Energy Calibration at the LHC

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Abstract

The TOTEM experiment at the LHC has requested a momentum determination with an accuracy of 0.05% at 7 TeV/c to minimize the contribution of the beam momentum uncertainty to the error on the total cross section measurement. This document discusses briefly possible calibration methods and their expected accuracy. The method based on the comparison of revolution frequencies of ion and proton beams is the most promising way to calibrate the LHC momentum, and it is explained in more details. First results from beam measurements using the Pb ion beams are presented.

INTRODUCTION

One of the aims of the TOTEM experiment [1] is the precise measurement of the total p-p cross-section that can later be used for luminosity measurements. The measurement principle relies on the optical theorem that relates the total cross-section to the imaginary part of the forward scattering amplitude. The total cross-section measurement of TOTEM depends quadratically on the beam momentum

$$\sigma_{tot} \propto \frac{1}{P^2} \tag{1}$$

through the measurement of the angular dependence of the electric scattering rates at small angles. The momentum error contributes to the final error on the cross-section measurement as $\frac{\Delta\sigma_{tot}}{\sigma_{tot}} = 2\frac{\Delta P}{P}$. To minimize the contribution of the momentum uncertainty on the cross-section measurement, TOTEM requires an uncertainty $\frac{\Delta P}{P} < 5 \cdot 10^{-4}$.

The 12 year long energy calibration program of LEP was extremely successful in providing accurate beam energies between 40 and 100 GeV/c [2, 3]. Although resonant depolarization, the workhorse of LEP energy calibration, is not available at the LHC, the experience gained on LEP is also relevant for LHC energy calibration. In particular the studies on the ring circumference variations are important in the context of the LHC.

This note first describes briefly the main ingredients to the machine energy, more details may be found in an earlier study performed before the LHC startup [4]. The calibration method based on the comparison of proton and ion beams is discussed, including first results obtained parasitically during proton and Pb ion operation in 2010 and 2011.

BEAM MOMENTUM AND FIELDS

In a storage ring the average beam momentum P is defined by the integral of the bending field B on the closed orbit

$$P = \frac{Ze}{2\pi} \oint B(s)\, ds \ , \tag{2}$$

where Ze is the total particle charge. s is the longitudinal position along the beam orbit. The contributions to the beam momentum can be decomposed into

$$P = P_d + P_q + P_\epsilon \tag{3}$$

where P_d and P_q are respectively the contributions of the dipoles and the quadrupoles to the field integral. Other elements (for example horizontal correctors) can give additional small contributions P_ϵ to the momentum. P_d depends on the integrated dipole field $(BL)_d$ and accounts usually for almost 100% of the beam energy since the dipoles define the nominal momentum. P_q depends on the orbit length C through

$$P_q = -\frac{1}{\alpha}\frac{C - C_c}{C} \ . \tag{4}$$

It is a function of the momentum compaction factor α, $\alpha \simeq 3.3 \cdot 10^{-4}$ for the LHC, of the actual orbit length (circumference) C and of the length of the central orbit C_c (the machine circumference). On the central orbit the beam is by definition centered on average in the quadrupoles and P_q vanishes. In general P_q does not account for more than few permill of the bending field integral.

The speed βc of a beam particle is related to the revolution frequency f_{rev} and to the RF frequency f_{RF},

$$\beta c = C f_{rev} = \frac{C f_{RF}}{h} \tag{5}$$

where h is the harmonic number of the RF system. c is the speed of light. For the LHC the nominal RF frequency is 400.87 MHz and $h = 35640$. The central RF frequency f^c_{RF} is defined with Eq. 5,

$$f^c_{RF} = \frac{h\beta c}{C_c} \ . \tag{6}$$

For a perfectly aligned machine the definition of the central frequency f^c_{RF} (and of the central orbit length) is unambiguous. It corresponds to the RF frequency (or orbit length) for which the beam is centered in *all* quadrupoles. In a real machine with misaligned magnets the beam is travelling on a closed orbit that is not centered in each quadrupole. In such a case the central frequency corresponds to the RF frequency for which the beam is centered *on average* in the quadrupoles. Methods the measure the central frequency are described in Ref. [2, 3, 6]

MAGNETIC MEASUREMENTS

The simplest way to estimate the beam momentum is to derive it from the magnetic calibration curves of the dipoles (also referred to as excitation curves or transfer functions). The same information is also used to define the machine settings for a given energy, where the desired momentum is translated into currents for the main dipoles. A sophisticated magnetic model has been developed for the LHC in order to predict as precisely as possible the field errors to speed up the machine commissioning and improve the machine turn-around time [5]. For the dipole field (b_1), the estimated accuracy of the model is 0.1% for absolute predictions and 0.01-0.1% (1-10 unit) for relative predictions (for example for the difference between 450 GeV and 4-7 TeV). Those values are close to the requirements, but a cross check with beam of the predictions is highly desirable.

