Part II: Waves, S-Parameters, Decibels and Smith Chart

Section A

- Forward and backward travelling waves

Section B

- S-Parameters
- The scattering matrix
- Decibels
- Measurement devices and concepts
- Superheterodyne Concept

Section C

- The Smith Chart
- Navigation in the Smith Chart
- Examples

Introduction of the S-parameters

- The first paper by Kurokawa
- Introduction of power waves instead of voltage and current waves using so far (1965)

Abstract—This paper discusses the physical meaning and properties of the waves defined by

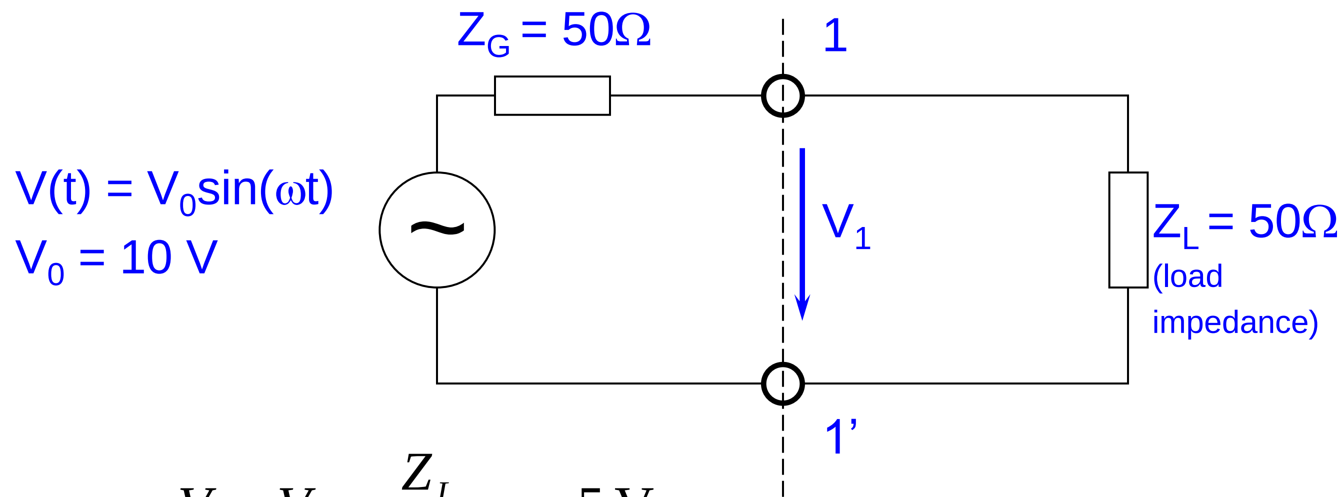
$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\mathrm{Re}\, Z_i|}}, \qquad b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{|\mathrm{Re}\, Z_i|}}$$

where V_i and I_i are the voltage at and the current flowing into the ith port of a junction and Z_i is the impedance of the circuit connected to the ith port. The square of the magnitude of these waves is directly related to the exchangeable power of a source and the reflected power. For this reason, in this paper, they are called the power waves. For certain applications where the power relations are of main concern, the power waves are more suitable quantities than the conventional traveling waves. The lossless and reciprocal conditions as well as the frequency characteristics of the scattering matrix are presented.

Then, the formula is given for a new scattering matrix when the Z_i's are changed. As an application, the condition under which an amplifier can be matched simultaneously at both input and output ports as well as the condition for the network to be unconditionally stable are given in terms of the scattering matrix components. Also a brief comparison is made between the traveling waves and the power waves.

K. Kurokawa, 'Power Waves and the Scattering Matrix,' IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-13, No. 2, March, 1965.

Example: A generator with a load



- Voltage divider: $V_1 = V_0 \dfrac{Z_L}{Z_L + Z_G} = 5\,\mathrm{V}$

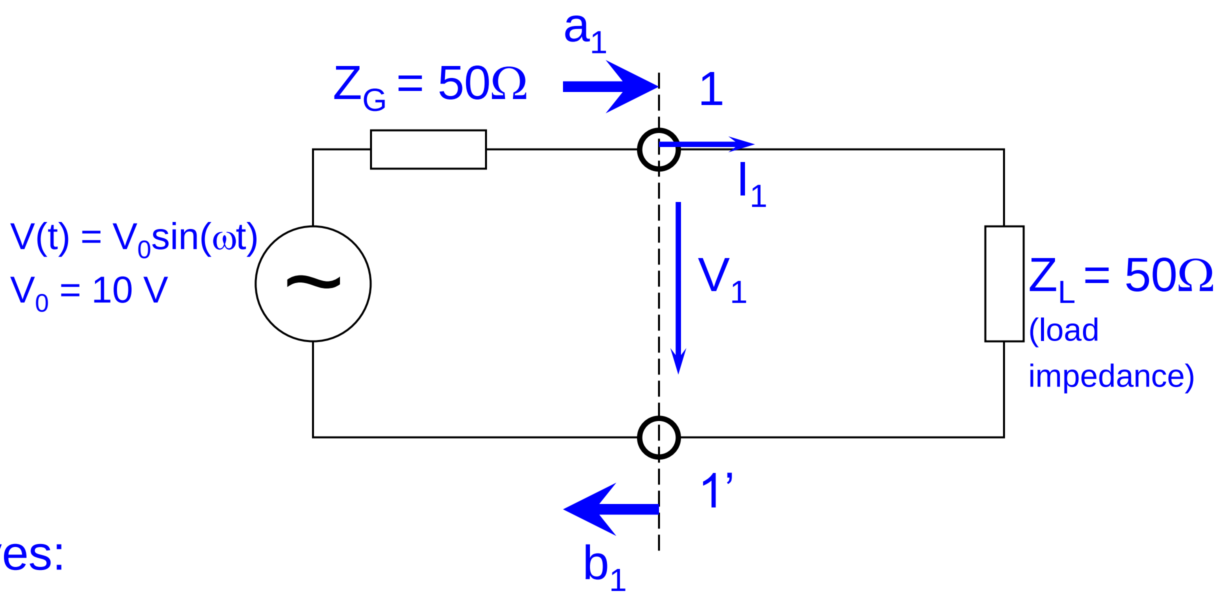
- This is the matched case, since $Z_G = Z_L$. Thus we have a forward travelling wave only, no reflected wave. Thus the amplitude of the forward travelling wave in this case is V_1=5V, a_1 returns as $5V/\sqrt{50\Omega}$ (forward power = $25V^2/50\Omega = 0.5W$)
- Matching means maximum power transfer from a generator with given source impedance to an external load
- In general, $Z_L = Z_G^*$

Power waves (1)

$$a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}},$$

$$b_1 = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}},$$

where the characteristic impedance $Z_0 = Z_G$



Definition of power waves:

- a_1 is the wave incident on the termination one-port (Z_L)
- b_1 is the wave running out of the termination one-port
- a_1 has a peak amplitude of 5 V/sqrt(50Ω)
- What is the amplitude of b_1? Answer: b_1 = 0.
- Dimension: [V/sqrt(Z)], in contrast to voltage or current waves

Caution! US notion: power = $|a|^2$ whereas European notation (often): power = $|a|^2/2$

Power waves (2)

This is the definition of a and b (see Kurokawa paper):

$$a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_1 = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

Here comes a probably more practical method for determination. Assume that the generator is terminated with an external load equal to the generator impedance. Then we have the matched case and only a forward travelling wave (no reflection). Thus, the voltage on this external resistor is equal to the voltage of the outgoing wave.

$$a_1 = \frac{U_0}{2\sqrt{Z_0}} = \frac{\textit{incident voltage wave}\left(\textit{port } 1\right)}{\sqrt{Z_0}} = \frac{U_1^{inc}}{\sqrt{Z_0}}$$

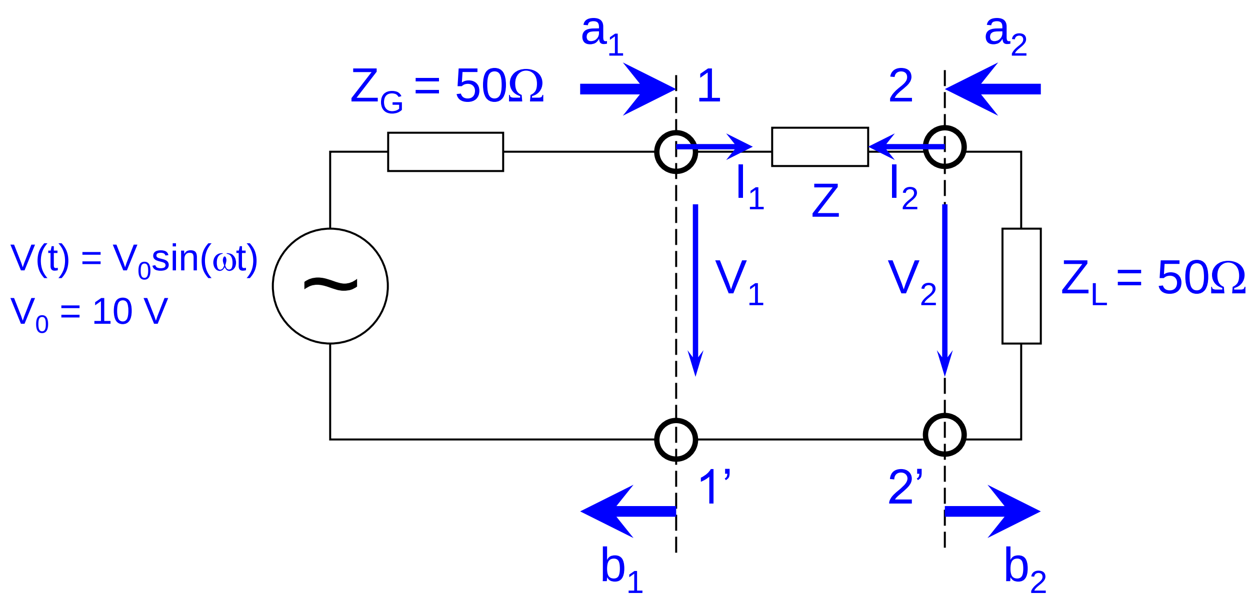
$$b_1 = \frac{U_1^{refl}}{\sqrt{Z_0}} = \frac{\textit{reflected voltage wave}\left(\textit{port } 1\right)}{\sqrt{Z_0}}$$

$$U_i = \sqrt{Z_0}\left(a_i + b_i\right) = U_i^{inc} + U_i^{refl}$$

$$I_i = \frac{1}{\sqrt{Z_0}}\left(a_i - b_i\right) = \frac{U_i^{refl}}{Z_0}$$

Caution! US notion: power = $|a|^2$ whereas European notation (often): power = $|a|^2/2$

Analysing a 2-port



- A 2-port or 4-pole is shown above between the generator impedance and the load
- Strategy for practical solution: Determine currents and voltages at all ports (classical network calculation techniques) and from there determine a and b for each port.
- Important for definition of a and b:

  The wave a always travels towards an N-port, the wave b always travels away from an N-port

3

Using S-Parameters

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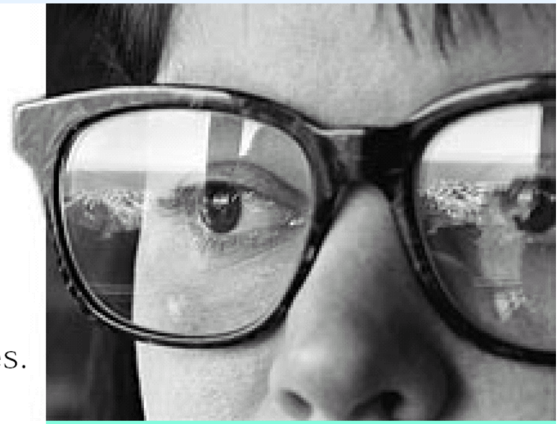
S-Parameter Techniques

Another important advantage of s-parameters stems from the fact that traveling waves, unlike terminal voltages and currents, do not vary in magnitude at points along a lossless transmission line. This means that scattering parameters can be measured on a device located at some distance from the measurement transducers, provided that the measuring device and the transducers are connected by low-loss transmission lines.

Derivation

Generalized scattering parameters have been defined by K. Kurokawa [Appendix A]. These parameters describe the interrelationships of a new set of variables (a_i , b_i). The variables a_i and b_i are normalized complex voltage waves incident on and reflected from the i^{th} port of the network. They are defined in terms of the terminal voltage V_i , the terminal current I_i , and an arbitrary reference impedance Z_i , where the asterisk denotes the complex conjugate:

$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\mathrm{Re}\, Z_i|}} \quad (4) \qquad b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{|\mathrm{Re}\, Z_i|}} \quad (5)$$



Transmission and Reflection
When light interacts with a lens, as in this photograph, part of the light incident on the woman's eyeglasses is reflected while the rest is transmitted. The amounts reflected and transmitted are characterized by optical reflection and transmission coefficients. Similarly, scattering parameters are measures of reflection and transmission of voltage waves through a two-port electrical network.

3 Using S-Parameters

For most measurements and calculations it is convenient to assume that the reference impedance Z_i is positive and real. For the remainder of this article, then, all variables and parameters will be referenced to a single positive real impedance, Z_0.

The wave functions used to define s-parameters for a two-port network are shown in Fig. 2.

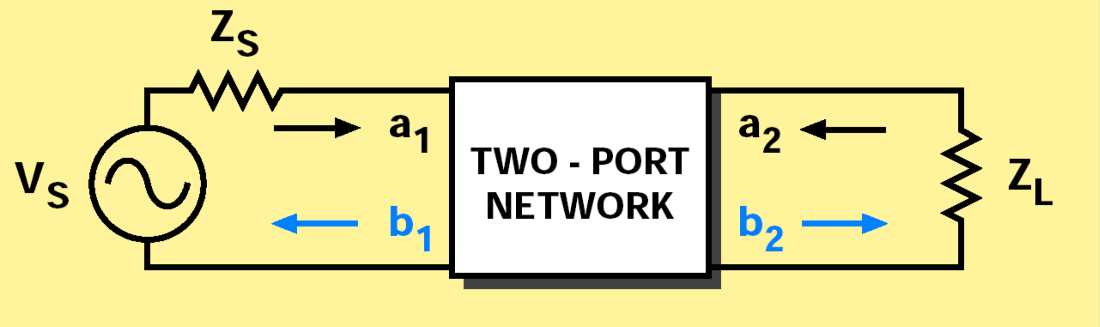


Figure 2
Two-port network showing incident waves (a_1, a_2) and reflected waves (b_1, b_2) used in s-parameter definitions. The flow graph for this network appears in Figure 3.

Scattering parameters relationship to optics
Impedance mismatches between successive elements in an RF circuit relate closely to optics, where there are successive differences in the index of refraction. A material's characteristic impedance, Z_0, is inversely related to the index of refraction, N:

$$Z_0\sqrt{\frac{\varepsilon}{377}} = \frac{1}{N}$$

The s-parameters s_{11} and s_{22} are the same as optical reflection coefficients; s_{12} and s_{21} are the same as optical transmission coefficients.

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The independent variables a_1 and a_2 are normalized incident voltages, as follows:

$$a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave incident on port 1}}{\sqrt{Z_0}} = \frac{V_{i1}}{\sqrt{Z_0}} \quad (6)$$

$$a_2 = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave incident on port 2}}{\sqrt{Z_0}} = \frac{V_{i2}}{\sqrt{Z_0}} \quad (7)$$

$$\sqrt{Z_0}$$

Dependent variables b_1, and b_2, are normalized reflected voltages:

$$b_1 = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave reflected from port 1}}{\sqrt{Z_0}} = \frac{V_{r1}}{\sqrt{Z_0}} \quad (8)$$

$$b_2 = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave reflected from port 2}}{\sqrt{Z_0}} = \frac{V_{r2}}{\sqrt{Z_0}} \quad (9)$$

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3 Using S-Parameters

The linear equations describing the two-port network are then:

$$b_1 = s_{11} a_1 + s_{12} a_2 \qquad (10)$$

$$b_2 = s_{21} a_1 + s_{22} a_2 \qquad (11)$$

The s-parameters s_{11}, s_{22}, s_{21}, and s_{12} are:

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad (12)$$
= Input reflection coefficient with the output port terminated by a matched load ($Z_L=Z_0$ sets $a_2=0$)

$$s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \qquad (13)$$
= Output reflection coefficient with the input terminated by a matched load ($Z_S=Z_0$ sets $V_s=0$)

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad (14)$$
= Forward transmission (insertion) gain with the output port terminated in a matched load.

