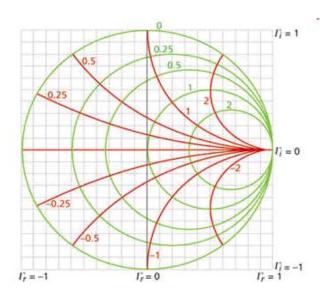
JUAS RF Course 2012

Cavities

RF Theory



Superconducting LEP cavity



The Smith Chart

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Part I: Passive and Active Elements

Section A

- Basics
- Cavity structures
- Equivalent circuit
- Behaviour in time and in frequency domain
- Beam-cavity interaction
- Scaling laws
- Simulation techniques

Section B

- Higher order modes (HOMs)
- Coupling and tuning
- Different forms of cavities
 - Voltage breakdown & Multipactor

Section C

- Groups of cavities
- Transmission lines
- Striplines, Microstriplines, Slotlines
- Waveguides
- Active elements
 - Transistors
 - Gridded tubes
 - Klystrons
 - IOTs

Part II: Waves, S-Parameters, Decibels and Smith Chart

Section A

 Forward and backward travelling waves

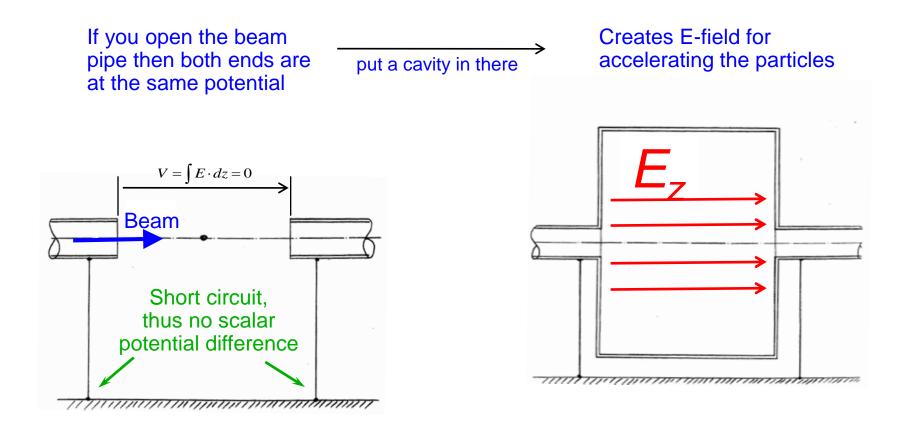
Section B

- S-Parameters
 - The scattering matrix
- Decibels
- Measurement devices and concepts
- Superheterodyne
 Concept

Section C

- The Smith Chart
- Navigation in the Smith Chart
- **Examples**

From L and C to a cavity

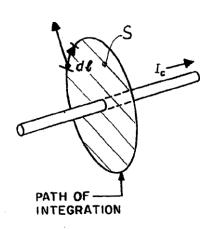


Capacitor at high frequencies, The Feynman Lectures on Physics Can the short-circuit be avoided?

Answer: No - but it doesn't bother us at high frequencies.

Basics

Maxwell's equations (1)



Ampere's Law : $\oint H \cdot dl = I = I_{conduction} + I_{displacement}$ $\partial \Phi_{r}$

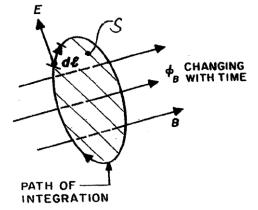
 $I_{displacement} = \frac{\partial \Phi_D}{\partial t}$

where the electric flux Φ_D is given by $\Phi_D = \int_{S} D \cdot dS = \mathcal{E} \int_{S} E \cdot dS$,

D designating the electric flux density.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

with the current density *J*
and the magnetic field *H*



Faraday's Law : $\oint E \cdot dl = -\frac{\partial \Phi_B}{\partial t}$

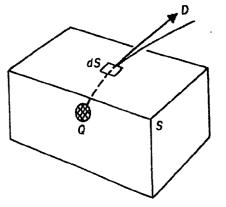
with the electric flux Φ_B $\Phi_B = \int_S BdS = \mu \int_S HdS$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

with the electric field *E*
and the magnetic field *B*

scalar vs. vector potential: path of integration makes a difference

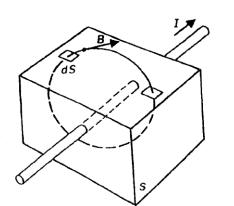
Maxwell's equations (2)



S = TOTAL SURFACE Q = TOTAL CHARGE INSIDE S Gauss' Law (Electrici ty): $\int_{S} D \cdot dS = Q$

with the electric displacement D

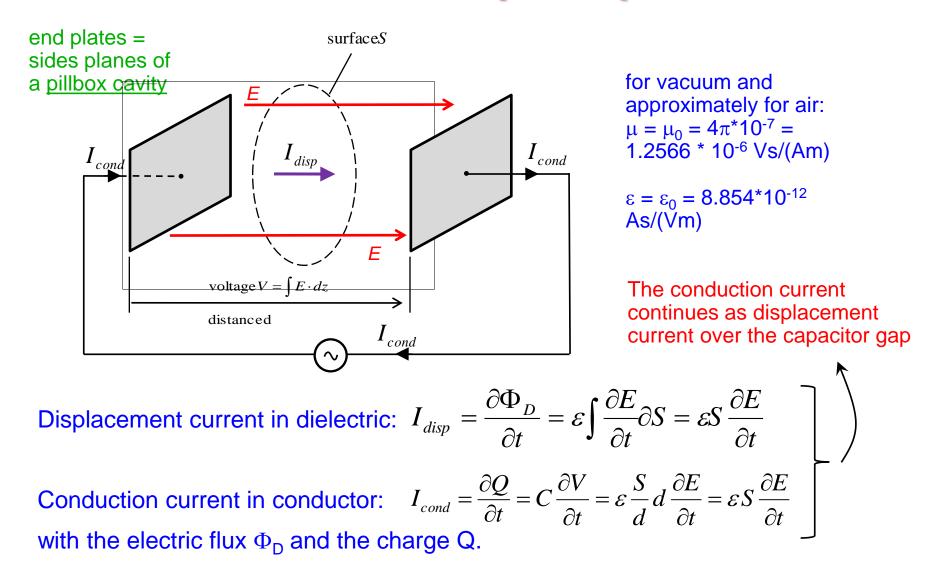
 $\nabla \cdot \mathbf{D} = \rho$ with the charge density ρ



Gauss' Law (Magnetism): $\int_{S} B \cdot dS = 0$ $\nabla \cdot \mathbf{B} = \mathbf{0}$

There are no magnetic charges

Displacement and conduction currents in a simple capacitor



General Solution for a Rectangular (brick-type) Cavity

When describing field components in a Cartesian coordinates system (assuming a homogeneous and isotropic material in a space charge free volume) with harmonic functions (angular frequency ω) then each Cartesian component needs to fulfill Laplace's equation:

 $\Delta \Psi + k_0^2 \varepsilon_r \mu_r \Psi = 0 \qquad \begin{array}{c} k_0^2 = \omega^2 \varepsilon_0 \mu_0 & k_0 & \text{free space wavenumber} \\ k_0 = 2\pi / \lambda_0 & \lambda_0 & \text{free space wavelength} \end{array}$

As a general solution we can use the product ansatz for Ψ

$$\Psi = X(x)Y(y)Z(z)$$

From this one obtains the general solution for Ψ (Ψ may be a vector potential or field) standing waves

$$\Psi = \begin{cases} A \cdot \cos(k_x x) + B \cdot \sin(k_x x) \\ A' \cdot e^{-jk_x x} + B' \cdot e^{jk_x x} \end{cases} \begin{cases} C \cdot \cos(k_y y) + D \cdot \sin(k_y y) \\ C' \cdot e^{-jk_y y} + D' \cdot e^{jk_y y} \end{cases} \begin{cases} E \cdot \cos(k_z z) + F \cdot \sin(k_z z) \\ E' \cdot e^{-jk_z z} + F' \cdot e^{jk_z z} \end{cases} \checkmark$$

travelling waves

with the separation condition

$$(k_x)^2 + (k_y)^2 + (k_z)^2 = (k_0)^2 \varepsilon_r \mu_r$$

see also: G. Dome, RF Theory Proceeding Oxford CAS, April 91 CERN Yellow Report 92-03, Vol. I

General Solution in Cylindrical Coordinates

As a general solution we can use the product ansatz for Ψ

 $\Psi = R(\rho)F(\varphi)Z(z)$

From this one obtains the general solution for Ψ (Ψ may be a vector potential or field) standing waves

$$\Psi = \begin{cases} A \cdot J_{m}(k_{\rho}\rho) + B \cdot N_{m}(k_{\rho}\rho) \\ A' \cdot H_{m}^{(2)}(k_{\rho}\rho) + B' \cdot H_{m}^{(1)}(k_{\rho}\rho) \end{cases} \begin{cases} C \cdot \cos(m\varphi) + D \cdot \sin(m\varphi) \\ C' \cdot e^{-jm\varphi} + D' \cdot e^{jm\varphi} \end{cases} \begin{cases} E \cdot \cos(k_{z}z) + F \cdot \sin(k_{z}z) \\ E' \cdot e^{-jk_{z}z} + F' \cdot e^{jk_{z}z} \end{cases} \end{cases}$$

and the functions

travelling waves

 J_m ... cylindrical harmonics of the Bessel function of order m

- N_m ... cylindrical harmonics of the Neumann function of order m
- $H_m^{(1)}$...Hankel function of the first kind of order *m* (outwardtravelling wave)
- $H_m^{(2)}$...Hankel function of the second kind of order *m* (inward travelling wave)

$$H_{m}^{(1)} = J_{m} + jN_{m}$$
$$H_{m}^{(2)} = J_{m} - jN_{m}$$

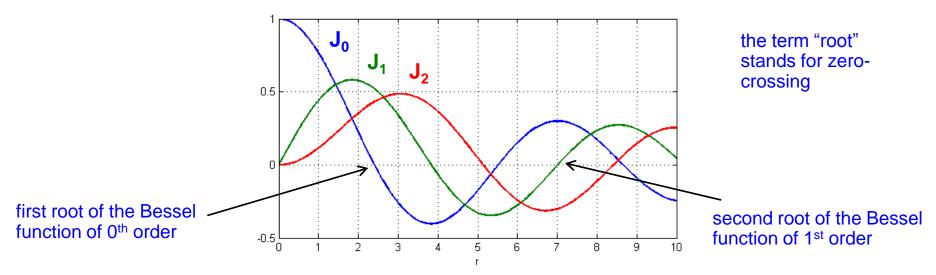
Here the separation condition is

$$(k_{\rho})^2 + (k_z)^2 = (k_0)^2 \varepsilon_r \mu_r$$

Hint: the index m indicating the order of the Bessel and Neumann function shows up again in the argument of the sine and cosine for the azimuthal dependency.

Bessel Functions (1)

A nice example of the derivation of a Bessel function is the solution of the cylinder problem of the capacitor given in the Feynman reference (Bessel function via a series expansion).



Comment: For the generalized solution of cylinder symmetrical boundary value problems (e.g. higher order modes on a coaxial resonator) Neumann functions are required. Standing wave patterns are described by Bessel- and Neumann functions respectively, radially travelling waves in terms of Hankel functions. Hint: Sometimes a Bessel function is called Bessel function of first kind, a Neumann function is Bessel function of second kind, and a Hankel function=Bessel function of third kind.

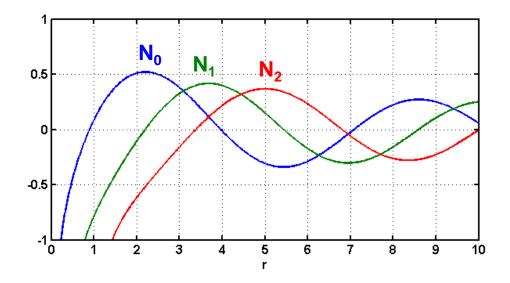
Bessel Functions (2)

Some practical numerical values:

k	$J_0(x)$	$J_1(x)$	$J_{2}\left(x ight)$	$J_{3}\left(x ight)$	$J_4(x)$	$J_5(x)$
1	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715
2	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386
3	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002
4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801
5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178

See: http://mathworld.wolfram.com/BesselFunctionZeros.html

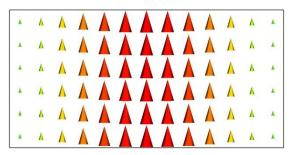
Neumann Functions



Neumann functions are often also denoted as $Y_m(r)$.

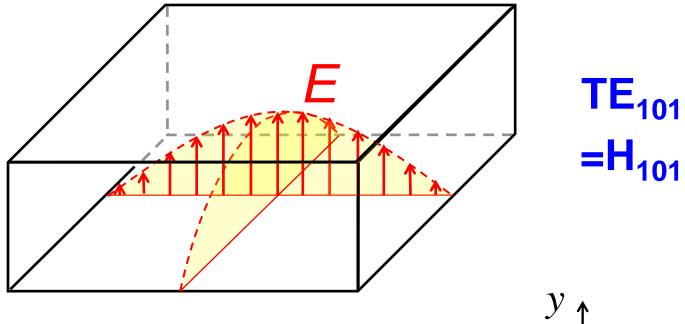
Electromagnetic waves

- Propagation of electromagnetic waves inside empty metallic channels is possible: there exist solutions of Maxwell's equations describing waves
- These waves are called waveguide modes
- There exist two types of waves,
 - Transverse electric (TE) modes:
 - \rightarrow the electric field has only transverse components
 - Transverse magnetic (TM) modes:
 - \rightarrow the magnetic field has only transverse components
- Propagate at above a characteristic cut-off frequency
- In a rectangular waveguide, the first mode that can propagate is the TE_{10} mode. The condition for propagation is that half of a wavelength can "fit" into the cross-section => cut-off wavelength $\lambda_c = 2a$
- The modes are named according to the number of field maxima they have along each dimension. The E field of the TE₁₀ mode for instance has 1 maximum along x and 0 maxima along the y axis.
- For circular waveguides, the maxima are counted in the radial and azimuthal direction

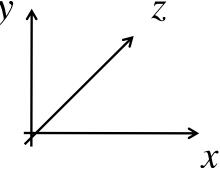


E field of the fundamental TE_{10} mode

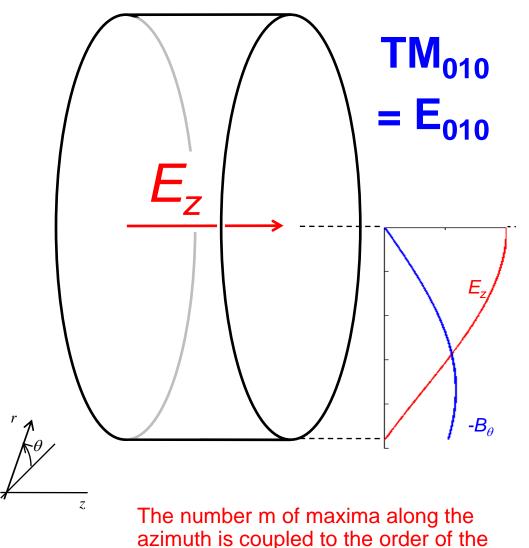
Mode Indices in Resonators (1)



For a structure in <u>rectangular coordinates</u> the mode indices simply indicate the number of half waves (standing waves) along the respective axis. Here we have one maximum along the x-axis, no maximum in vertical dimension (y-axis), and one maximum along the z-axis. TE_{101} corresponds to TE_{xvz}



Mode Indices in Resonators (2)



Bessel function (see slide on theory).

For a structure in <u>cylindrical</u> <u>coordinates:</u>

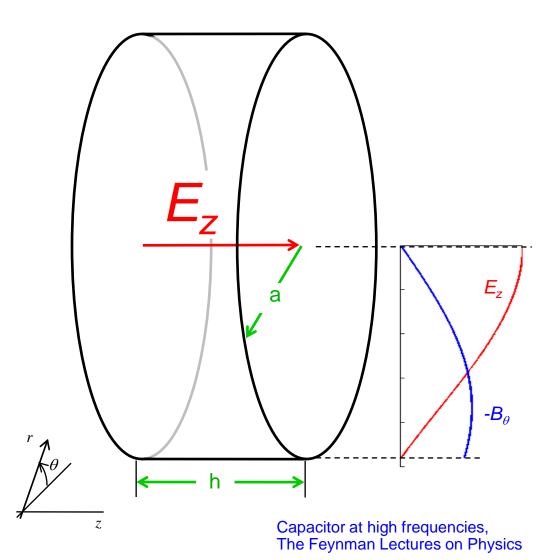
The first index is the order of the Bessel function or in general cylindrical function.

The second index indicates "the root" of the cylindrical function which is the number of zero-crossings.

The third index is the number of half waves (maxima) along the z-axis.

Hint: In an empty pillbox there will be <u>no</u> Neumann function as it has a pole in the center (conservation of energy). However we need Bessel and Neumann functions for higher order modes of coaxial structures.

Fields in a pillbox cavity



Cavity height: h cavity radius: a

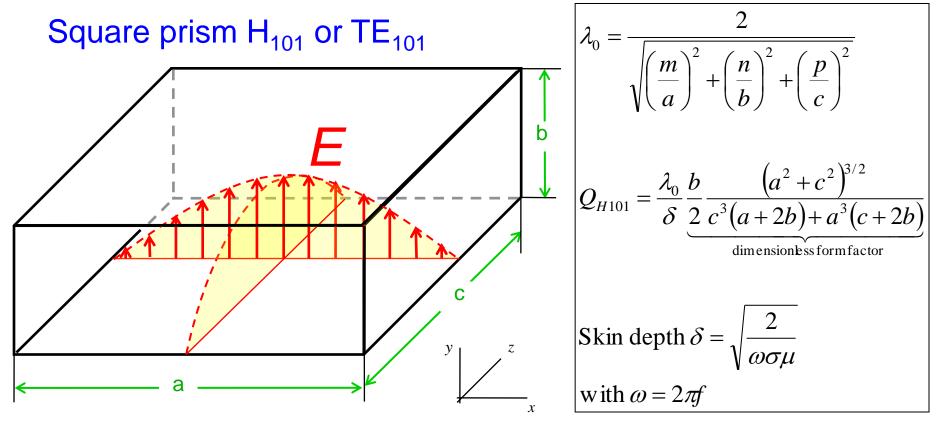
 TM_{010} mode resonance = E_{010} mode resonance for

$$a = 0.383\lambda = 1.53\lambda/4$$

TM₀₁₀ resonance frequency independent of h!!!

In the cylindrical geometry the E and H fields are proportional to Bessel functions for the radial dependency.

