Beam dynamics for cyclotrons

Frédéric Chautard

GANIL, Caen, France

OUTLINE

Chapter 1
- History
- Cyclotron review
- Transverse dynamics

Chapter 2
- Longitudinal dynamics
- Acceleration
- Injection \ Extraction

Chapter 3
- RF modelisation & Computation
- B modelisation & Computation
- Beam transport computation

Chapter 4
- Beam Diagnostics
- Cyclotron as a mass separator
- Few cyclotrons examples

Avant-propos

- Fortunately, the beam dynamics in cyclotrons obeys to the same laws than for the other accelerators.
- The courses from Ph. Bryant, A. Lombardi etc … are obviously to keep entirely and to be applied to the cyclotron dynamics.
- In this following, I will admit the previous lessons as understood and will attach more importance to the application of the formalism (focalisation, stability, acceleration …) to the cyclotron case.

CYCLOTRON HISTORY

In 1919 *Lord Ernest Rutherford* (1871 - 1937) discovered that nitrogen can be brought to emit protons by bombardment with alpha particles, according to the nuclear-reaction equation:

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

This discovery meant the initiation of a new era in natural sciences.

How then would it be possible, by some method other than the use of radioactive substances, to make available projectiles with sufficient energies to bring about nuclear reactions ?

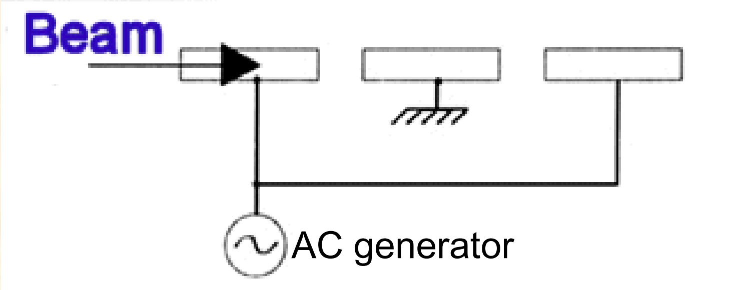
In this case, use was made of a high electrical voltage, up to about 600 kV, to accelerate protons which, upon bombarding lithium, caused a nuclear reaction:

$$^{1}_{1}\mathrm{H} + ^{7}_{3}\mathrm{Li} \rightarrow 2 \times ^{4}_{2}\mathrm{He}$$

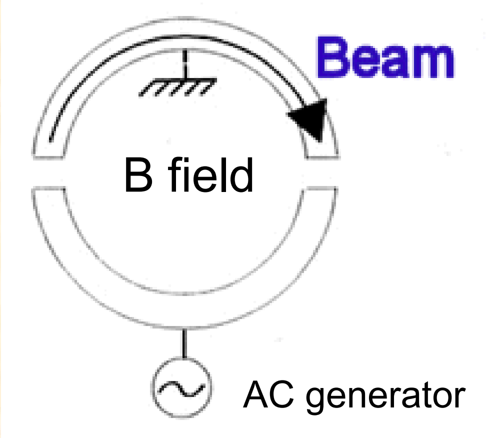
IMPROVEMENTS:


- **Idea 1**: Large potential difference (Cockroft-Walton, Van de Graaf). But high voltage limit around 1.5 MV (Breakdown)

- **Idea 2**: Linacs (Wideröe). Successive drift tubes with alternative potential (sinusoidal). Large dimensions



- **Idea 3**: Another brilliant idea (E. Lawrence, Berkeley, 1929). The device is put into a magnetic field, curving the ion trajectories and only one electrode is used several times.

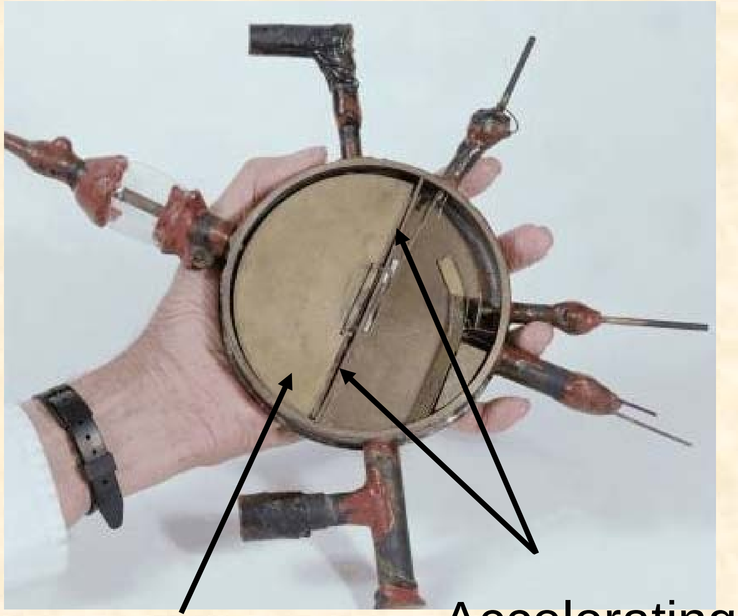


FIRST EXPERIMENTAL DEVICE

- A circular copper box is cut along the diameter:
  - A half is at the ground
  - the other « Dee » is connected to an AC generator.

4 inch first cyclotron

- Insert all under vacuum (the first vessel was in glass) and slip it into the magnet gap



- At the centre, a heated filament ionises an injected gas: This is the ion source.

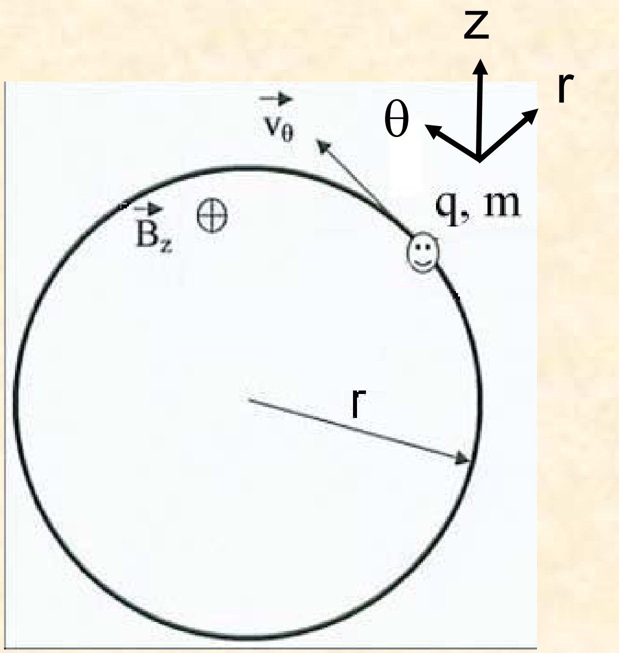
Cyclotrons

1. Uniform field cyclotron
2. Azimuthally Varying Field (AVF) cyclotron
3. Separated sector cyclotron
4. Spiral cyclotron
5. Superconducting cyclotron
6. Synchrocyclotron
7. FFAG

Conventional cyclotron

Let us consider an ion with a charge ***q*** and a mass ***m*** circulating at a speed $\boldsymbol{v_\theta}$ in a uniform induction field ***B***. The motion equation can be derived from the **Newton's law** and the **Lorentz force *F***:

$$\frac{d(m\vec{v})}{dt} = \vec{F} \qquad \text{and} \qquad \vec{F} = q(\vec{v} \times \vec{B})$$



<u>In cylindrical coordinates **(r,θ,z)**</u>

$$\begin{cases} \dfrac{d(m\dot{r})}{dt} - mr\dot{\theta}^2 = q[r\dot{\theta}B_z - \dot{z}B_\theta] \\ \dfrac{d(mr\dot{\theta})}{dt} + m\dot{r}\dot{\theta} = q[\dot{z}B_r - \dot{r}B_z] \\ \dfrac{d(m\dot{z})}{dt} = q[\dot{r}B_\theta - r\dot{\theta}B_r] \end{cases}$$

Conventional cyclotron

Taking the magnetic field B_z along the negative z-axis: $B_z = -B_0$, the equations become:

$$\begin{cases} m_0(\ddot{r} - r\dot{\theta}^2) = -qr\dot{\theta}B_0 \\ m_0(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = q\dot{r}B_0 \\ m_0\ddot{z} = 0 \end{cases}$$

with the beam initial conditions: $(\dot{r} = 0, \dot{\theta}, \dot{z} = 0)$

The trajectory is a circle. It is called ***closed orbit*** in the **median plane** (r,θ). The radius is ***r*** and the angular velocity $\dot{\theta} = \omega_{rev}$ (f_{rev}=Larmor frequency):

