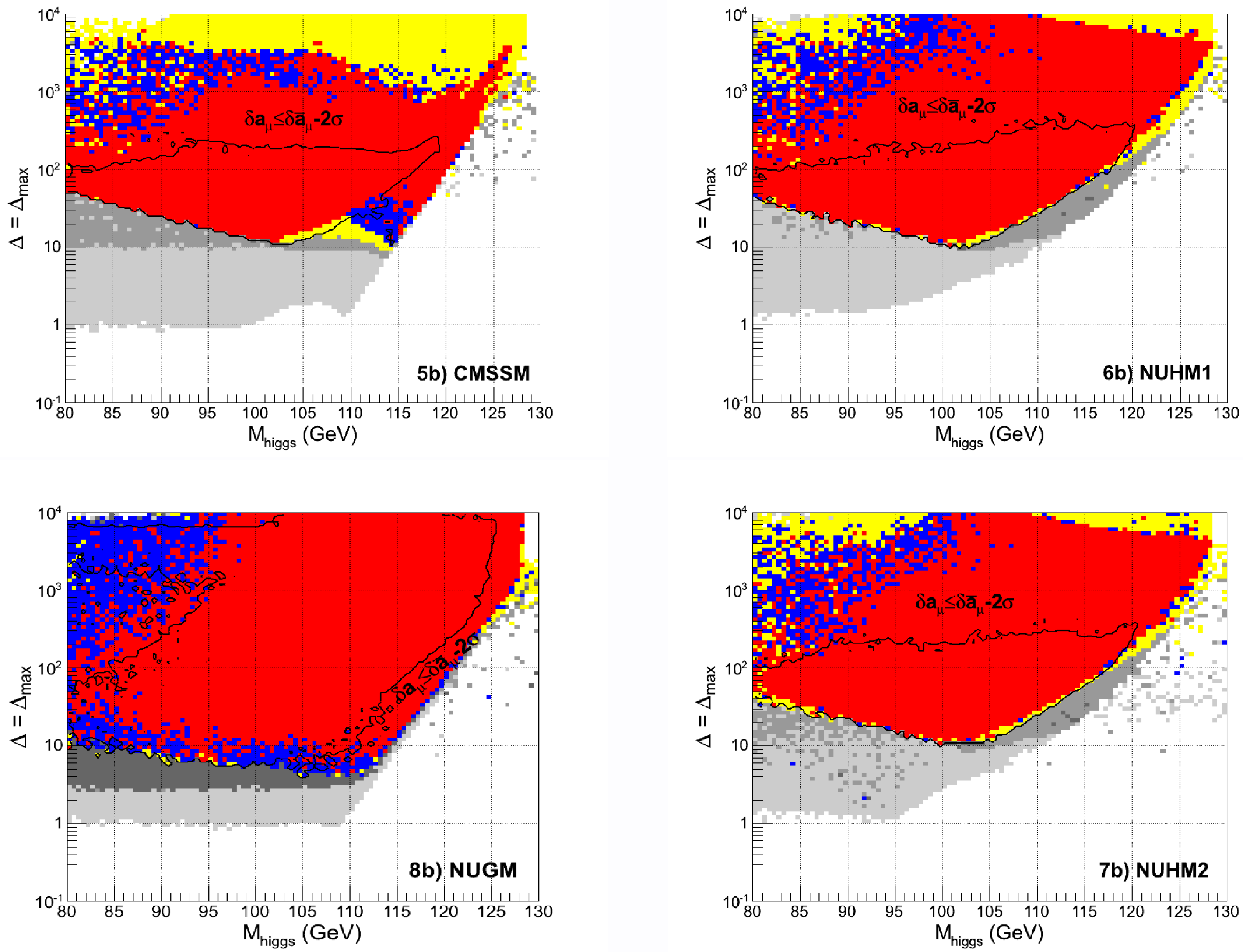
• NUGM better behaviour if δa_μ considered, near 125 GeV for M_{higgs}.

• Impact of M_{higgs} and dark matter bounds on Δ_q: [all γ_i, $\tan\beta$]



• blue: consistent with Ωh^2; Red: saturate Ωh^2 within 3σ. Grey areas as before.

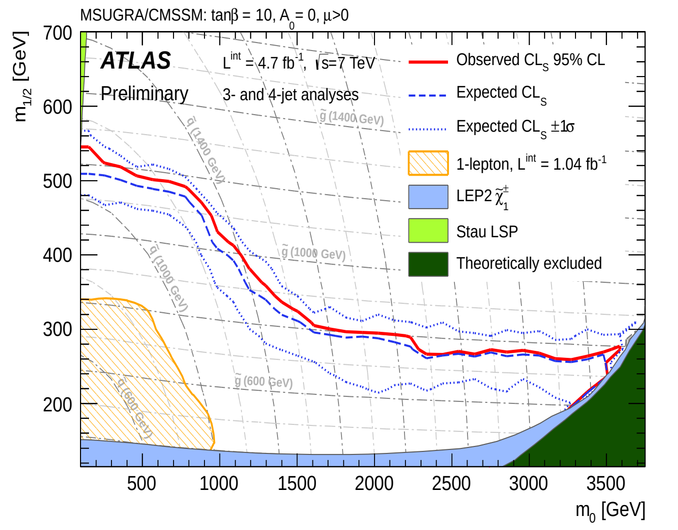
- Impact of M_{higgs} and dark matter bounds on Δ_{max}: [all γ_i, $\tan\beta$]