Common cavity geometries (1)



Comment: For a brick-shaped cavity (the structure is described in Cartesian coordinates) the E and H fields would be described by sine and cosine distributions. The mode indices indicate the number of half waves along the x-,y-, and z-axis, respectively.

this simplifies in the case a=c:

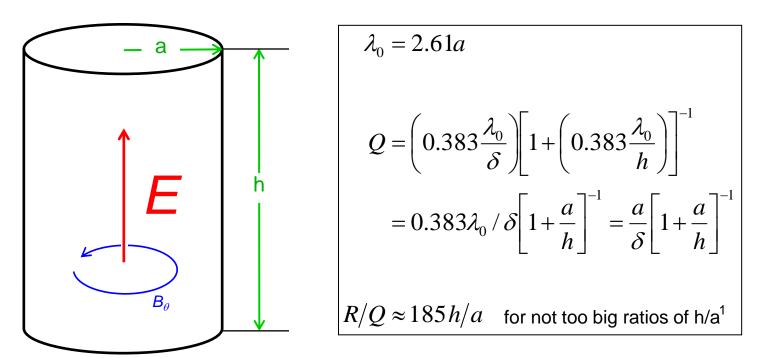
$$\lambda_0 = \sqrt{2a}$$
$$Q = \frac{1}{\delta} \frac{ab}{a+2b}$$

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Cavity structures

Common cavity geometries (2)

Circular cylinder: E_{010} , = TM_{010}



Note: h denotes the **full** height of the cavity In some cases and also in certain numerical codes, h stands for the half height

1: formula uses Linac definition and includes time transit factor F. Caspers, M. Betz; JUAS 2012 RF Engineering

Cavity structures

R/Q for cavities

The full formula for calculating the R/Q value of a cavity is $\sin^2(\frac{\chi_{01}}{h})$

$$\frac{R}{Q} = \frac{4\eta}{\chi_{01}^{3} \pi J_{1}^{2}(\chi_{01})} \frac{\sin^{2}(\frac{\chi_{01}}{2} \frac{h}{a})}{\frac{h}{a}} s$$

ee lecture: RF cavities, E. lensen, Varna CAS 2010

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\mu_0^2 c_0^2} = 4\pi \times 10^{-7} \times 3 \times 10^8 = 377\Omega$$

 $\chi_{01} = 2.4048$ (First zero of the Bessel function of 0th order) $J_1(\chi_{01}) = 0.5192$

h

This leads to

$$\frac{R}{Q} = 128 \frac{\sin^2(1.2024\frac{n}{a})}{\frac{h}{a}}$$

The sinus can be approximated by sinx = x (for small values of x) leading to

$$\frac{R}{Q} \approx 128 \frac{(1.2024 \frac{h}{a})^2}{\underline{h}} = 185 \frac{h}{a}$$

Common cavity geometries (3)

Circular cylinder:

H₀₁₁

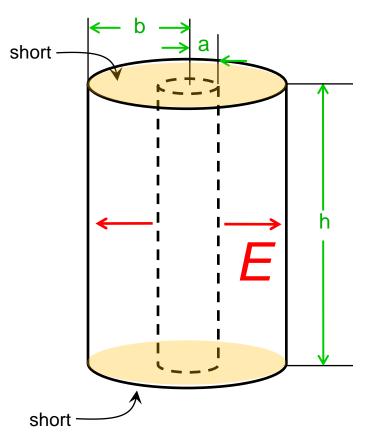
$$Q = 0.61 \frac{\lambda_0}{\delta} \frac{\left[1 + 0.17 \left(\frac{2a}{h}\right)^2\right]^{3/2}}{1 + 0.17 \left(\frac{2a}{h}\right)^3}$$

H₁₁₁

$$Q = 0.206 \frac{\lambda_0}{\delta} \frac{\left[1 + 0.73 \left(\frac{2a}{h}\right)^2\right]^{3/2}}{1 + 0.22 \left(\frac{2a}{h}\right)^2 + 0.51 \left(\frac{2a}{h}\right)^3}$$

Common cavity geometries (4)

Coaxial TEM



$$\lambda_{0} = 2h \text{ or } h = \lambda_{0} / 2$$

$$Q = \frac{\lambda_{0}}{\delta} \frac{1}{4 + \frac{h}{b} \cdot \frac{1 + b / a}{\ln(b / a)}}$$
Optimum Q for $(b/a) = 3.6 \quad (Z_{0} = 77\Omega)$

$$Q_{optimum} = \frac{\lambda_{0}}{\delta} \frac{1}{4 + 7.2 \frac{h}{b}}$$

Coaxial line with minimum loss → slide TEM transmission lines (3)

Taken from S. Saad et.al., Microwave Engineers' Handbook, Volume I, p.180

Cavity structures

Common cavity geometries (5)

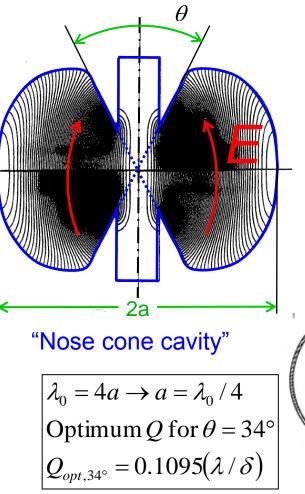
Sphere

Sphere with cones

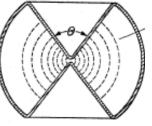


"Energy storage in LEP"

$$\lambda_0 = 2.28a$$
$$Q = 0.318(\lambda / \delta)$$



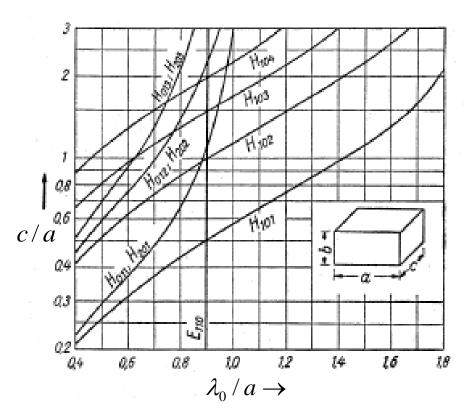
a spherical "λ/4-resonator" Cavity structures



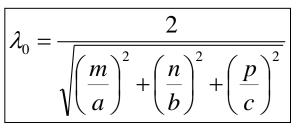
the tips of the cone don't touch

Mode chart of a brick-shaped cavity

Carol G. Montgomery, 1st ed., 1947; by permission, McGraw-Hill Book Co., N. Y. and Reprinted from Meinke, H. and Gundlach, F. W., 1968) Microwave Measurements by Taschenbuch der Hochfrequenztechnik,S Springer-Verlag, Berlin Erste Auflage Techniques of

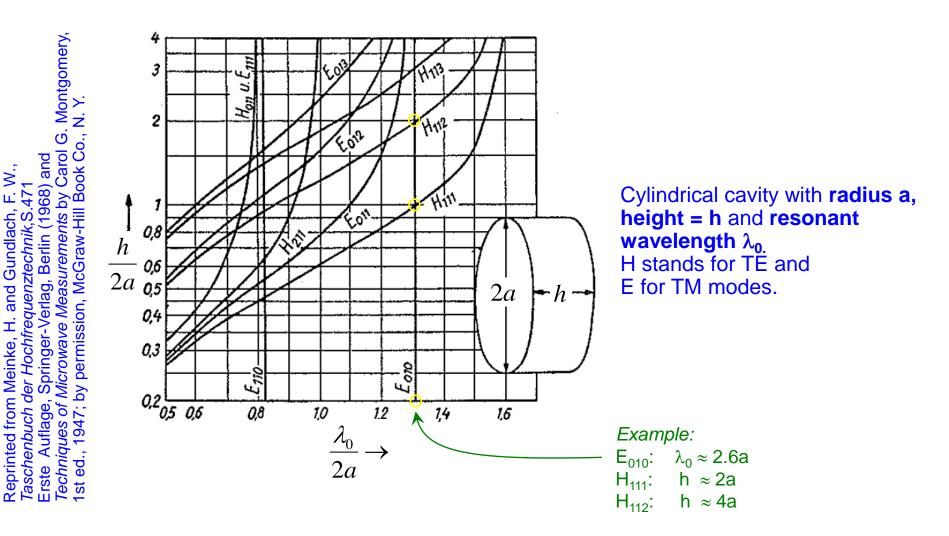


The resonant wavelength of the H_{mnp} resonance calculates as

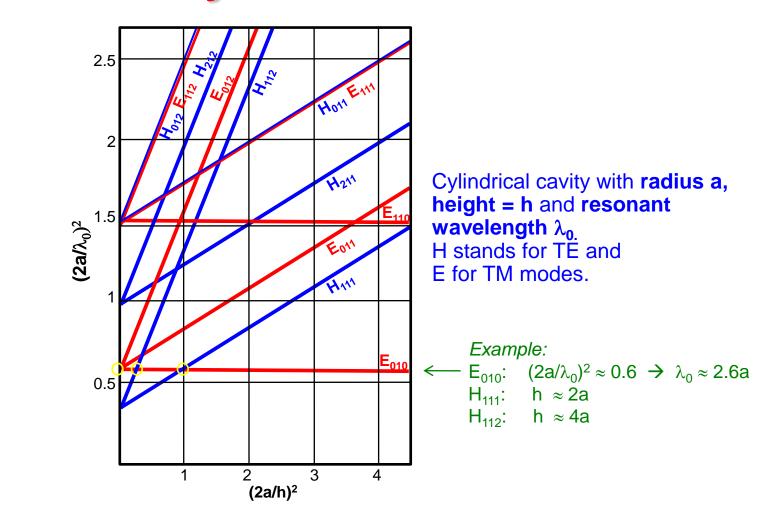


for a $E_{\rm mn}$ or a $H_{\rm mn}$ wave with p half waves along the c-direction.

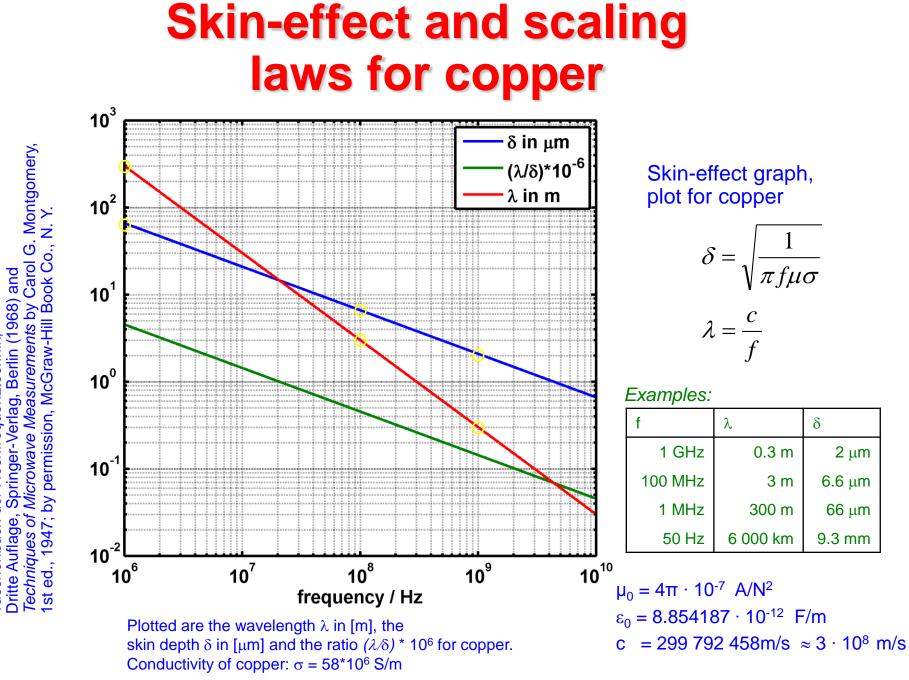
Mode chart of a Pillbox cavity – Version 1



Mode chart of a Pillbox cavity – Version 2



Carol G. Montgomery, 1st ed., 1947; by permission, McGraw-Hill Book Co., N. Y. and Reprinted from Meinke, H. and Gundlach, F. W., Taschenbuch der Hochfrequenztechnik,S.471 1968) Microwave Measurements by Springer-Verlag, Berlin Erste Auflage Techniques of

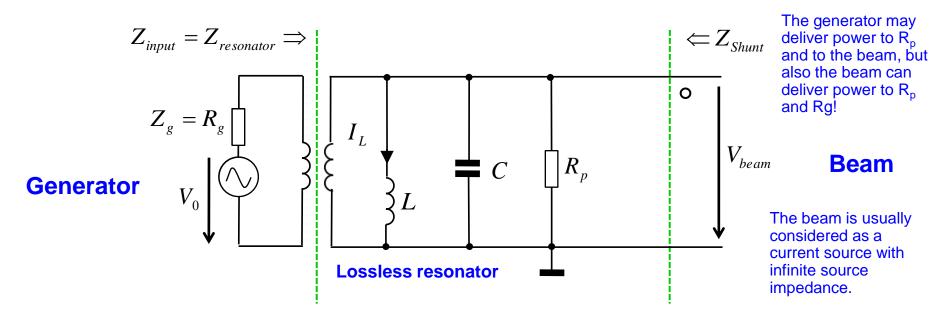


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Reprinted from Meinke, H. and Gundlach, F. W., Taschenbuch der Hochfrequenztechnik,

Cavity structures

Equivalent circuit (1)



 R_p = resistor representing the losses of the parallel RLC equivalent circuit

We have Resonance condition, when
$$\omega L = \frac{1}{\omega C}$$

 \Rightarrow Resonance frequency: $\omega_{res} = 2\pi f_{res} = \frac{1}{\sqrt{LC}} \Rightarrow \int f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$

Equivalent circuit

Equivalent circuit (2)

 $X = \frac{R}{Q} = \omega_{res}L = \frac{1}{\omega_{res}C} = \sqrt{L/C}$ Characteristic impedance "R upon Q" (R/Q) is independent of Q and a pure geometry factor for any cavity or resonator! This formula assumes a HOMOGENEOUS field in the capacitor ! $W = \frac{CV^2}{2} = \frac{LI_L^2}{2}$ Stored energy at resonance $P = \frac{V^2}{2R}$ Dissipated power $Q = \frac{R}{X} = \frac{\omega_{res}W}{P}$ Q-factor $R = \frac{V^2}{2P}$ Shunt impedance (circuit definition) $\frac{\Delta f}{f} = -\frac{1}{2} \frac{\Delta C}{C}$ Tuning sensitivity Coupling parameter (shunt impedance $k^2 = \frac{R}{R_{innut}}$ over generator or feeder impedance Z)

Equivalent circuit

The Quality Factor (1)

 The quality (Q) factor of a resonant circuit is defined as the ratio of the stored energy W over the energy dissipated P in one cycle.

$$Q = \frac{\omega_{res}W}{P}$$

- The Q factor can be given as
 - Q₀: Unloaded Q factor of the unperturbed system, e.g. a closed cavity
 - Q_L: Loaded Q factor with measurement circuits etc connected
 - Q_{ext}: External Q factor of the measurement circuits etc
- These Q factors are related by

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

Reprinted from Madhu S

Number . Dome,

Volume 1 See also:

The Quality Factor (2)

Q as defined in a Circuit Theory Textbook:

$$Q = \frac{\omega_{res}L}{R}$$

Q as defined in a Field Theory Textbook:

 $Q = 2\pi \frac{\text{energy stored in the resonator}}{\text{energy dissipated per cycle}}$

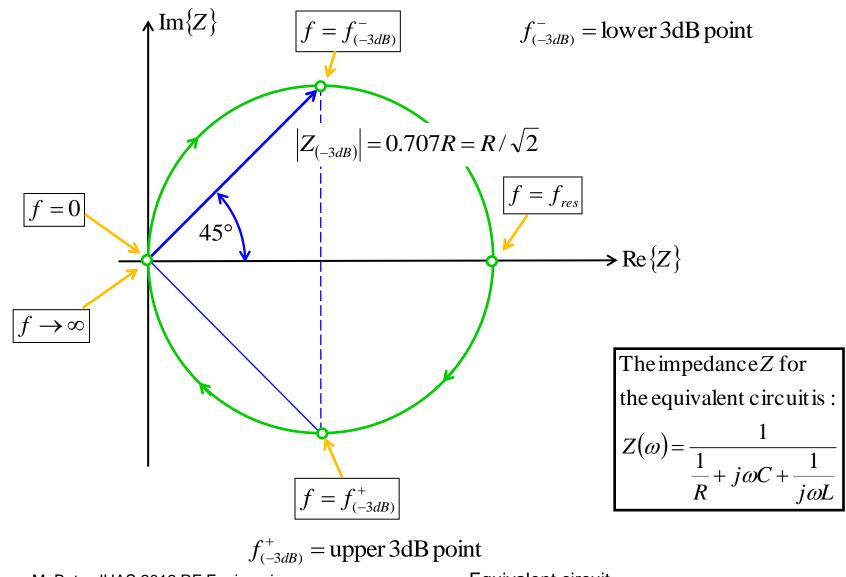
Q as defined in an optoelectronics Textbook:

$$Q = \frac{V_0}{V_{1/2}}$$

 $v_0 =$ the resonant frequency

 $v_{1/2}$ = "full - width at half power maximum" (FWHM)

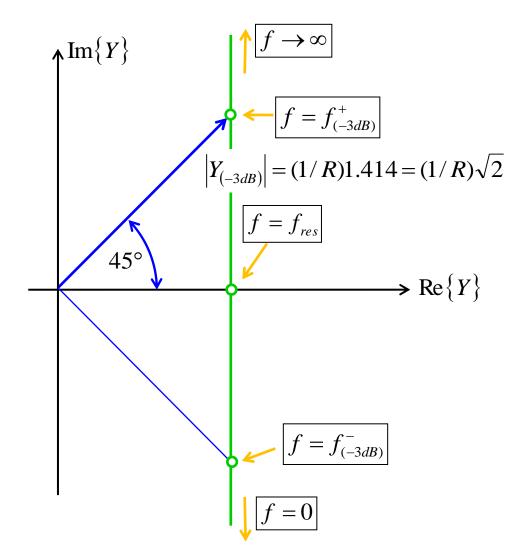
Input Impedance: Z-plane



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Equivalent circuit

Input Admittance: Y-plane

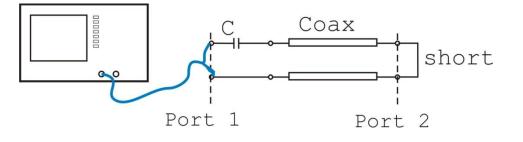


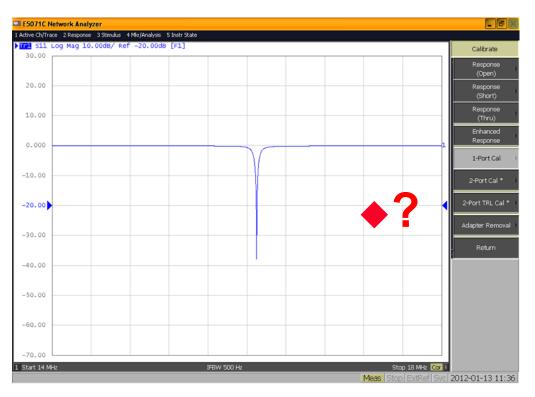
Evaluating the admittance *Y* for the equivalent circuit we get

$$Y = \frac{1}{Z} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$
$$= \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$
$$= \frac{1}{R} + j\frac{1}{R/Q}(\frac{f}{f_{res}} - \frac{f_{res}}{f})$$

Equivalent circuit

Example: Measurement of Q with VNA in Reflection





But how?