$$\omega_{rev} = 2\pi . f_{rev} = \frac{qB_0}{m_0}$$

The magnetic rigidity is defined by: $B_0 r = \dfrac{p(momentum)}{q}$

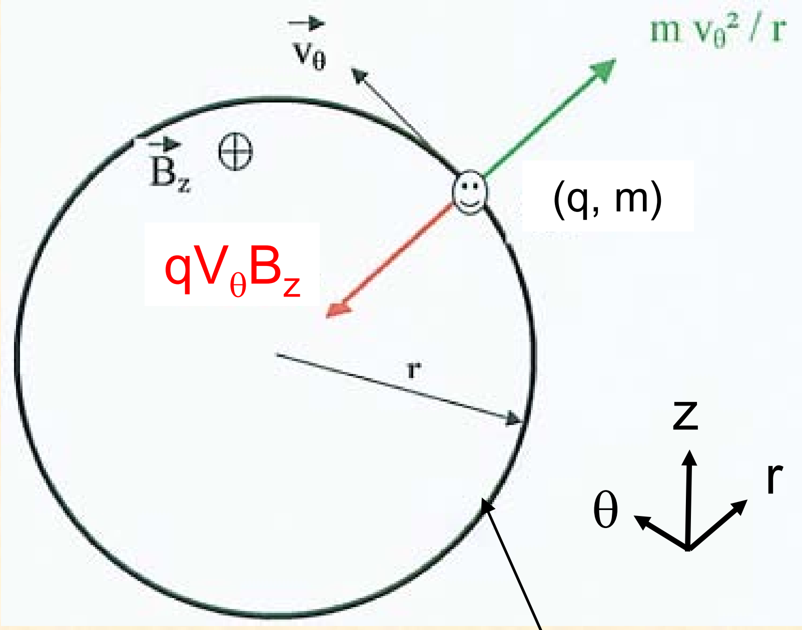
Conventional cyclotron II

Centrifugal force = Magnetic force

$$\boxed{\frac{mv_\theta^2}{r} = qv_\theta B_z}$$

The Lamor frequency is derived such as:

$$\boxed{\omega_{rev} = \frac{d\theta}{dt} = \frac{v_\theta}{r} = \frac{qB_z}{m}}$$



Closed orbit

Conventional cyclotrons means *non relativistic* cyclotrons

low energy ⇨ $\gamma \sim 1$ ⇨ $m / m_0 \sim 1$

In this domain $$\boxed{\omega_{rev} = \frac{qB_z}{m} = \quad const}$$

We can apply between the Dees a RF <u>accelerating</u> voltage:

$$\boxed{V \;=\; V_0 \cos \omega_{RF}\, t}$$

with

$$\boxed{\omega_{RF} = h\omega_{rev}}$$

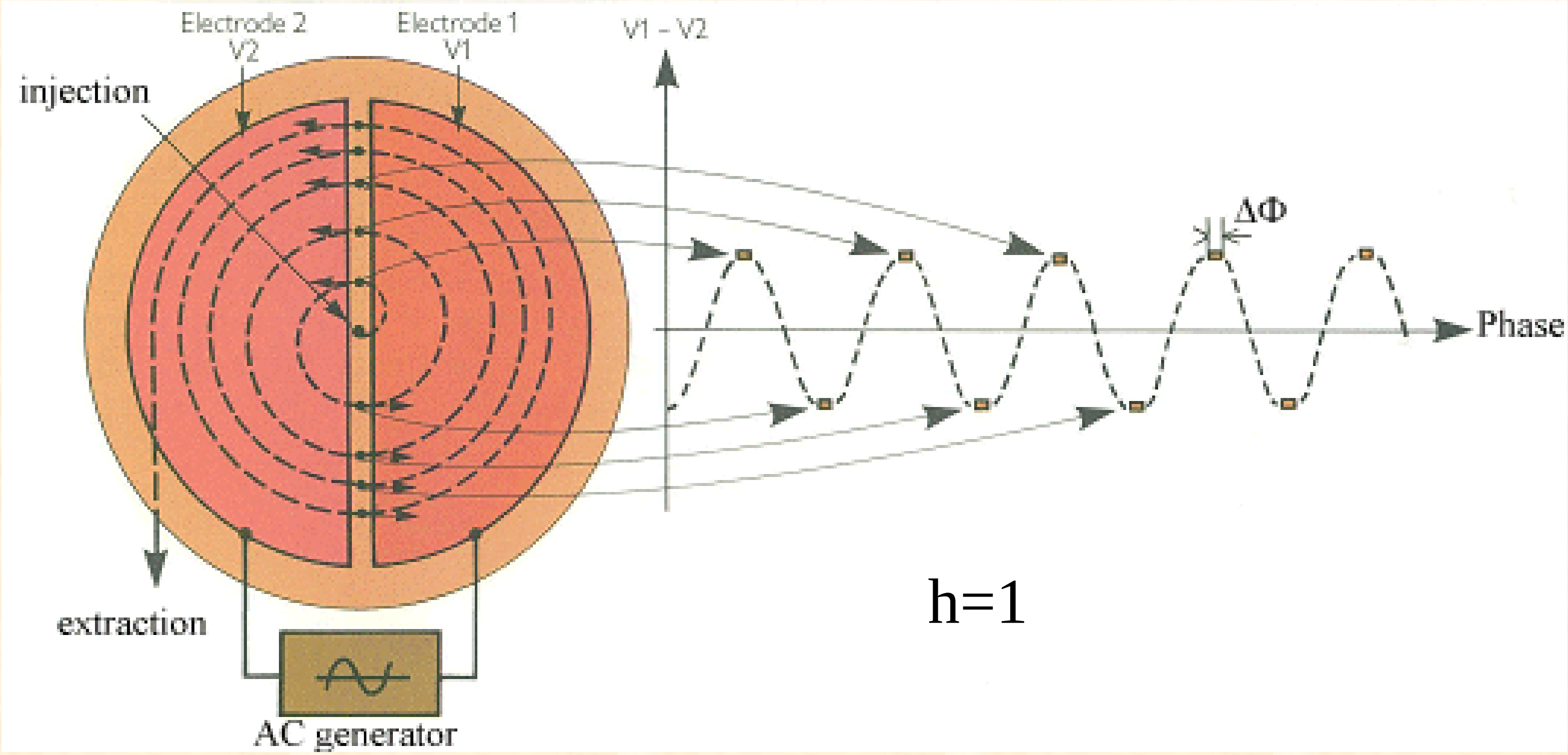
h = 1, 2, 3, … called the RF harmonic number

Isochronism condition: The particle takes the same amount of time to travel one turn

and

with $\omega_{rf} = h\ \omega_{rev}$, the particle is synchronous with the RF wave.

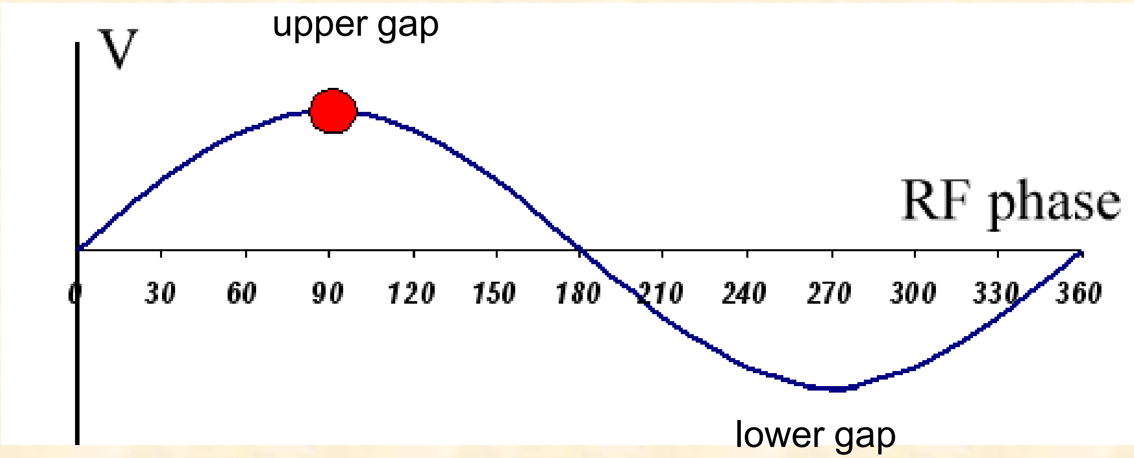
In other words, the particle arrives always at the same RF phase in the middle of the accelerating gap.