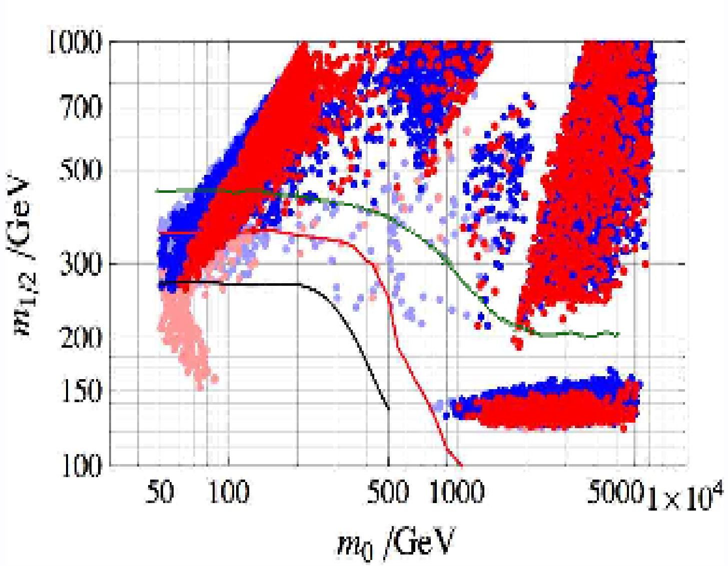
$\Rightarrow$ Δ_{max}: identical behaviour to Δ_q, marginally smaller for same Higgs mass.

- blue: consistent with Ωh^2; Red: saturate Ωh^2 within 3σ. Grey areas as before.

- SUSY searches from ATLAS and impact on parameter space (CMSSM only):



ATLAS-CONF-2012-041

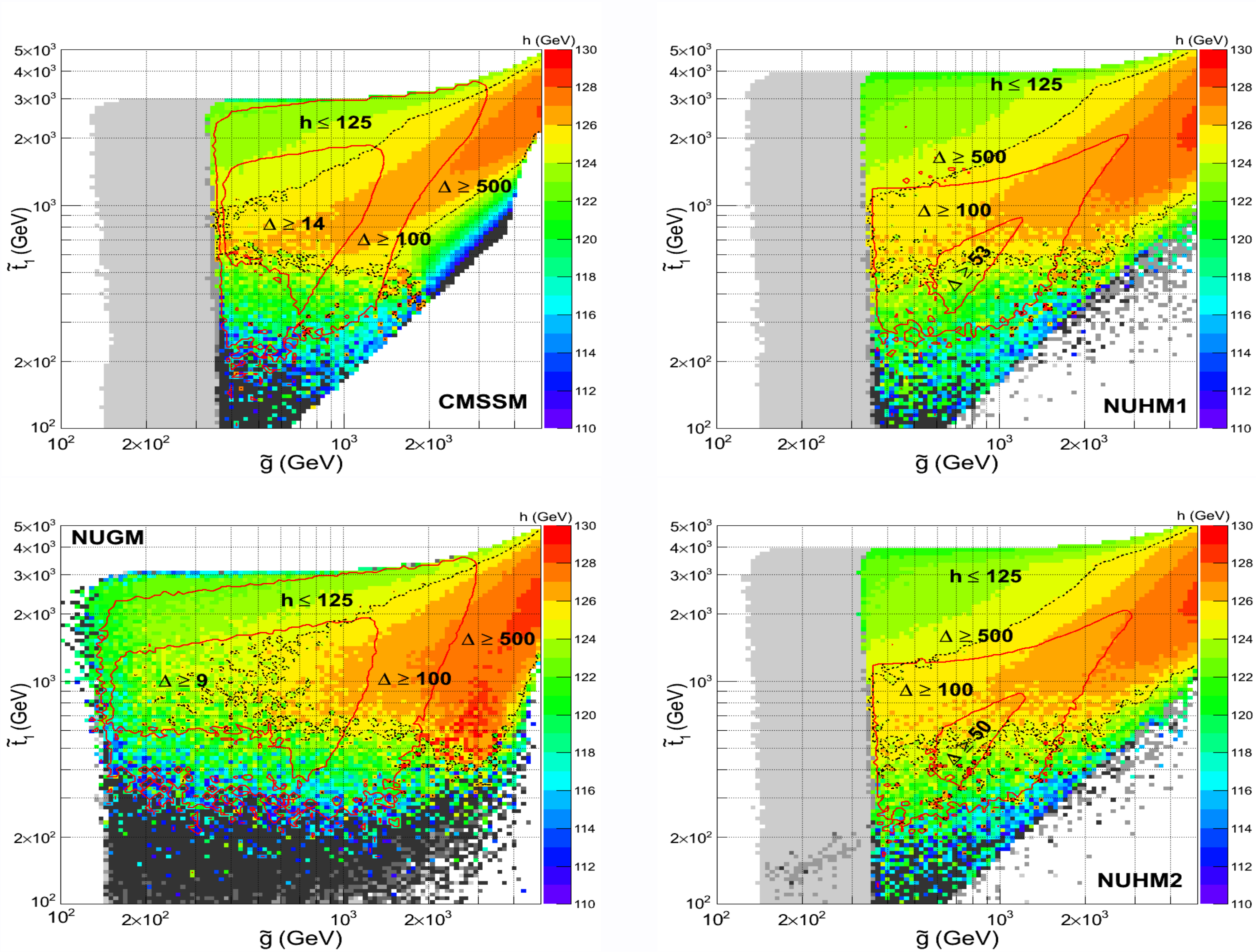


CMSSM, $\Delta < 1000$, $m_{higgs} > 111.5$ GeV.

blue: consistent with Ωh^2. Red: saturate Ωh^2 (3σ).