This is the recipe:

→ Get the resonance frequency and read out the 3dB-points

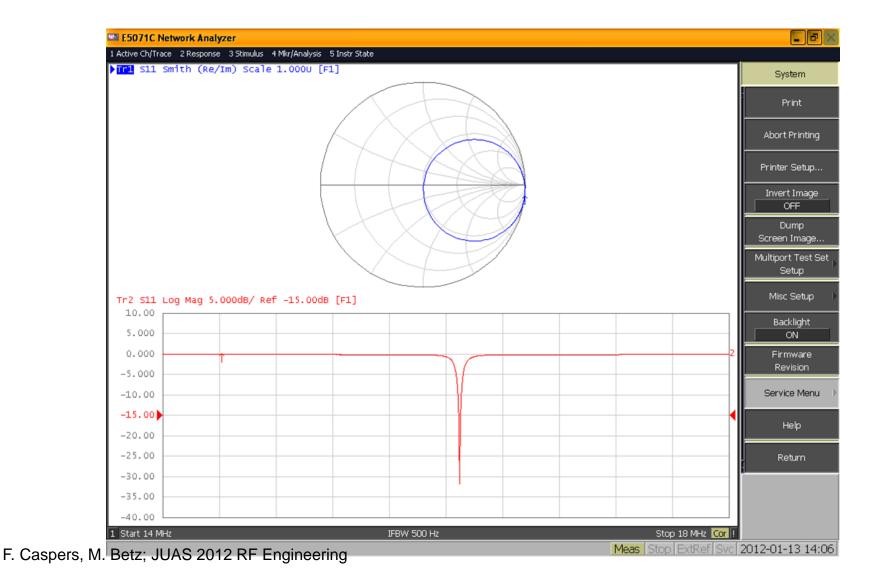
→ Calculate Q = $f_{res}/\Delta f$.

Ooops? Not so straightforward?

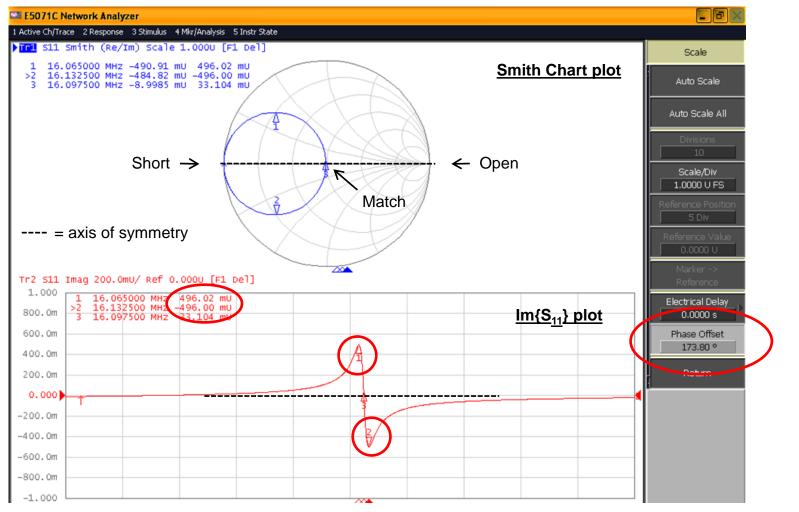


This is "our" recipe: Determine Q in the Smith-chart!

1. Put your network analyzer in Smith-chart mode.



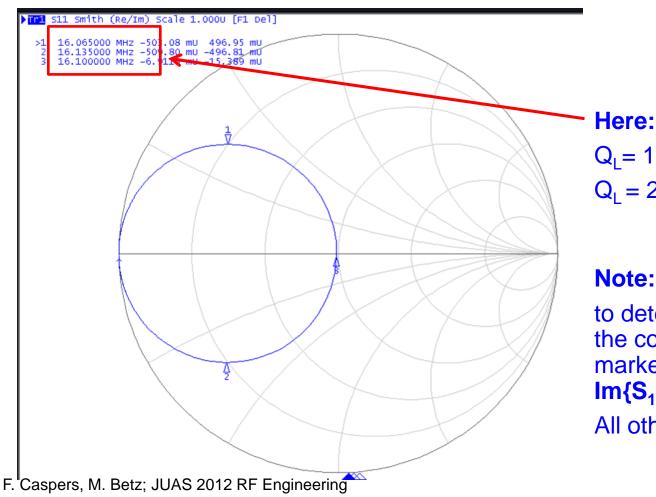
- 2. Move your graph in the Smith-chart to the so-called "detuned short position".
- 3. For this, you display the **imaginary part** of S11 and change the **phase offset** so that the graph is **symmetric to the abscissa**.
 - Hint: Put Markers on the plot to make sure that your graph is symmetric



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- Use **markers** in the Smith-chart to read out the frequencies at points (1) and (2), in the 3. upper and lower halves of the circle, this is the **minimum and maximum of Im{S**₁₁}.
- Calculate the difference in frequency Δf , this is the 3 dB bandwidth of the loaded cavity. 4.
- Read-out the resonant frequency f_{res} at point (3) 5.
- Now your formulae will give you the loaded Q: 4.

$$Q_L = f_{res} / \Delta f$$





 $Q_1 = 16.1 \text{ MHz} / 70 \text{ kHz}$ $Q_1 = 230$

Note:

to determine the unloaded Q_0 , the condition for placing the markers in Step 3 is: $Im{S_{11}} = Re{S_{11}}$

All other steps stay the same.

3 dB bandwidth

In the Z-plane (= impedance) |Z| reduces to 0.707 to the value at resonance.

The real part of Z becomes 50% of the real part of that at resonance.

The phase deviates +- 45 degrees from the phase at resonance.

0.707 in voltage = unit voltage – 3dB (decibel)

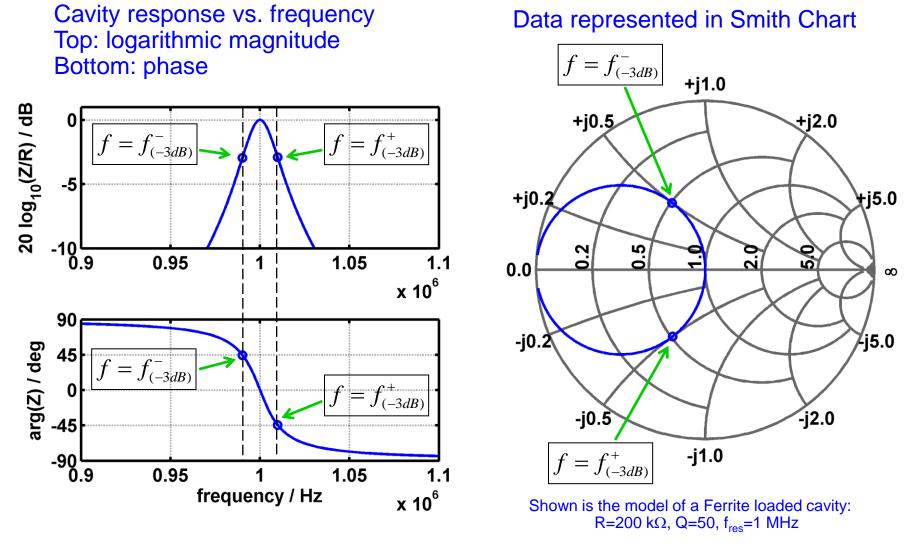
0.707 in voltage = 50% in power since power ~ V^2

The Q factor of a resonance peak or dip can be calculated from the center frequency f_{res} and the 3 dB bandwidth $\Delta f = f^+_{(-3dB)} - f^-_{(-3dB)}$ as $Q = f_{res}/\Delta f$.

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Equivalent circuit

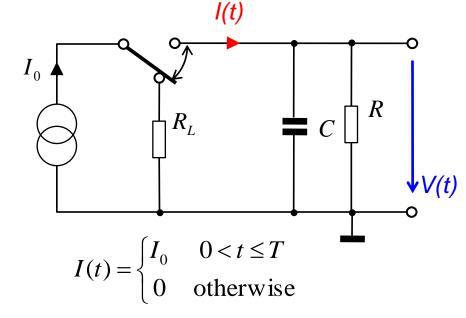
Simulated data



Decibels and Smith Chart are discussed in detail in Part II.

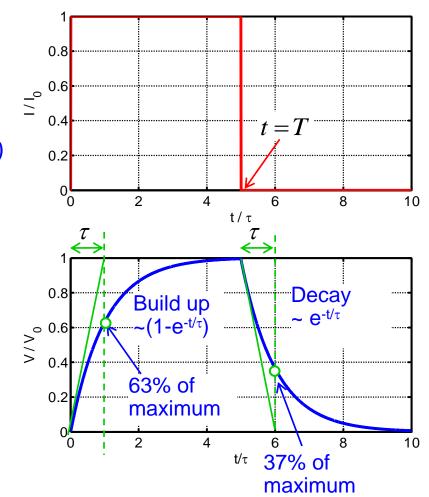
Equivalent circuit

Transients on an RC-Element (1)



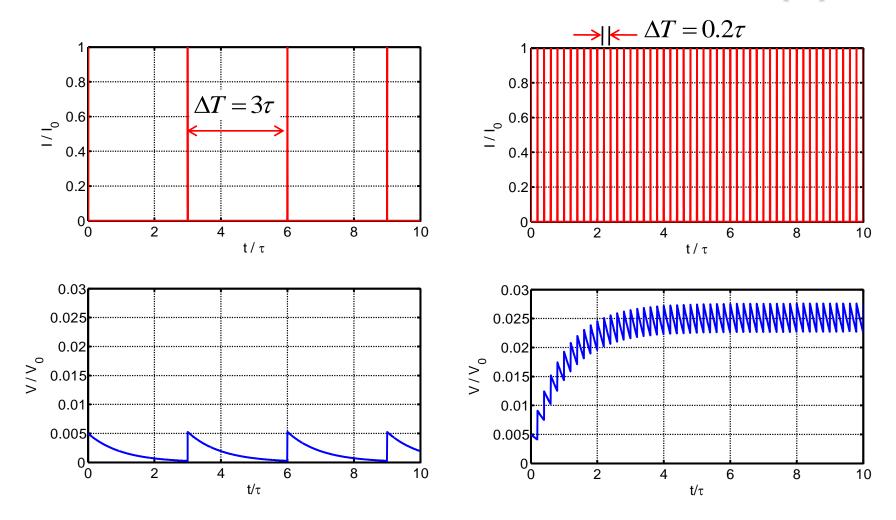
A voltage source would not work here! Explain why.

$$\tau = RC \dots$$
 time constant
 $V_0 = I_0 R \dots$ maximum voltage



Behaviour in time and in frequency domain

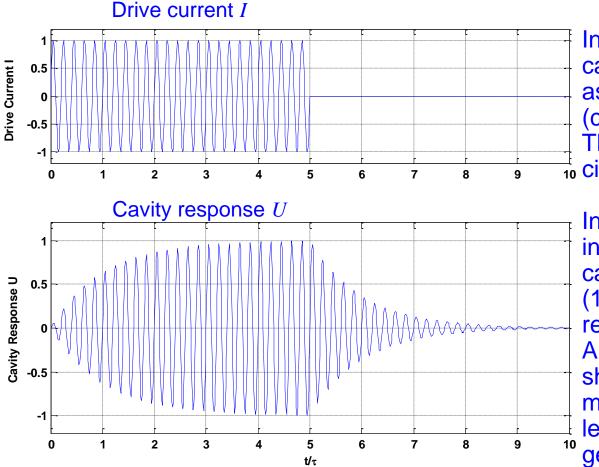
Transients on an RC-Element (2)



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Behaviour in time and in frequency domain

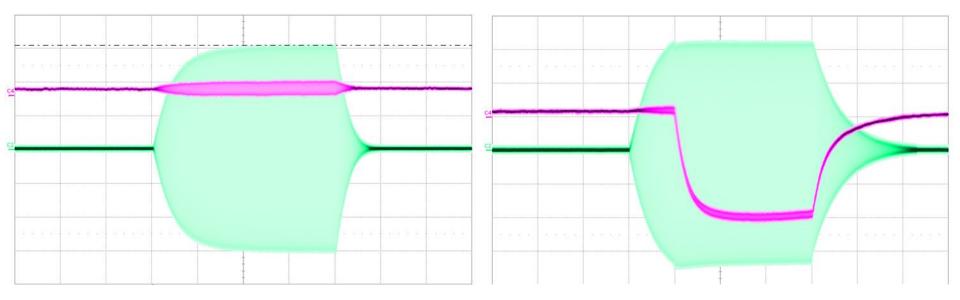
Response of a tuned cavity to sinusoidal drive current (1)



In the first moment, the
 cavity acts like a capacitor,
 as seen from the generator
 (compare equivalent circuit).
 The RF is therefore short circuited

In the stationary regime, the inductive (ωL) and capacitive reactances (1/(ωC)) cancel (operation at resonance frequency!). All the power goes into the shunt impedance R => no more power reflected, at least for a matched generator...

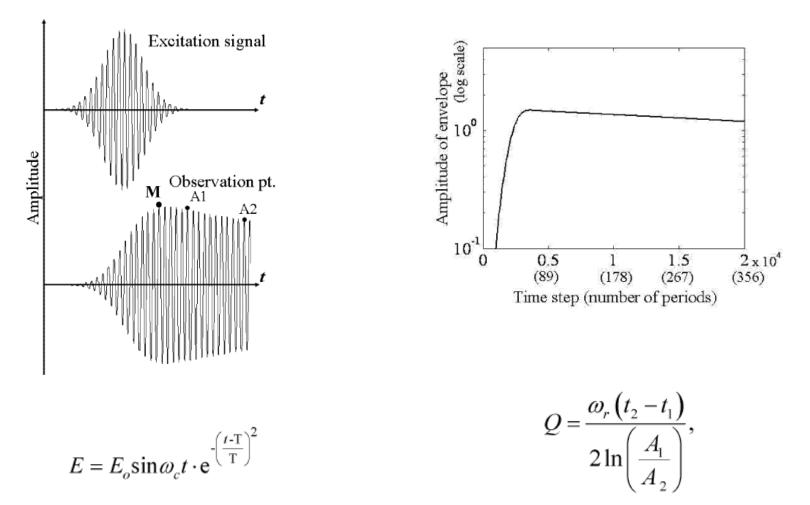
Measured time domain response of a cavity



♦ Cavity E field (green trace) and electron probe signal (red trace) with and without multipacting. 200 µs RF burst duration.

see: O. Heid, T Hughes, COMPACT SOLID STATE DIRECT DRIVE RF LINAC EXPERIMENTAL PROGRAM, IPAC Kyoto, 2010

Numerically calculated response of a cavity in the time domain



see: I. Awai, Y. Zhang, T. Ishida, Unified calculation of microwave resonator parameters, IEEE 2007 F. Caspers, M. Betz; JUAS 2012 RF Engineering

Response of a tuned cavity to sinusoidal drive current (2)

Differential equation of the envelope

(shown without derivation)

$$\dot{V} = \frac{1}{2C} = (I - \frac{V}{Z}) = \frac{1}{2ZC}(IZ - V)$$

V, V, I, Z are complex quantities, evaluated at the stimulus (drive) frequency.

For a tuned cavity all quantities become real. In particular Z = R, therefore

$$\dot{V} = \frac{1}{2RC} (IR - V)$$

 \rightarrow time constant becomes

$$\underline{\tau} = 2RC = 2\frac{R}{Q}QC = \frac{2Q}{\omega_0} = \frac{Q}{\frac{\pi f}{\omega_0}} = \frac{QT}{\frac{\pi}{\omega_0}} = \frac{QT}{\frac{\pi}{\omega_0}}$$
"Q over π periods"

V... envelope amplitudeC... cavity capacitanceI... drive currentZ... cavity impedanceR... real part of cavityimpedance

This τ value refers to the 1/e decay of the <u>field</u> in the cavity. Sometimes one finds τ_w referring to the energy with $2\tau_w = \tau$.

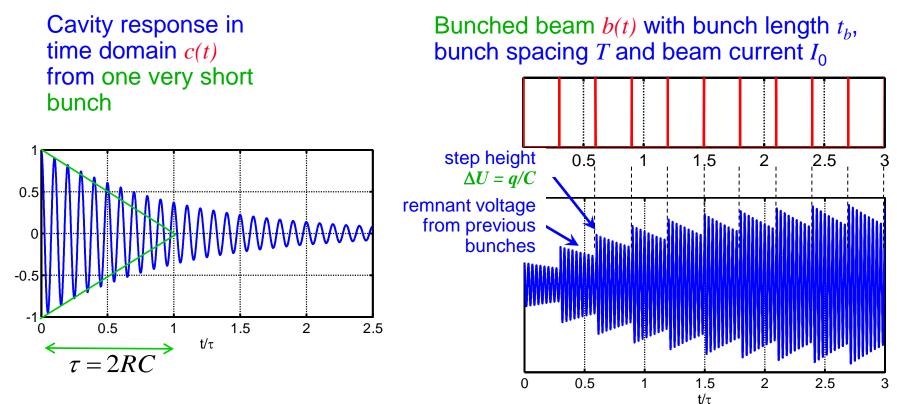
The voltage (or current) decreases to 1/e of the initial value within the time τ .

see also: H. Klein, Basic concepts I Proceeding Oxford CAS, April 91 CERN Yellow Report 92-03, Vol. I

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Behaviour in time and in frequency domain

Beam-cavity interaction (1)



Resulting response for bunched beam obtained by convolution of the bunch sequence with the cavity response $r(t) = b(t) \otimes c(t)$ Condition that the induced signals in the cavity add up: cavity resonant frequency f_{res} must be an integer multiple of bunch frequency 1/T

Beam-cavity interaction (2)

For a quantitative evaluation the worst case is considered with the induced signals adding up in phase.