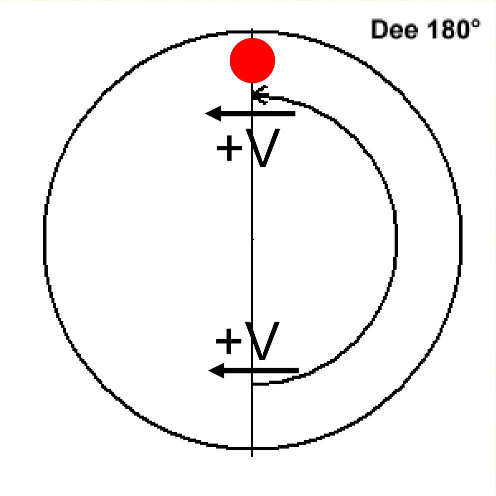


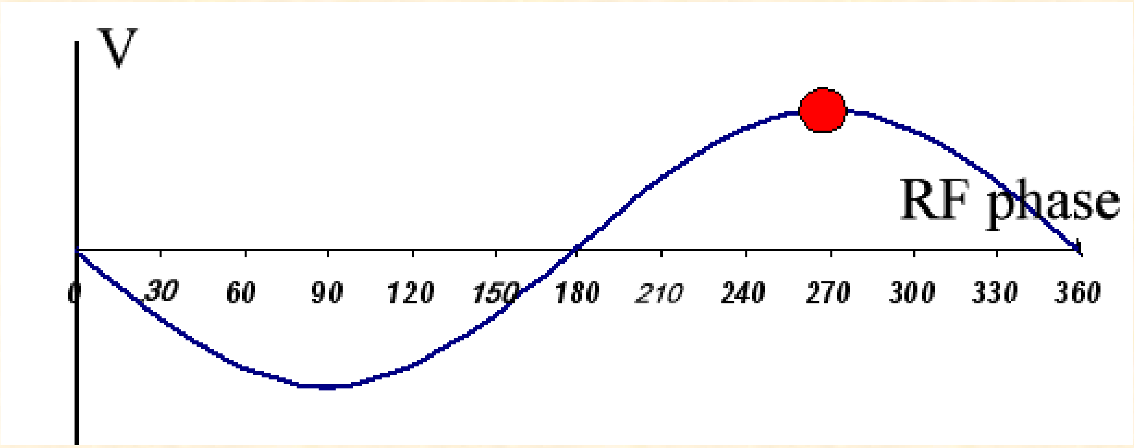
Harmonic notion

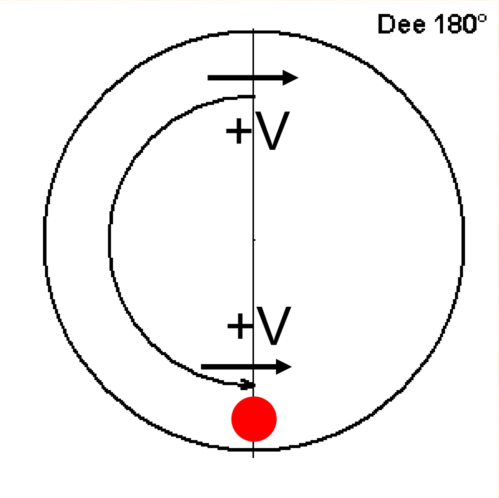
h = 1

1 beam by turn $\omega_{rf} = \omega_{rev}$









Harmonic notion

h = 3

3 beams by turn $\omega_{rf} = 3\omega_{rev}$

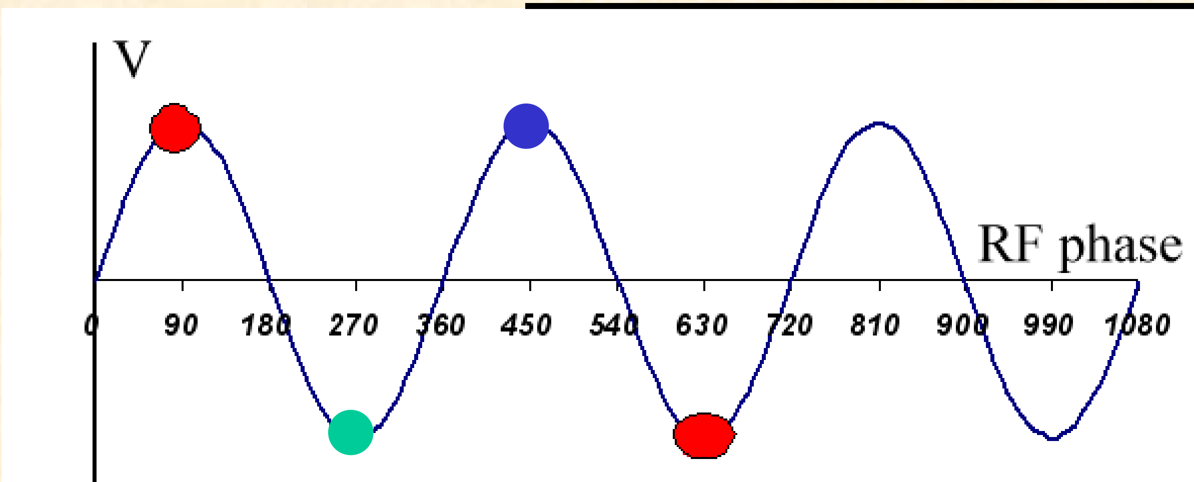
The beam goes 3 times slower than the RF frequency.

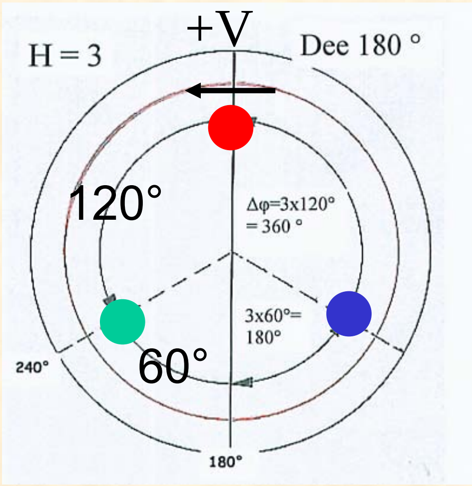
Over 360 the 3 beams are separated by 360 /3 = 120 (beam phase)

$\omega_{rf} = 3\omega_{rev}$, then while the beam travels 60 , the RF shifts of 3x60 = 180 .

Therefore, for the 1st beam at the maximum accelerating field (90 RF phase), the second beam is 120 further and the 60 remaining to the gap will be travelled to an equivalent of 3x60 =180 RF. The electric field is inverted and accelerates the second beam.

Idem for the 3rd beam. **<u>ONLY 1 BEAM IS ACCELERATED AT A TIME</u>**





Why several harmonic numbers are important

- RF cavities have a fixed and reduced frequency range : $f_{min} < f_{rf} < f_{max}$

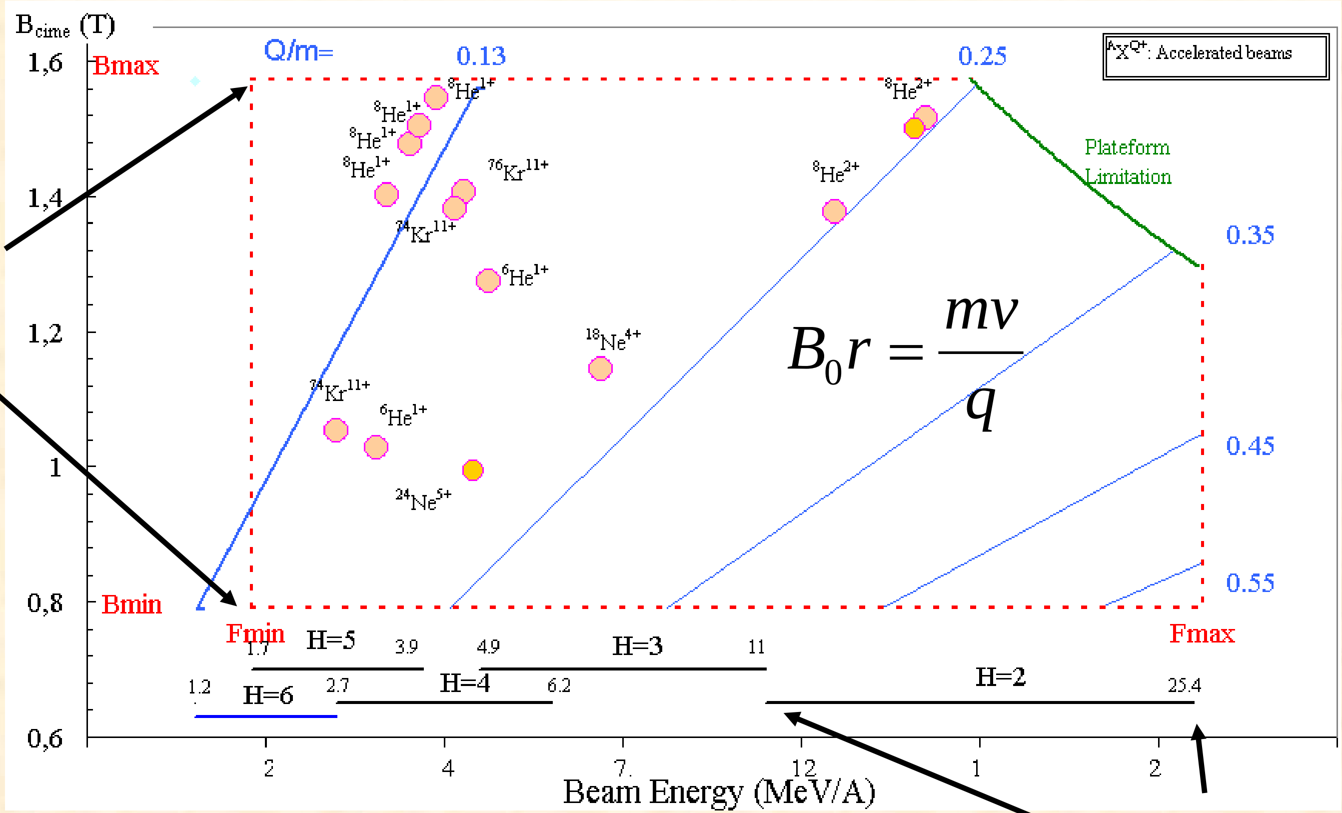
And

- The energy is proportional to f_{rev}^2 ! ($W_{particle}=mv^2/2 = m(w_{rev}R)^2/2 \propto f_{rev}^2 = f^2_{rf}/h^2$)