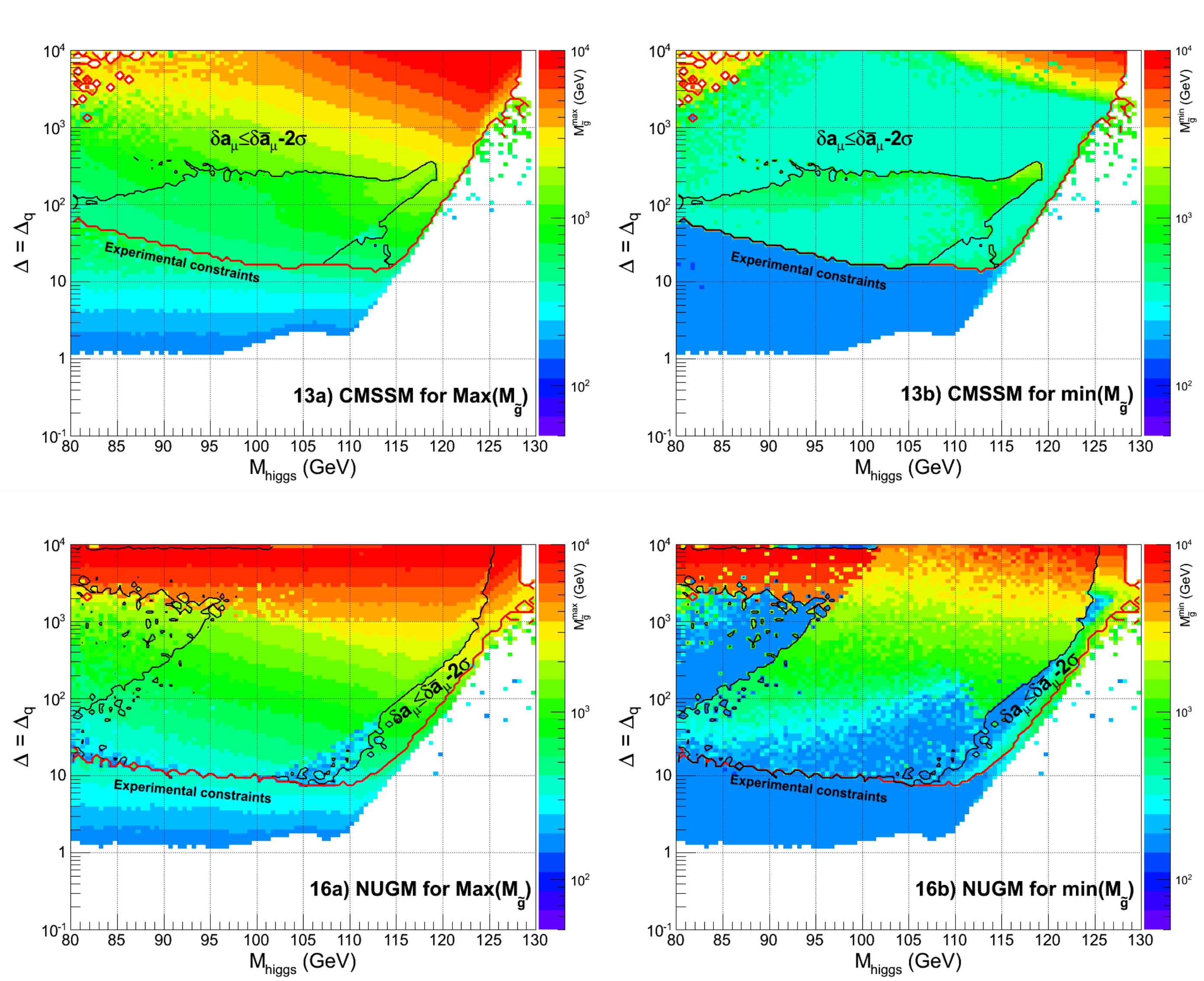
[all γ_i, all $\tan\beta$].

• Stop vs Gluino with largest M_{higgs} and mininal Δ_q [all γ_i, $\tan\beta$]



⇒ excluding $\Delta \geq 100$ or constraints on M_{higgs} strongly reduce the viable regions.