BEAM MEASUREMENTS

Resonant depolarization

For electron machines resonant depolarization provides a high precision measurement technique which relies on the precession frequency of the electron spins in the magnetic lattice of the storage ring. Electron beams polarize spontaneously in a ring up to a theoretical limit of 92% due to synchro radiation, and the spin precession frequency is proportional to the energy [2, 3]. Unfortunately this technique does not work for proton beams as the polarization time at the LHC is too long.

RF frequency of different ions

For beam that are *not too relativistic*, RF (or revolution) frequency measurements of two different ion species provide a means of accurate momentum measurements for hadrons [6]. The measurement technique works only if the frequency measurements are performed under exactly the same field conditions. This technique is the only option for precise calibration with beams that we have with the present LHC, it will be described in the following sections.

Spectrometers

Momentum measurements using spectrometer systems require a precisely calibrated and monitored dipole magnet that has to fit into the machine lattice. Open drift space of $\sim$ 10 m is required on either side of the magnet to determine the angles with beam position monitors (BPMs). The spectrometer system usually requires precise cross calibrations at some energies, and is mainly used for extrapolation to the desired target energy. The difficulty at the LHC is too find a suitable location, as for practical reason the spectrometer magnet should be normal-conducting to ease the installation and maintenance of precision instrumentation to monitor the magnetic field (for example Nuclear Magnetic Resonance probes).

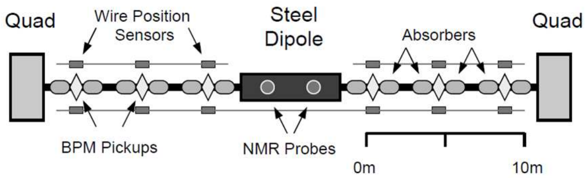

Figure 1: Schematic layout of the LEP spectrometer system.

One of the three energy calibration techniques used at LEP2 for high beam energies consisted of such a magnetic spectrometer [3]. The core of the spectrometer was a dedicated, individually powered dipole magnet with a very accurate calibration. This dipole was surrounded on either side by 3 dedicated BPMs installed in a 20 meter long field-free region, se Fig. 1. The BPMs were and surveyed by a wire positioning system and provided a precise relative beam angle measurements on both sides of the magnet. The BPM and alignment system resolutions were in the range of 1 μm. The magnet was equipped with in situ Nuclear Magnetic Resonance probes to survey the field.

ENERGY CALIBRATION WITH ION BEAMS

This precise absolute momentum calibration method takes advantage of the fact that the revolution frequency (and therefore f^c_{RF}) is different for ions and protons due to the different ratio of charge over rest mass. Using this technique, a precise energy calibration was performed at LEP with protons and positrons at 20 GeV/c [7]. In 2002 the LHC beam momentum was calibrated in the SPS at 450 GeV/c using proton and Pb^{53+} beams [6].

The speed βc of a particle is related to the revolution frequency f_{rev} and the RF frequency f_{RF} by Eq. 5. To determine β and therefore the particle momentum, both the machine circumference and the revolution frequency must be known at the same time. If the nominal machine momentum and the circumference are stable (within a certain tolerance) over a sufficiently long time interval, it is possible to determine those parameters by measuring the revolution frequency for two beam of particles with different charge over mass ratio. The two species must be injected successively into the same magnetic machine and on the same orbit.

The speed $\beta_p c$ of the proton beam is related to its momentum P and its rest mass m_p by the well known relation

$$\beta_p^2 = \frac{P^2}{P^2 + (m_p c)^2} \ . \tag{7}$$

An ion with charge Ze, injected into the same magnetic machine and on the same orbit than the proton beam has a momentum $P_i = ZP$. The speed $\beta_i c$ of the ions is

$$\beta_i^2 = \frac{P^2}{P^2 + (m_i c/Z)^2} \tag{8}$$

with m_i the ion rest mass. The two equations can be solved for the proton beam momentum P, yielding

$$P = m_p c \sqrt{\frac{\kappa^2 \mu^2 - 1}{1 - \kappa^2}} \tag{9}$$

with

$$\kappa = \beta_i / \beta_p = f^i_{RF} / f^p_{RF} \tag{10}$$

and

$$\mu = \frac{m_i}{Z m_p} \; . \tag{11}$$

μ is the number of nucleons per charge of the ion, $\mu \simeq 2.5$ for fully stripped Pb^{82+} Lead ions. Eq. 9 can be approximated by

$$P \cong m_p c \sqrt{\frac{f^p_{RF}}{2\Delta f}(\mu^2 - 1)} \tag{12}$$

where $\Delta f = f^p_{RF} - f^i_{RF}$ is the RF frequency difference between the proton and ion beams.