Then

- Working on various harmonics extend the energy range of the cyclotron

Magnetic field limit



Frequency/energy limits for h=2

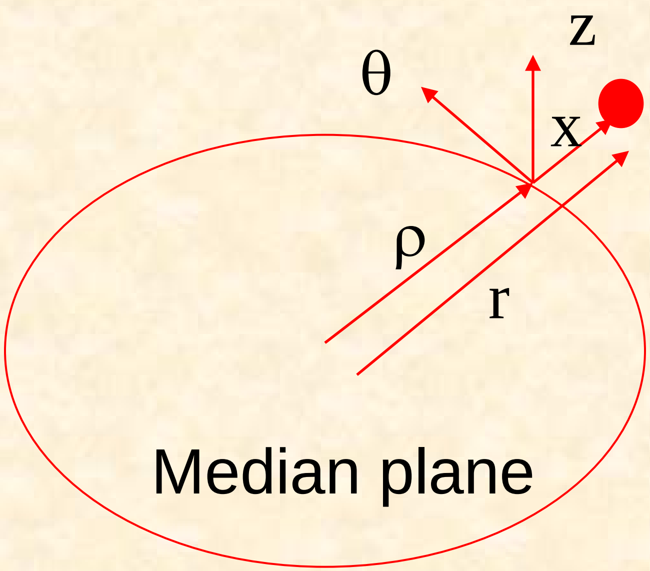
Transverse dynamics

Steenbeck 1935, Kerst and Serber 1941

Horizontal stability : cylindrical coordinates **(r,θ,z)**

and

define **x** a small orbit deviation



Closed orbit

$$r = \rho + x = \rho\,(1 + \frac{x}{\rho})$$

$$x << \rho$$

$(Paraxial\ or\ Gauss\ \ conditions)$

• Taylor expansion of the field B_z around the median plane:

$$B_z = B_{0z} + \frac{\partial B_z}{\partial x} x = B_{0z}\left(1 + \frac{\rho}{B_{0z}} \frac{\partial B_z}{\partial x} \frac{x}{\rho}\right) = B_{0z}\left(1 - n\frac{x}{\rho}\right)$$

$$\boxed{with \quad n = -\frac{\rho}{B_{0z}} \frac{\partial B_z}{\partial x} \qquad the \;\; field \;\; index}$$

• Not on Closed Orbit (Centrifugal force ≠ Magnetic force)

Horizontal restoring force = Centrifugal force - Magnetic force

$$F_x = \frac{m v_\theta^2}{r} - q\, v_\theta B_z \quad \Rightarrow \quad F_x = \frac{m v_\theta^2}{\rho}\left(1 - \frac{x}{\rho}\right) - q\, v_\theta B_{0z}\left(1 - n\frac{x}{\rho}\right)$$

$$F_x = \frac{mv_\theta^2}{\rho}(1-\frac{x}{\rho}) - qv_\theta B_{0z}(1-n\frac{x}{\rho})$$

and $$\omega_{rev} = \frac{v_\theta}{\rho} = \frac{qB_{0z}}{m}$$

After simplification the restoring force is: $$F_x = -\frac{mv_\theta^2}{\rho}\frac{x}{\rho}(1-n)$$

Motion equation under the restoring force $F_x = m\ddot{x}$

$$\ddot{x} + \frac{v_\theta^2}{\rho^2}(1-n)x = 0 \Rightarrow \ddot{x} + \omega^2 x = 0 \qquad \omega = \frac{v_\theta^2}{\rho^2}(1-n)$$

Harmonic oscillator with the frequency $\boxed{\omega = \sqrt{1-n}\ \omega_0}$

Horizontal stability condition: **n < 1**

Vertical stability

Vertical restoring force requires B_x : $F_z = m\ddot{z} = q\, v_\theta\, B_x$

(no centrifugal force)

Because $\nabla \times B = 0$ $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = 0$ $B_x = -n \frac{B_{oz}}{\rho} z$

Motion equation $\boxed{\ddot{z} + \omega^2 z = 0}$

Harmonic oscillator with the frequency $\boxed{\omega = \sqrt{n}\,\omega_0}$

Vertical stability condition : **n > 0**

Betatron oscillation

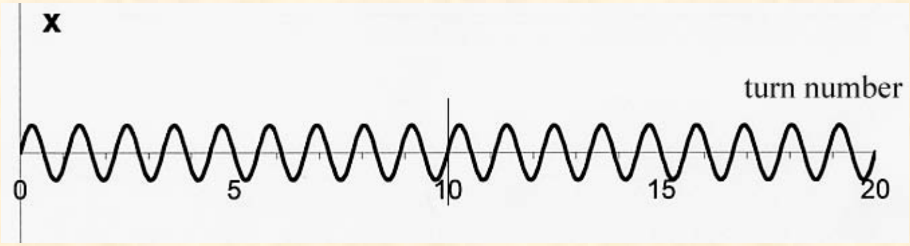
A selected particular solution in the median plane for harmonic oscillator:

$\ddot{x} + \omega^2 x = 0$ $\quad$ $x(t) = x_0 \cos(\omega_x t) = x_0 \cos(\nu_r \omega_0 t)$ $\quad$ $\boxed{\nu_r = \sqrt{1-n}}$

$\ddot{z} + \omega^2 z = 0$ $\quad$ $z(t) = z_0 \cos(\omega_z t) = z_0 \cos(\nu_z \omega_0 t)$ $\quad$ $\boxed{\nu_z = \sqrt{n}}$

EXAMPLES:

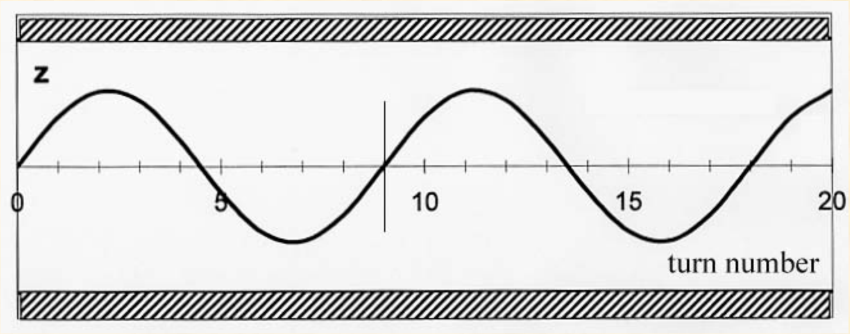
Horizontal oscillation



10 turns in the cyclotron for 9 horizontal oscillations $\nu_r \omega_0 < \omega_0$

$(9/10 = 0.9 = \nu_r)$

Vertical oscillation



1 vertical oscillation for 9 turns in the cyclotron $\nu_z \omega_0 < \omega_0$

$(1/9 = 0.11 = \nu_z)$

Weak focusing

with $n = -\frac{\rho}{B_{0z}}\frac{\partial B_z}{\partial x}$ the field index

Horizontal stability, $\nu_r = \sqrt{1-n}$, with n < 1 means :

- 0 < n < 1 Bz can slightly decrease
- n < 0 Bz can increase as much as wanted

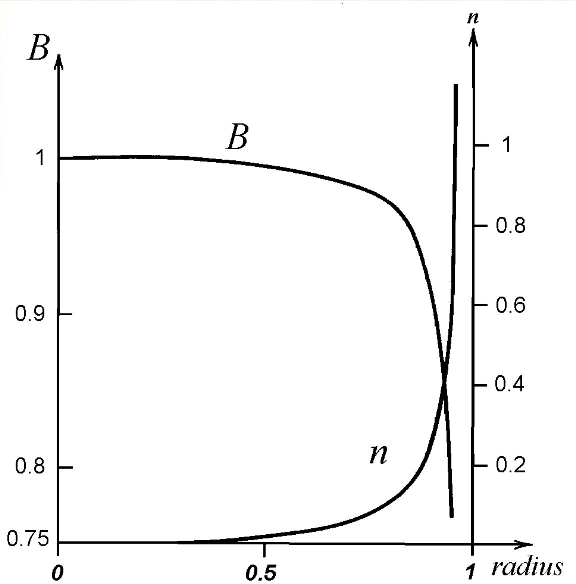
Vertical stability, $\nu_z = \sqrt{n}$, with n > 0 means :