Two approaches:

• Equilibrium condition: Voltage drop between two bunch passages compensated by newly induced voltage

$$U_{end} e^{-T/\tau} = U_{step} = U_{end} - \frac{q}{C} \implies U_{end} = \frac{q}{C} \frac{1}{1 - e^{-T/\tau}}$$

Summing up individual stimuli

$$U_{end} = \frac{q}{C} (1 + e^{-T/\tau} + e^{-2T/\tau} + \dots) = \frac{q}{C} \frac{1}{1 - e^{-T/\tau}}$$

Approximation for $T/\tau \ll 1$:

$$1 - e^{-T/\tau} = 1 - (1 - T/\tau + ...) \approx T/\tau$$

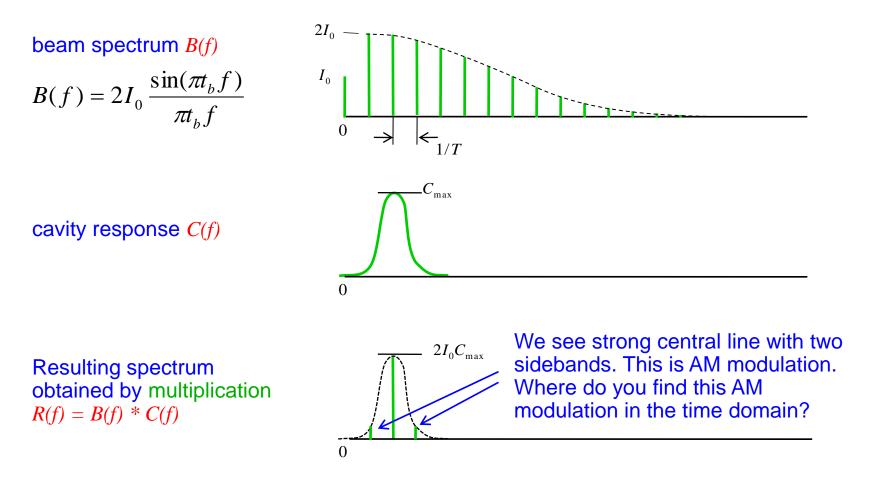
$$\underline{U_{end}} = \frac{q}{C} \frac{1}{T/\tau} = \frac{q}{C} \frac{2RC}{T} = 2R\frac{q}{T} = \frac{2RI_0}{T}$$

where I_0 is the mean beam current.

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Beam-cavity interaction in Frequency domain

• Frequency domain



Typical parameters for different cavity technologies

Cavity type	<i>R/Q</i>	Q	R
Ferrite loaded cavity (low frequency, rapid cycling)	4 kΩ	50	200 kΩ
Room temperature copper cavity (type 1 with nose cone)	192 Ω	30 * 10 ³	5.75 M Ω
Superconducting cavity (type 2 with large iris)	50 Ω	$1 * 10^{10}$	500 GΩ

Different definitions of the shunt impedance r

Four different parameters => confusion can be maximized by using $2^4 = 16$ different definitions...

Example: Pillbox cavity $r = 3.3 M\Omega$ L = 0.2 m $cos(\phi) = 0.866$ transit-time factor T = 0.756 (defined later, see transit time factor slides!)

Linac and electrical definition most often used.

Linac definition:

$$P = \frac{\hat{U}^2}{R}$$
 with the peak voltage \hat{U}

Electrical (or circuit) definition for circular machines uses the <u>effective</u> voltage U => factor 2

 $P = \frac{\hat{U}^2}{2R}$

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Full correct shunt impedance designation	cos(φ) included	T included	L included	LINAC definition	Value
r (electrical def.)	0	0	0	0	3.3 MΩ
R (Linac def.)	0	0	0	1	6.6MΩ
r/L	0	0	1	0	16.5 MΩ/m
R/L (effective shunt impedanceZ)	0	0	1	1	33.0 MΩ/m
rT ² (electrical def. with T)	0	1	0	0	1.88 MΩ
RT ² (Linac def. with T)	0	1	0	1	3.77 MΩ
rT ² /L	0	1	1	0	2.86 MΩ/m
RT ² /L	0	1	1	1	5.72 MΩ/m
$r \cos^{2(\phi)}$	1	0	0	0	2.47 MΩ
$R \cos^{2(\phi)}$	1	0	0	1	4.95 MΩ
$r \cos^{2(\phi)}/L$	1	0	1	0	12.37 MΩ/m
$R \cos^{2(\phi)}/L$	1	0	1	1	24.75 <i>M</i> Ω/m
$rT^2\cos^2(\phi)$	1	1	0	0	1.41 MΩ
$RT^2\cos^2(\phi)$	1	1	0	1	2.83 MΩ
$rT^2\cos^2(\phi)/L$	1	1	1	0	7.07 MΩ/m
$RT^2\cos^2(\phi)/L$	1	1	1	1	14.14 <i>M</i> Ω/m

Electromagnetic scaling laws

A cavity of a given geometry can be scaled using three rules:

- The ratio of any cavity dimension to λ is constant. To put it another way, all cavity dimensions are inversely proportional to frequency
- Characteristic impedance R/Q = const.

•
$$Q * \delta / \lambda = \text{const.}$$

The skin depth δ is given by

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}}$$

with the conductivity σ , the permeability μ , and the angular frequency $\omega = 2\pi f$.

Note that it is proportional to $\frac{1}{\sqrt{f\sigma}}$ For instance, in copper ($\sigma_{copper} = 5.8^{*}10^{7}$ S/m) the skin depth is ≈9 mm at 50 Hz, while it decreases to ≈ 2 µm at 1 GHz.

Scaling laws

Scaling of a pillbox-type cavity

<u>Starting point:</u> SUPERFISH simulation results for a cavity of a given geometry with copper walls. Parameters: f = 3030 MHz, $Q_0 = 9625$ and R = 631 k Ω

<u>Question:</u> What are the characteristic parameters (Q, R/Q, λ) of a cavity of similar shape, that operates at a frequency of 814.5 MHz, built with steel walls? ($\sigma_{copper} = 58 \text{ MS/m}$, here we assume $\sigma_{steel} \approx 2 \text{ MS/m}$)

<u>Answer:</u> For the first cavity we find Skin depth $\sigma_1 = 1.195 \ \mu m$, Resonant wavelength $\lambda_1 = c/f_1 = 98.97 \ mm$, $Q_1 * \sigma_1 / \lambda_1 = 0.1162$

For the larger steel cavity all dimensions have to be scaled by the inverse frequency ratio f_1/f_2 , which gives a factor of 3030/814.5 = 3.72 = $3.72 \lambda_1 = 3.68 \text{ mm}$

The characteristic impedance remains unchanged. $R_2/Q_2 = R_1/Q_1 = 632 * 10^3 / 9625 = 65.56 \Omega$

The skin depth for steel at 814.5 MHz is $\sigma_2 = 12.5 \ \mu\text{m}$. Using $Q_1 * \sigma_1 / \lambda_1 = Q_2 * \sigma_2 / \lambda_2$ we find $Q_2 = 3420$

Finally, the shunt impedance gets $\underline{R}_2 = (R_1/Q_1) * Q_2 = 65.56 * 3420 = \underline{224 \text{ k}\Omega}$

Simulation Tools

- Poisson Superfish (poisson equation; poisson = fish in french)
- Microwave Studio, Mafia (Maxwell's finite integration algorithm), <u>http://www.cst.com</u>
- Ansoft HFSS (High frequency structure simulator), <u>http://www.ansoft.com</u>
- GdfidL ("Gitter drauf fertig ist die Laube" no joke, really true!)

Simulation Techniques (1)

Frequency domain analysis

- CST Microwave Studio 2009, HFSS 12.0
- Uses a tetrahedral mesh
- Maxwell's equations solved in frequency domain for one frequency point at a time
- Frequency sweeps take very long time, very powerful PC or computer cluster needed!
- Applications: quite universal

Time domain analysis

- Microwave Studio
- Space is discretized by a rectangular mesh
- Excitation of structure with time domain pulse
- Transformation to frequency domain by Fourier Transform => entire frequency range with only one run => fast!!!
- Bad convergence for resonant structures, since pulse does not decay fast
- Applications: Waveguide transitions, connectors, antennas, but no resonant structures such as cavities!!!

Simulation techniques

Simulation Techniques (2)

• Eigenmode analysis

- Microwave Studio, Mafia, HFSS, Superfish …
- Allows to calculate eigenmodes of resonant structures
- Used for instance to determine resonant frequencies of cavities, including higher order modes

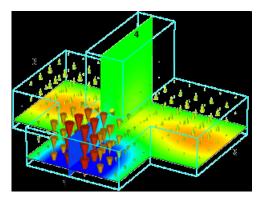
The Mesh

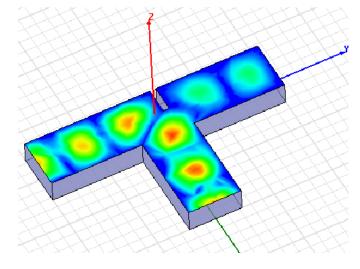
- Space discretized by a mesh
- Mesh width in the order of a tenth of the wavelength in the material
- Successive mesh refinement to improve precision
- Expert systems or user determine critical regions where mesh needs to be denser
- Magic T shown below: Roughly 10⁵ mesh cells and a few seconds to minutes simulation time on a presentday PC

3D Simulation examples

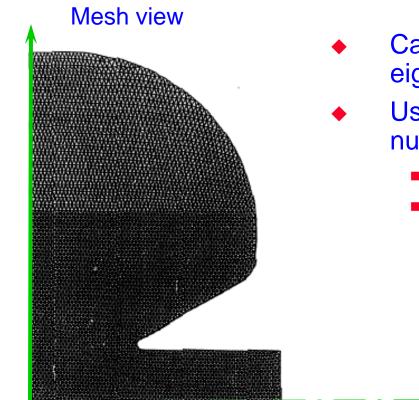
- A Magic T with Microwave Studio 4.3
 - Arrows show the E field of the TE₁₀ mode
 - Power goes in at the front port
 - How much power gets out by the other ports?
- A T-junction with HFSS 9.0
 - Junction with conducting iris
 - Magnitude of TE₁₀ electric field







Superfish: 2 ¹/₂ D simulation

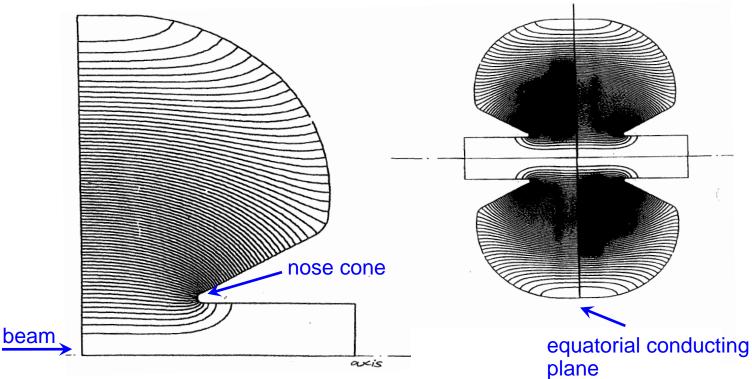


- Calculate resonant modes using eigenmode analysis
- Use symmetries to reduce the number of mesh points!!!
 - rotational symmetry around axis
 - left-right symmetry by defining metallic boundary (electric field lines perpendicular to this plane)

Field pattern

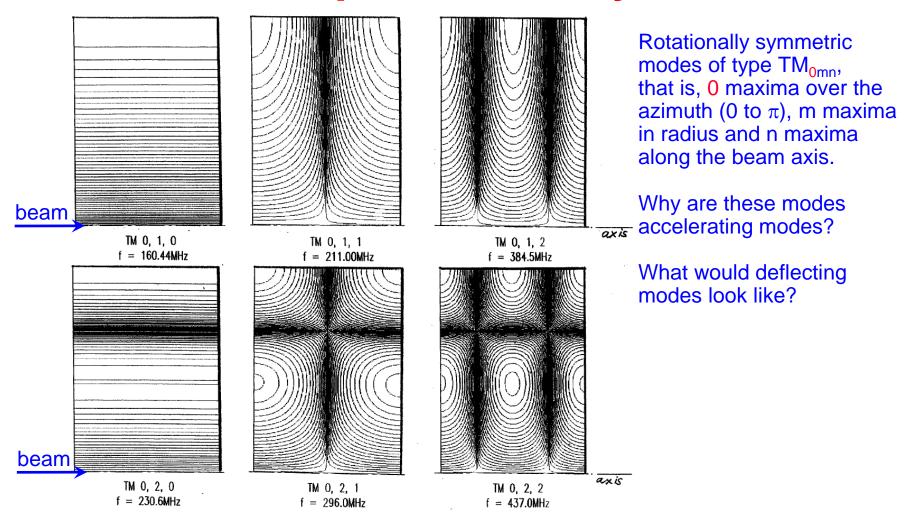
type-1 cavity with all symmetries exploited

entire type-1 cavity



The electric field lines are plotted

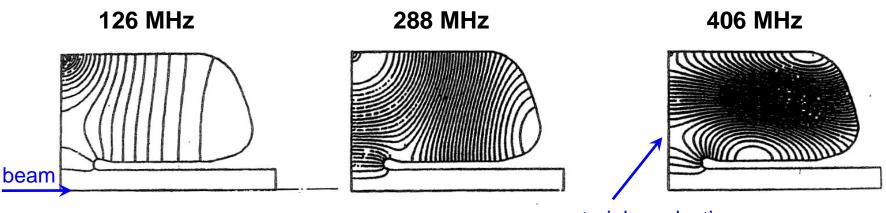
Accelerating modes in a pillbox cavity



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Higher order modes

Higher order modes (HOMs)



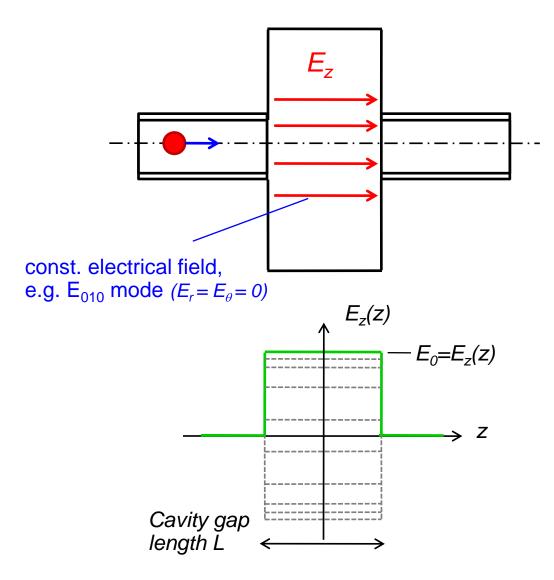
equatorial conducting plane

Higher order modes in a 100-MHz cavity. All these modes are TM type modes. This is due to the boundary condition: electric wall in equatorial plane. references: G. Rogner, CERN report SPS/SME/Note 86-65

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Higher order modes

Transit time factor (1)



The "voltage" in a cavity along the particle trajectory (which coincides with the axis of the cavity) is given by the integral along this path for a fixed moment in time:

$$V = \int_{L} E_{z}(z) \, dz$$

But: the field in the cavity is varying in time:

$$E_{z}(z,t) = E_{z}(z)f(t)$$
$$= E_{z}(z)\cos(\omega t + \varphi)$$

Thus, the field seen by the particle is

$$V = E_0 \int_{-L/2}^{L/2} \cos(\omega t + \varphi) dz$$

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Transit time factor (2)

The transit time factor describes the amount of the supplied RFenergy that is effectively used to accelerate the traversing particle.

Usually, as a reference the moment of time is taken when the longitudinal field strength of the cavity is at its maximum, i.e. $\cos(\varphi)=1$. A particle with infinite velocity passing through the cavity at this moment would see

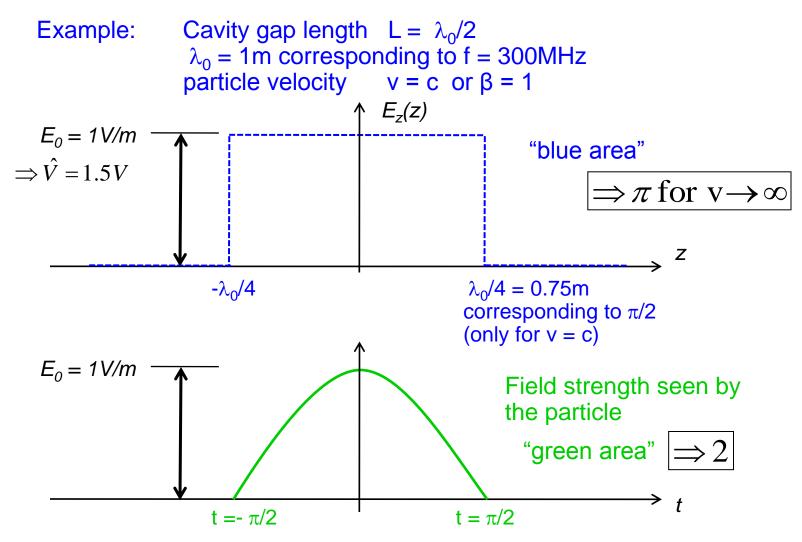
$$\hat{V} = E_0 L$$

Now the particle is sampling this field with a <u>finite velocity</u>. This velocity is given by $v = \beta c$. The resulting transit time factor returns therefore as

$$T = \sin\left(\frac{L}{2}\frac{\omega}{\beta c}\right) / \left(\frac{L}{2}\frac{\omega}{\beta c}\right)$$

Transit time factor, p.565f. ,Alexander Wu Chao, Handbook of Accelerator Physics and Engineering

Transit time factor (3)



Acceleration

We have "slow" particles with β significantly below 1. They become faster when they gain energy and in a circular accelerator with fixed radius we must tune the cavity (increase its resonance frequency).

When already highly relativistic particles become accelerated (gaining momentum) they cannot become significantly faster as they are already very close to c, but they become heavier. Here we can see very nicely the conversion of energy into mass. In this case no or little tuning of the resonance frequency of the cavity is required. It is sufficient to move the frequency of the RF generator within the 3dB bandwidth of the cavity.

Fast tuning (fast cycling machines) can only be done electronically and is implemented in most cases by varying the inductance via the effective μ of a ferrite.