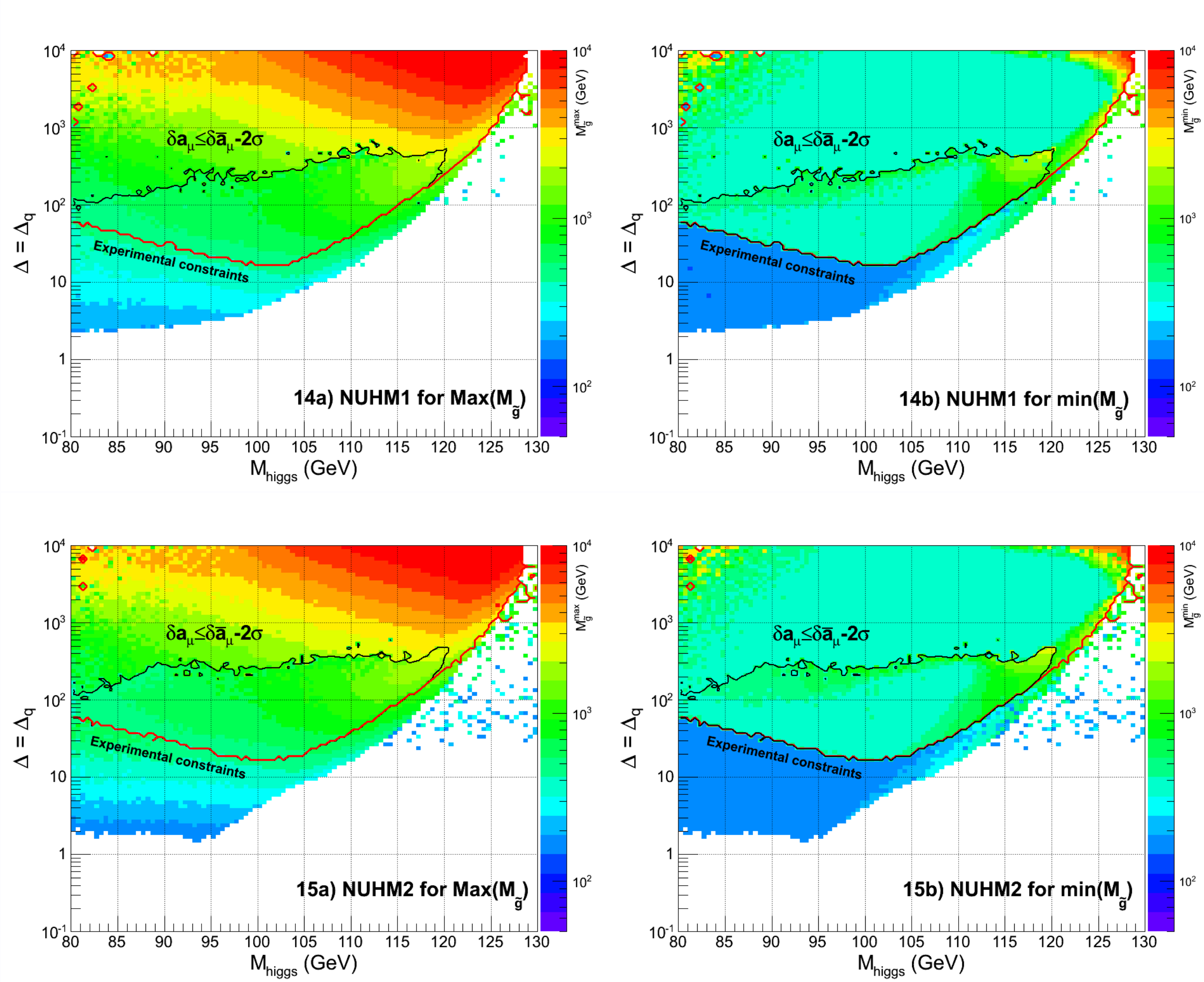
- Fine tuning vs M_{higgs} and gluino mass range, δa_μ:



largest gluino mass   lowest gluino mass.

$\Rightarrow$ gluino mass range smaller near 125 GeV.

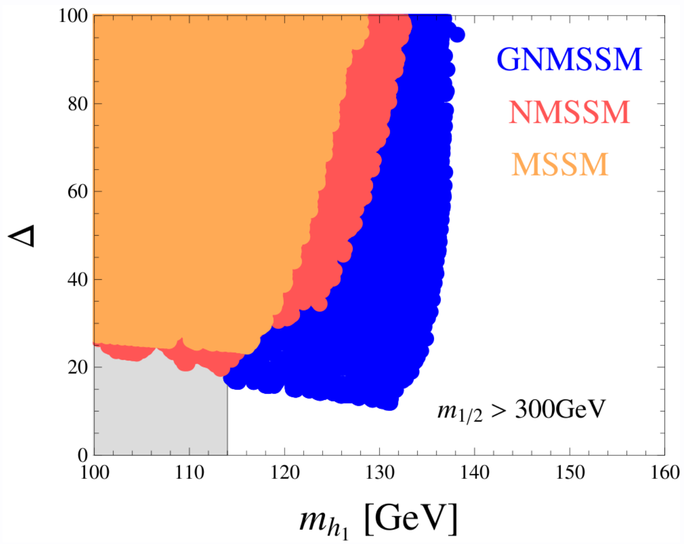
- Fine tuning vs M_{higgs} and gluino mass range, δa_μ:



largest gluino mass

lowest gluino mass.