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \qquad (15)$$
= Reverse transmission (insertion) gain with the input port terminated in a matched load.

Limitations of lumped models

At low frequencies most circuits behave in a predictable manner and can be described by a group of replaceable, lumped-equivalent black boxes. At microwave frequencies, as circuit element size approaches the wavelengths of the operating frequencies, such a simplified type of model becomes inaccurate. The physical arrangements of the circuit components can no longer be treated as black boxes. We have to use a distributed circuit element model and s-parameters.

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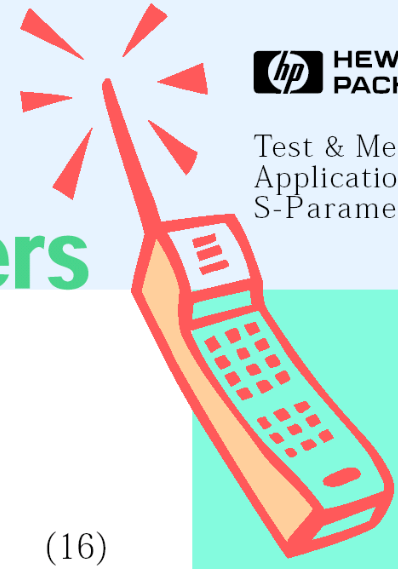
3 Using S-Parameters



Notice that

$$s_{11} = \frac{b_1}{a_1} = \frac{\frac{V_1}{I_1} - Z_0}{\frac{V_1}{I_1} + Z_0} = \frac{Z_1 - Z_0}{Z_1 + Z_0} \qquad (16)$$

and $$Z_1 = Z_0 \frac{(1 + s_{11})}{(1 - s_{11})} \qquad (17)$$

where $Z_1 = \frac{V_1}{I_1}$ is the input impedance at port 1.

This relationship between reflection coefficient and impedance is the basis of the Smith Chart transmission-line calculator. Consequently, the reflection coefficients s_{11} and s_{22} can be plotted on Smith charts, converted directly to impedance, and easily manipulated to determine matching networks for optimizing a circuit design.

S-parameters

S-parameters and distributed models provide a means of measuring, describing, and characterizing circuit elements when traditional lumped-equivalent circuit models cannot predict circuit behavior to the desired level of accuracy. They are used for the design of many products, such as cellular telephones.

3 Using S-Parameters

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Techniques

Another advantage of s-parameters springs from the simple relationship between the variables a_1, a_2 , b_1, and b_2, and various power waves:

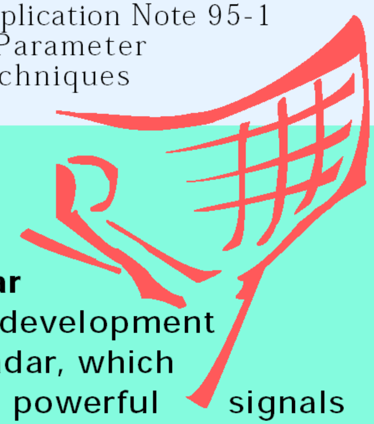
$$|a_1|^2 = \text{Power incident on the input of the network.}$$
$$= \text{Power available from a source impedance } Z_0.$$

$$|a_2|^2 = \text{Power incident on the output of the network.}$$
$$= \text{Power reflected from the load.}$$

$$|b_1|^2 = \text{Power reflected from the input port of the network.}$$
$$= \text{Power available from a } Z_0 \text{ source minus the power delivered to the input of the network.}$$

$$|b_2|^2 = \text{Power reflected from the output port of the network.}$$
$$= \text{Power incident on the load.}$$
$$= \text{Power that would be delivered to a } Z_0 \text{ load.}$$

Here the US notion is used, where power = $|a|^2$.
European notation (often): power = $|a|^2/2$
These conventions have no impact on S parameters, only relevant for absolute power calculation



Radar
The development of radar, which uses powerful signals at short wavelengths to detect small objects at long distances, provided a powerful incentive for improved high frequency design methods during World War II. The design methods employed at that time combined distributed measurements and lumped circuit design. There was an urgent need for an efficient tool that could integrate measurement and design. The Smith Chart met that need.

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3 Using S-Parameters

The previous four equations show that s-parameters are simply related to power gain and mismatch loss, quantities which are often of more interest than the corresponding voltage functions:

$$|s_{11}|^2 = \frac{\text{Power reflected from the network input}}{\text{Power incident on the network input}}$$

$$|s_{22}|^2 = \frac{\text{Power reflected from the network output}}{\text{Power incident on the network output}}$$

$$|s_{21}|^2 = \frac{\text{Power delivered to a } Z_0 \text{ load}}{\text{Power available from } Z_0 \text{ source}}$$

$$= \text{Transducer power gain with } Z_0 \text{ load and source}$$

$$|s_{12}|^2 = \text{Reverse transducer power gain with } Z_0 \text{ load and source}$$

Here the US notion is used, where power = $|a|^2$.
European notation (often): power = $|a|^2/2$
These conventions have no impact on S parameters, only relevant for absolute power calculation

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The Scattering-Matrix (1)

The abbreviation S has been derived from the word *scattering*. For high frequencies, it is convenient to describe a given network in terms of waves rather than voltages or currents. This permits an easier definition of reference planes. For practical reasons, the description in terms of in- and outgoing waves has been introduced.

Waves travelling towards the n-port: $(a) = (a_1, a_2, a_3, \ldots a_n)$

Waves travelling away from the n-port: $(b) = (b_1, b_2, b_3, \ldots b_n)$

The relation between a_i and b_i (i = 1..n) can be written as a **system of n linear equations** (a_i being the independent variable, b_i the dependent variable):

$$
\begin{array}{ll}
\text{one-port} & b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 + \ldots \\
\text{two-port} & b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{44}a_4 + \ldots \\
\text{three-port} & b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{44}a_4 + \ldots \\
\text{four-port} & b_4 = S_{41}a_1 + S_{42}a_2 + S_{33}a_3 + S_{44}a_4 + \ldots
\end{array}
$$

In compact matrix notation, these equations are equivalent to

$$\boxed{(b) = (S)(a)}$$

The Scattering Matrix (2)

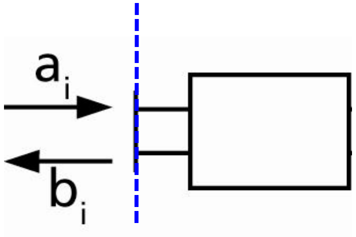
The simplest form is a passive **one-port (2-pole)** with some reflection coefficient Γ.

$$(S) = S_{11} \quad \rightarrow \quad b_1 = S_{11} a_1$$

With the reflection coefficient Γ it follows that

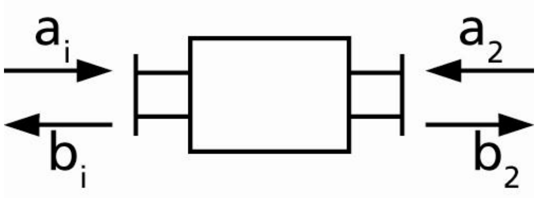
$$S_{11} = \frac{b_1}{a_1} = \Gamma$$

Reference plane



Two-port (4-pole)

$$(S) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 \\ b_2 &= S_{21} a_1 + S_{22} a_2 \end{aligned}$$



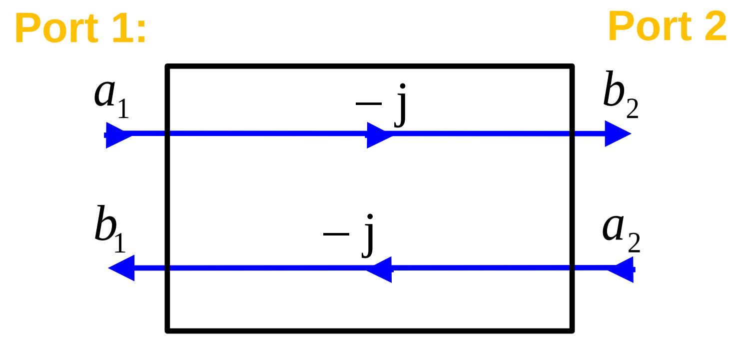
A non-matched load present at port 2 with reflection coefficient Γ_{load} transfers to the input port as

$$\boxed{\Gamma_{in} = S_{11} + S_{21} \frac{\Gamma_{load}}{1 - S_{22}\Gamma_{load}} S_{12}}$$

Examples of 2-ports (1)

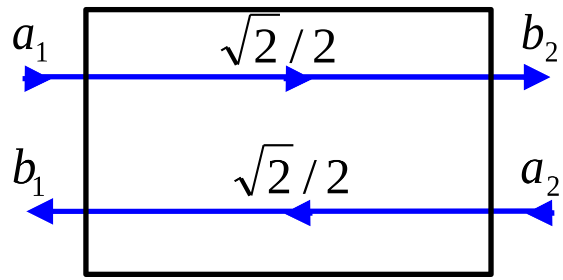
Line of Z=50Ω, length l=λ/4

$$(S)=\begin{bmatrix}0 & -j\\ -j & 0\end{bmatrix}\qquad \begin{aligned} b_1 &= -j a_2\\ b_2 &= -j a_1\end{aligned}$$



Attenuator 3dB, i.e. half output power

$$(S)=\frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\qquad \begin{aligned} b_1 &= \frac{1}{\sqrt{2}}a_2 = 0.707a_2\\ b_2 &= \frac{1}{\sqrt{2}}a_1 = 0.707a_1\end{aligned}$$



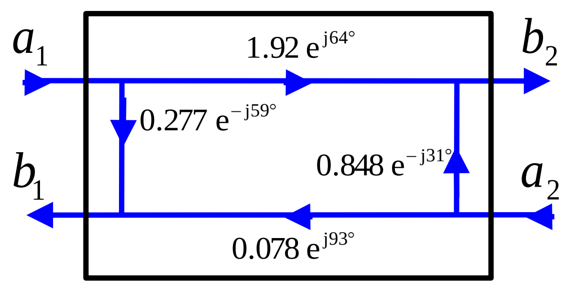
RF Transistor

backward transmission

$$(S)=\begin{bmatrix}0.277e^{-j59°} & 0.078e^{j93°}\\ 1.92e^{j64°} & 0.848e^{-j31°}\end{bmatrix}$$

forward transmission

non-reciprocal since $S_{12} \neq S_{21}$!
=different transmission forwards and backwards

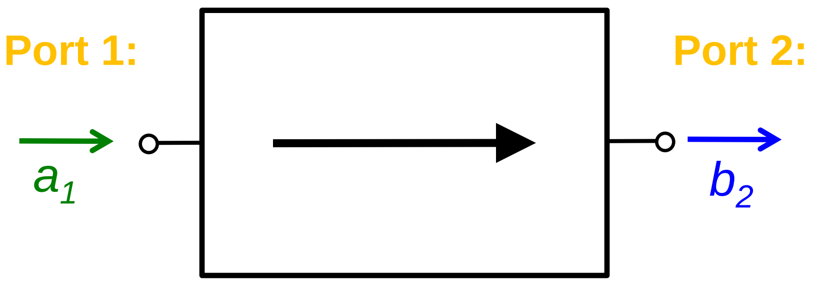


Examples of 2-ports (2)

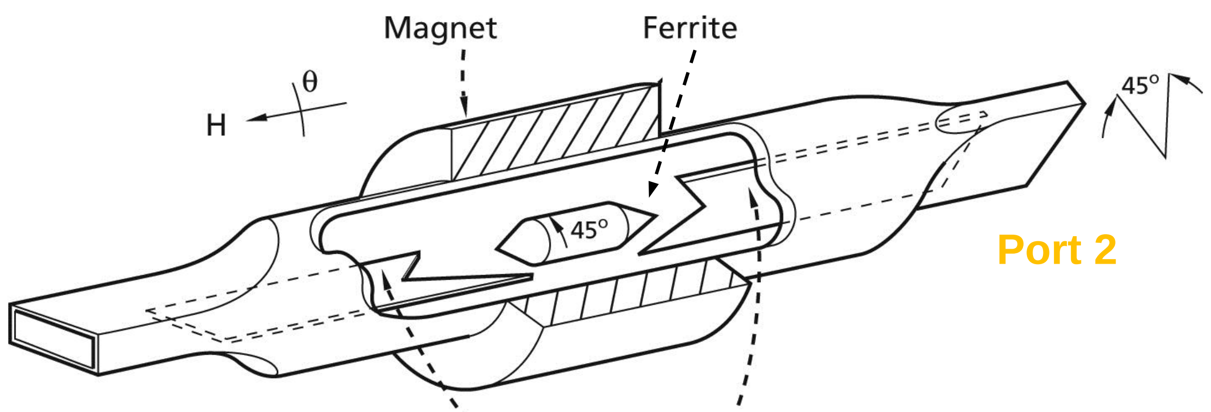
Ideal Isolator

$$(S)=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix} \qquad b_2 = a_1$$

only forward transmission



Faraday rotation isolator



The left waveguide uses a TE_{10} mode (=vertically polarized H field). After transition to a circular waveguide, the polarization of the mode is rotated counter clockwise by 45° by a ferrite. Then follows a transition to another rectangular waveguide which is rotated by 45° such that the forward wave can pass unhindered. However, a wave coming from the other side will have its polarization rotated by 45° clockwise as seen from the right hand side.

Examples of 3-ports (1)

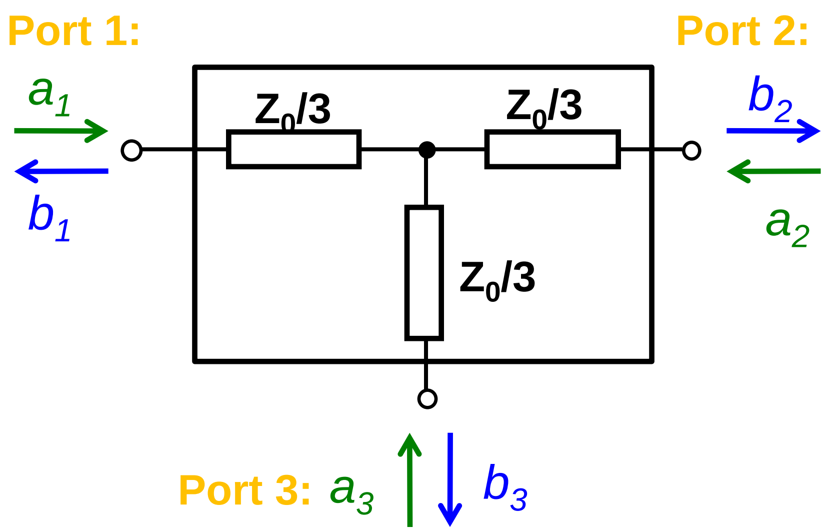
Resistive power divider

$$(S) = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$b_1 = \frac{1}{2}(a_2 + a_3)$$

$$b_2 = \frac{1}{2}(a_1 + a_3)$$

$$b_3 = \frac{1}{2}(a_1 + a_2)$$



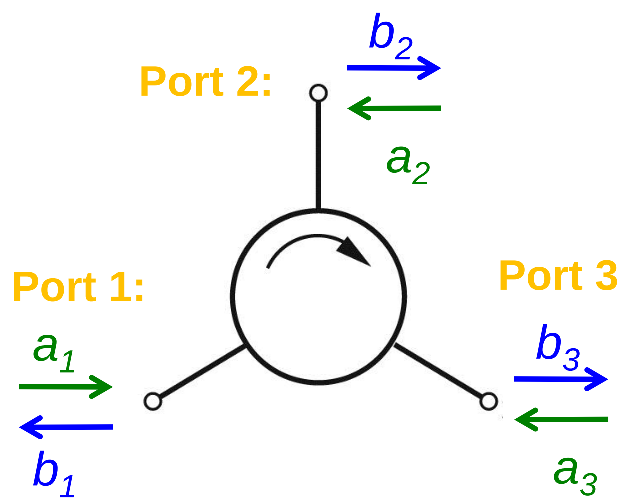
3-port circulator

$$(S) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$b_1 = a_3$$

$$b_2 = a_1$$

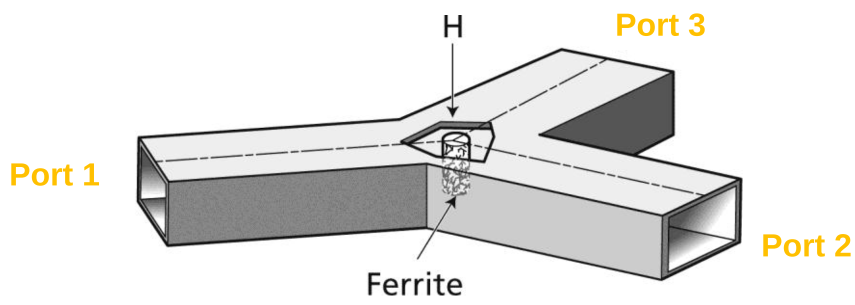
$$b_3 = a_2$$



The ideal circulator is lossless, matched at all ports, but not reciprocal. A signal entering the ideal circulator at one port is transmitted exclusively to the next port in the sense of the arrow.