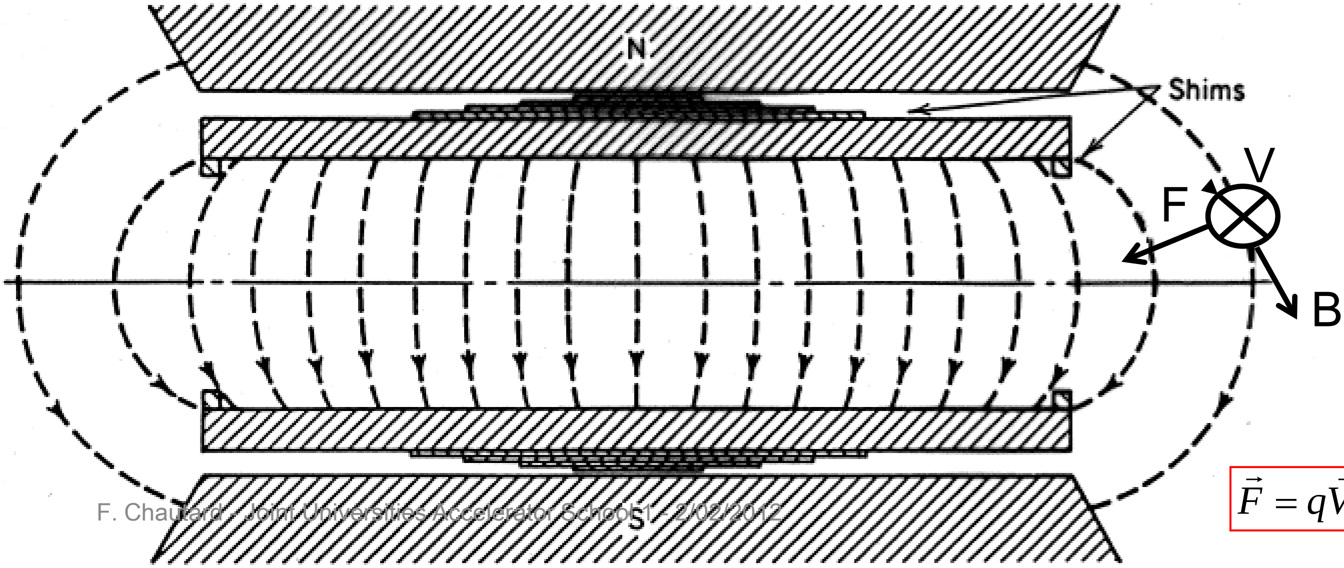
- Bz should decrease with the radius

Simultaneous radial and axial focusing : Weak focusing

$$0 \le n \approx -\frac{\partial B_z}{\partial x} \le 1$$

slightly decreasing field

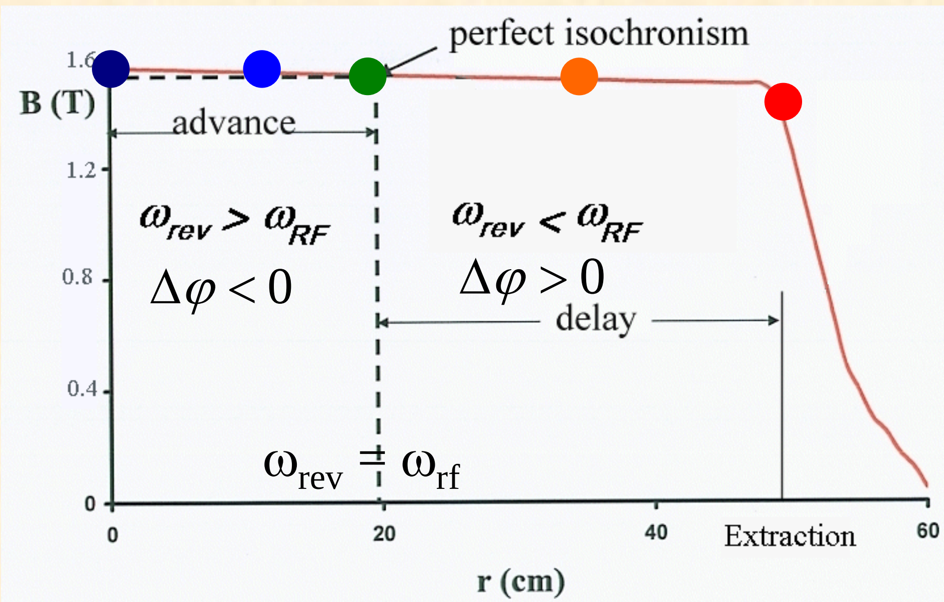


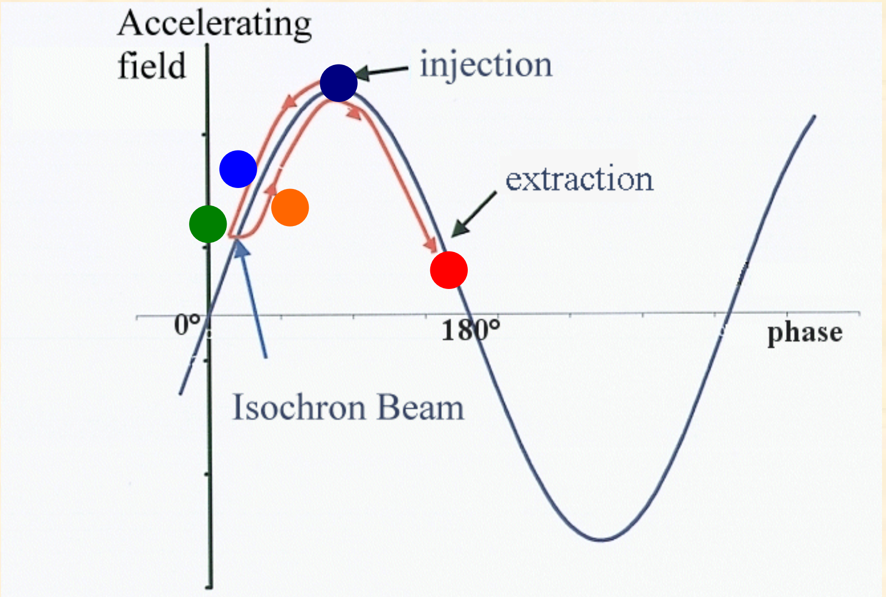


$\vec{F} = q\vec{V} \times \vec{B}$

First limit

Decreasing field (0 < n < 1) gives 1 point with a perfect isochronism $\boxed{\omega_{RF} = \omega_{rev}}$





$$\boxed{\omega_{rev} = \frac{qB_z}{m}}$$

$$\boxed{\Delta\varphi = \pi \cdot \left(\frac{\omega_{RF}}{\omega_{rev}} - 1 \right)}$$

Limited acceleration

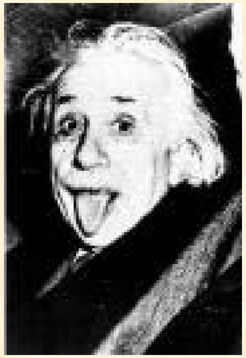
Relativistic case

Isochronism and Lorentz factor

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1-\beta^2}}, \qquad \beta = \frac{v}{c}$$

$$\boxed{\omega_{rev} = \frac{qB(r)}{\gamma(r) m_0}}$$

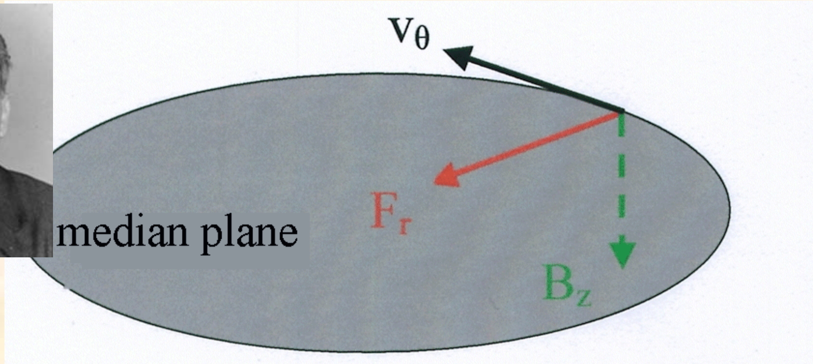
ω_{rev} constant if $B(r)=\gamma(r)B_0 \nearrow$ increasing field (n < 0)



For high energy B(r) should increase. Not compatible with a decreasing field for vertical stability

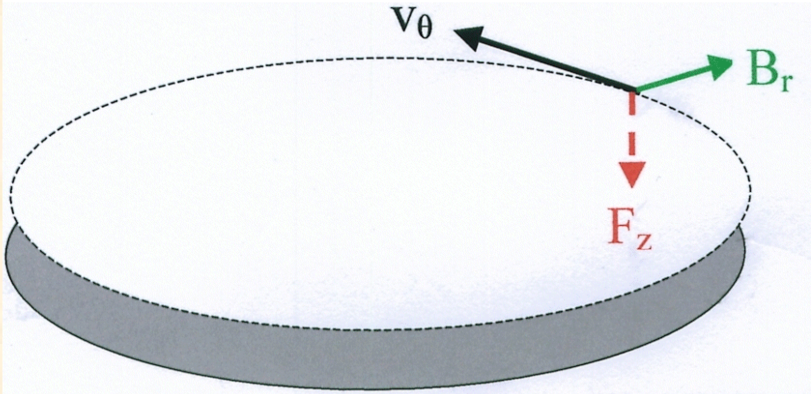
Vertical focusing

AVF or Thomas focusing (1938)



We need to find a way to increase the vertical focusing :

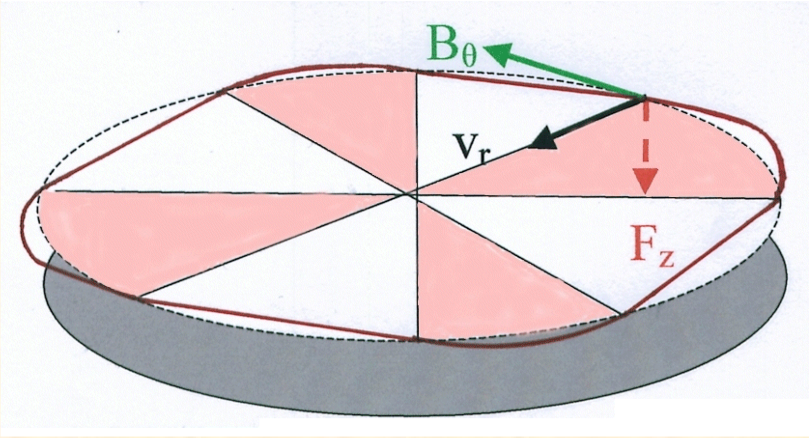
- $F_r(v_\theta , B_z)$: keep ion on the circle
- $F_z(v_\theta , B_r)$: vertical focusing (not enough)