The measurement error on P is dominated by the accuracy of the RF frequency determination since all other parameters entering Eq. 9 and 12 are known with high accuracy. The measurement error σ_P on P is dominated by the term

$$\frac{\sigma_P}{P} \simeq \frac{\sqrt{\sigma^2_{f^p_{RF}} + \sigma^2_{f^i_{RF}}}}{2\,\Delta f} \tag{13}$$

with $\sigma_{f^p_{RF}}$ and $\sigma_{f^i_{RF}}$ the measurement errors on the central RF frequencies of the proton and ion beams. For highly relativistic beams, the difference between β_p and β_i decreases rapidly. The frequency difference Δf between the beams follows from Eq. 12,

$$\Delta f \cong \left(\frac{m_p c}{P}\right)^2 \frac{f^p_{RF}}{2}(\mu^2 - 1) \; . \tag{14}$$

The dependence on $1/P^2$ makes the measurement very difficult at the highest energies. As the speeds of both beams approach c the differences tend to vanish.

Calibration at the LHC

The frequency difference between protons and Pb^{82+} Lead ions at LHC is shown in Fig. 2. Due to the dependence on $1/P^2$ the frequency difference shrinks by more than 2 orders of magnitude between 450 GeV/c and 7 TeV/c, and Δf becomes extremely small. for a momentum determination at 7 TeV/c with a relative accuracy of 10^{-3}, the central frequency difference must be determined to 20 mHz, which corresponds to a relative accuracy on the circumference of 4×10^{-11} or to an absolute accuracy of 1 μm on the LHC circumference. Such values seem at present beyond the experimental possibilities (systematic errors), in particular in the light of the LEP experience [2, 3] and given the temperature issues affecting the LHC BPM system.

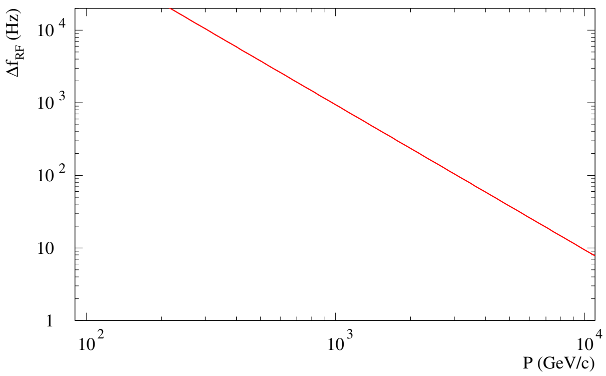

Figure 2: Expected central RF frequency difference between proton and Pb^{82+} beams as a function of the beam momentum at the LHC. At 7 TeV/c the frequency difference is only $\sim$ 20 Hz.

Due to the slow LHC machine cycle a measurement of the frequency with protons and Pb ions in the same ring will cover a time span of at least 3 hours and include two distinct ramps from 450 GeV/c to 4-7 TeV/c. The difficulty of the measurement is due to the systematic changes of the machine circumference on the time scale of a few hours - they clearly exceed the required accuracy for 7 TeV/c.

The experience gained on the stability of the LEP machine and tunnel applies directly to the LHC. The LEP circumference was submitted to changes due to periodic and aperiodic strains. The most commonly known periodic effect is the influence of Earth Tides that changes the circumference at high tides by 1 mm over 12 hours [8, 2, 3]. The tidal effects are modeled and experimentally verified to an accuracy of $\sim$ 5% from the LEP data. In addition the LEP circumference was subject to circumference variations related apparently to the underground water conditions (water table height under the Jura). Figure 3 shows the long term evolution of the LEP central RF frequency in 1999. The circumference was measured in every fill using the radial beam position obtained from 240 arc BPMs that were cross-calibrated against direct central frequency measurements. The evolution of f^c_{RF} was similar for every year. First the central frequency decreases (and the circumference increases) between spring and summer and increases again in the autumn. The "spikes" where the frequency increases rather sharply before it relaxes again follow periods of heavy rainfall.