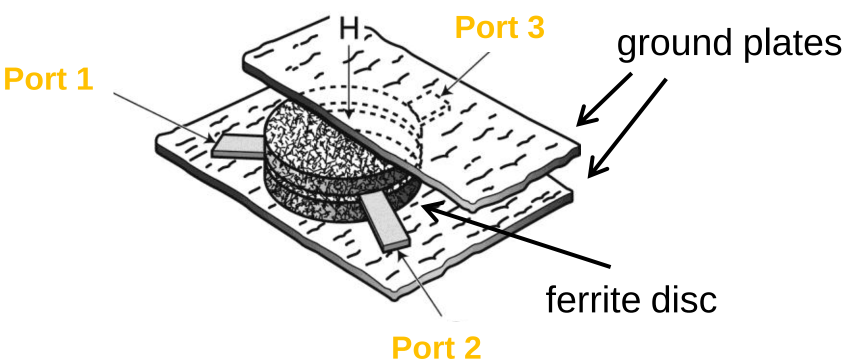
Examples of 3-ports (2)

Practical implementations of circulators:



Waveguide circulator

Stripline circulator



A circulator contains a volume of ferrite. The magnetically polarized ferrite provides the required non-reciprocal properties, thus power is only transmitted from port 1 to port 2, from port 2 to port 3, and from port 3 to port 1.

Examples of 4-ports (1)

Ideal directional coupler

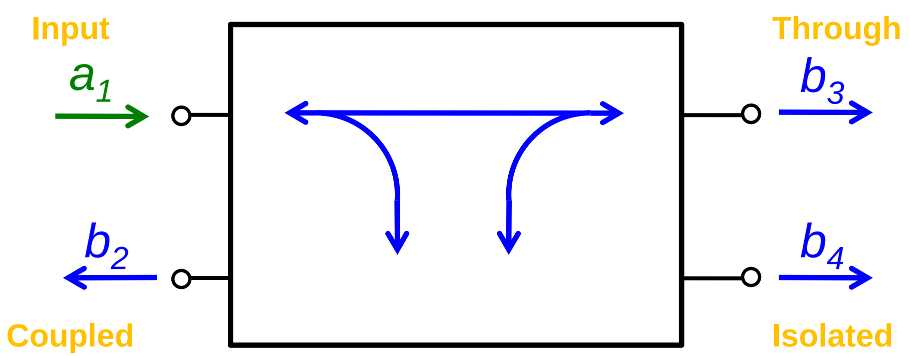
$$(S)=\begin{bmatrix} 0 & jk & \sqrt{1-k^2} & 0 \\ jk & 0 & 0 & \sqrt{1-k^2} \\ \sqrt{1-k^2} & 0 & 0 & jk \\ 0 & \sqrt{1-k^2} & jk & 0 \end{bmatrix} \text{ with } k=\left|\frac{b_2}{a_1}\right|$$

To characterize directional couplers, three important figures are used:

the coupling $$C=-20\log_{10}\left|\frac{b_2}{a_1}\right|$$

the directivity $$D=-20\log_{10}\left|\frac{b_4}{b_2}\right|$$

the isolation $$I=-20\log_{10}\left|\frac{a_1}{b_4}\right|$$

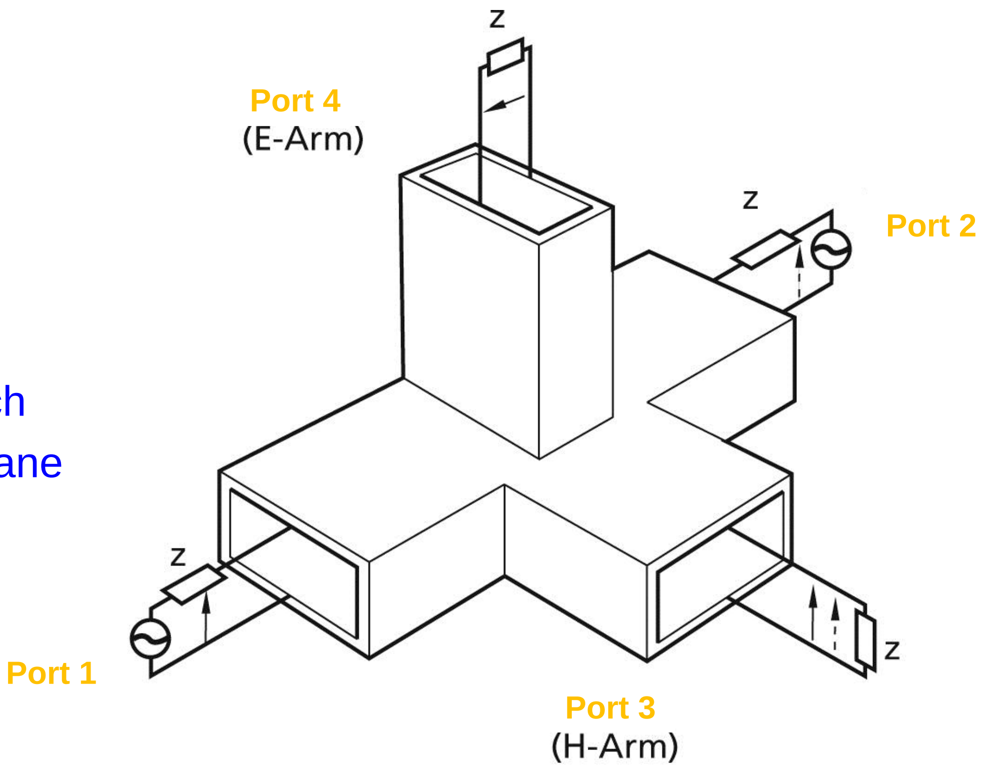


Examples of 4-ports (2)

Magic-T also referred to as **180º hybrid**:

$$(S) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

The H-plane is defined as a plane in which the magnetic field lines are situated. E-plane correspondingly for the electric field.



Can be implemented as waveguide or coaxial version.
Historically, the name originates from the waveguide version where you can “see” the horizontal and vertical “T”.

Evaluation of scattering parameters (1)

Basic relation:

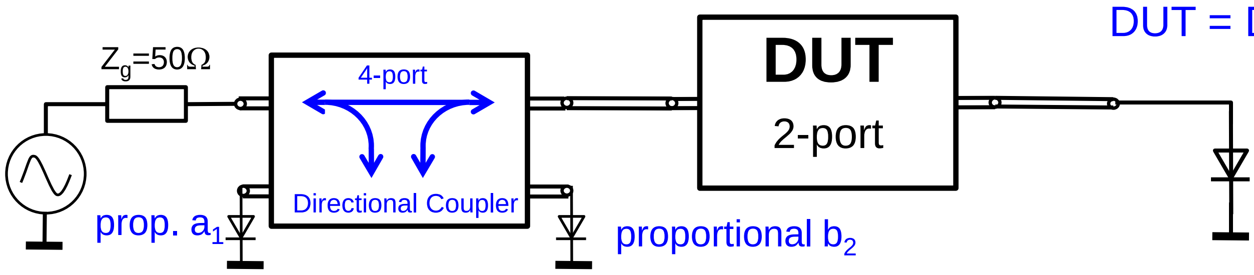
$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

Finding S_{11}, S_{21}: ("forward" parameters, assuming port 1 = input, port 2 = output e.g. in a transistor)

- connect a generator at port 1 and inject a wave a_1 into it
- connect reflection-free absorber at port 2 to assure $a_2 = 0$
- calculate/measure
  - wave b_1 (reflection at port 1)
  - wave b_2 (generated at port 2)
- evaluate

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \quad \text{"input reflection factor"}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \quad \text{"forward transmission factor"}$$



DUT = Device Under Test

Matched receiver or detector

Evaluation of scattering parameters (2)

Finding S_{12}, S_{22}: ("backward" parameters)

- interchange generator and load
- proceed in analogy to the forward parameters, i.e. inject wave a_2 and assure $a_1 = 0$
- evaluate

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \quad \text{"backward transmission factor"}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \quad \text{"output reflection factor"}$$

For a proper S-parameter measurement all ports of the Device Under Test (DUT) including the generator port must be terminated with their characteristic impedance in order to assure that waves travelling away from the DUT (b_n-waves) are not reflected back and convert into a_n-waves.

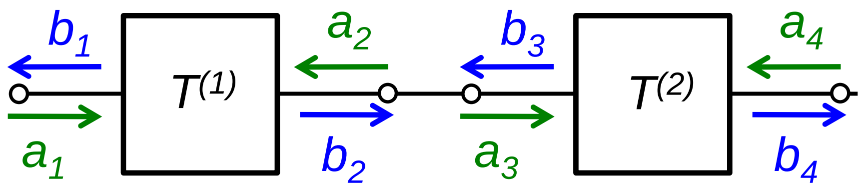
Scattering transfer parameters

The T-parameter matrix is related to the incident and reflected normalised waves at each of the ports.

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

T-parameters may be used to determine the effect of a cascaded 2-port networks by simply multiplying the individual T-parameter matrices:

$$[T] = \left[T^{(1)}\right]\left[T^{(2)}\right]\ldots\left[T^{(N)}\right] = \prod_N \left[T^{(i)}\right]$$



T-parameters can be directly evaluated from the associated S-parameters and vice versa.

From S to T:

$$[T] = \frac{1}{S_{21}} \begin{bmatrix} -\det(S) & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

From T to S:

$$[S] = \frac{1}{T_{22}} \begin{bmatrix} T_{12} & \det(T) \\ 1 & -T_{21} \end{bmatrix}$$

Decibel (1)

- The Decibel is the unit used to express relative differences in signal power. It is expressed as the base 10 logarithm of the ratio of the powers of two signals:

$$P\,[dB] = 10 \cdot \log(P/P_0)$$

- Signal amplitude can also be expressed in dB. Since power is proportional to the square of a signal's amplitude, the voltage in dB is expressed as follows:

$$V\,[dB] = 20 \cdot \log(V/V_0)$$

- P_0 and V_0 are the reference power and voltage, respectively.
- A given value in dB is the same for power ratios as for voltage ratios
- There are no “power dB” or “voltage dB” as dB values always express a ratio!!!

Decibel (2)

- Conversely, the absolute power and voltage can be obtained from dB values by

$$P = P_0 \cdot 10^{\frac{P\,[\mathrm{dB}]}{10}}, \quad V = V_0 \cdot 10^{\frac{V\,[\mathrm{dB}]}{20}}$$

- Logarithms are useful as the unit of measurement because (1) signal power tends to span several orders of magnitude and (2) signal attenuation losses and gains can be expressed in terms of subtraction and addition.

Decibel (3)

- The following table helps to indicate the order of magnitude associated with dB:
- Power ratio = voltage ratio squared!
- S parameters are defined as ratios and sometimes expressed in dB, no explicit reference needed!

	power ratio	V, I, E or H ratio, S_{ij}
-20 dB	0.01	0.1
-10 dB	**0.1**	**0.32**
-3 dB	0.50	0.71
-1 dB	0.74	0.89
0 dB	**1**	**1**
1 dB	1.26	1.12
3 dB	2.00	1.41
10 dB	**10**	**3.16**
20 dB	100	10
n * 10 dB	10^n	$10^{n/2}$

Decibel (4)

- Frequently dB values are expressed using a special reference level and not SI units. Strictly speaking, the reference value should be included in parenthesis when giving a dB value, e.g. +3 dB (1W) indicates 3 dB at P_0 = 1 Watt, thus 2 W.
- For instance, dBm defines dB using a reference level of P_0 = 1 mW. Often a reference impedance of 50Ω is assumed.
- Thus, 0 dBm correspond to -30 dBW, where dBW indicates a reference level of P_0=1W.
- Other common units:
  - dBmV for the small voltages, V_0 = 1 mV
  - dBμV/m for the electric field strength radiated from an antenna, E_0 = 1 μV/m

Measurement devices (1)

- There are many ways to observe RF signals. Here we give a brief overview of the four main tools we have at hand

- Oscilloscope: to observe signals in time domain
  - periodic signals
  - burst signal
  - application: direct observation of signal from a pick-up, shape of common 230 V mains supply voltage, etc.
- Spectrum analyser: to observe signals in frequency domain
  - sweeps through a given frequency range point by point
  - application: observation of spectrum from the beam or of the spectrum emitted from an antenna, etc.

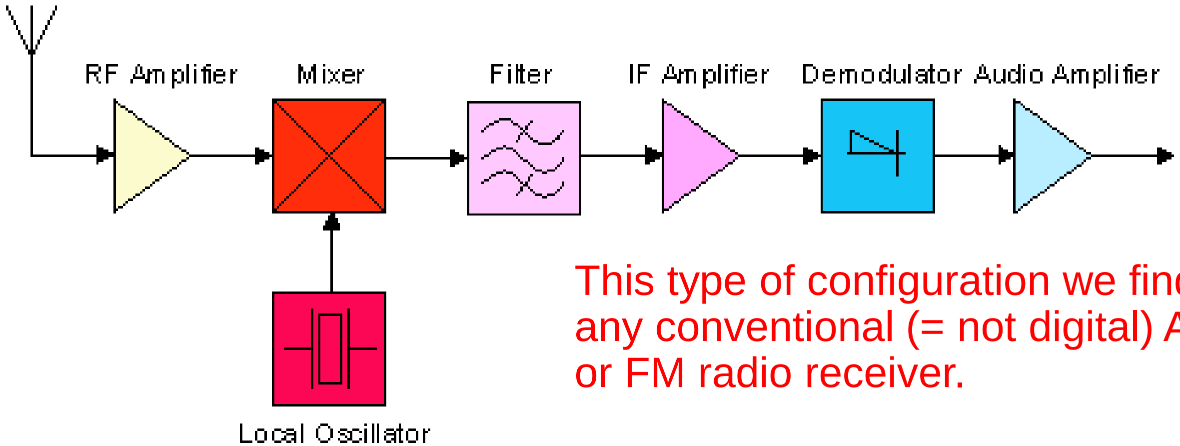
Measurement devices (2)

- Dynamic signal analyser (FFT analyser)
  - Acquires signal in time domain by fast sampling
  - Further numerical treatment in digital signal processors (DSPs)
  - Spectrum calculated using Fast Fourier Transform (FFT)
  - Combines features of a scope and a spectrum analyser: signals can be looked at directly in time domain or in frequency domain
  - Contrary to the SPA, also the spectrum of non-repetitive signals and transients can be observed
  - Application: Observation of tune sidebands, transient behaviour of a phase locked loop, etc.
- Network analyser
  - Excites a network (circuit, antenna, amplifier or such) at a given CW frequency and measures response in magnitude and phase => determines S-parameters
  - Covers a frequency range by measuring step-by-step at subsequent frequency points
  - Application: characterization of passive and active components, time domain reflectometry by Fourier transforming reflection response, etc.