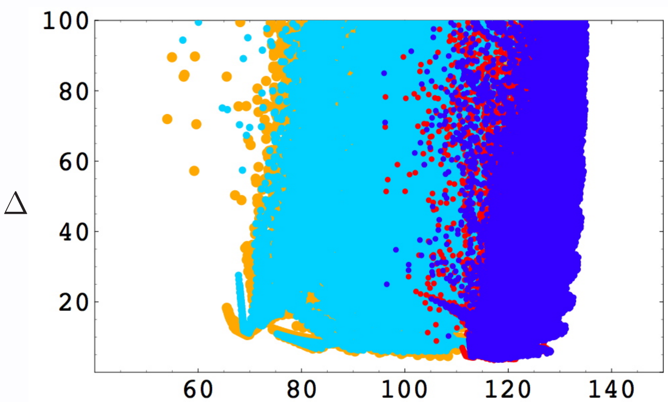
• Fine tuning $\leftrightarrow$ New physics ? NMSSM, GNMSSM, beyond MSSM ....



$$\begin{aligned} \text{NMSSM:} \quad & W = W_Y + \lambda S H_1 H_2 + \kappa S^3 \\ \text{GNMSSM:} \quad & W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2 + \kappa S^3 \\ & \Delta < 20 \ \text{ for } \ m_{higgs} \le 130\, GeV. \end{aligned}$$

G. Ross, K. Schmidt-Hoberg, arXiv:1108.1284

U. Ellwanger et al arXiv:1107.2472



m_{h_1} [GeV]

Decoupling limit: CMSSM with a massive gauge singlet:

$\Rightarrow$ CMSSM + d=5 operator (dark blue):

$$\begin{aligned} W &= W_Y + \mu\, H_1 H_2 + \lambda\, S\, H_1\, H_2 + M_*\, S^2 \\ \Rightarrow W &= W_Y + \mu\, H_1 H_2 + \zeta_0\, (H_1 H_2)^2, \quad \zeta_0 \sim \lambda/M_* \\ & \Delta < 20 \ \text{ for } \ m_{higgs} \le 130\, GeV. \end{aligned}$$

S. Cassel, D.G., G. Ross, arXiv:0903.1115

• Final Remarks:

⇒ Fine tuning $\tilde{\Delta}_q$: mathematical support: $\mathcal{P}(\gamma_i|D) \sim \mathcal{L}(D|\gamma_i)\,/\tilde{\Delta}_q$, Bayesian $p(D) \sim 1/\tilde{\Delta}_q$.

⇒ study fine-tuning in relation to $p(D)$, Bayesian $\mathcal{P}$, likelihood $\mathcal{L}$?

⇒ of all exp data so far: M_{higgs} strongest impact on fine-tuning (exponential).

⇒ δa_μ: very significant impact, if included. NUGM better scenario.

⇒ Δ_q, Δ_{max}: similar, large: ~ 1000 in CMSSM, NUHM1, NUHM2, NUGM for $M_{higgs} \sim 125$ GeV.

⇒ GNMSSM preferable?. Similar to CMSSM + massive singlet (few TeV) decoupled.

⇒ Fine tuning ↔ "new physics" beyond MSSM ? MSSM + d=5 and d=6 operators.

Can increase λ, M_{higgs}, reduce Δ. M_{higgs}, dark matter constrain these operators significantly.

More....→

- Fine tuning $\leftrightarrow$ New physics? beyond MSSM Higgs: d=6 operators:

$$\begin{aligned}
\mathcal{O}_j &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_j\,(H_j^\dagger\,e^{V_j}\,H_j)^2, \qquad j=1,2.\\
\mathcal{O}_3 &= \frac{1}{M_*^2}\int d^4\theta\ \ \mathcal{Z}_3\,(H_1^\dagger\,e^{V_1}\,H_1)\,(H_2^\dagger\,e^{V_2}\,H_2),\\
\mathcal{O}_4 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_4\,(H_2\,H_1)\,(H_2\,H_1)^\dagger,\\
\mathcal{O}_k &= \frac{1}{M_*^2}\int d^4\theta\ \ \mathcal{Z}_k\,(H_k^\dagger\,e^{V_k}\,H_k)\,(H_2\,H_1+h.c.), \quad k=5,6.\\
\mathcal{O}_7 &= \frac{1}{M_*^2}\int d^2\theta\ \frac{1}{16g_i^2}\,\mathcal{Z}_7\,\mathrm{Tr}\,W_i^\alpha\,W_{i,\alpha}\,(H_2\,H_1)+h.c.,
\end{aligned}$$

where

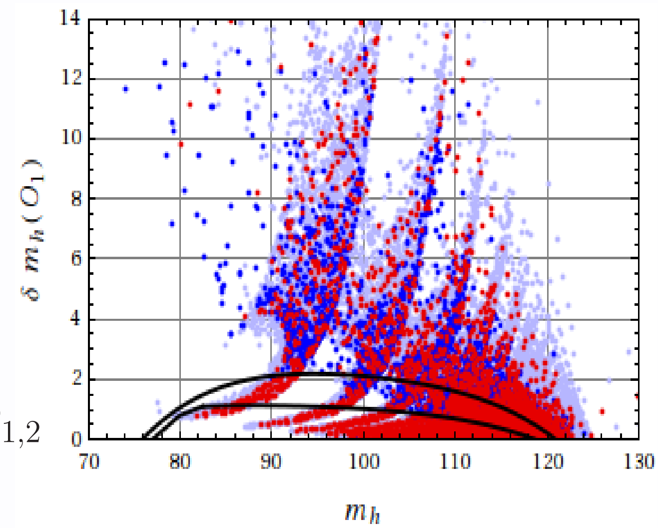
$$(1/M_*^2)\,\mathcal{Z}_j(\mathcal{S},\mathcal{S}^\dagger)=\alpha_{j0}+\alpha_{j1}\,m_0\,\theta\theta+\alpha_{j1}^*\,m_0\,\overline{\theta\theta}+\alpha_{j2}\,m_0^2\,\theta\theta\overline{\theta\theta}, \quad \alpha_{jk}\sim 1/M_*^2,\ \ \mathcal{S}=m_0\,\theta\theta$$