Both at the SPS and at LEP, central frequency measurements were limited in accuracy to $\pm$ 1 Hz (or 100 μm on the circumference) due to instrumental effects. It has to be demonstrated that those accuracy can be reduced below

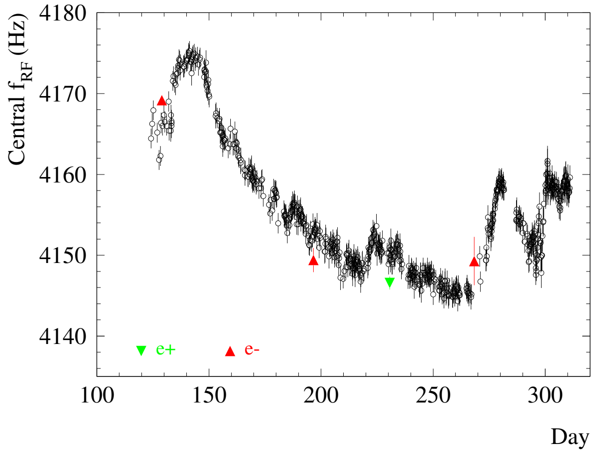

Figure 3: Evolution of the LEP central RF frequency during the 1999 LEP run [3, 4]. The open points are obtained from BPM measurements and are normalized to the actual central frequency measurements (filled triangles). Only the last four digits of the central frequency are given, f^c_{RF} = 352'25.'... Hz. $\Delta f = +1$ Hz corresponds to $\Delta C = -100$ μm.

0.1 Hz.

For a precise measurement of the energy the changes of the LHC circumference due to geological effects must be known with an accuracy much better than $\simeq 1 \mu$m on the circumference ($\simeq$ 0.01 Hz on the central RF frequency), since the data must be corrected for those effects.

The new mixed mode of operation with proton and Lead beams opens the possibility to determine the central frequencies for both beams at the same time (but not for the same ring!), reducing drastically the errors induced by corrections of the time dependence of the circumference of the LHC. The only residual error is due to differences between the rings. Since the new mode offers the possibility to swap the beams between the two rings, this may be used to further reduce the systematic errors. The uncertainties could be assessed more precisely by beam tests at injection in the fall of 2012 during the first mixed p-Pb running period.

FIRST LHC MEASUREMENTS

Injection

A momentum estimate for the beams at injection has been obtained from the 2010 and 2011 Pb ion runs. Because no dedicated time was devoted to the calibration, the information has been extracted from parasitic measurements, but with limited accuracy of ± 10 Hz on the RF frequency difference between Pb and p beams. The measured frequency differences were $\Delta f = 4652$ Hz in 2010 and $\Delta f = 4638$ Hz in 2011. The corresponding beam energies are given in Table 1. At injection it is also possible to obtain a cross-calibration from the SPS by observing the energy offset of the beam injected from the SPS on the first turn. The corresponding energy is consistent with the estimates from Pb-p comparisons and is also given in Table 1. The average momentum is 450.3 ± 0.2 GeV/c, very close to the nominal momentum (error of less than 10 unit $= 10^{-3}$ relative field error).

Table 1: Momentum estimates for the LHC beam at injection.

Source	P (GeV/c)
Pb-p calibration, 2010 run	449.90 ± 0.35
Pb-p calibration, 2011 run	450.58 ± 0.35
SPS-LHC cross-calibration	450.35 ± 0.25
Average	450.3 ± 0.2

Top energy

The only combined ramp with Pb and p beams performed in October 2011 provides a rough calibration at 3.5 TeV based on the radial offsets of the two beam orbits. The resulting momentum is 3.47 ± 0.10 TeV/c. If on the other hand the results obtained at injection are extrapolated assuming an extrapolation error of 10 units ($= 10^{-3}$) one obtains 3.50 ± 0.02 TeV/c. It must however be noted that the error on the extrapolation requires further studies.

DISCUSSION AND OUTLOOK

A momentum calibration entirely based on the knowledge of the LHC dipole field model is expected to provide a momentum uncertainty at the level of 0.1% at 7 TeV/c. Such a method relies however on the long term stability of the magnetic field in the dipoles, and an independent and direct momentum measurement of the beam may be desired as a cross-check.

A first calibration based on proton and Pb beams has been obtained at injection using data acquired parasitically during proton and Pb beam operation. The accuracy is at the level of 0.05%, it could be improved further with dedicated studies. The calibration at injection has then to be extrapolated to higher energies.

The new mixed mode of operation with proton and Lead beams opens the possibility to determine the central frequencies for both beams at the same time, offering possibly a drastic reduction of the systematic errors. The uncertainties could be assessed more precisely by beam tests at injection in the fall of 2012 during the first mixed p-Pb running period.

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