Superheterodyne Concept (1)

Design and its evolution

The diagram below shows the basic elements of a single conversion superhet receiver. The essential elements of a local oscillator and a mixer followed by a fixed-tuned filter and IF amplifier are common to all superhet circuits. [super ετερω δυναμισ] a mixture of latin and greek … it means: *another force becomes superimposed.*

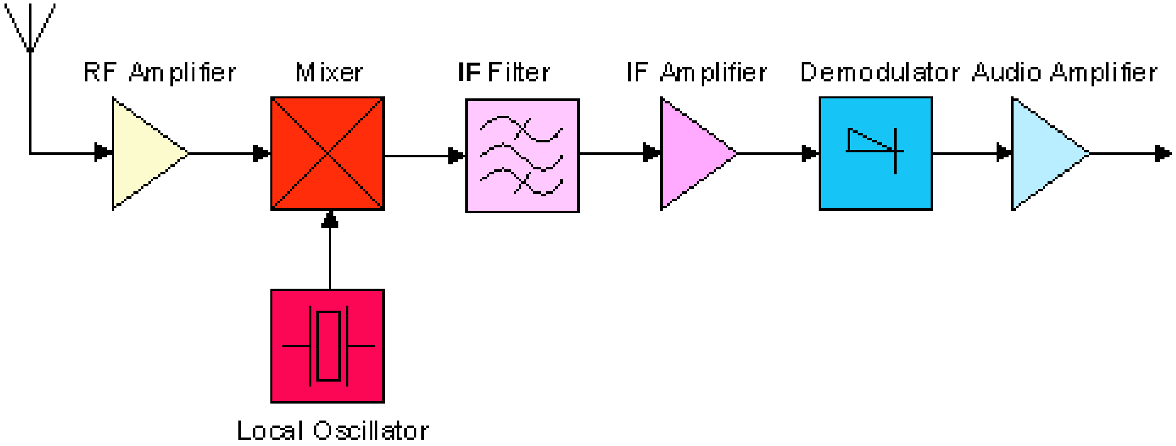


This type of configuration we find in any conventional (= not digital) AM or FM radio receiver.

The advantage to this method is that most of the radio's signal path has to be sensitive to only a narrow range of frequencies. Only the front end (the part before the frequency converter stage) needs to be sensitive to a wide frequency range. For example, the front end might need to be sensitive to 1–30 MHz, while the rest of the radio might need to be sensitive only to 455 kHz, a typical IF. Only one or two tuned stages need to be adjusted to track over the tuning range of the receiver; all the intermediate-frequency stages operate at a fixed frequency which need not be adjusted.

en.wikipedia.org

Superheterodyne Concept (2)



RF Amplifier = wideband frontend amplification (RF = radio frequency)

The Mixer can be seen as an analog multiplier which multiplies the RF signal with the **LO** (local oscillator) signal.

The local oscillator has its name because it's an oscillator situated in the receiver locally and not far away as the radio transmitter to be received.

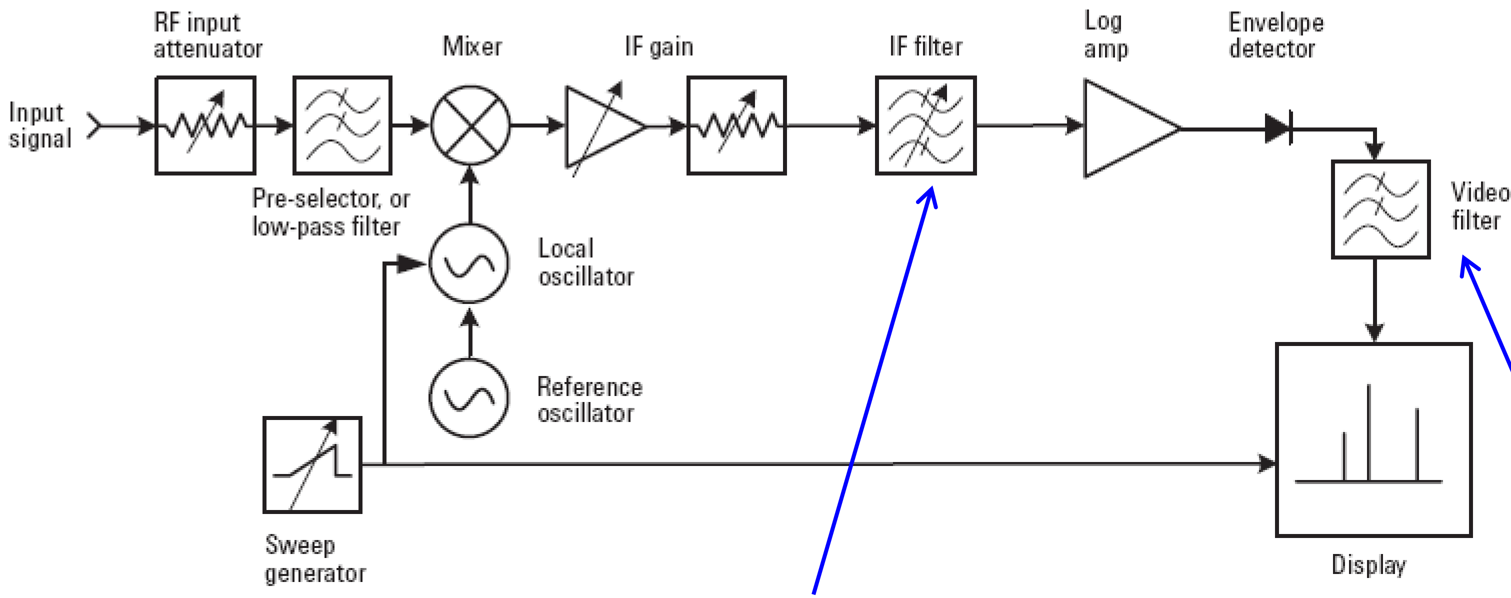
IF stands for intermediate frequency.

The demodulator can be an amplitude modulation (AM) demodulator (envelope detector) or a frequency modulation (FM) demodulator, implemented e.g. as a PLL (phase locked loop).

The tuning of a normal radio receiver is done by changing the frequency of the LO, not of the IF filter.

en.wikipedia.org

Example for Application of the Superheterodyne Concept in a Spectrum Analyzer



The center frequency is fixed, but the bandwidth of the IF filter can be modified.

The video filter is a simple low-pass with variable bandwidth before the signal arrives to the vertical deflection plates of the cathode ray tube.

Agilent, 'Spectrum Analyzer Basics,' Application Note 150, page 10 f.

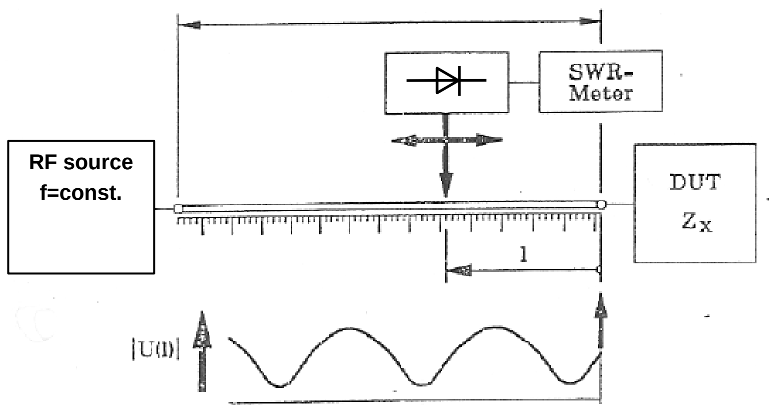
Voltage Standing Wave Ratio (1)

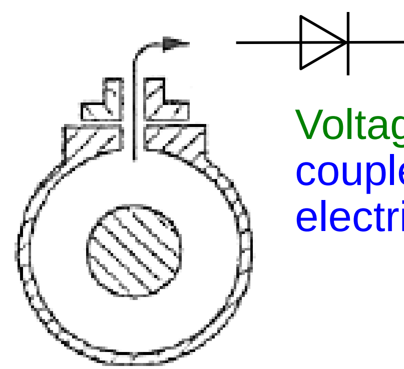
Origin of the term *"VOLTAGE Standing Wave Ratio – VSWR"*:

In the old days when there were no Vector Network Analyzers available, the reflection coefficient of some DUT (device under test) was determined with the coaxial measurement line.

Was is a coaxial measurement line?

This is a coaxial line with a narrow slot (slit) in length direction. In this slit a small voltage probe connected to a crystal detector (detector diode) is moved along the line. By measuring the ratio between the maximum and the minimum voltage seen by the probe and the recording the position of the maxima and minima the reflection coefficient of the DUT at the end of the line can be determined.





Voltage probe weakly coupled to the radial electric field.

Cross-section of the coaxial measurement line

Voltage Standing Wave Ratio (2)

VOLTAGE DISTRIBUTION ON LOSSLESS TRANSMISSION LINES

For an ideally terminated line the magnitude of voltage and current are constant along the line, their phase vary linearly.

In presence of a notable load reflection the voltage and current distribution along a transmission line are no longer uniform but exhibit characteristic ripples. The phase pattern resembles more and more to a staircase rather than a ramp.

A frequently used term is the "Voltage Standing Wave Ratio VSWR" that gives the ratio between maximum and minimum voltage along the line. It is related to load reflection by the expression

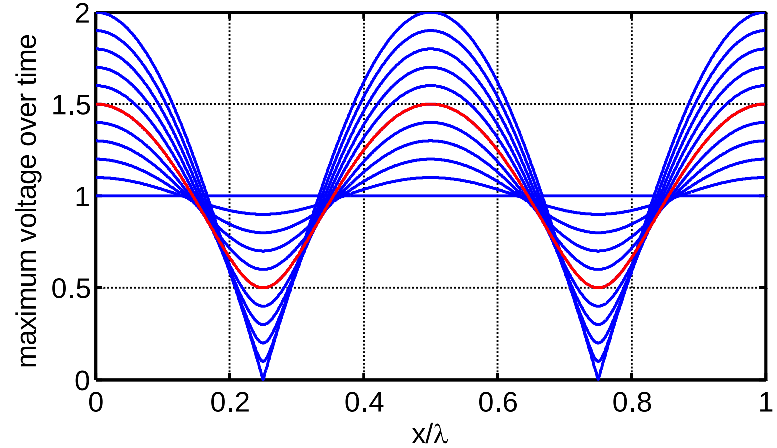
$$V_{\max} = |a| + |b| \qquad VSWR = \frac{V_{\max}}{V_{\min}} = \frac{|a| + |b|}{|a| - |b|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

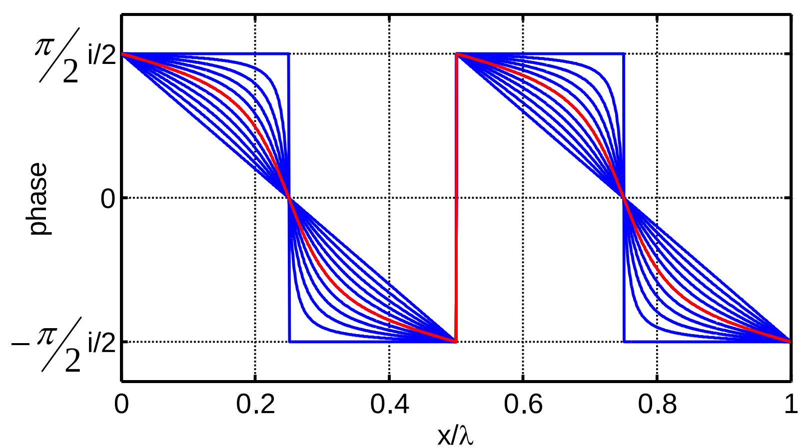
$$V_{\min} = |a| - |b|$$

Remember: the reflection coefficient Γ is defined via the ELECTRIC FIELD of the incident and reflected wave. This is historically related to the measurement method described here. We know that an open has a reflection coefficient of Γ=+1 and the short of Γ=-1. When referring to the magnetic field it would be just opposite.

Voltage Standing Wave Ratio (3)

Γ	**VSWR**	**Refl. Power $\|-\Gamma\|^2$**
0.0	1.00	1.00
0.1	1.22	0.99
0.2	1.50	0.96
0.3	1.87	0.91
0.4	2.33	0.84
0.5	**3.00**	**0.75**
0.6	4.00	0.64
0.7	5.67	0.51
0.8	9.00	0.36
0.9	19	0.19
1.0	∞	0.00





With a simple detector diode we cannot measure the phase, only the amplitude.

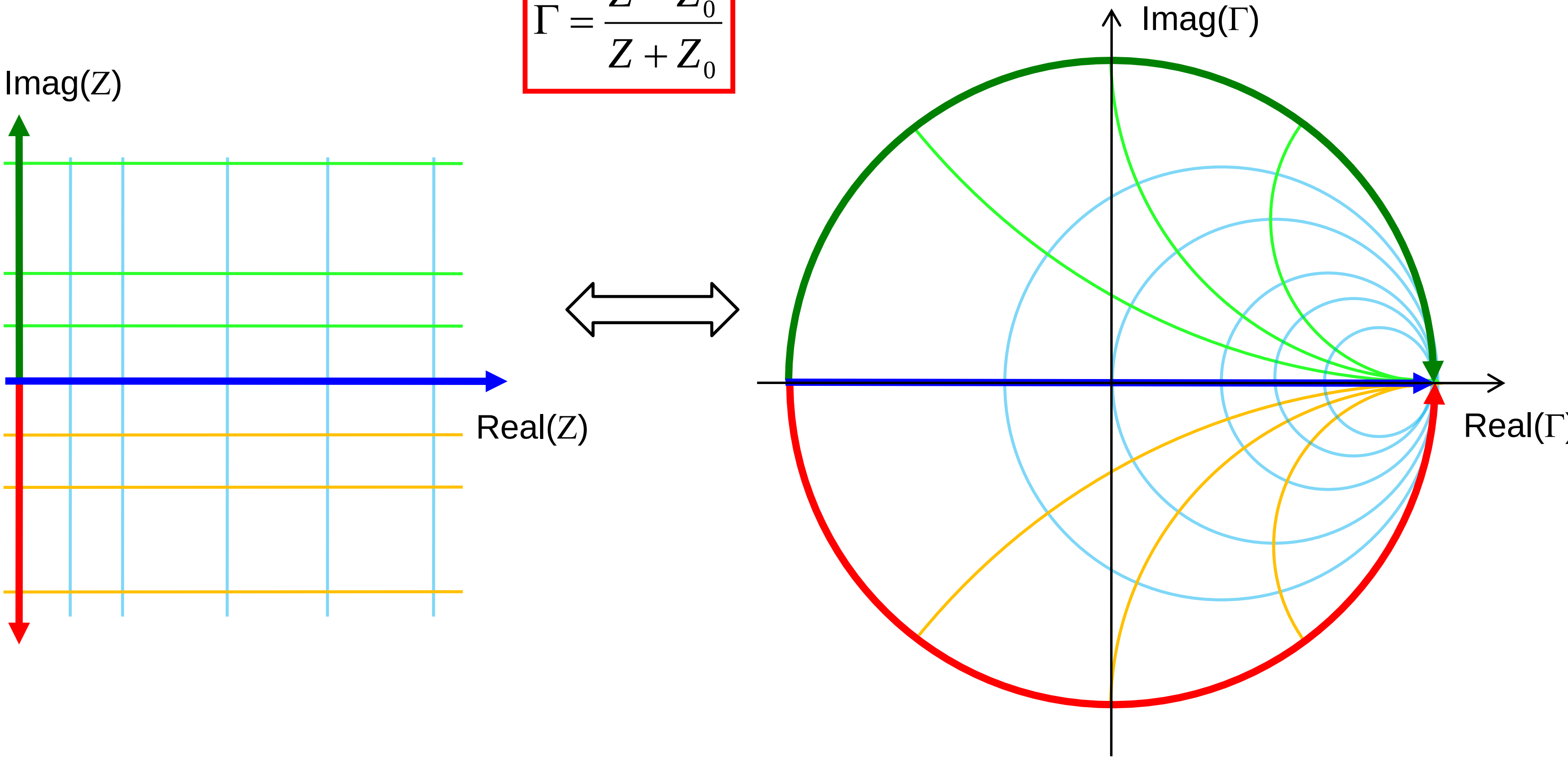
Why? – What would be required to measure the phase?