Tuning of cavities (1)

Slater's perturbation theorem: $\frac{2}{3}$

$$\frac{\Delta f}{f} = -\frac{1}{2} \frac{\Delta W}{W}$$

with W designating the energy stored in the cavity



- In regions of high magnetic field
- increases resonant frequency (△W < 0)

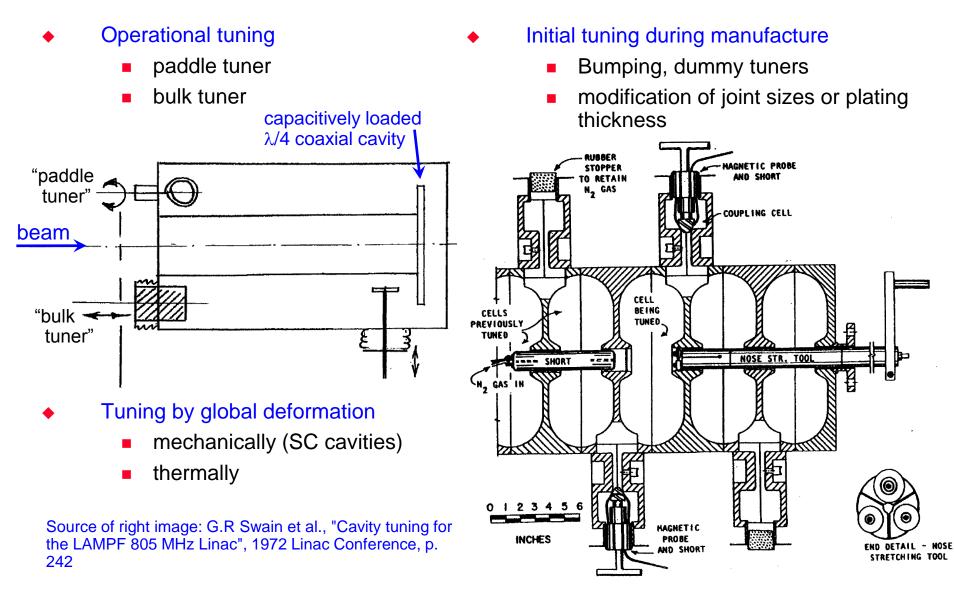
$$\Delta W = -\frac{LI^{2}}{2}; \ dW = -\frac{\mu_{0}\mu_{r}H^{2}}{2}dV$$

Capacitive tuner

- In regions of high electric field
- decreases resonant frequency (△W > 0)

$$\Delta W = \frac{CI^2}{2}; \quad dW = \frac{\varepsilon_0 \varepsilon_r E^2}{2} dV$$

Tuning of cavities (2)

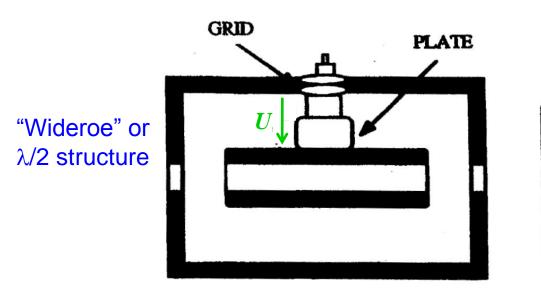


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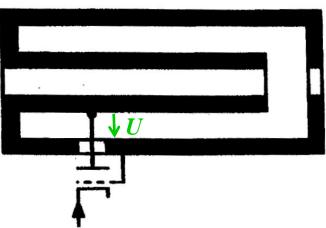
Coupling and Tuning

Coupling cavities to the outside world (1)

Direct coupling (DC coupling)
 Generator (tube) has to "see" a certain voltage U



basic $\lambda/4$ - resonator



Source: M. Puglisi: "Conventional RF cavity design" CAS "RF engineering for Particle Accelerators", CERN 92-03, Vol. 1

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Coupling and Tuning

Coupling cavities to the outside world (2)

• Inductive coupling Generator requirement:

$$U = \sqrt{2PZ}$$

P ... required power Z ... optimum load resistance Induced voltage in loop:

$$U = \mu_0 \frac{d}{dt} \int_{S} H ds$$

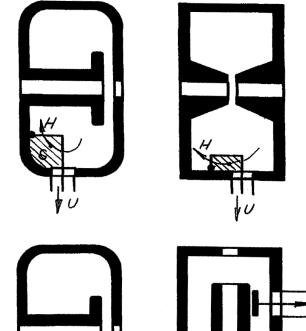
• Capacitive coupling Generator requirement:

$$I = \sqrt{2P / Z}$$

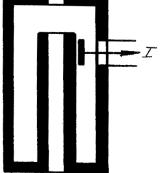
Induced displacement current

$$U = \varepsilon_0 \frac{d}{dt} \int_{S} E ds$$

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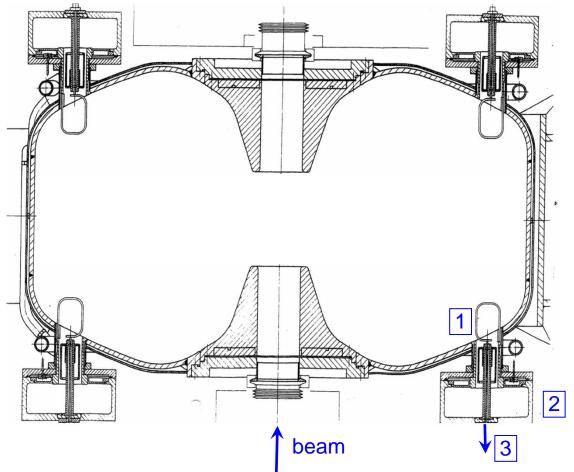




Coupling and Tuning

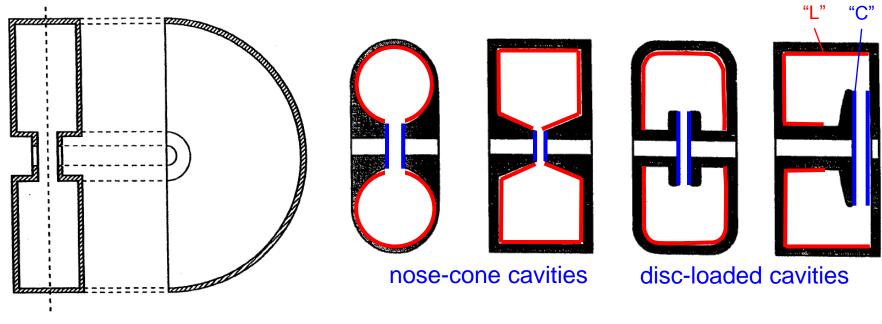
General example

A single-cell configuration: 114-MHz room temperature cavity of CERN PS. Type I profile with nose-cone to optimize shunt impedance



1: higher order mode (HOM) coupling loop which serves for eliminating beaminduced power 2: HOM filter 3: HOM power guided towards load and dissipated

Different forms of the pillbox cavity



Cross section of a radial cavity

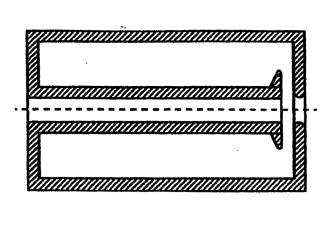
Four different cross sections of fundamentally similar cavities. In spite of their similarity they have been given different names...

Source: M. Puglisi: "Conventional RF cavity design" CAS "RF engineering for Particle Accelerators", CERN 92-03, Vol. 1

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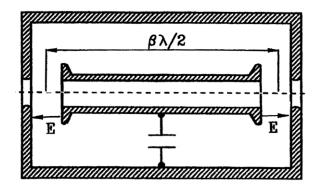
Different forms of cavities

The coaxial (TEM-mode) cavity



basic $\lambda/4$ - resonator

 $L=\beta\lambda/2$



"Wideroe" or $\lambda/2$ structure

works for β =0.2-0.4 does not work for β =1 as we are in a quasi-static case

modified $\lambda/4$ - resonator for acceleration in $\beta\lambda/2$ -mode

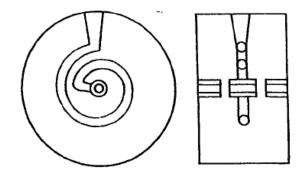
Source: M. Puglisi: "Conventional RF cavity design" CAS "RF engineering for Particle Accelerators", CERN 92-03, Vol. 1

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Different forms of cavities

Spiral resonators

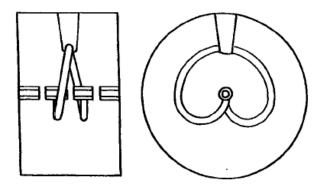
- For small β relatively low RF
 frequencies have to be used
- Drift tubes are mounted on λ/4 lines acting as λ/4 resonators (will be treated in second part of lectures)
- Long λ/4 lines coiled up to make structure smaller



Spiral resonator.



A β = 5.4 % power resonator

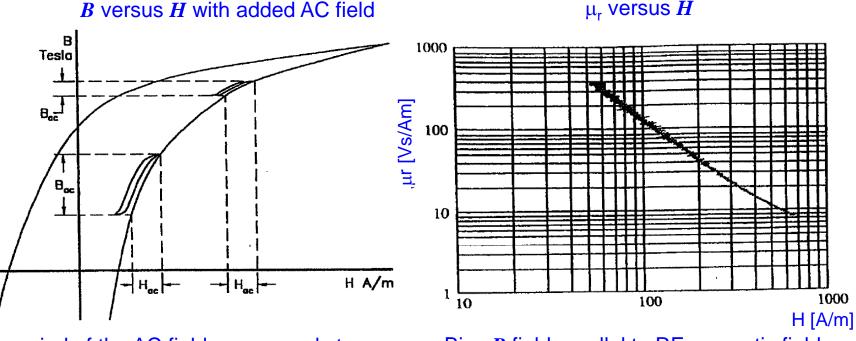


Split-ring resonator.

Different forms of cavities

Ferrite loaded cavities (1)

 Tuning possible by choosing an appropriate static or slowly varying magnetic bias field => differential μ adjustable. Bias field and RF field are parallel.



A period of the AC field corresponds to one round in one of the little hysteresis loops.

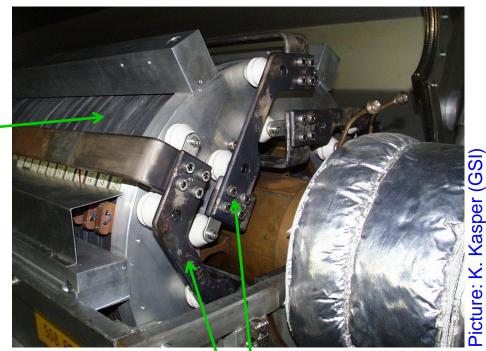
Bias *B* field parallel to RF magnetic field in above plot

Ferrite loaded cavities (2)

This is essentially a $\lambda/4$ cavity with magnetically variable length.

Ferrite toroidal discs, interleaved with copper sheets on "equipotential lines" for cooling.

ceramic window



Source: H. Damerau (CERN), private communication Cavity in the SIS (Schwerlonen Synchrotron) at GSI, frequency range from 0.8 – 5.4 MHz Bus bars to supply the DC bias, the **DC field is parallel (azimuthally)** to the RF field.

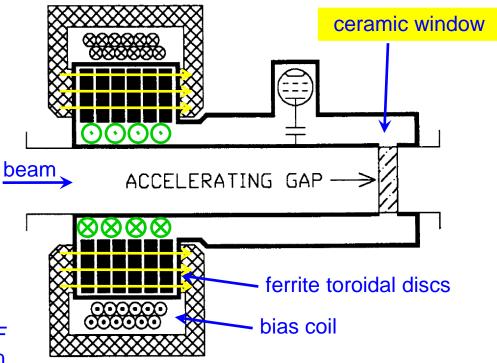
Different forms of cavities

Ferrite loaded cavities (3)

- Ferrite loading makes line electrically longer => cavity size can be reduced
- Bias *B* field in ferrite orthogonal to RF magnetic field
- tunable between 46 and 61 MHz by variable bias field

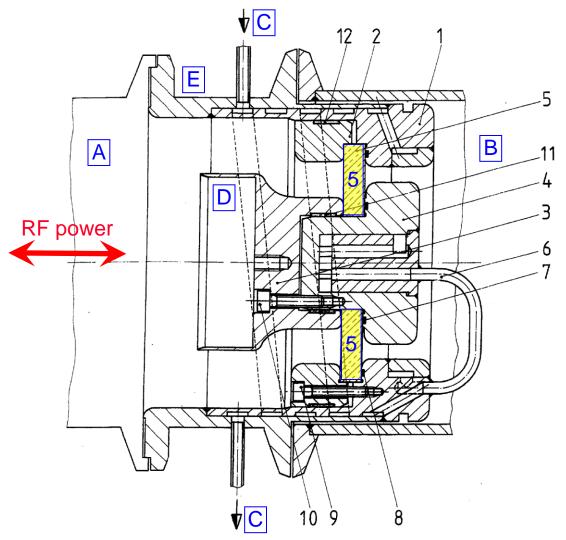
In this mode of operation of orthogonal bias, the DC field is orthogonal to the RF field. The μ of the ferrite can be varied in a more efficient way as compared to parallel magnetic bias.

Tunable cavity for TRIUMF (Three Universities Meson Facility, Vancouver) designed by LANL (Los Alamos).



From: ISK Gardner: "Ferrite dominated cavities" CERN 92-03, Vol. II [2]

RF window



An RF window for a 114 MHz LEP cavity

On which side is the vacuum?

How does the structure continue on the left side?

5: Ceramic disc 6: Coupling loop 7: Vacuum seal

A: Pressurized side (air)

- B: Cavity side (vacuum)
- C: Cooling water ducts
- D: Inner conductor
- E: Outer conductor

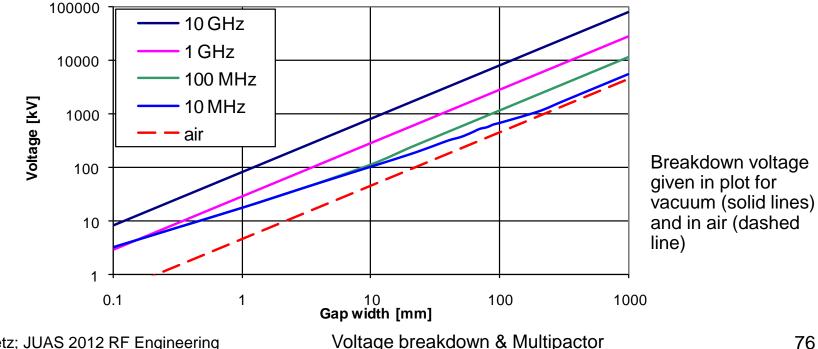
"Kilpatrick" voltage breakdown (1)

The maximum E field achievable is limited by a process known as RF breakdown. The Breakdown voltage is given by

 $W \cdot E^2 \cdot e^{\frac{-1.7 \cdot 10^5}{E}} = 1.8 \cdot 10^{14}$

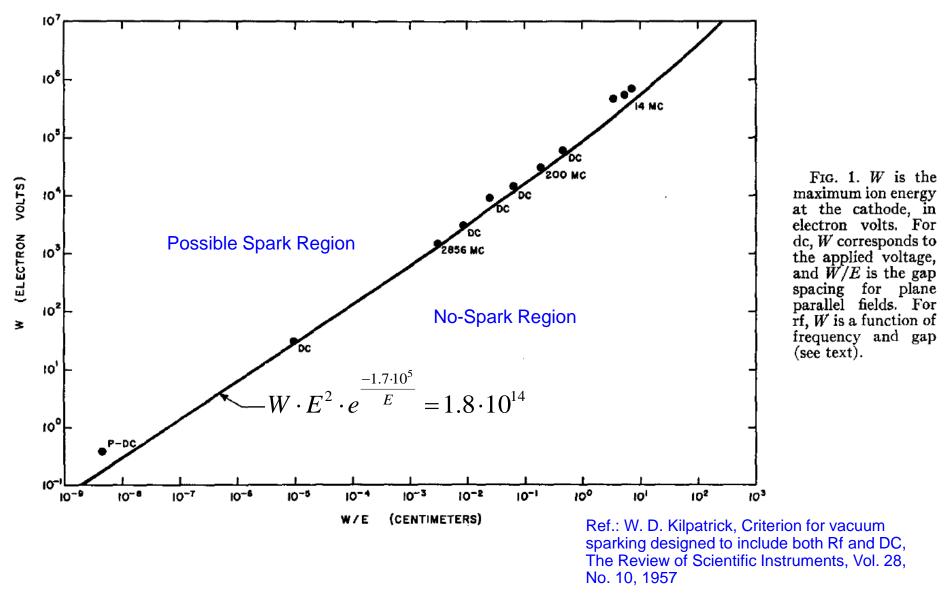
where W [eV] is the impact energy of the electrons and E the electric field [V/m] (W = E * gap width for DC, W < E * gap width for RF).

- High power effect
- Destructive!!
 - Breakdown voltage proportional to square root of frequency



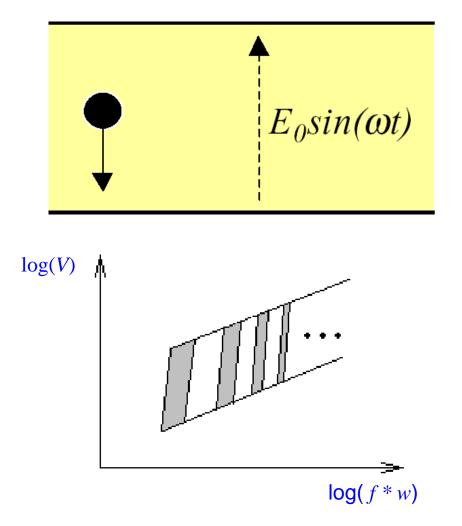
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"Kilpatrick" voltage breakdown (2)



Voltage breakdown & Multipactor

Multipactor (1)



Basically,

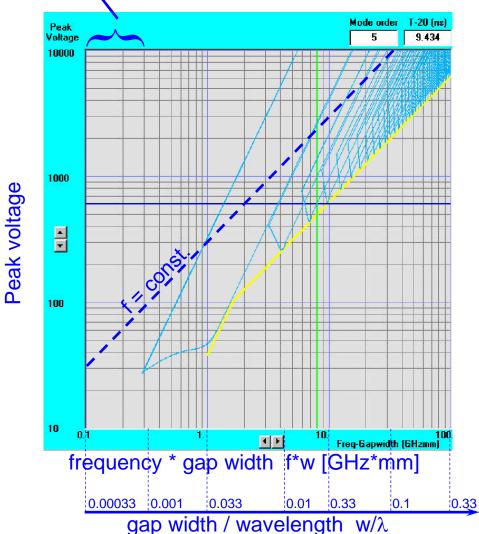
- Electrons get accelerated in an electric field
- When they hit the wall, secondary electrons are freed
- If the electric field changes sign as it is the case for RF fields, the secondary electrons will eventually see an accelerating field
- Therefore, at least for some distinct frequency bands and accelerating voltages, resonance effects can be expected

V ... gap voltage *f* ... RF frequency *w* ... gap width

Voltage breakdown & Multipactor

No multipactor for very small gap width or very high frequencies

Multipactor (2)

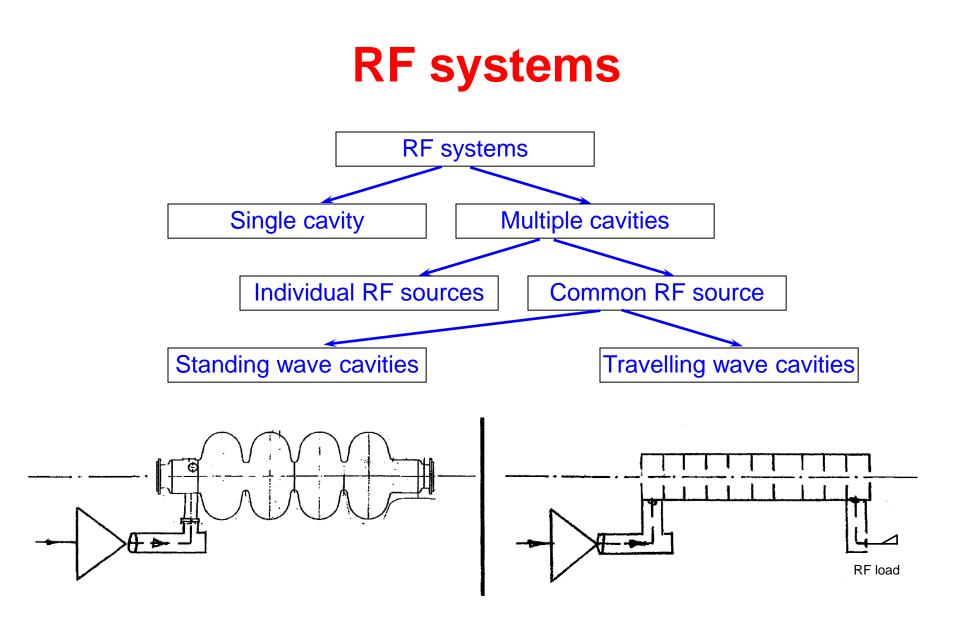


More formally,

- Multipactor is a resonant avalanche discharge, typically a low-power effect.
- The basic "two-point" resonance condition is met if the time of flight of an electron between electrodes equals an odd number of RF half cycles
- Other necessary condition: The coefficient of secondary electron emission must be larger than 1. This corresponds to an energy range between 50 eV and 5000 eV for copper surfaces