<u>Remains</u>

- F_z with v_r , B_θ : one has to find an azimuthal component $\mathbf{B_\theta}$ and a radial component $\mathbf{v_r}$ (meaning a non-circle trajectory)

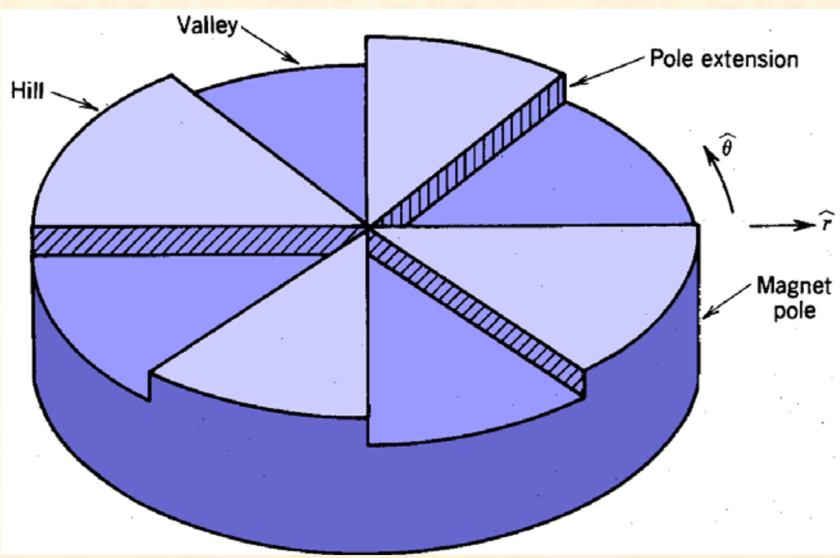
→ Sectors

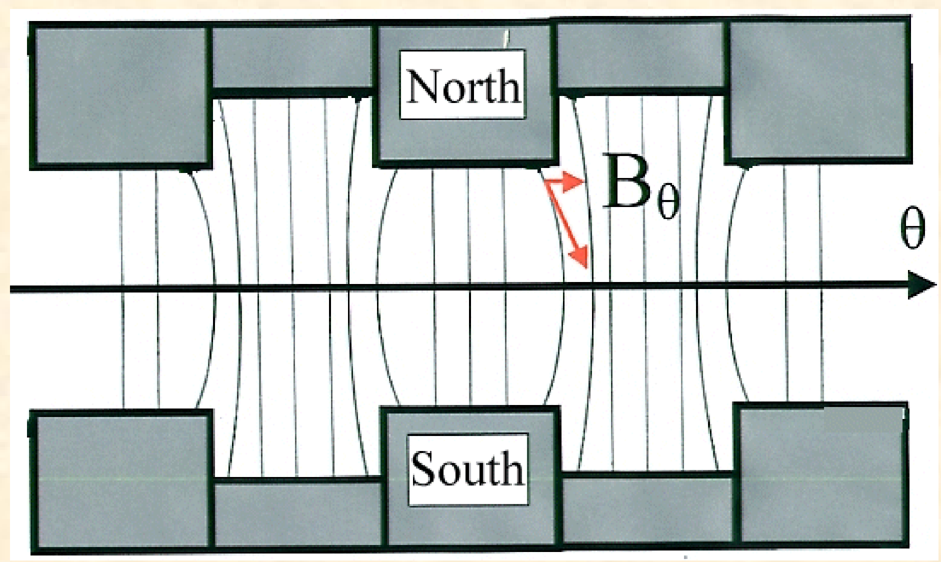


Azimuthally varying Field (AVF)

B_θ created by:

- Succession of high field and low field regions
- B_q appears around the median plane
  - Valley : large gap, weak field
  - Hill : small gap, strong field





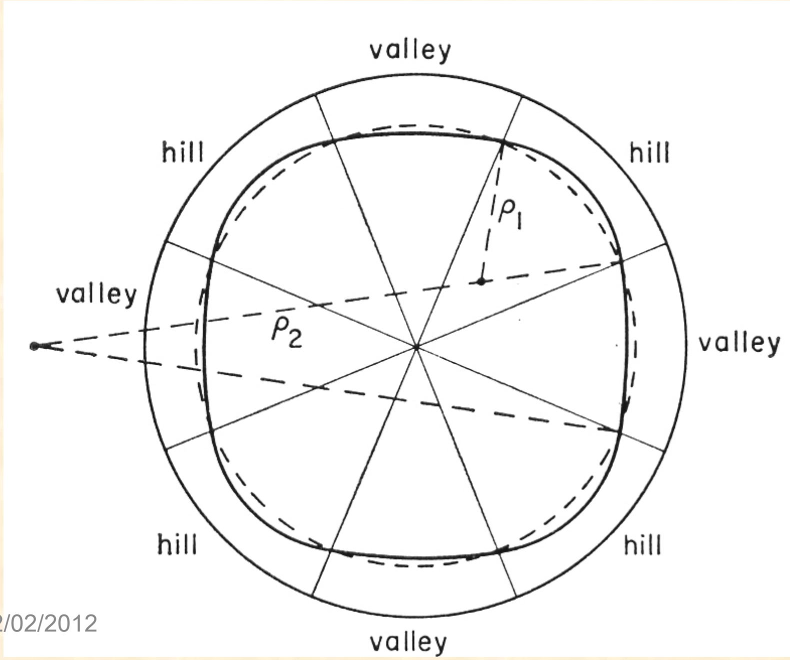
V_r created by :

- Valley: weak field, large trajectory curvature
- Hill : strong field, small trajectory curvature

→ Trajectory is not a circle

- Orbit not perpendicular to hill-valley edge

→ Vertical focusing $F_z \propto v_r \, . \, B_\theta$



Vertical focusing and isochronism for AVF

2 conditions to fulfill

• **<u>Increase the vertical focusing force strength:</u>** $|Fz| \approx \omega^2 z = \nu_z^2 \omega_0^2 z \approx \nu_z^2$

• AVF Field modulation or Flutter:

$$\boxed{F_l = \frac{\langle B^2\rangle - \langle B\rangle^2}{\langle B\rangle^2} \approx \frac{\left(B_{hill} - B_{val}\right)^2}{8\langle B\rangle^2}}$$

where <B> is the average field over 1 turn

• For a cyclotron with N sectors, the betatron number depends on the flutter term such as :

$$\boxed{\nu_z^2 = \mathrm{n} + \frac{N^2}{N^2-1}F_l + \ldots > 0}$$

F_l enhances the initial weak focusing term *n*

• **<u>Keep the isochronism condition true:</u>** $\boxed{\omega_{rev} = \frac{QB(r)}{\gamma(r)m_0}}$

For high energies, $\gamma(r) \nearrow$ $\quad then \ \bar{B}_z(r) = \gamma(r)\bar{B}_z(0) \Rightarrow \frac{\partial B_z(r)}{\partial r} > 0$

Leading to: $\boxed{n = 1 - \gamma^2 < 0}$

Vertical focusing and isochronism for AVF

2 conditions to fulfil

- **Increase the vertical focusing force strength:**

$$\nu_z^2 = \mathrm{n} + \frac{N^2}{N^2 - 1} F_l + \ldots > 0$$

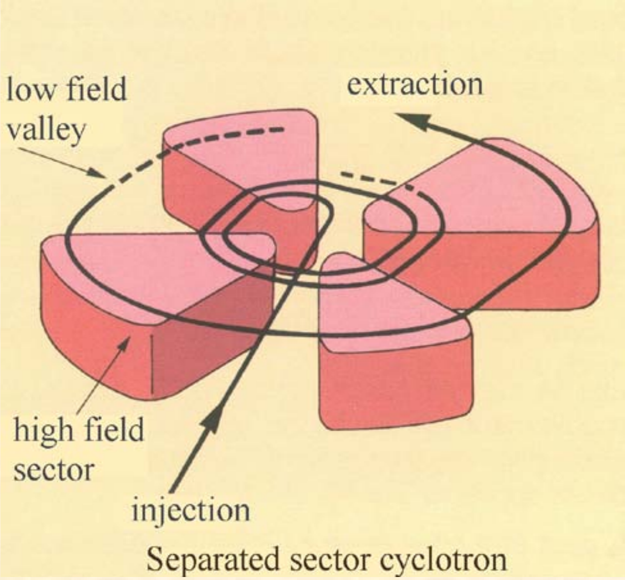
- **Keep the isochronism condition true:**

$$n = 1 - \gamma^2 < 0$$

The focusing limit is:

$$\frac{N^2}{N^2 - 1} F_l > -n = \gamma^2 - 1$$

Separated sector cyclotron



Focusing condition limit:

$$\frac{N^2}{N^2-1}F_l > -n = \gamma^2 - 1$$

If we aim to high energies:

γ ↗ then -n >> 0

➢ Increase the flutter F_l, using separated sectors where $B_{val} = 0$

$$F_l = \frac{\left(B_{hill} - B_{val}\right)^2}{8\langle B\rangle^2}$$

⇨ High energies

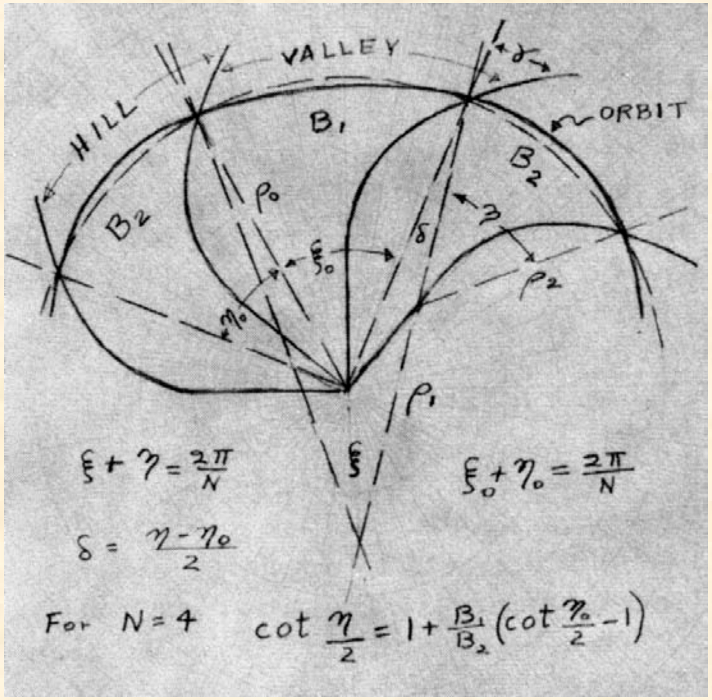
Spiralled sectors

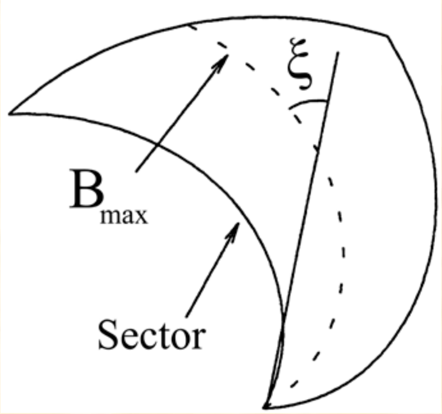
In 1954, Kerst realised that the sectors need not be symmetric. By tilting the edges (ξ angle) :

- The valley-hill transition became more focusing
- The hill-valley less focusing.

But by the strong focusing principle (larger betatron amplitude in focusing, smaller in defocusing), the net effect is focusing (cf F +D quadripole).

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l (1 + 2\tan^2 \xi)$$





Superconducting cyclotron (1985)

- Most existing cyclotrons utilise room temperature magnets B_{max} = 2T (iron saturation)
- Beyond that, superconducting coils must be used: B_{hill} ~ 6 T
    1. Small magnets for high energy
    2. Low operation cost

Energy and focusing limits

1. For conventional cyclotron, F_l increases for small hill gap (B_{hill} ↗) and deep valley (B_{val} ↘) but <u>does not depend</u> on the magnetic field level:

$$\boxed{F_l = \frac{\left(B_{hill} - B_{val}\right)^2}{8\langle B\rangle^2}}$$

2. For superconducting cyclotron, the iron is saturated, the term $(B_{hill}\text{-}B_{val})^2$ is constant, hence $F_l \propto 1/\langle B\rangle^2$

⇨ consequences on W_{max}

Max Energy for Conventional Cyclotrons

A cyclotron is characterised by its K_b *bending* factor giving its max capabilities

$$\boxed{W_{\max}(MeV/nucleon) = K_b \left\{\frac{Q}{A}\right\}^2} \quad \text{with } K_b = 48{,}244\left(\langle B\rangle r_{ext}\right)^2$$

- $W_{max} \propto r_{ext}^2$ but iron volume $\propto r^3$!
- for compact cyclotron $r_{extraction}$~2 m.
- For a same ion (or isobar A=Cst), W_{max} grows with Q^2 : great importance of the ion sources.

Max Energy for Superconducting Cyclotrons

Because of the focusing limitation due to the Flutter dependence on the B field, the max energy is given as a function of K_f the so-called *focusing* factor:

$$W_{\max}(MeV/nucleon) = K_f \left\{ \frac{Q}{A} \right\}$$

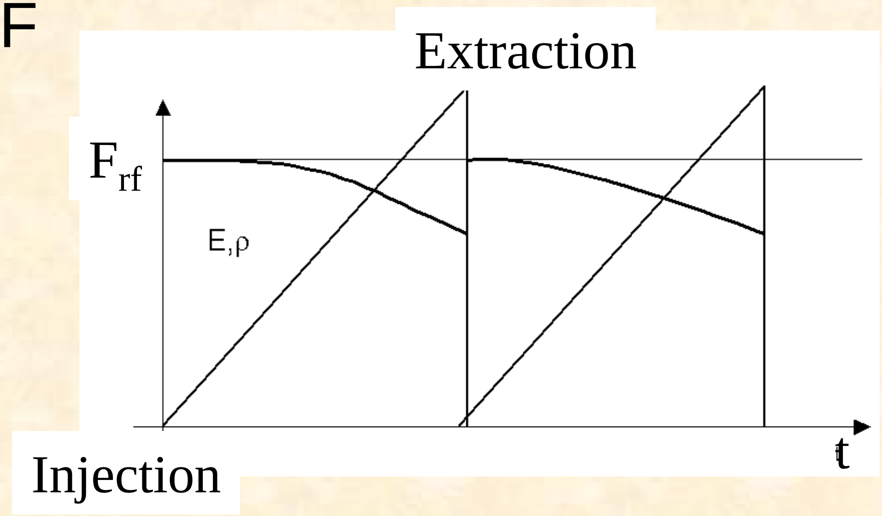
Synchrocyclotron

• Machine : n > 0 and uniform magnetic field. (only 4 machines remain around the world)

• The RF frequency is varied to keep the synchronism between the beam and the RF

$$\omega_{rev} = QB/\gamma m_0 = \omega_{rf} \searrow$$

• Cycled machine
(continuous for cyclotrons)



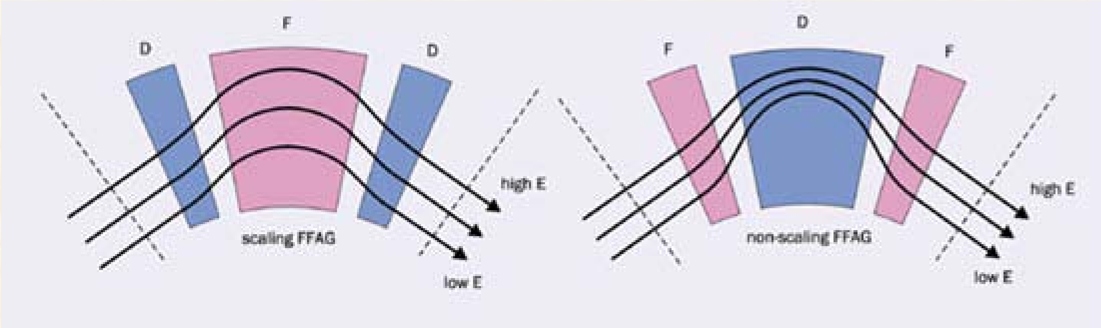
• 10000 to 50000 turns (RF variation speed limitation) ➔ low Dee voltage ➔ small turn separation
• W ~ few MeV to GeV

Fixed-field alternating-gradient (FFAG)

- Following the discovery of alternating gradient focusing in 1952, FFAGs were proposed (T. Ohkawa, K. Symon and A. Kolomensky)
- Principle :
  - A FFAG is a type of circular particle accelerator being developed for potential applications in physics, medicine, national security, and energy production
  - Features of cyclotrons and synchrotrons.
    - Cyclotron's advantage of continuous, unpulsed operation,
    - Synchrotron's relatively inexpensive small magnet ring, of narrow bore.
- Since 2000, a great activity around this accelerator design appeared.
- Fixed magnetic fields, modulated radiofrequency (RF) and pulsed beams, FFAGs operate just like synchrocyclotrons

Fixed-field alternating-gradient (FFAG)