Answer: Because there is no reference. With a mixer which can be used as a phase detector when connected to a reference this would be possible.

The Smith Chart (1)

The Smith Chart represents the complex Γ-plane within the unit circle. It is a conform mapping of the complex Z-plane onto itself using the transformation

$$\boxed{\Gamma = \frac{Z - Z_0}{Z + Z_0}}$$



→The real positive half plane of Z is thus transformed into the interior of the unit circle!

The Smith Chart (2)

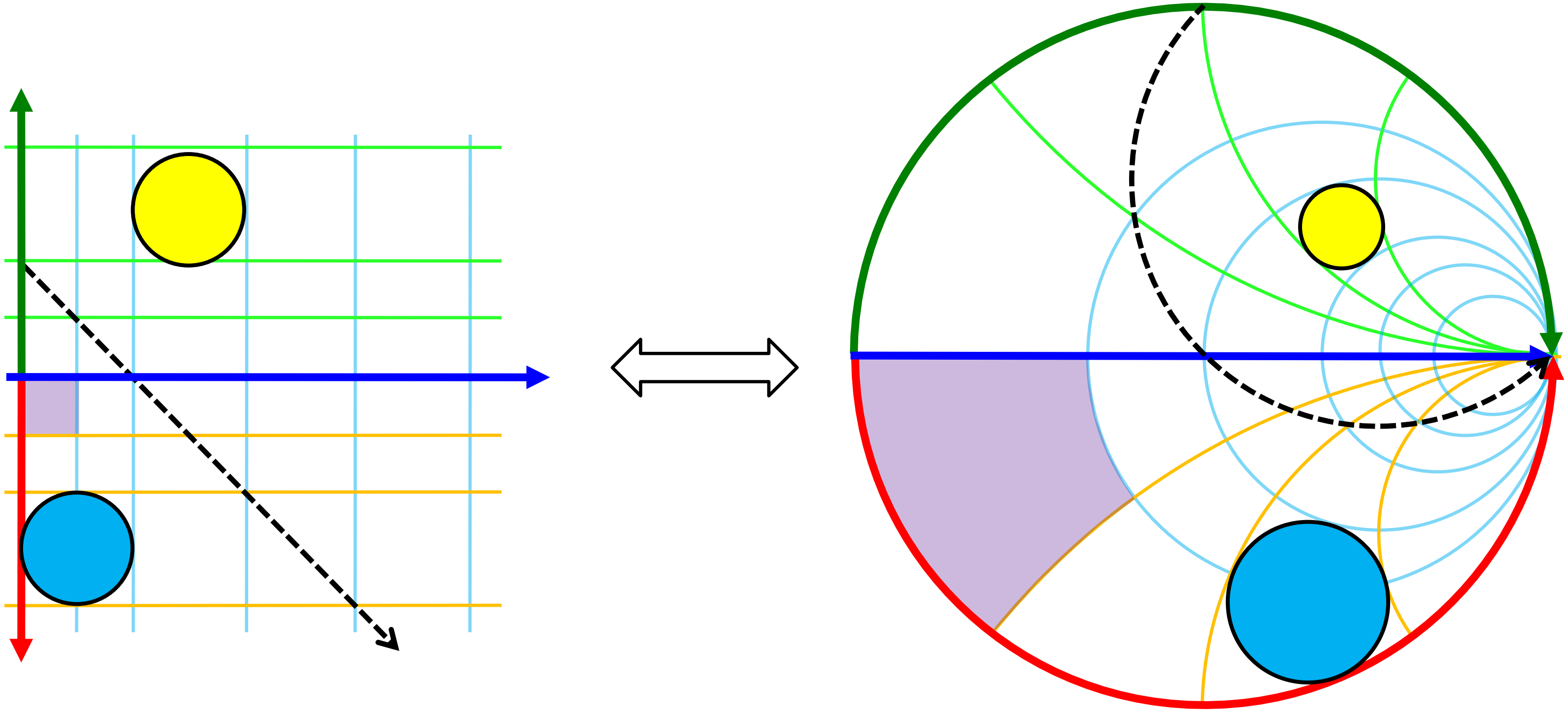
This is a "bilinear" transformation with the following properties:

- generalized circles are transformed into generalized circles
  - circle → circle
  - straight line → circle
  - circle → straight line
  - straight line → straight line
- angles are preserved locally

a straight line is nothing else than a circle with infinite radius

a circle is defined by 3 points

a straight line is defined by 2 points



The Smith Chart (3)

Impedances Z are usually first normalized by $$z = \frac{Z}{Z_0}$$

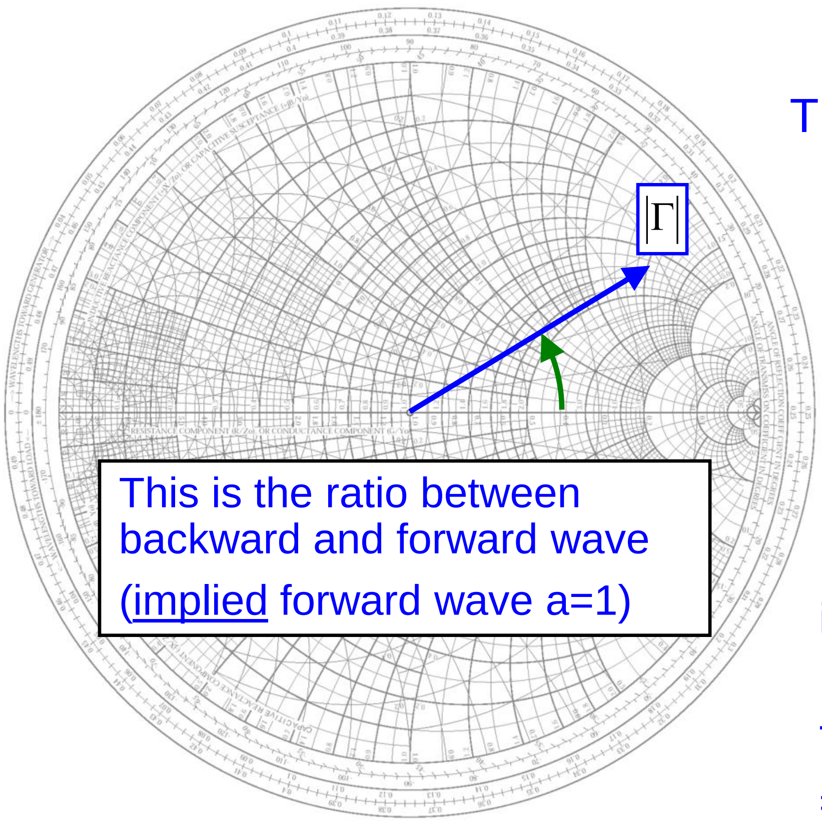
where Z_0 is some characteristic impedance (e.g. 50 Ohm). The general form of the transformation can then be written as

$$\Gamma = \frac{z-1}{z+1} \quad resp. \quad z = \frac{1+\Gamma}{1-\Gamma}$$

<u>This mapping offers several practical advantages:</u>

1. The diagram includes all "passive" impedances, i.e. those with positive real part, from zero to infinity in a handy format. Impedances with negative real part ("active device", e.g. reflection amplifiers) would be outside the (normal) Smith chart.
2. The mapping converts impedances or admintances into reflection factors and vice-versa. This is particularly interesting for studies in the radiofrequency and microwave domain where electrical quantities are usually expressed in terms of "direct" or "forward" waves and "reflected" or "backward" waves. This replaces the notation in terms of currents and voltages used at lower frequencies. Also the reference plane can be moved very easily using the Smith chart.

The Smith Chart (4)



The Smith Chart (*Abaque Smith* in French) is the linear representation of the complex reflection factor

$$\boxed{\Gamma = \frac{b}{a}}$$

i.e. the ratio backward/forward wave.

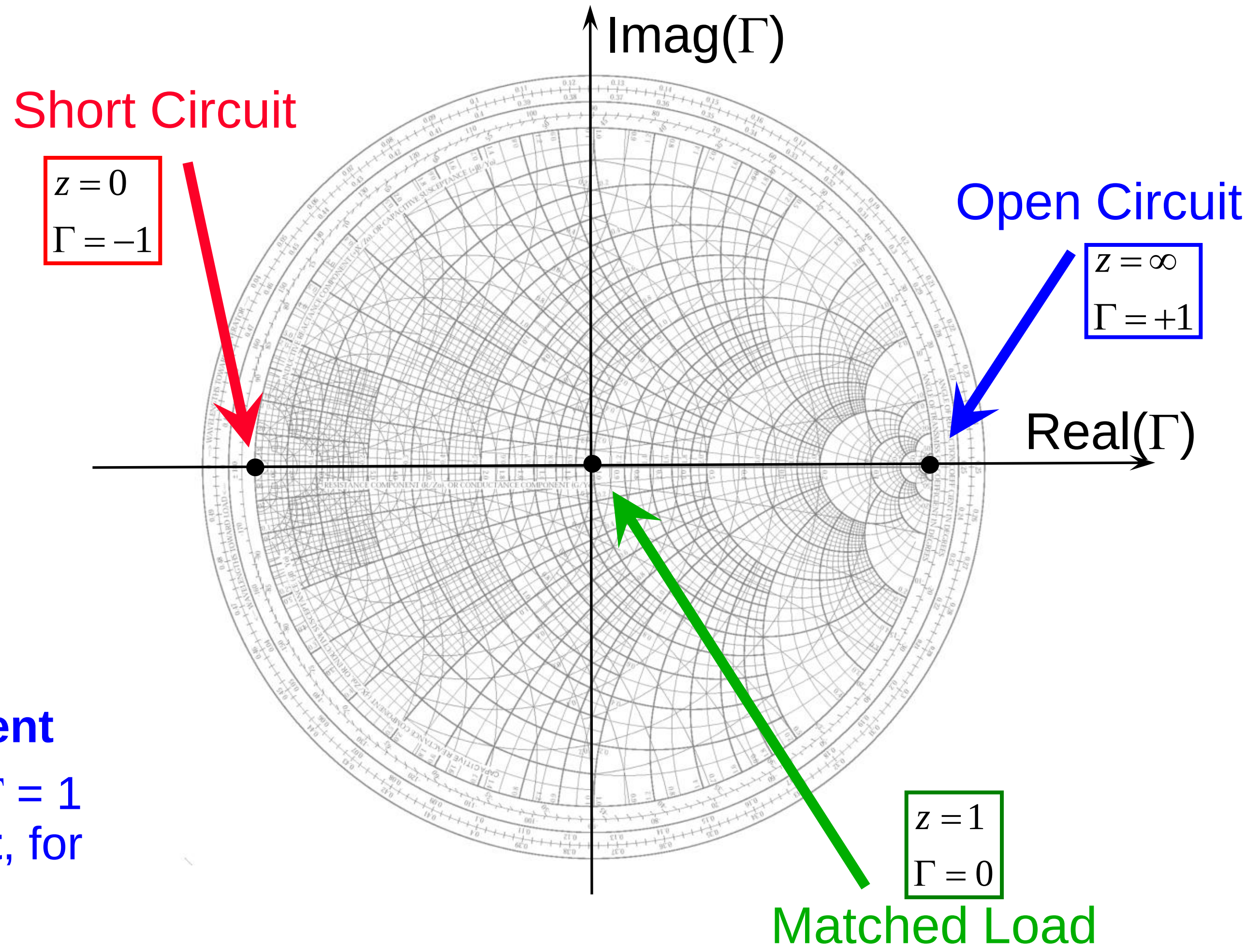
The upper half of the Smith-Chart is “inductive” = positive imaginary part of impedance, the lower half is “capacitive” = negative imaginary part.

Important points

Important Points:

- Short Circuit
  Γ = -1, z = 0
- Open Circuit
  Γ = 1, $z \to \infty$
- Matched Load
  Γ = 0, z = 1

- On circle Γ = 1
  lossless element
- Outside circle Γ = 1
  active element, for instance tunnel diode reflection amplifier

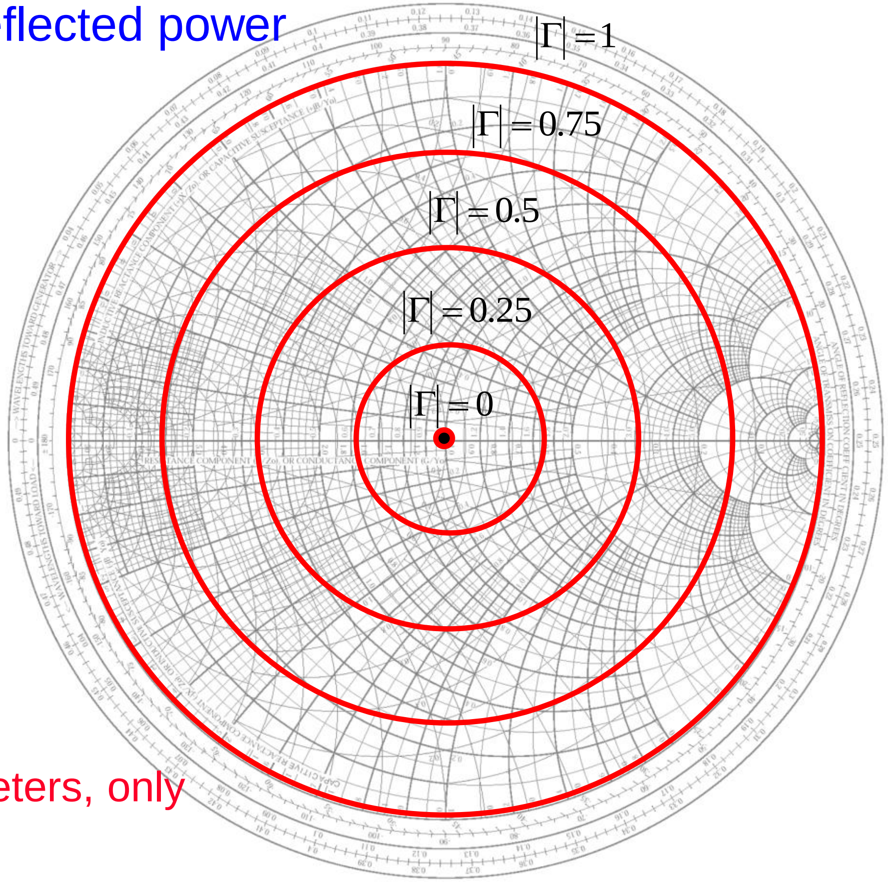


The Smith Chart (5)

3. The distance from the center of the diagram is directly proportional to the magnitude of the reflection factor. In particular, the perimeter of the diagram represents full reflection, $|\Gamma|=1$. Problems of matching are clearly visualize.