- Also 8 derivative op's eliminated by field redefinitions ($\Rightarrow$ wavefunction ren, soft terms, μ)
- $\mathcal{O}_{1,2}$: generated by extra T, U(1); $\mathcal{O}_3$: T, U(1); $\mathcal{O}_4$: singlet S. $\mathcal{O}_{5,6}$: 2 D of SU(2), singlet S.
- $\mathcal{O}_{1,\dots 7}$ "perturbation" of CMSSM: increase (decrease) M_{higgs}: reduce (worsen) fine-tuning.

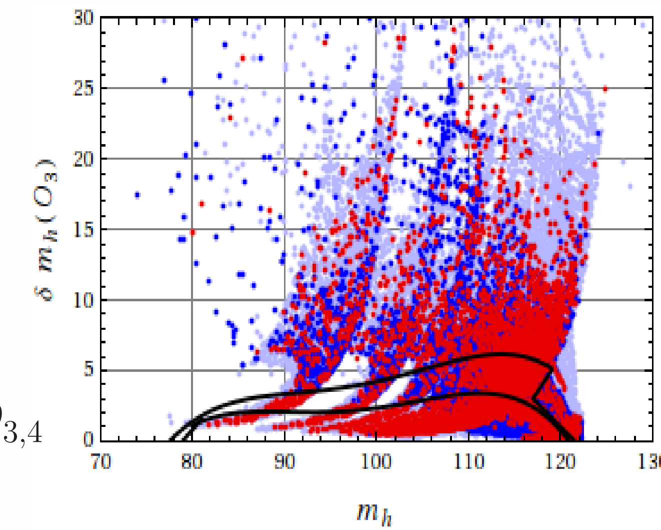
D.G. et al arXiv:1012.5310, 0910.1100, M. Carena et al arXiv:0909.5434, 1005.4887, F. Boudjema et al,1112.1434

- Corrections to m_h from CMSSM + d=6 operators

$\mathcal{O}_{1,2}$



$\mathcal{O}_{3,4}$



$$\delta m_h^2 = -2\,v^2\left[(\alpha_{30}+\alpha_{40})\mu_0^2 - \alpha_{20}\, m_Z^2\right] - \frac{(2\,\zeta_0\,\mu_0)^2\; v^4}{m_A^2 - m_Z^2} + \frac{v^2 \cot\beta}{m_A^2 - m_Z^2}\Big[4\, m_A^2\, \mu_0^2\;(2\alpha_{50}+\alpha_{60})$$
$$- (2\alpha_{60} - 3\alpha_{70})\, m_A^2\, m_Z^2 - (2\alpha_{60}+\alpha_{70})\, m_Z^4\Big] + \mathcal{O}\big(1/(M_*^2 \tan^2\beta)\big)$$

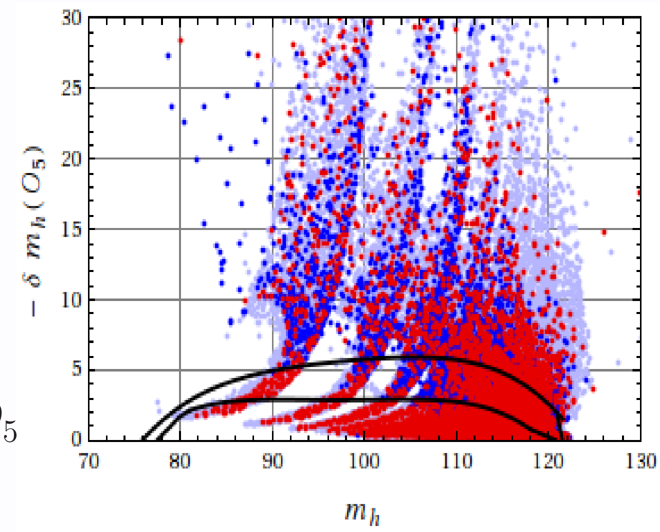
$$\Rightarrow\;\; \delta m_h = \big(m_h^2 + \delta m_h^2\big)^{1/2} - m_h; \qquad m_h : \text{2-loop LL value CMSSM}$$