Multipactor calculator available at http://www.estec.esa.nl/multipac/

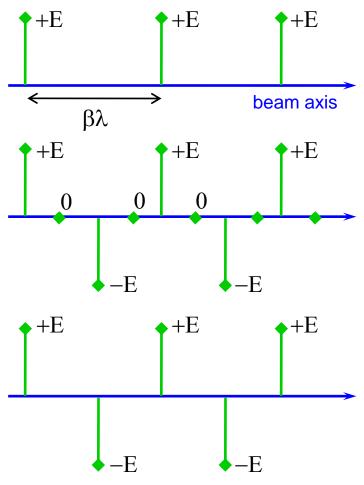
Voltage breakdown & Multipactor



Standing wave cavities

- The only possible phase differences between the SW fields in lossless cells are 0° or 180°.
- For N cells there are N possible longitudinal modes. Practically used modes:
 - 0° (zero mode): Gap distance βλ with β = v/c. Structures: Alvarez Drift Tube Line (DTL)
 - 90° (π/2 mode): Distance active cell to coupling cell βλ/4 or βλ/2. Structures: Side coupled, Disk and Washer

180° (π mode): Gap distance βλ/2.
 Structures: Wideroe, superconducting cavities (LHC, TESLA), Interdigital (IH)



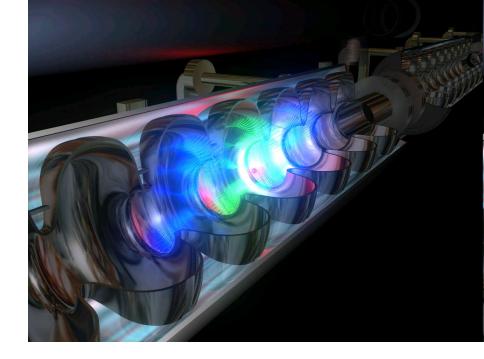
Groups of cavities

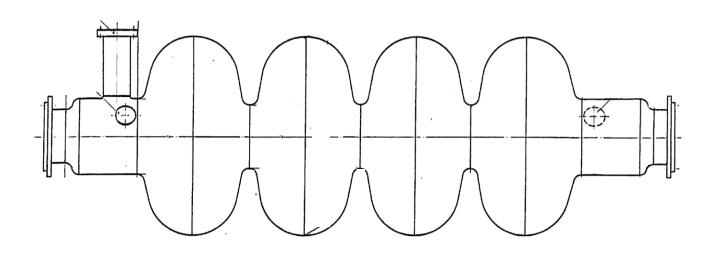
Travelling wave cavities

- Phase difference between adjacent cells can be chosen arbitrarily to assure synchronism with beam. Values around $2\pi/3$ give best compromise between structure length, group velocity (filling time) and overall dissipation.
- Without beam loading, almost the full input power is dissipated in the absorber.
- The field decay due to attenuation of the structure can be taken into account by designing "constant gradient" rather than constant geometry structures.
- There exists a specific amount of beam loading for which all RF power is transmitted to the beam, resulting in zero power dissipated in the absorber =>"fully loaded" structure.
- Repercussion of beam loading and structure transients on generator is minimised.
- Structures
- Loaded waveguide (generally used in electron linacs, e.g. CERN LIL, CLIC ...), Parallel bar

Examples (1)

- Standing wave cavity in multicell configuration
- This superconducting cavity was used in CERN's LEP
- "Type II" profile without nosecone to avoid multipactor and reduce r/Q

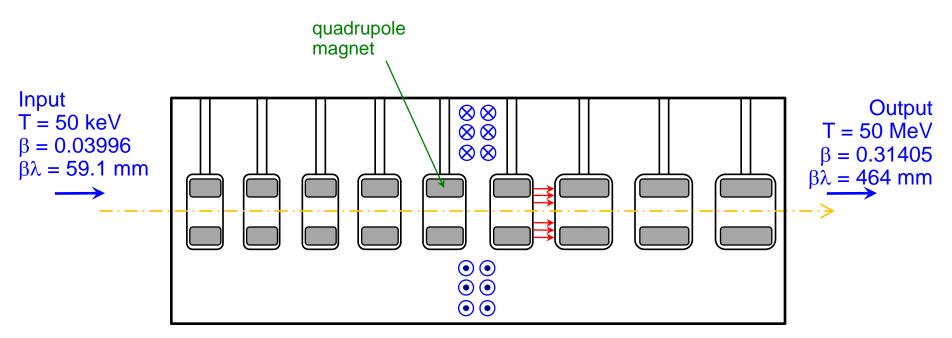




Examples (2)

- Travelling wave cavity
- Below: An ALVAREZ structure (Drift tubes with interposed quadrupole magnets), used in the CERN 50 MeV Proton LINAC Frequency: 202.56 MHz



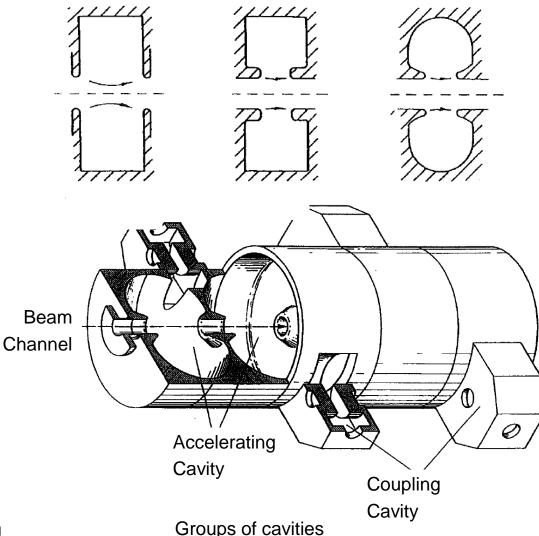


Groups of cavities

Side coupled structures

- Cavity geometry changes to optimize shunt impedance
- Higher shunt impedance => higher accelerating gradient
- Side coupled cavity configuration for optimum shunt impedance

Source: E.A. Knapp and W Shlaer: Design and initial performance of a 20MeV highcurrent side-coupled cavity electron accelerator, 1968 Linac Conference Proceedings, p. 635 to 649



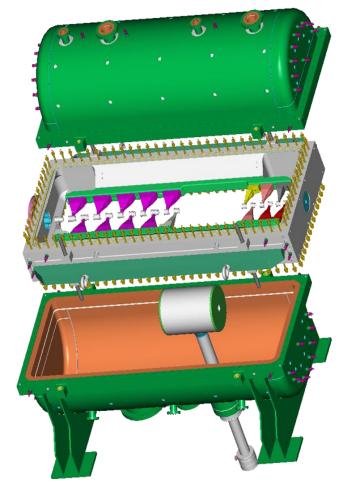
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The IH structure

- IH stands for <u>Interdigital H</u> mode
- Interleaved fingers "adapt" the deformed H (TE) mode that is usually deflecting
- Inside the resonator tank cylindrical cavity drift tubes of varying length (matching the ion velocity) are mounted alternating on opposite sides. The magnetic field lines are parallel to the beam axis and the induced currents flow azimuthally on the wall, creating electric fields of alternating direction between the drift tubes. This field forces the ions forward.
- The big pot is necessary for transverse focusing.

Properties of the structure on the right: $E_{in} = 300 \text{ keV}, E_{out} = 1.1-1.2 \text{ MeV}$ Electrode voltage $V_{eff} = 4.05 \text{ MV}$ Tank length L = 1.5 m Number of gaps = 20 Peak power consumption $W_{peak} = 36 \text{ kW}$



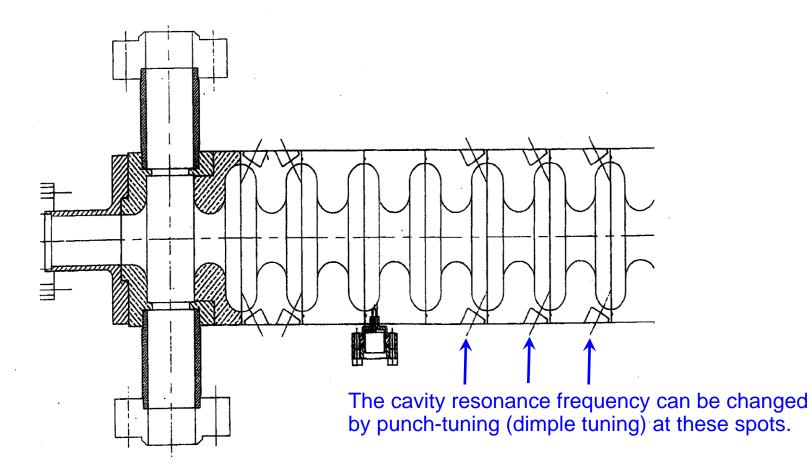


Source: fy.chalmers.se/subatom/f2bfw/poster97_ps_pic/ ihstructure.ppt

Groups of cavities

A disc loaded waveguide structure

CERN LIL (Linear Injector for LEP), operating frequency 2.98 GHz



TEM transmission lines (1)

Transverse electric modes (TEM) can propagate on any structure with at least two conductors

Given a structure with C' ... capacitance per unit length [F/m] L' ... inductance per unit length [H/m]

It then follows

characteristic impedance
$$Z = \frac{V_{\text{wave}}}{I_{\text{wave}}} = \sqrt{\frac{L'}{C'}}$$
 $[Z] = \Omega$
velocity of propagation $v = \frac{1}{\sqrt{L'C'}} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}}$ $[v] = m/s$

If $\mu_r = \varepsilon_r = 1$ (vacuum or approximately air), then the velocity of propagation is equal to the velocity of light $c_0 = 2.998 \text{ E8 } m/s$

TEM transmission lines (2)

Formulae for the characteristic impedance Z can be found in many textbooks (e.g. "Reference Data for Radio Engineers" or others). From a known Z the values for C' and L' can be deduced by

$$C' = \frac{1}{vZ}$$

$$L' = \frac{Z}{v}$$
for "normal" cable ($\mu_r = 1$)

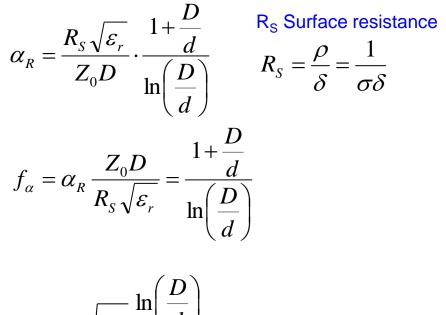
$$C' = \frac{100\sqrt{\varepsilon_r}}{3Z} \qquad [C'] = \frac{pF}{cm}$$
$$L' = \frac{\sqrt{\varepsilon_r}}{30}Z \qquad [L'] = \frac{nH}{cm}$$

For coaxial cables:
$$Z = \sqrt{2}$$

$$= \sqrt{\frac{\mu_r}{\varepsilon_r}} 60 \ln\left(\frac{R}{r}\right)$$

TEM transmission lines (3)

Coaxial cable with minimum loss:



$$Z_L = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\operatorname{III}\left(\frac{d}{d}\right)}{2\pi\sqrt{\varepsilon_r}}$$

Reprinted from O.Zinke, H.Brunswig, Lehrbuch der Hochfrequenztechnik, p.222

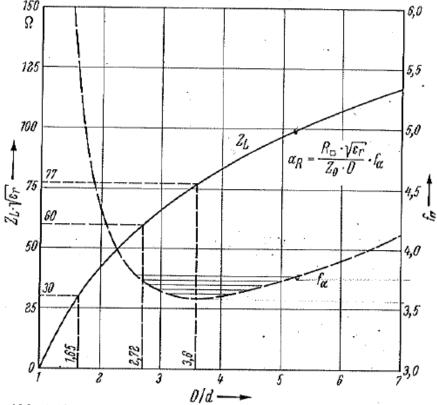
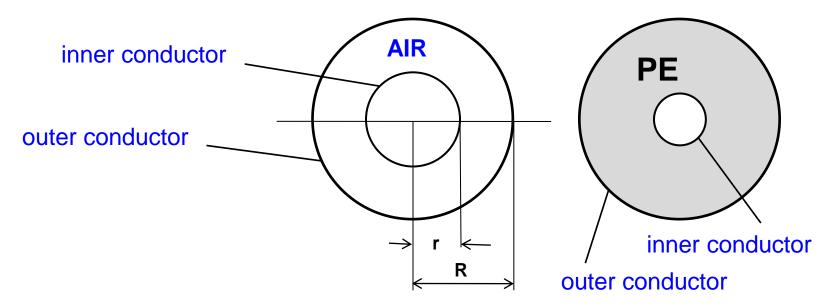


Abb. 4.6/2. Wellenwiderstand und Dämpfung α_R eines Koaxialkabels in Abhängigkeit vom Durchmesserverhältnis D/d. In dem eingezeichneten Toleranzfeld bedeutet eine Linie jeweils 1 % Abweichung vom Optimum

TEM transmission lines (4)

Applied to 50-Ohm-lines (the impedance mostly used) one finds

	Vacuum or air	Polyethylene (PE)
ε _r	1	2.26
v (m/sec)	$3E8 = c_0$	0.665 c ₀
L' (nH/m)	166.7	250.6
C' (pF/m)	66.7	100.2
R/r	2.30	3.50



Transmission lines (1)

• Coaxial lines

- frequency range: 0...10 GHz
- largest practical size: 350 mm for outer conductor, 150 mm for the inner conductor
- power rating: for CW operation at 200 MHz: 1 MW
- low-pass line, upper frequency limit given by moding
- relatively high attenuation
- power limited by inner conductor (high field => thermal load)
- in general easier to handle than waveguides

Waveguides

- frequency range 0.32...325 GHz (standard guides)
- largest practical size: 590 mm x 298 mm
- power rating: 150 MW peak at 310 MHz
- Iow attenuation
- bandpass, low frequency cut-off determined by dimension

Transmission lines

Transmission lines (2)

Standard RF coax cables

single screen

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		_	_

H+S type	Item no.		Center cond	uctor 1		Dielectr	ic 🛛	Screen	1 3		Scree	n 2 4	Jacket 5						Cable (group*
		Curves see page	Design	Mat.	Dim. mm	Mat.	Dim. mm	Mat.	Dim. mm	Cover %	Dim. mm	Cover %	Mat.	Dim. mm	Colour	Weight kg/ 100 m	Operating voltage kV	Max. operation frequency	crimp	clamp
G_03212-01	22610095		Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	95	-	-	PUR ¹⁾	4.95	black	3.60	2.5	1	U7	U7
RG_58_C/U	22510015		Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	PVC2(LM)	4.95	black	3.70	2.5	1	U7	U7
RG_58_C/U-01	22510350		Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	PVC2(LM)	4.95	black	3.70	2.5	1	U7	U7
RG_58_C/U-05	22511239		Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	PVC2(LM)	4.95	blue	3.70	2.5	1	U7	U7
RG_58_C/U-06	22510017	1	Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	PVC	4.95	black	3.70	2.5	1	U7	U7
RG_58_C/U-07	22511244	1	Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	PVC2(LM)	4.95	9то у	3.70	2.5	1	U7	U7
RG_58_C/U-22	22511607	1	Strand-19	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	PVC2(LM)	4.95	red	3.70	2.5	1	U7	U7
RG_58_C/U-62b)	23024284	5	Strand-19	CuAg	0.90	PE	2.95	CuAg	3.60	96	-	-	PVC(UL)	4.95	black	3.70	2.5	1	U7	U7
G_03232	22510128	and	Strand-7	Cu	0.95	PE	2.95	Cu	3.60	95	-	-	PVC	5.00	black	3.70	2.5	1	U7	U7
G_03262-1	22512108	6	Strand/7	CuSn	0.90	PE	2.95	CuSn	3.60	96	-	-	LSFH ¹⁾	4.95	black	3.90	2.5	1	U7	U7
G_03272	22511434	1	Strand-7	Cu	0.95	PE	2.95	Cu	3.60	95	-	-	PE1)	5.00	black	3.50	2.5	2	U7	U7
G_05232	22510176	1	Strand-7	Cu	1.50	PE	4.80	Cu	5.60	92	-	-	PVC2(LM)	7.40	black	7.70	3.5	1	-	U19
RG_213_U	22510052	1	Strand-7	Cu	2.25	PE	7.25	Cu	8.10	96	-	-	PVC2(LM)	10.30	black	15.30	5.0	1	U29	U28
RG_213_U-01ª)	22510053	1	Strand-7	Cu	2.25	PE	7.24	Cu	8.10	96	-	-	PVC2(LM)	10.30	black	15.30	5.0	1	U29	U28
RG_213_U-04	22510055	1	Strand-7	Cu	2.25	PE	7.25	Cu	8.10	96	-	-	PVC	10.30	black	15.30	5.0	1	U29	U28
G_07262	22511836	1	Strand-7	Cu	2.25	PE	7.28	Cu	8.10	96	-	-	LSFH ¹	10.30	black	15.30	5.0	1	U29	U28
RG_218_U	22510066	1	Wire	Cu	5.00	PE	17.30	Cu	18.40	96	-	-	PVC2(LM)	22.10	black	66.90	11.0	1	-	U44

* for suitable connectors

a) precision type: impedance $50 \pm 1 \Omega$

b) UL recognised (see UL types page 117)

1) Low Smoke Free of Halogen (LSFH) acc. waste electrical and electronic equipment (WEEE) and restriction of the use of certain hazardous substances (RoHS) directive.