Power into the load = forward power – reflected power

$$P = |a|^2 - |b|^2$$
$$= \underbrace{|a|^2}_{\text{max source power}} \underbrace{\left(1 - |\Gamma|^2\right)}_{\text{"(mismatch)" loss}}$$



Here the US notion is used, where power = $|a|^2$.
European notation (often): power = $|a|^2/2$
These conventions have no impact on S parameters, only relevant for absolute power calculation

The Smith Chart (6)

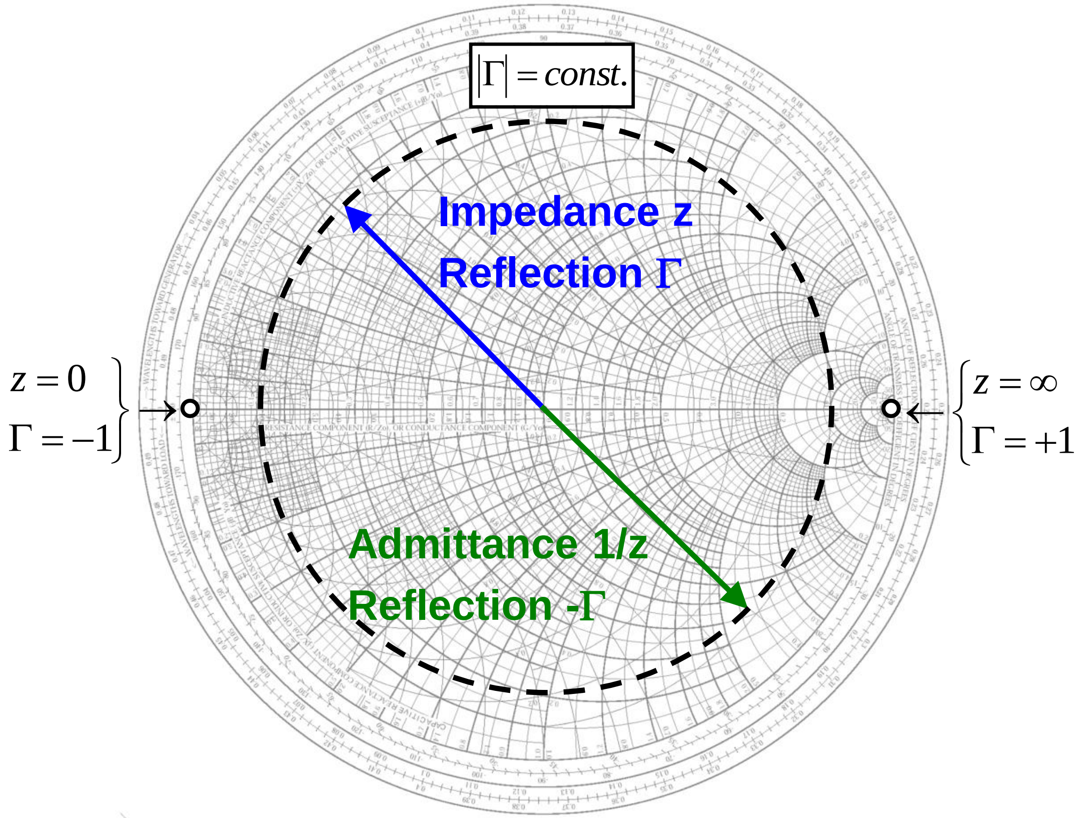
4. The transition

impedance ⇔ admittance

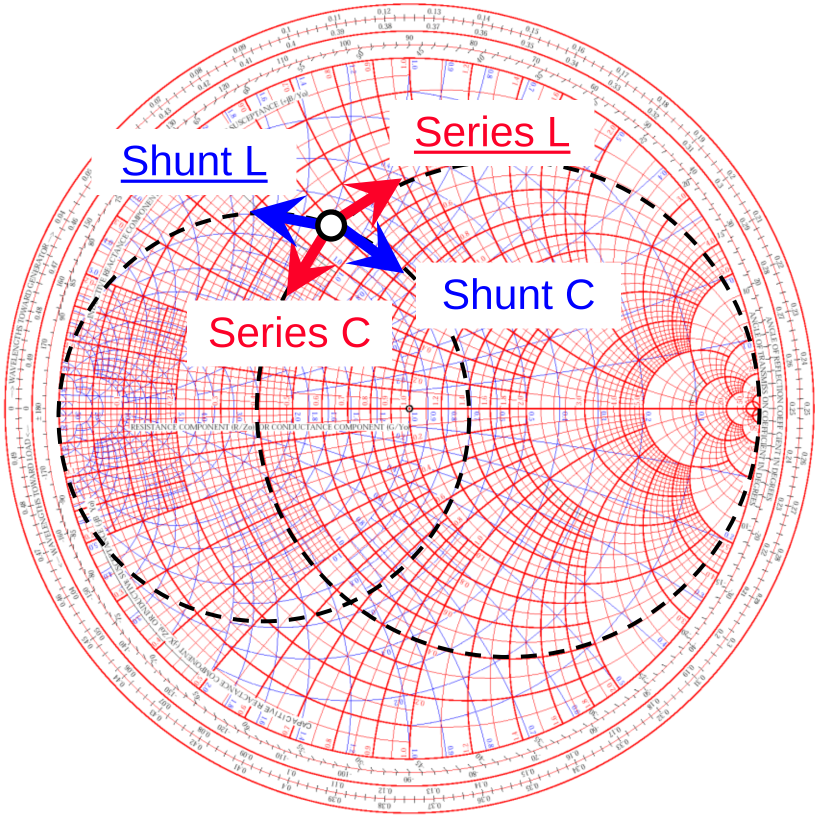
and vice-versa is particularly easy.

$$\Gamma(1/z) = \frac{1/z - 1}{1/z + 1} = \frac{1 - z}{1 + z} = -\left(\frac{z-1}{z+1}\right)$$

$$\underline{\Gamma(1/z) = -\Gamma(z)}$$



Navigation in the Smith Chart (1)

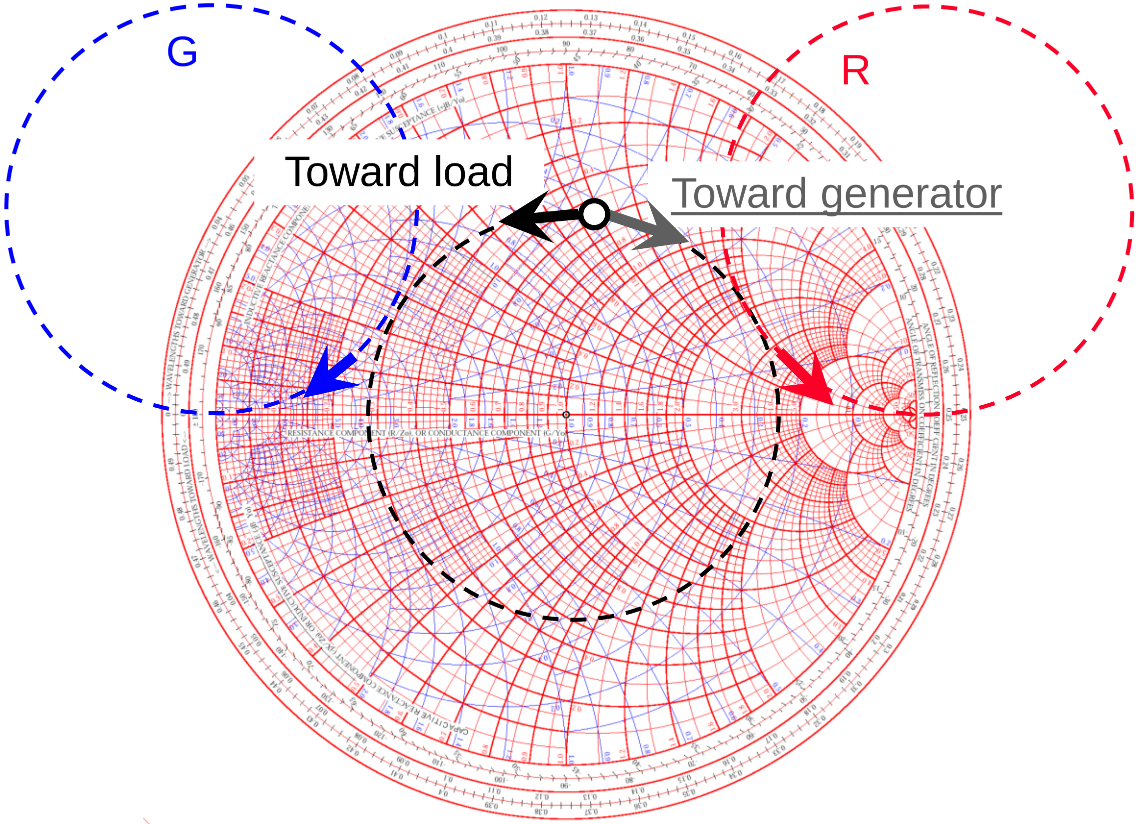


in blue: Impedance plane (=Z)

in red: Admittance plane (=Y)

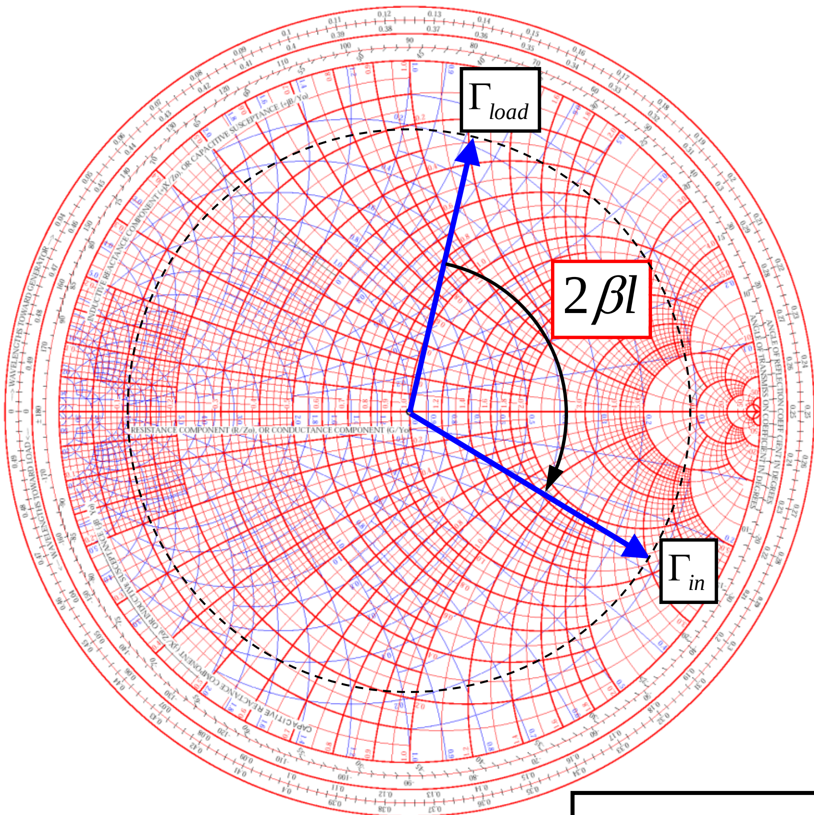
	Up	Down
Red circles	**Series L**	**Series C**
Blue circles	**Shunt L**	**Shunt C**

Navigation in the Smith Chart (2)



Red arcs	**Resistance R**
Blue arcs	**Conductance G**
Con-centric circle	**Transmission line going Toward load** **Toward generator**

Impedance transformation by transmission lines



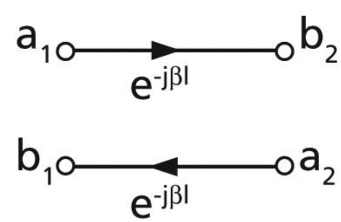
$$\Gamma_{in} = \Gamma_{load}\, e^{-j2\beta l}$$

The S-matrix for an ideal, lossless transmission line of length l is given by

$$\mathbf{S} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

where $\beta = 2\pi / \lambda$

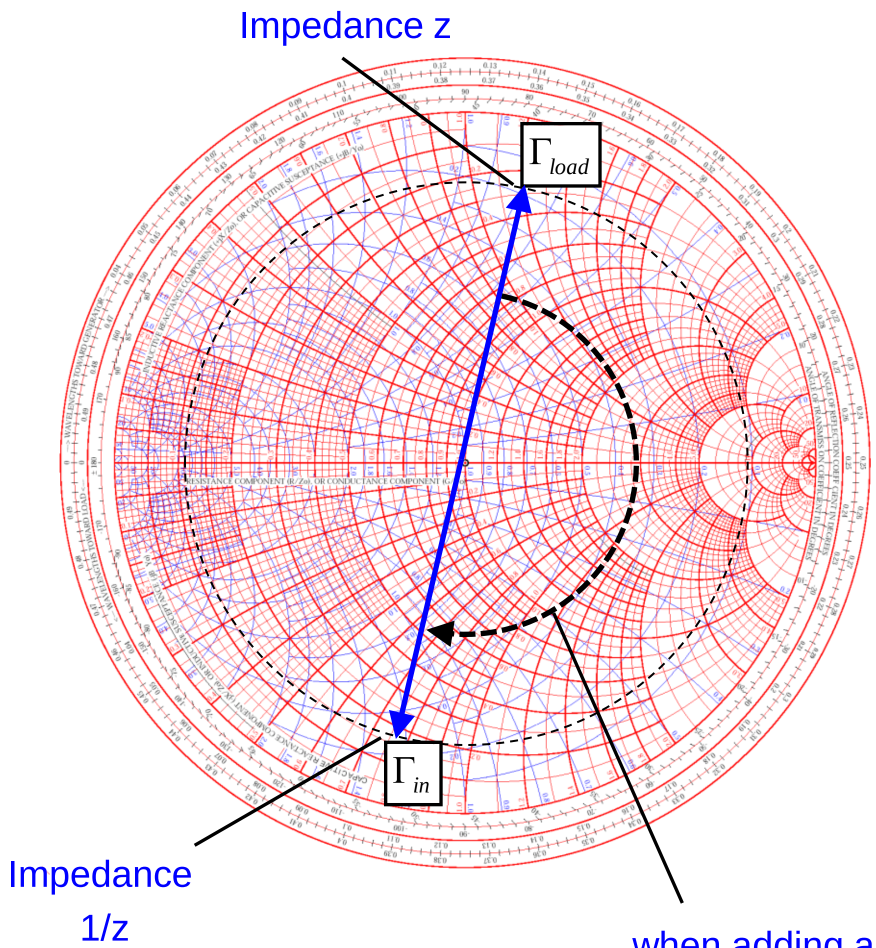
is the propagation coefficient with the wavelength λ (this refers to the wavelength on the line containing some dielectric).



How to remember that when adding a section of line we have to turn clockwise: assume we are at Γ=-1 (short circuit) and add a very short piece of coaxial cable. Then we have made an inductance thus we are in the upper half of the Smith-Chart.

N.B.: It is supposed that the reflection factors are evaluated with respect to the characteristic impedance Z_0 of the line segment.

λ/4 - Line transformations



A transmission line of length

$$l = \lambda / 4$$

transforms a load reflection Γ_{load} to its input as

$$\Gamma_{in} = \Gamma_{load}\, e^{-j2\beta l} = \Gamma_{load}\, e^{-j\pi} = \underline{-\Gamma_{load}}$$

This means that normalized load impedance z is transformed into 1/z.

In particular, a short circuit at one end is transformed into an open circuit at the other. This is the principle of λ/4-resonators.

when adding a transmission line
to some terminating impedance we move
clockwise through the Smith-Chart.

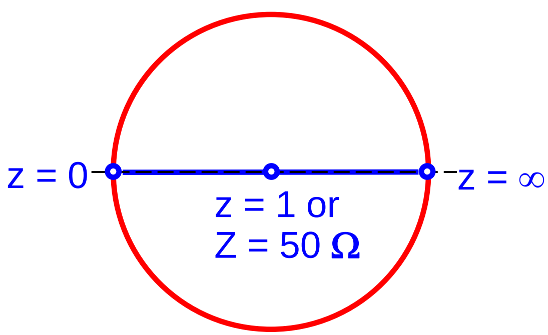
Looking through a 2-port (1)

In general:

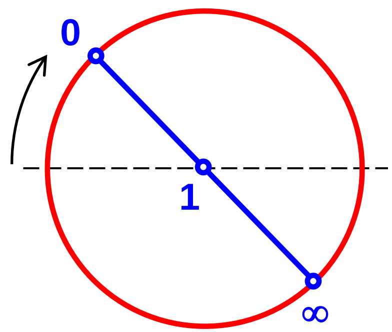
$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

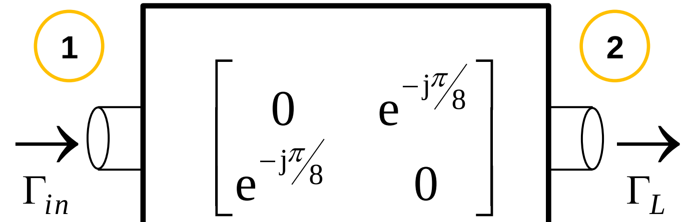
were Γ_{in} is the reflection coefficient when looking through the 2-port and Γ_{load} is the load reflection coefficient.