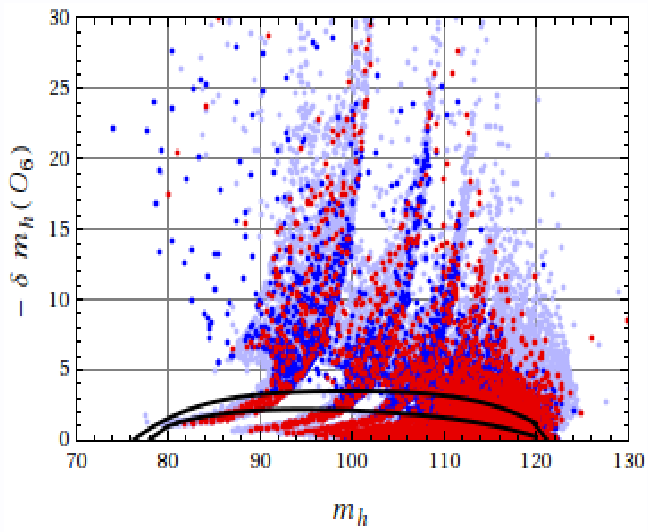
$\Rightarrow$ lower curve: $\Delta < 100$: $m_h < 121$ GeV, $\delta m_h < 4$ GeV. $M_* = 8$ TeV. take: $\alpha_{j0} \leq 1/4$

$\Rightarrow$ top curve: $\Delta < 200$: $m_h < 122$ GeV, $\delta m_h < 6$ GeV. $\pm 1\,\mathrm{GeV}\;(\delta m_h) \leftrightarrow \mp 1$ TeV (δM_*).

D.G. et al arXiv:1012.5310, 0910.1100, M. Carena et al arXiv:0909.5434, 1005.4887, F. Boudjema et al,1112.1434

- Corrections to m_h from CMSSM + d=6 operators

$\mathcal{O}_5$ 

$\mathcal{O}_6$ 

- constraint: $\rho - 1 = -v^2 \big[\alpha_{10} \cos^4\beta + \alpha_{20} \sin^4\beta - \alpha_{30} \sin^2\beta \cos^2\beta\big] + \mathcal{O}(v^4/M_*^4)$, $M_* \sim 8$ TeV

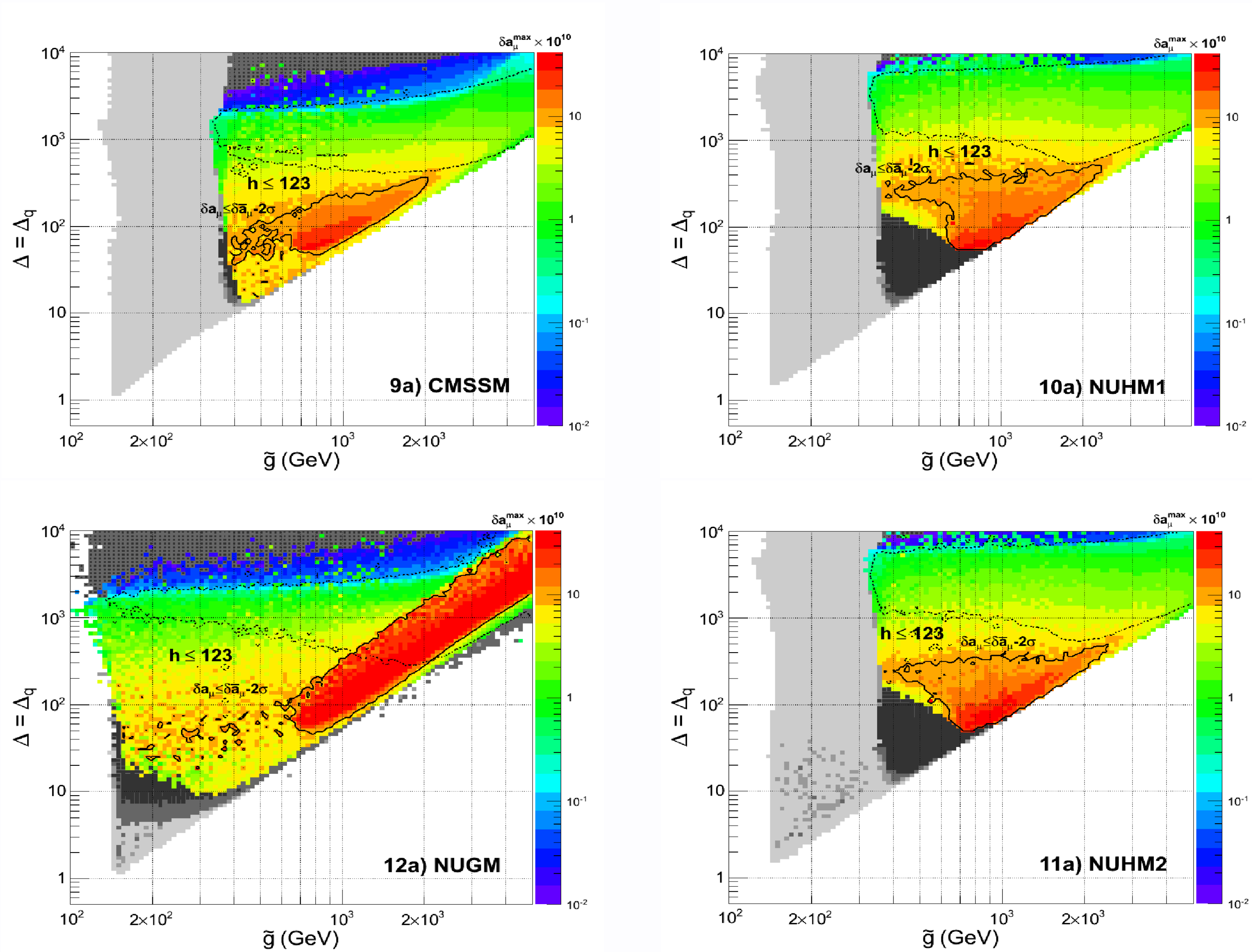
- large $\tan\beta$: larger α_{30}, α_{10} allowed $\Rightarrow$ α_{30}, α_{40} largest correction to m_h