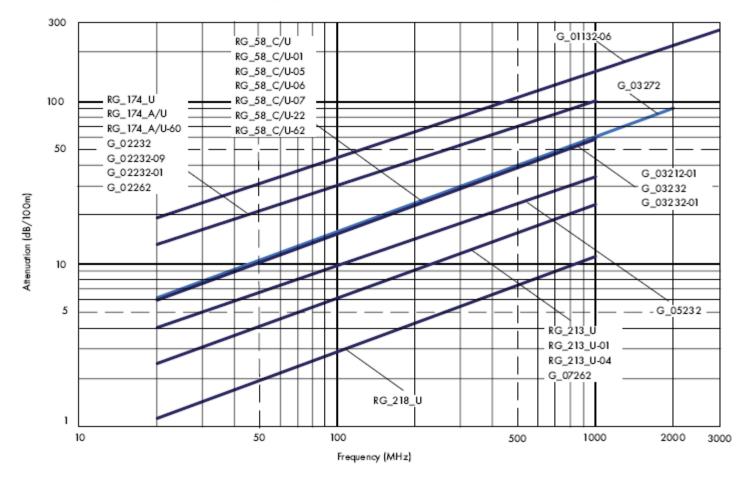
Transmission lines

Transmission lines (3)

Attenuation

Standard RF coax cables, single screen, 50 Ω

typical values at +20 °C ambient temperature

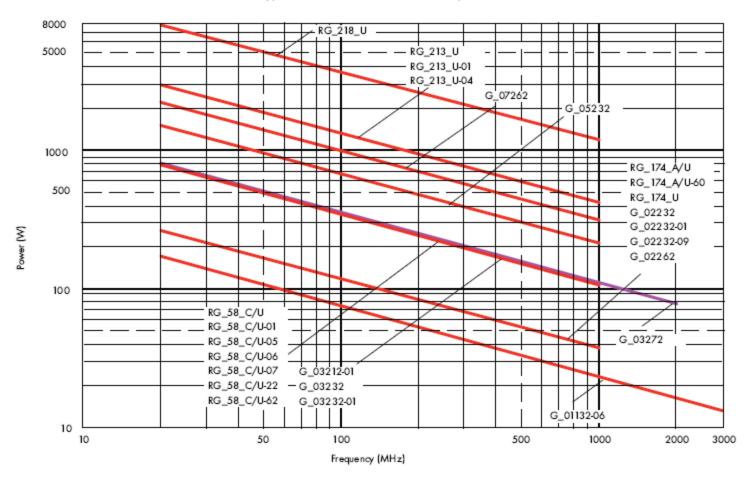


Transmission lines (4)

Power

Standard RF coax cables, single screen, 50 Ω

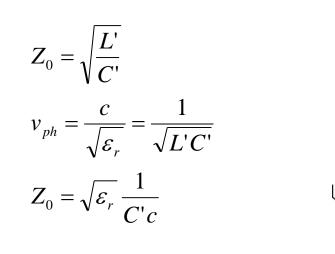
typical values at +40 °C ambient temperature

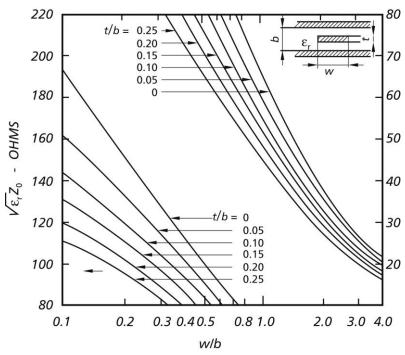


Transmission lines

Striplines (1)

A stripline is a **flat conductor** between a top **and** bottom ground plane. The space around this conductor is filled with a homogeneous dielectric material. This line propagates a pure TEM mode. With the static capacity per unit length, **C'**, the static inductance per unit length, **L'**, the relative permittivity of the dielectric, ε_r and the speed of light **c** the characteristic impedance **Z**₀ of the line is given by



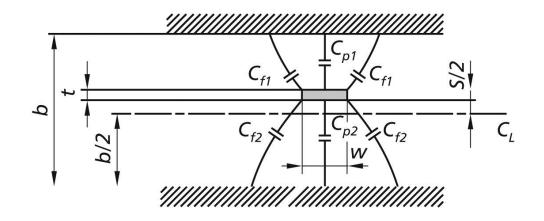


Characteristic impedance of striplines

Striplines, Microstriplines, Slotlines

Striplines (2)

For a mathematical treatment, the effect of the fringing fields may be described in terms of static capacities. The total capacity is the sum of the principal and fringe capacities Cp and Cf.



$$C_{tot} = C_{p1} + C_{p2} + 2C_{f1} + 2C_{f2}$$

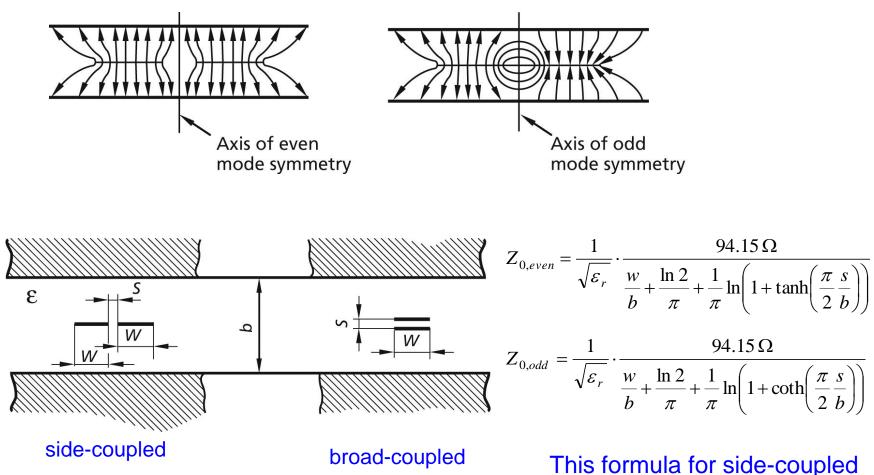
Cf stands for fringe field capacity, Cp stands for principal capacity

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Striplines, Microstriplines, Slotlines

Striplines (3)

Coupled striplines (in odd and even mode):

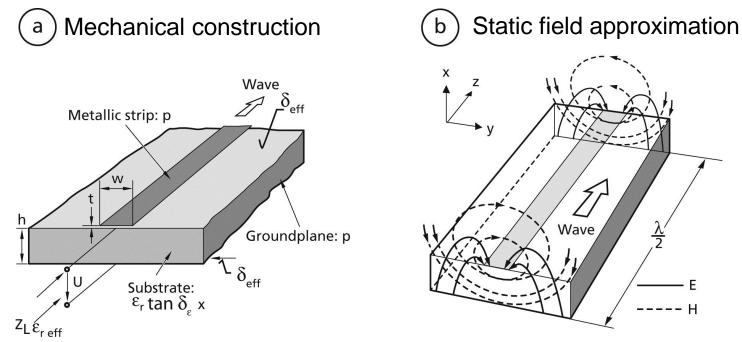


Striplines, Microstriplines, Slotlines

structure only.

Microstriplines (1)

A microstripline may be visualized as a stripline with the top cover and the top dielectric layer taken away. It is thus an asymmetric open structure, and only part of its cross section is filled with a dielectric material. Since there is a transversely inhomogeneous dielectric, only a quasi-TEM wave exists. This has several implications such as a frequency-dependent characteristic impedance and a considerable dispersion.



Note: Quasi-TEM wave due to different dielectric constants in different parts of the cross-section. We do get longitudinal field components.

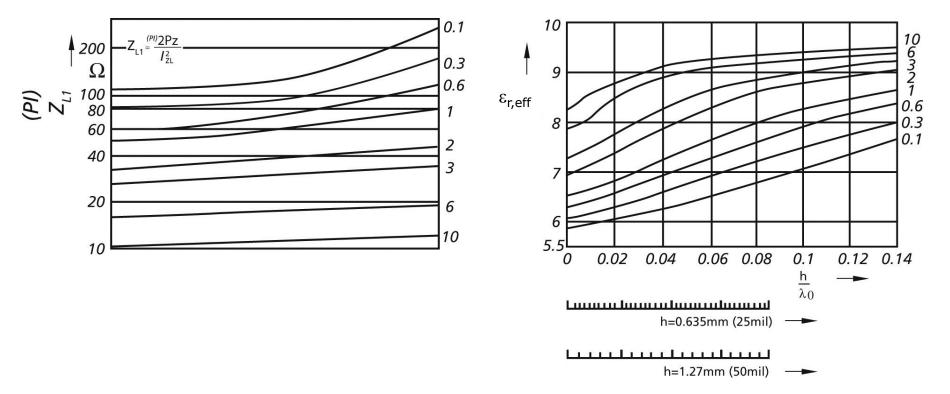
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Striplines, Microstriplines, Slotlines

Microstriplines (2)

Frequency-dependent characteristic impedance

Effective permittivity

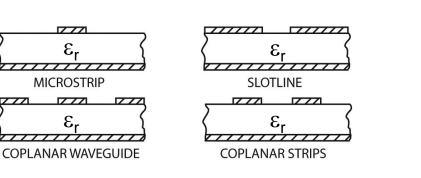


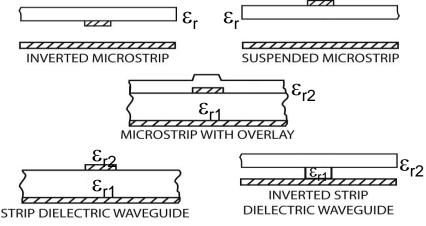
Striplines, Microstriplines, Slotlines

Microstriplines (3)

Planar transmission lines used in MIC (microwave integrated circuits)

Various transmission lines derived from microstrip

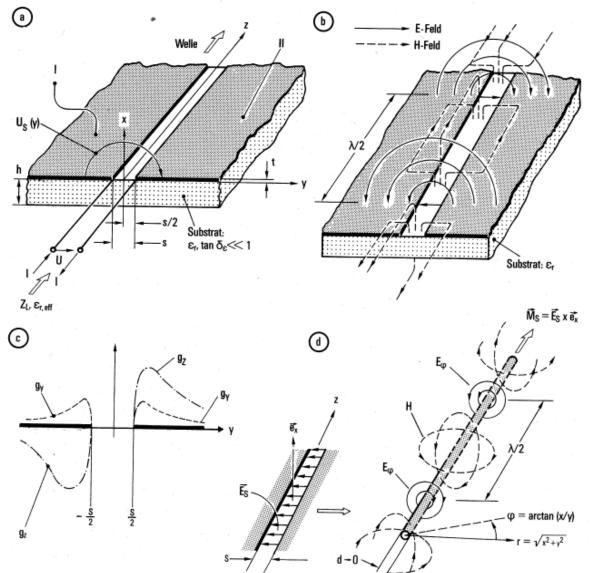




Slotlines (1)

The slotline may be considered as the dual structure of the microstrip. It is essentially a slot in metallization the of а dielectric substrate. The characteristic impedance and the effective dielectric constant exhibit similar dispersion properties to those of the microstrip line.

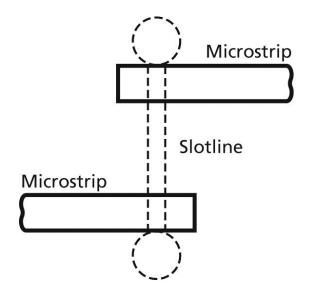
- (a) Mechanical construction
- (b) Field pattern (TE approximation)
- (c) Longitudinal and transverse current densities
- (d) Magnetic line current model.



Striplines, Microstriplines, Slotlines

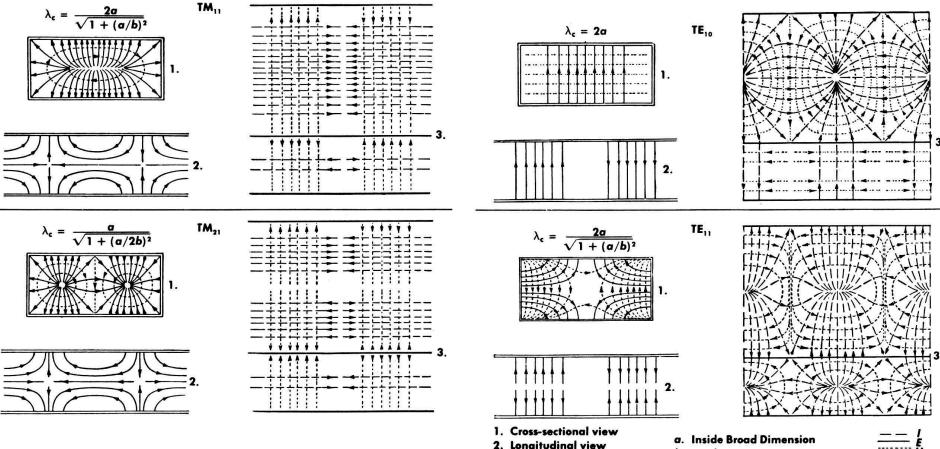
Slotlines (2)

A broadband (decade bandwidth) pulse inverter. Assuming the upper microstrip to be the input, the signal leaving the circuit on the lower microstrip is inverted since this microstrip ends on the opposite side of the slotline compared to the input.



Two microstrip-slotline transitions connected back to back for 180° phase change.

Rectangular waveguide



Modes in a rectangular waveguide with dimensions a and b. solid lines: E field, dotted lines: H field

3. Surface view

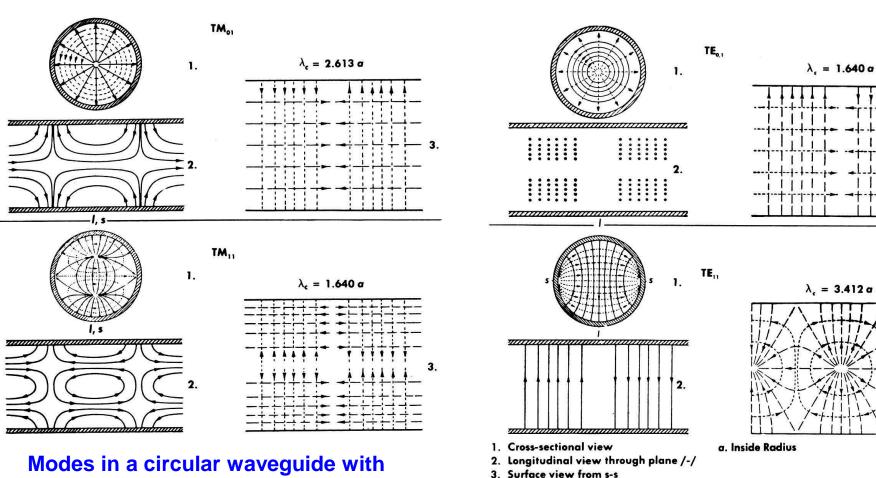
a. Inside Broad Dimension b. Inside Narrow Dimension



Reprinted from Saad, T S, Microwave Engineers' Handbook, Artech House

Waveguides

Circular waveguide



radius a solid lines: E field, dotted lines: H field Please note the similarity to the pillbox cavity!

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Waveguides

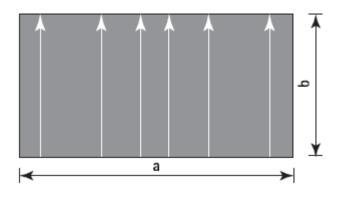
Reprinted from Saad, T S, Microwave

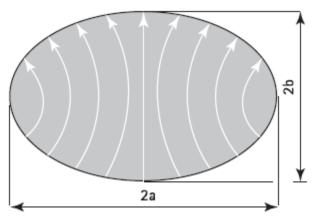
Engineers' Handbook, Artech House

Elliptical waveguide (1)

E field lines for TE₁₀ mode







The small index c stands for the polarization and refers to sine (s) or cosine (c).

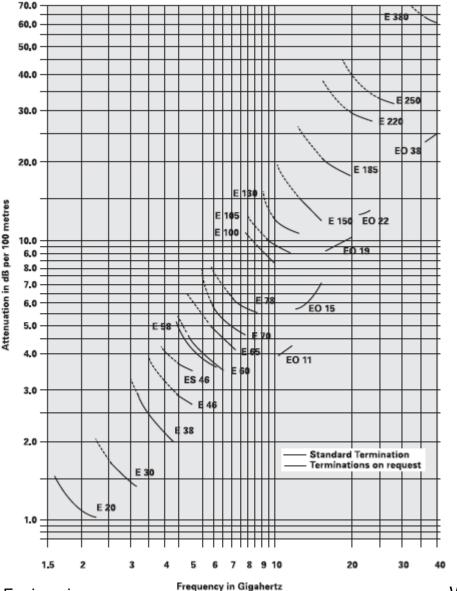
The cut-off wavelengths of the various modes that can propagate in an elliptical waveguide can be found analytically using rather complicated methods or numerically.

Elliptical waveguide (2)

Reference Temperature 20'C

Typical attenuation values for flexible elliptical waveguides:

Rigid rectangular cross-section waveguides are rather seldom used in industry.



EO stands for overmoded waveguide

Reprinted from Flexwell, Elliptical Waveguides **RFS Datasheet**

Elliptical waveguide (3)