The outer circle and the real axis in the simplified Smith diagram below are mapped to other circles and lines, as can be seen on the right.



Line λ/16:

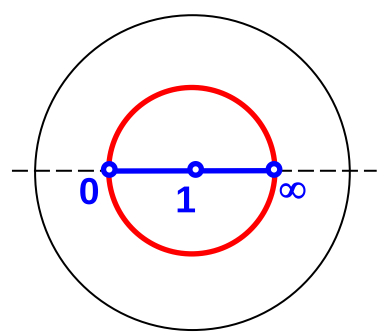


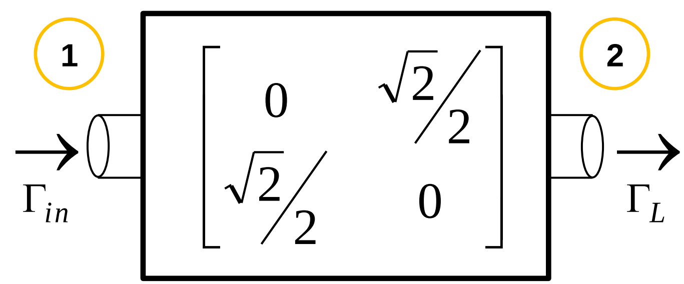


$$\begin{bmatrix} 0 & e^{-j\pi/8} \\ e^{-j\pi/8} & 0 \end{bmatrix}$$

$$\Rightarrow \Gamma_{in} = \Gamma_L \, e^{-j\pi/4}$$

Attenuator 3dB:



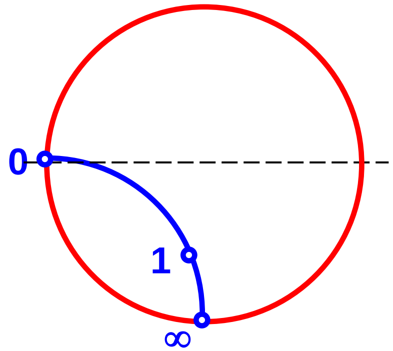


$$\begin{bmatrix} 0 & \sqrt{2}/2 \\ \sqrt{2}/2 & 0 \end{bmatrix}$$

$$\Rightarrow \Gamma_{in} = \Gamma_L/2$$

Looking through a 2-port (2)

Lossless
Passive
Circuit



1

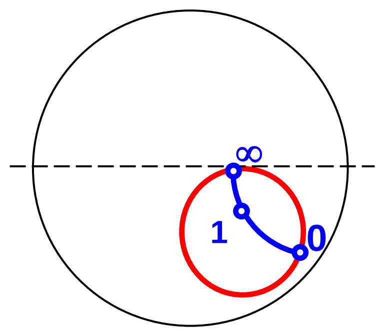
If S is unitary

$$S^{*}S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

→ Lossless Two-Port

2

Lossy
Passive
Circuit



1

Lossy Two-Port:

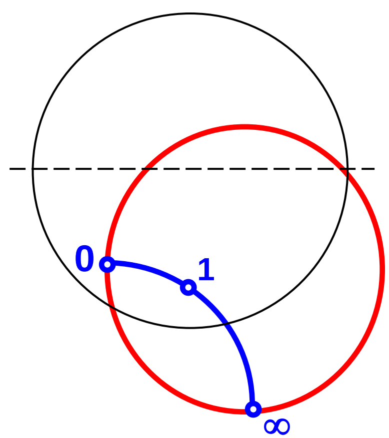
If $K_{LINVILL} < 1$

$K_{ROLLET} > 1$

unconditionally stable

2

Active
Circuit



1

Active Circuit:

If $K_{LINVILL} \geq 1$

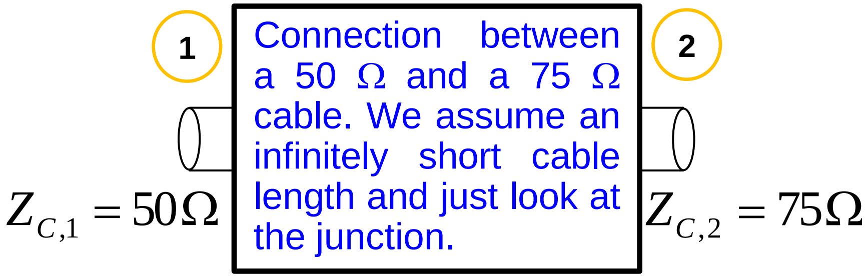
$K_{ROLLET} \leq 1$

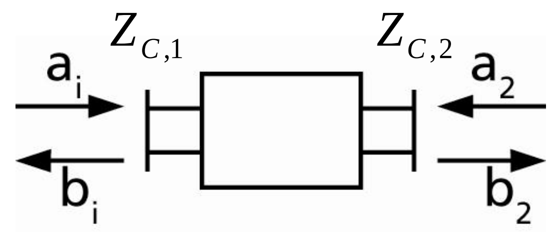
potentially unstable

2

Example: a Step in Characteristic Impedance (1)

Consider a connection of two coaxial cables, one with $Z_{C,1}$ = 50 Ω characteristic impedance, the other with $Z_{C,2}$ = 75 Ω characteristic impedance.





<u>Step 1:</u> Calculate the reflection coefficient and keep in mind: all ports have to be terminated with their respective characteristic impedance, i.e. 75 Ω for port 2.

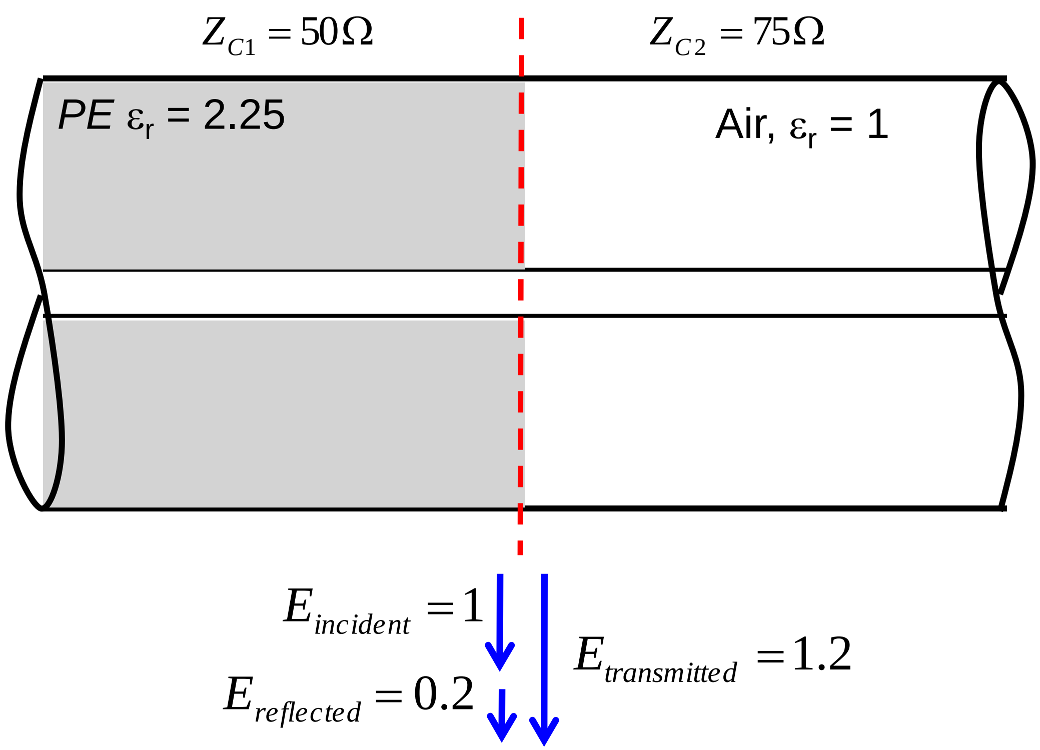
$$\Gamma_1 = \frac{Z - Z_{C,1}}{Z + Z_{C,1}} = \frac{75-50}{75+50} = 0.2$$

Thus, the voltage of the reflected wave at port 1 is 20% of the incident wave and the reflected power at port 1 (proportional Γ^2) is $0.2^2 = 4\%$. As this junction is lossless, the transmitted power must be 96% (conservation of energy). From this we can deduce $b_2^2 = 0.96$. But: how do we get the voltage of this outgoing wave?

Example: a Step in Characteristic Impedance (2)

Step 2: Remember, a and b are power-waves and defined as voltage of the forward- or backward travelling wave normalized to $\sqrt{Z_C}$.

The tangential electric field in the dielectric in the 50 Ω and the 75 Ω line, respectively, must be continuous.



t = voltage transmission coefficient $t = 1 + \Gamma$ in this case.

This is counterintuitive, one might expect 1-Γ. Note that the voltage of the transmitted wave is higher than the voltage of the incident wave. But we have to normalize to $\sqrt{Z_C}$ to get the corresponding S-parameter. $S_{12} = S_{21}$ via reciprocity! But $S_{11} \neq S_{22}$, i.e. the structure is NOT symmetric.

Example: a Step in Characteristic Impedance (3)

Once we have determined the voltage transmission coefficient, we have to normalize to the ratio of the characteristic impedances, respectively. Thus we get for

$$S_{12} = 1.2\sqrt{\frac{50}{75}} = 1.2 \cdot 0.816 = 0.9798$$

We know from the previous calculation that the reflected power (proportional Γ^2) is 4% of the incident power. Thus 96% of the power are transmitted.

Check done $S_{12}{}^2 = 1.44\frac{1}{1.5} = 0.96 = (0.9798)^2$

$$S_{22} = \frac{50-75}{50+75} = -0.2$$ To be compared with S11 = +0.2!

Example: a Step in Characteristic Impedance (4)

Visualization in the Smith chart

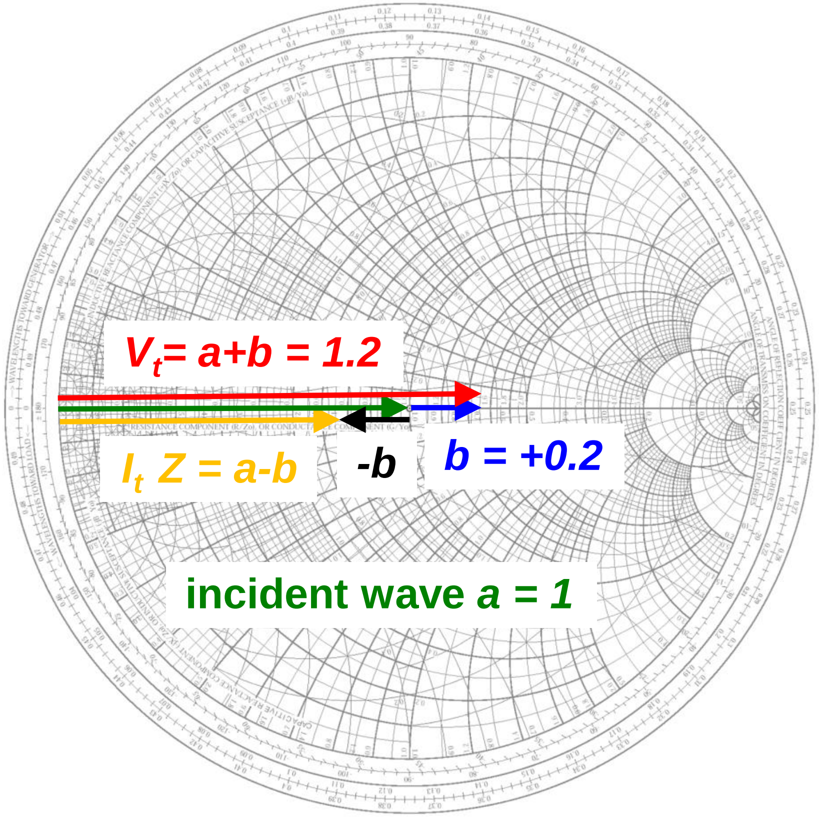
As shown in the previous slides the voltage of the transmitted wave is with

$t = 1 + \Gamma$

$V_t = a + b$ and subsequently the current is

$I_t Z = a - b$.

Remember: the reflection coefficient Γ is defined with respect to voltages. For currents the sign inverts. Thus a positive reflection coefficient in the normal definition leads to a subtraction of currents or is negative with respect to current.

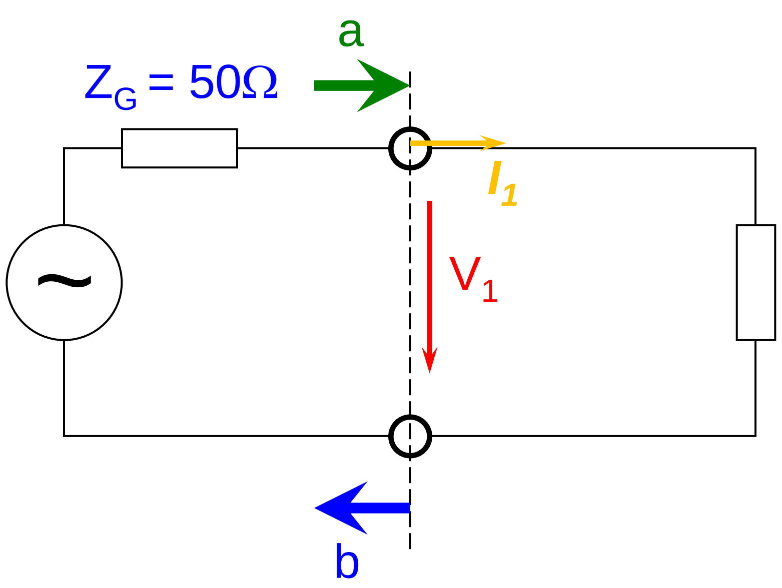


Note: here Z_{load} is real

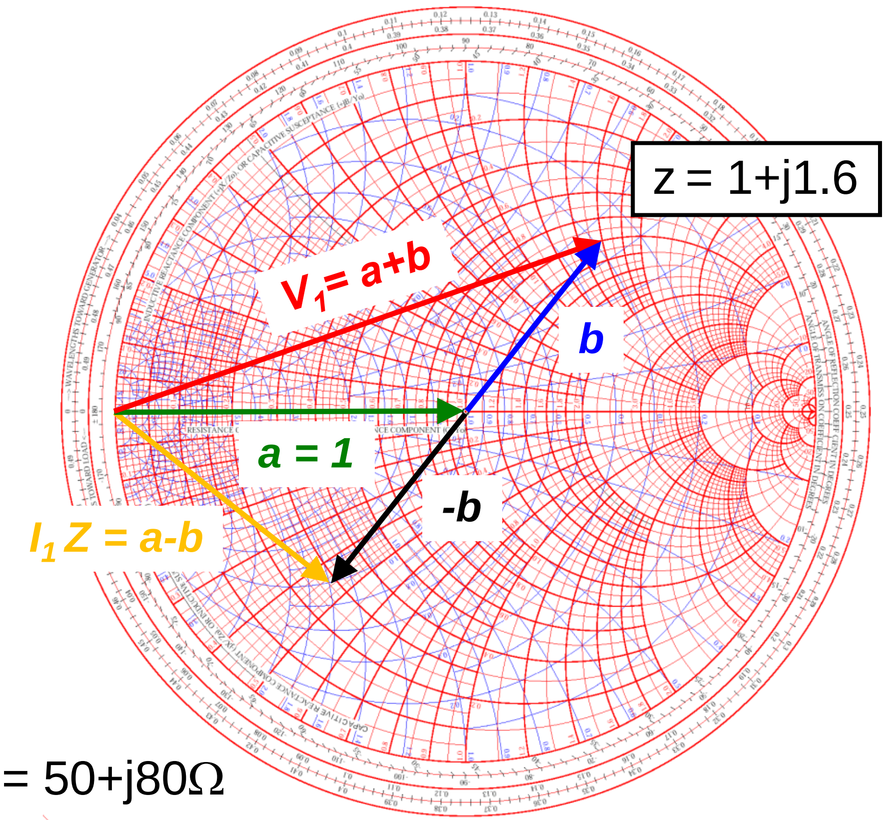
Example: a Step in Characteristic Impedance (5)

General case

Thus we can read from the Smith chart immediately the amplitude and phase of voltage and current on the load (of course we can calculate it when using the complex voltage divider).