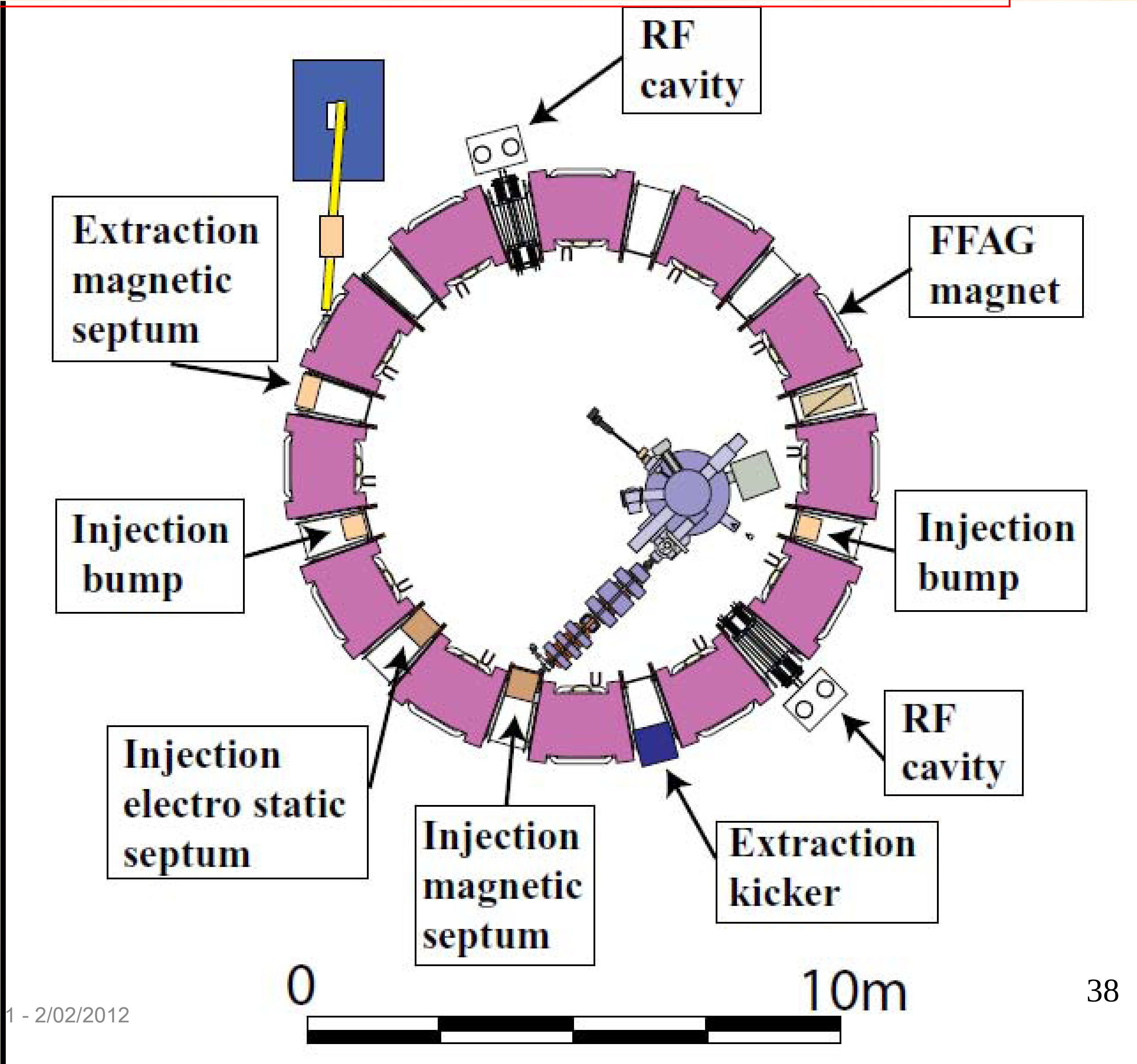
- The fixed magnetic field leads to a spiral orbit, so the vacuum chamber and magnets tend to be larger than for a synchrotron, but the repetition rate (and hence beam intensity) can be much higher, as it is set purely by RF considerations. High repetition rate and large momentum acceptance are the two features where FFAGs offer advantages over synchrotrons

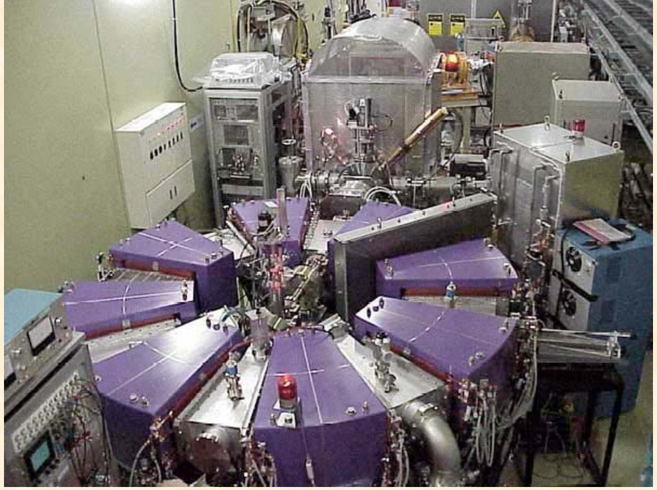


The orbit shape is invariant in scaling FFAG cells, but varies with energy (E) in non-scaling ones (F = focusing, D = defocusing).

Fixed-field alternating-gradient (FFAG)

- Energy 10 - 125 MeV(proton)
- Type of magnet Triplet radial (DFD)
- Number of Cell 12
- Average radius 4.47 - 5.20 m
- Betatron tune (injection) 3.62 (Horizontal) 1.45 (Vertical)
- Magnetic field Focus: 1.63 T
- Defocus: 0.78 T
- Revolution Freq. 1.5 - 4.2 MHz
- Repetition 100 Hz / 2 cavities
- Beam Current 1.5 nA (In the first stage)





150 MeV FFAG accelerator complex in Research Reactor Institute, Kyoto University (KURRI). ADS experiments using this accelerator has been started in March of 2009.

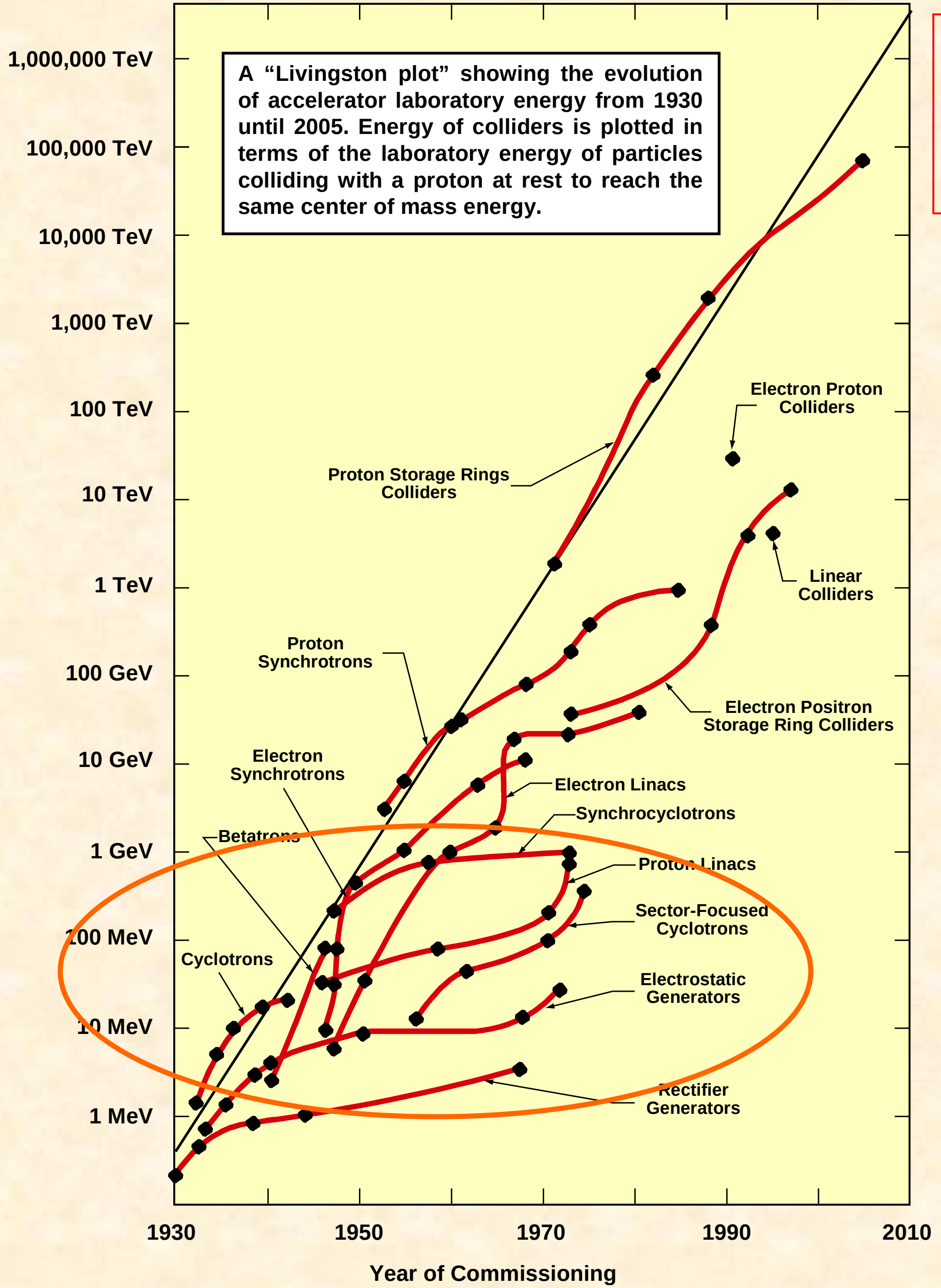
1 MeV proton beam
2.5 m diameter experiment at the Japanese KEK laboratory has demonstrated fixed-field proton acceleration.

Few K_b

$$W_{\max}(MeV / nucleon) = K_b \left\{ \frac{Q}{A} \right\}^2$$

Laboratories	Cyclotron name/type	K (MeV/n) (or proton energy Q/A =1)	$R_{extraction}$(m)
GANIL(FR)	C0	28	0.48
NAC (SA)	SSC	220	4.2
GANIL (FR)	CIME	265	1.5
GANIL (FR)	SSC2	380	3
RIKEN (JP)	RING	540	3.6
PSI (CH)	Ring	592	4.5
DUBNA (RU)	U400	625	1.8
MSU (USA)	K1200(cryo)	1200 (K_f=400)	1

42 more http://accelconf.web.cern.ch/accelconf/c01/cyc2001/ListOfCyclotrons.html



Livingston chart