Datasheet

	ODED	CUTOFF	MAN MONDA			. (64)	AVG.	GROUP	GROUP
	OPER.	CUT OFF	MAX. VSWR/		ATTENUATION dB/100m (ft) IN THE OPERATING FREQUENCY BAND				DELAY
WVG.	FREQ.	FREQ.	RETURN		-		POWER kW	%с	ns/100m (ft)
TYPE	GHz	GHz	LOSS, dB	LOW BAND	MID BAND	HIGH BAND	MID BAND	MID BAND	MID BAND
E30	2.7 - 3.1	1.8	1.128/24.4	1.61 (0.49)	1.49 (0.45)	1.4 (0.43)	30.37	78.4	425.4 (129.7)
E38	3.6 - 4.2	2.4	1.15/23.1	2.37 (0.72)	2.20 (0.67)	2.08 (0.63)	16.27	78.8	423.2 (129.0)
EP38	3.6 - 4.2	2.4	1.083/28.0	2.37 (0.72)	2.20 (0.67)	2.08 (0.63)	16.27	78.8	423.2 (129.0)
E46	4.4 - 5.0	2.88	1.15/23.1	2.92 (0.89)	2.80 (0.85)	2.73 (0.83)	10.93	79.0	422.1 (128.7)
EP46	4.4 - 5.0	2.88	1.083/28.0	2.92 (0.89)	2.80 (0.85)	2.73 (0.83)	10.93	79.0	422.1 (128.7)
ES46	4.4 - 5.0	3.08	1.15/23.1	3.69 (1.12)	3.55 (1.08)	3.49 (1.06)	8.39	75.5	441.6 (134.6)
ESP46	4.4 - 5.0	3.08	1.073/29.1	3.69 (1.12)	3.55 (1.08)	3.49 (1.06)	8.39	75.5	441.6 (134.6)
EP58	4.4 - 6.2	3.56	1.083/28.0	5.10 (1.55)	3.96 (1.21)	3.60 (1.10)	6.54	74.1	450.3 (137.2)
E60	5.6 - 6.425	3.65	1.15/23.1	4.15 (1.27)	3.95 (1.20)	3.80 (1.16)	7.24	79.4	420.3 (128.1)
EP60	5.6 - 6.425	3.65	1.062/30.5	4.15 (1.27)	3.95 (1.20)	3.80 (1.16)	7.24	79.4	420.3 (128.1)
E65	5.9 - 7.125	4.01	1.15/23.1	4.9 (1.50)	4.5 (1.37)	4.25 (1.30)	5.26	78.7	423.8 (129.2)
EP65	5.9 - 7.125	4.01	1.062/30.5	4.9 (1.50)	4.5 (1.37)	4.25 (1.30)	5.26	78.7	423.8 (129.2)
EP70	6.4 - 7.75	4.34	1.062/30.5	5.5 (1.68)	5.0 (1.52)	4.8 (1.46)	4.65	79.1	421.5 (128.5)
E78	7.1 - 8.5	4.72	1.15/23.1	6.2 (1.89)	5.8 (1.77)	5.6 (1.71)	3.67	79.6	419.0 (127.7)
EP78	7.1 - 8.5	4.72	1.062/30.5	6.2 (1.89)	5.8 (1.77)	5.6 (1.71)	3.67	79.6	419.0 (127.7)
EP100	9.0 - 10.0	6.43	1.105/26.0	9.5 (2.90)	8.9 (2.71)	8.4 (2.56)	1.91	73.6	453.1 (138.1)
E105	10.0 - 11.7	6.49	1.15/23.1	9.6 (2.92)	9.2 (2.79)	8.9 (2.71)	1.77	79.9	417.3 (127.2)
EP105	10.0 - 11.7	6.49	1.062/30.5	9.6 (2.92)	9.2 (2.79)	8.9 (2.71)	1.77	79.9	417.3 (127.2)
E130	10.7 - 13.25	7.43	1.15/23.1	12.6 (3.84)	11.5 (3.52)	11.1 (3.39)	1.22	78.5	424.8 (129.5)
EP130	10.7 - 13.25	7.43	1.083/28.0	12.6 (3.84)	11.5 (3.52)	11.1 (3.39)	1.22	78.5	424.8 (129.5)
E150	13.4 - 15.35	8.64	1.15/23.1	14.6 (4.44)	14.0 (4.26)	13.7 (4.16)	0.88	79.7	418.6 (127.6)
EP150	13.4 - 15.35	8.64	1.083/28.0	14.6 (4.44)	14.0 (4.26)	13.7 (4.16)	0.88	79.7	418.6 (127.6)
E185	17.3 - 19.7	11.06	1.15/23.1	20.3 (6.17)	19.4 (5.92)	18.9 (5.75)	0.51	80.2	416.1 (126.8)
EP185	17.3 - 19.7	11.06	1.083/28.0	20.3 (6.17)	19.4 (5.92)	18.9 (5.75)	0.51	80.2	416.1 (126.8)
E220	21.2 - 23.6	13.36	1.105/26.0	28.8 (8.77)	28.3 (8.63)	28.1 (8.56)	0.31	80.3	415.6 (126.7)
E250	24.25 - 26.5	15.06	1.15/23.1	33.2 (10.1)	32.4 (9.88)	32.0 (9.75)	0.31	80.5	414.2 (126.3)
E300	27.5 - 33.4	19.05	1.15/23.1	50.0 (15.2)	46.0 (14.0)	44.4 (13.5)	0.14	78.1	427.1 (130.2)
E380	37.0 - 39.5	23.45	1.15/23.1	61.3 (18.7)	60.7 (18.5)	60.0 (18.3)	0.09	79.1	421.9 (128.6)

Amplifiers (1)

Semiconductors

- Bipolar transistors
- Field effect transistors
- many others
- Frequency range: 0...100 GHz
- Power range: from close to thermal noise level to many kW
- High reliability, but lifetime not infinite (thermal fatigue, metal migration, etc.)
- Often unforgiving, failure is normally definitive
- Inherently low-voltage, high current devices compared to tubes
- Low to medium gain

Amplifiers (2)

- Gridded Tubes (electron tubes)
- Frequency range: 0...0.5 GHz (tetrodes), 0...3 GHz (triodes)
- Power range:
 - for CW (continuous wave) up to 30 MHz: 1 MW
 - at 300 MHz: 200 kW
 - pulsed at 200 MHz: 4 MW
- Medium reliability, lifetime cathode limited to 5000...40000 hours
- Relatively robust
- Inherently medium to high voltage, low current devices
- Density modulated
- High gain at low frequencies, medium gain at high frequencies

Amplifiers (3)

• Klystrons

- Frequency range: 0.3...10 GHz
- Power range:
 - CW at 350 MHz: 1 MW
 - pulsed at 3 GHz: 30 MW
- Medium reliability, lifetime cathode limited
- Needs expert care
- Inherently very high voltage device
- Velocity modulated
- ◆ Very high gain (≈40 to 60 dB, about 10 dB per passive resonator)
- Tend to be noisy (acoustically and electrically)

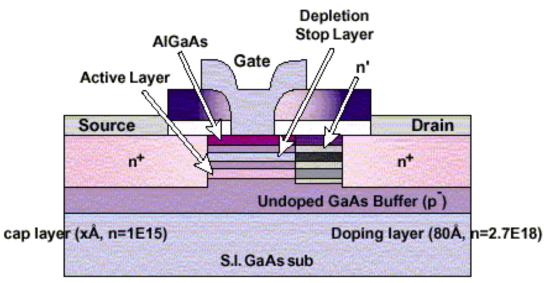
Others

- Travelling wave tubes, magnetrons (Microwave ovens!!), Gyrotrons
- 2-beam accelerators (CLIC)

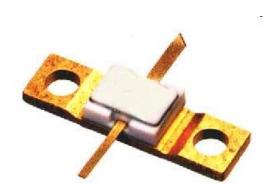
Transistors (1)

Example: a field effect transitor (FET)

Structure of an advanced pulse-doped MESFET



High Power and Low Distortion GaAs FET



Transistors (2)

A typical data sheet of a Medium Power GaAs FET

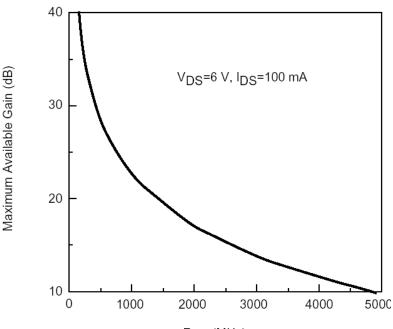
- Up to 2.5 GHz frequency band
- Beyond 22 dBm output power
- Low distortion characteristics
- Low power consumption
- High power gain
- Low-cost plastic mold package
- Low thermal resistance lead

Applications

• Driver amplifier preceding final power amplifier for DECT



Maximum Available Gain VS. Frequency



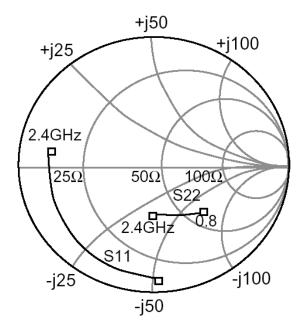
Active elements

Transistors (3)

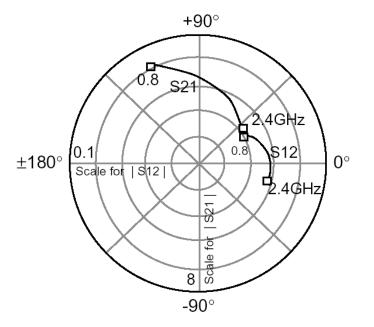
Transistor scattering parameters

They will be covered in detail in the second part of this lecture...



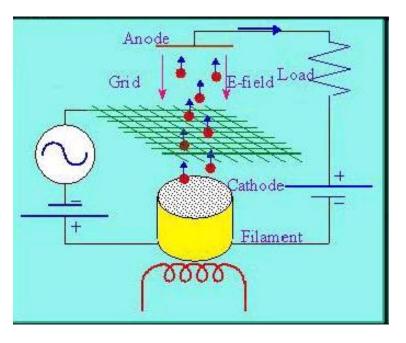


The forward transmission $S_{\rm 21}$ and the backward transmission $S_{\rm 12}$

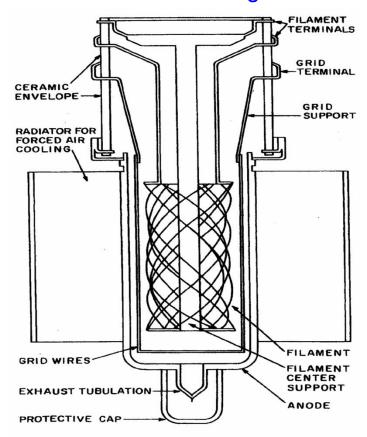


Gridded tubes

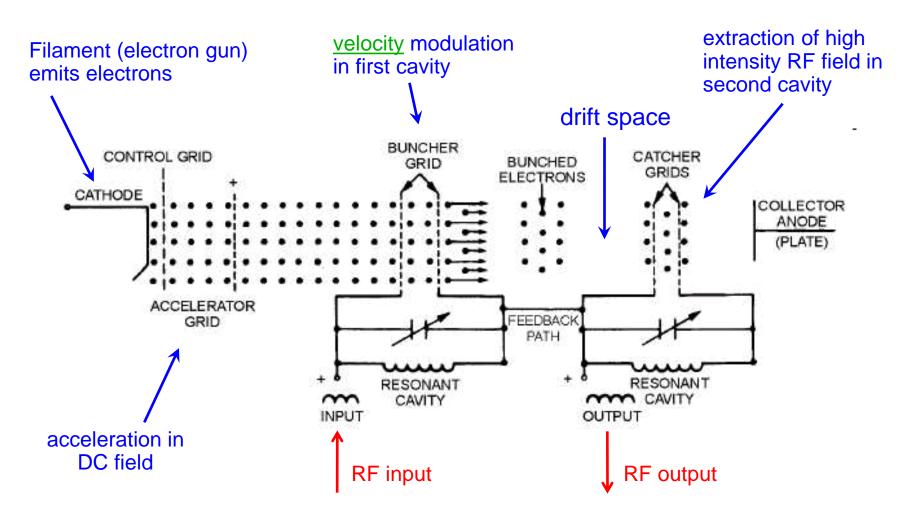
- Filament burns off electrons
- acceleration in DC field
- <u>density</u> modulation by grid
- => voltage controlled current source



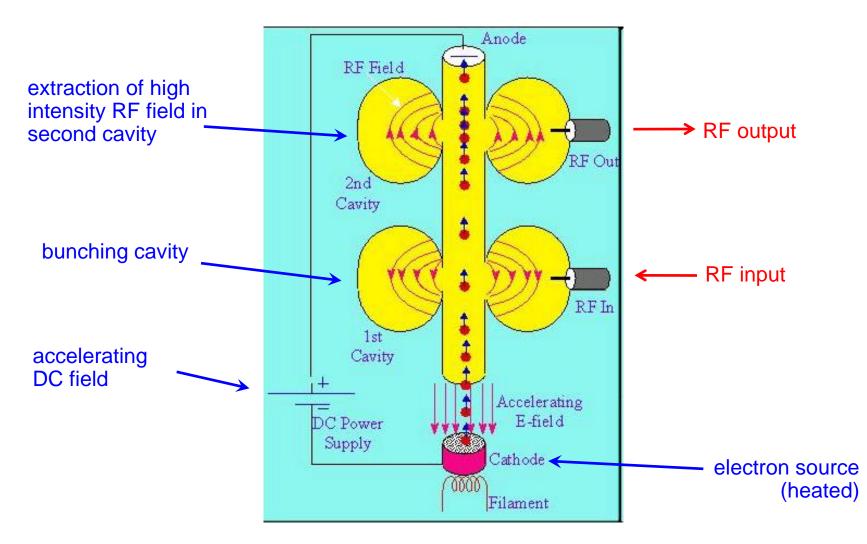
A medium power external anode transmitting tube



Klystrons (1)



Klystrons (2)



IOT – Inductive Output Tubes (1)



A Light for Science

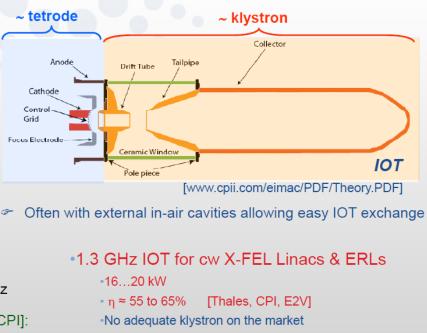
IOT - Inductive Output Tubes or klystrodes



- TV IOT: typically 60 kW at 460 860 MHz
- IOT developed for accelerators [Thales, CPI]:
 - 80 kW CW at 470 760 MHz
 - η ≈ 70% *⊶* operation in class B
 - Intrinsic low Gain = 20 ... 22 dB \Rightarrow P_{in} = 1 kW
 - Compact, external cavity \Rightarrow easy to handle
 - BUT: low unit power ⇒ power combiners







Superiority of IOTs:

- Higher efficiency
- Less amplitude & phase sensitivity to HV ripples
- The collector overheating after loss of drive
- Expected lower costs

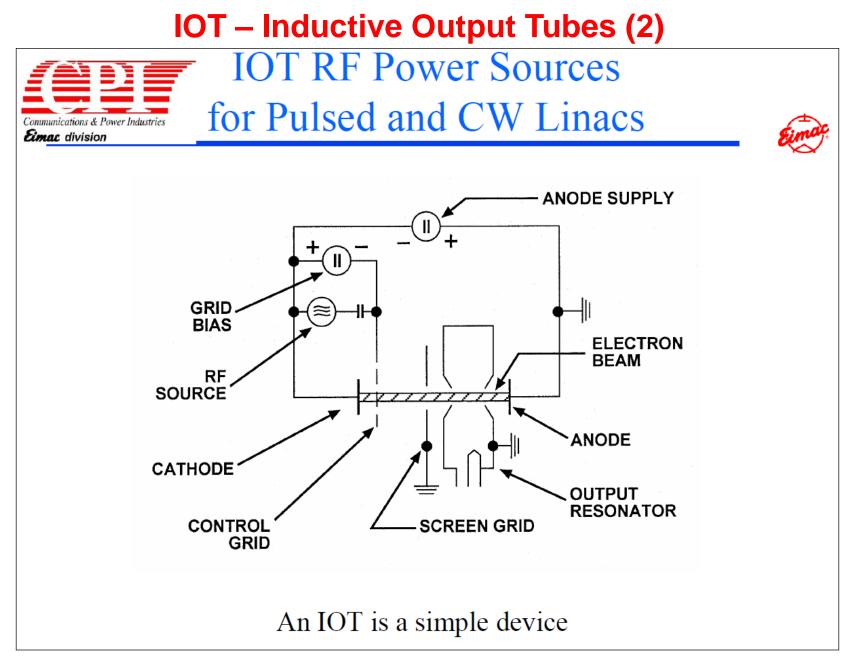
Tutorial: RF Power Sources J. Jacob, slide 15

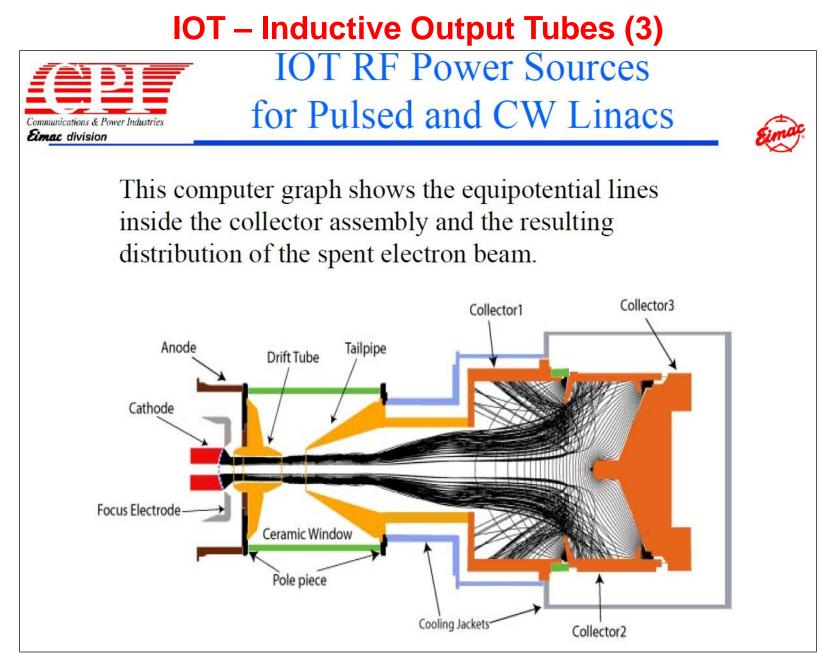
History of the IOT:

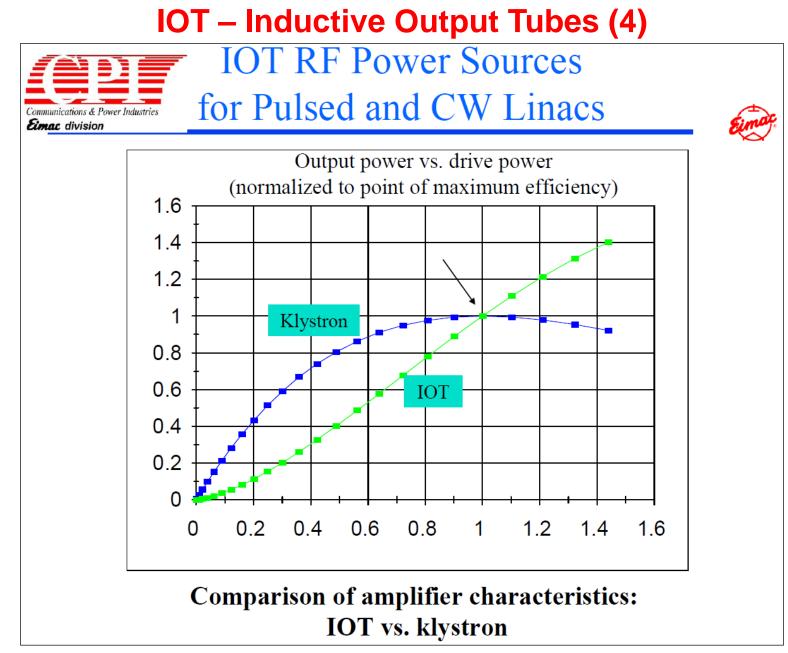
www.bext.com/iot.htm

F. Caspers, M. Betz; JUAS 2012 RF Engineering

Active elements





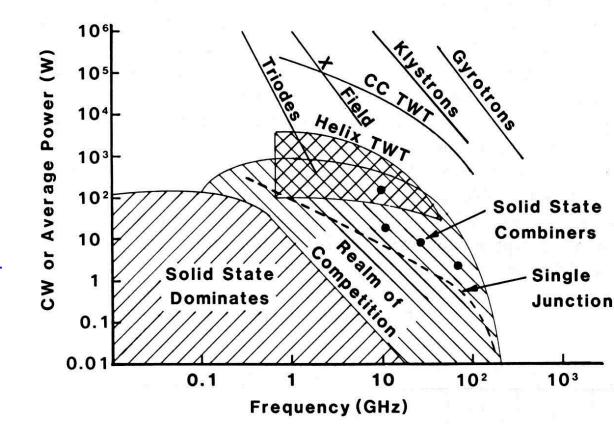


Comparison of solid state and vacuum technology for RF power generation (1986)

Solid state devices move steadily up in frequency

Abbreviations:

X Field: crossed field, especially magnetrons TWT: Travelling wave tubes CC TWT: coupled cavity TWT





Active elements

Comparison of solid state and vacuum technology for RF power generation (2009)