$$\mathcal{O}_3 \sim \alpha_{30} \int d^4\theta \ \ (H_1^\dagger \, e^{V_1} \, H_1) \, (H_2^\dagger \, e^{V_2} \, H_2), \qquad \mathcal{O}_4 \sim \alpha_{40} \int d^4\theta \ \ (H_2 \, H_1) \, (H_2 \, H_1)^\dagger,$$

- difficult to generate α_{30}, α_{40} with the "right" sign (by integrating massive $T, U(1), S$ in ren model)

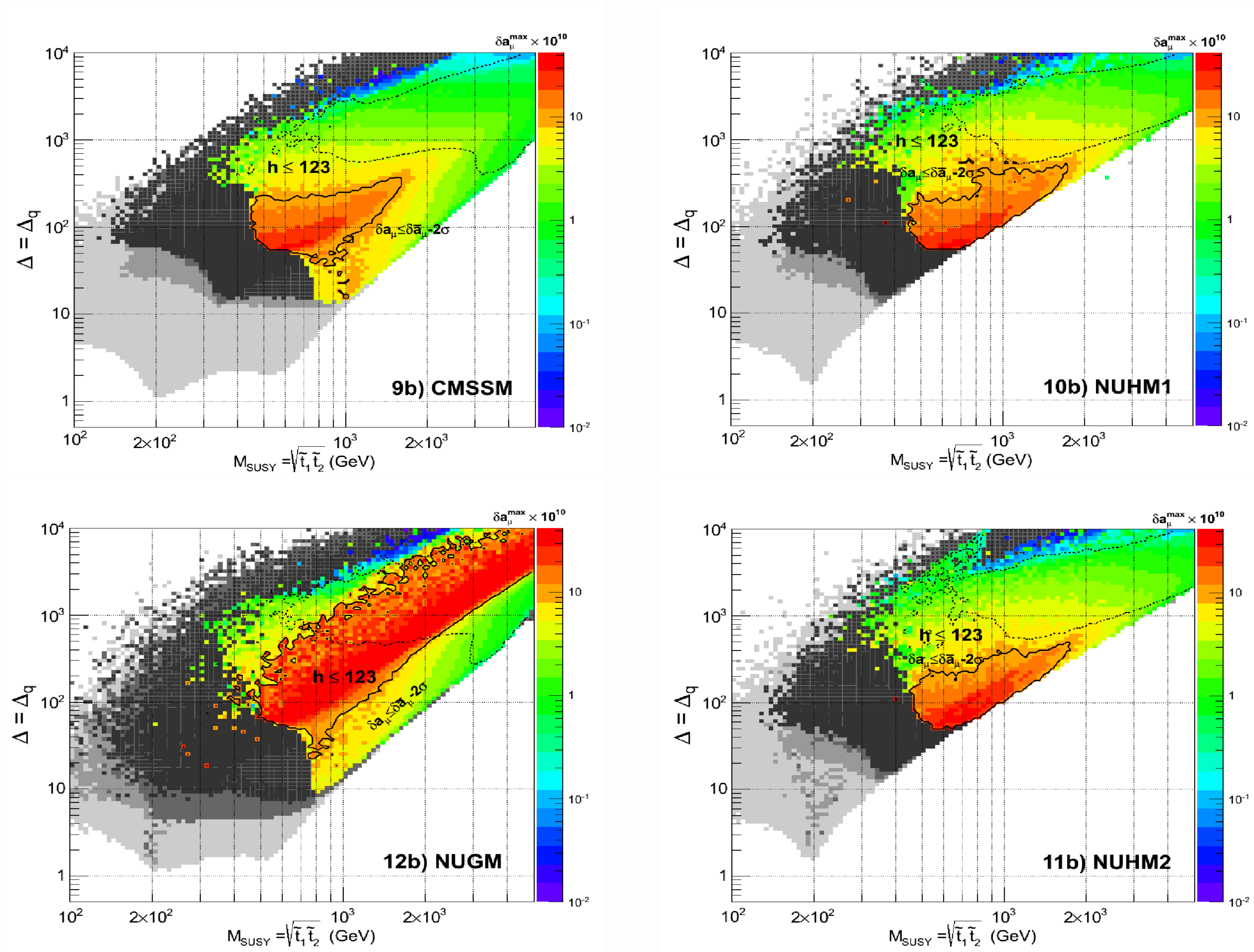
- impact on fine tuning measure: $\Delta_{new} \sim \Delta_{old} \ m_h^2/(m_h + \delta m_h)^2$

- Δ_q versus gluino mass and δa_μ:



[all γ_i, $\tan\beta$ allowed]

- Δ_q versus SUSY scale $M_{SUSY} = (m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2}$ with δa_μ:



$\Rightarrow$ excluding $M_{higgs} < 123$ GeV has a strong impact. [all γ_i, $\tan\beta$ allowed]