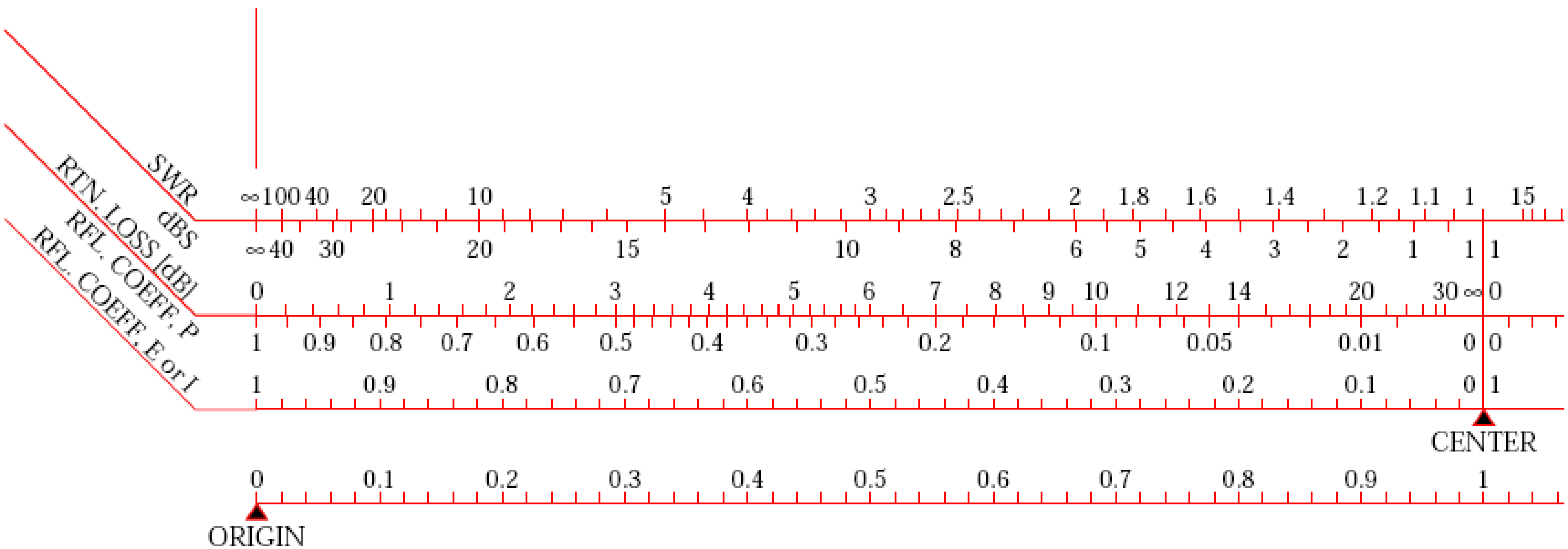
Z = 50+j80Ω
(load impedance)



What about all these rulers below the Smith chart (1)

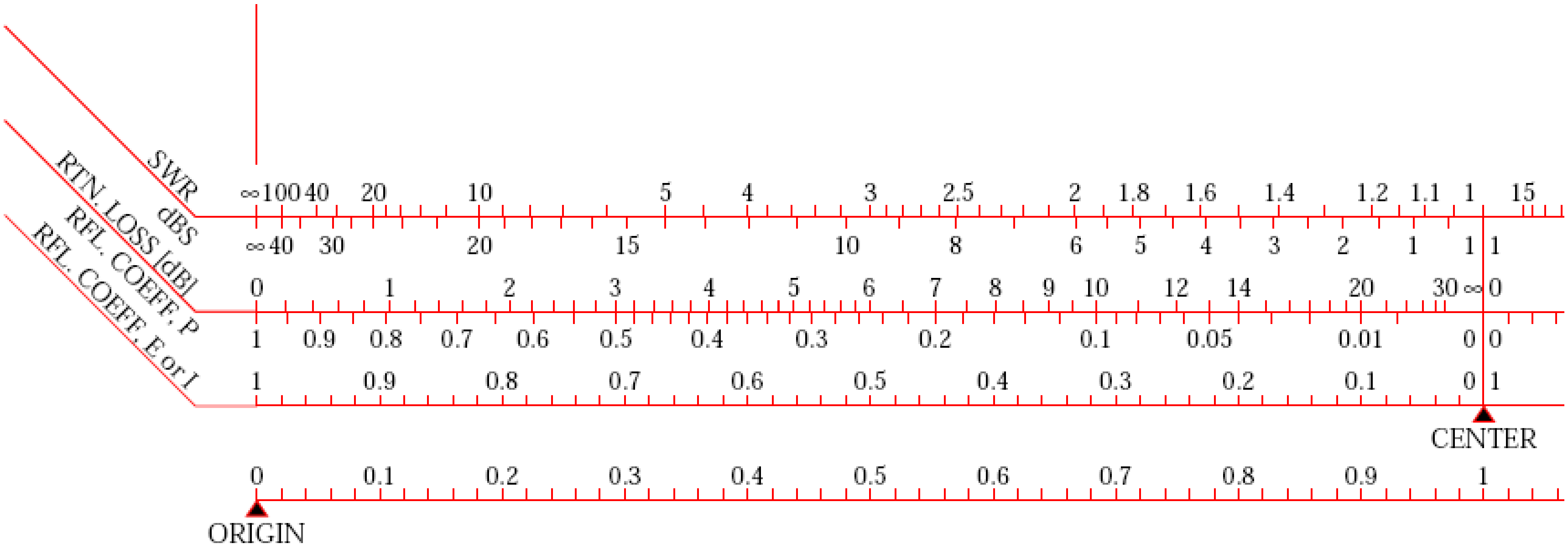
How to use these rulers:

You take the modulus of the reflection coefficient of an impedance to be examined by some means, either with a conventional ruler or better take it into the compass. Then refer to the coordinate denoted to CENTER and go to the left or for the other part of the rulers (not shown here in the magnification) to the right except for the lowest line which is marked ORIGIN at the left.



What about all these rulers below the Smith chart (2)

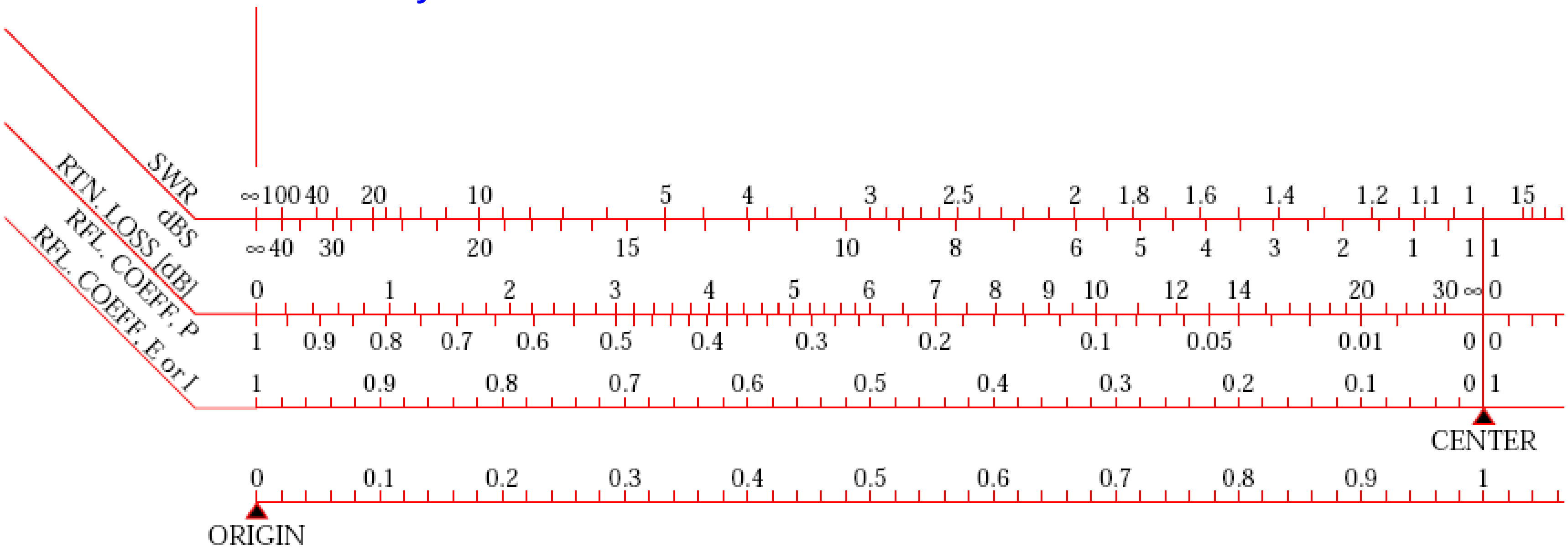
First ruler / left / upper part, marked SWR. This means VSWR, i.e. Voltage Standing Wave Ratio, the range of value is between one and infinity. One is for the matched case (center of the Smith chart), infinity is for total reflection (boundary of the SC). The upper part is in linear scale, the lower part of this ruler is in dB, noted as dBS (dB referred to Standing Wave Ratio). Example: SWR = 10 corresponds to 20 dBS, SWR = 100 corresponds to 40 dBS [voltage ratios, not power ratios].



Example

What about all these rulers below the Smith chart (3)

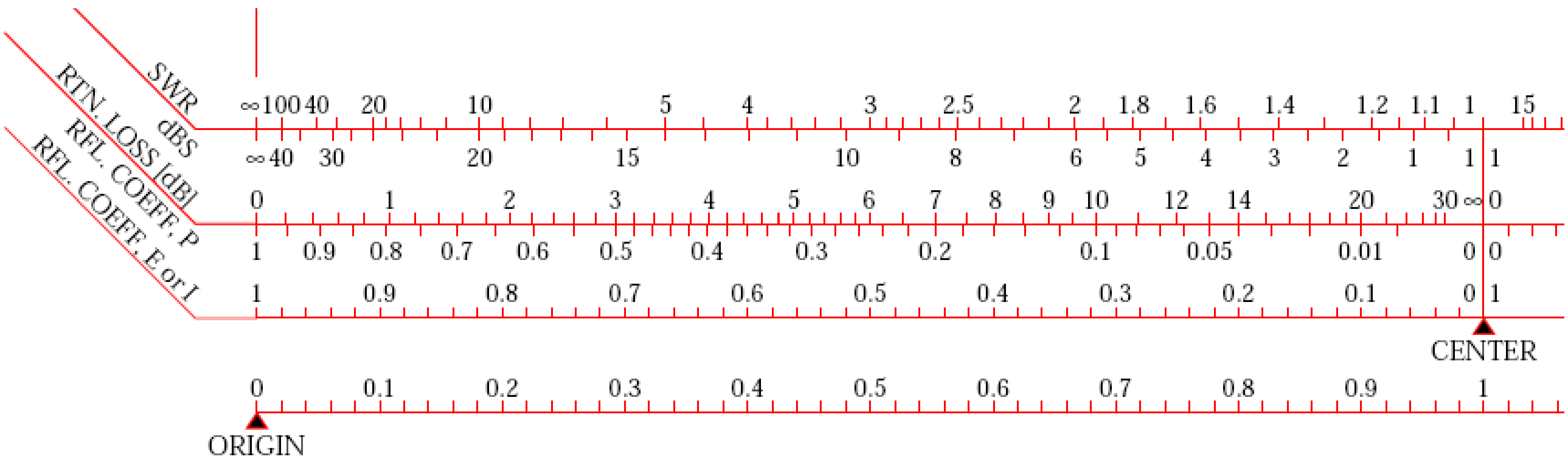
Second ruler / left / upper part, marked as RTN.LOSS = return loss in dB. This indicates the amount of reflected wave expressed in dB. Thus, in the center of SC nothing is reflected and the return loss is infinite. At the boundary we have full reflection, thus return loss 0 dB. The lower part of the scale denoted as RFL.COEFF. P = reflection coefficient in terms of POWER (proportional $|\Gamma|^2$). No reflected power for the matched case = center of the SC, (normalized) reflected power = 1 at the boundary.



What about all these rulers below the Smith chart (4)

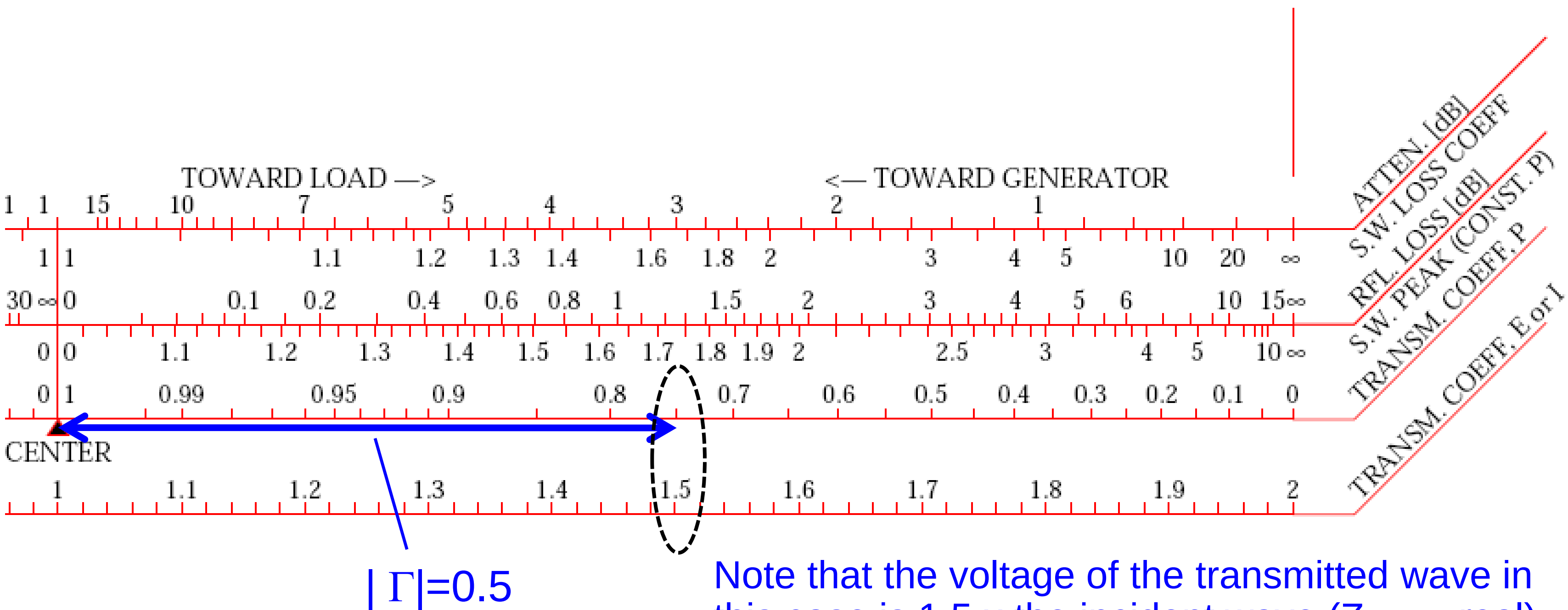
Third ruler / left, marked as RFL.COEFF,E or I = gives us the modulus (= absolute value) of the reflection coefficient in linear scale. Note that since we have the modulus we can refer it both to voltage or current as we have omitted the sign, we just use the modulus. Obviously in the center the reflection coefficient is zero, at the boundary it is one.

The fourth ruler has been discussed in the example of the previous slides: Voltage transmission coefficient. Note that the modulus of the voltage (and current) transmission coefficient has a range from zero, i.e. short circuit, to +2 (open = 1+Γ with Γ=1). This ruler is only valid for Z_{load} = real, i.e. the case of a step in characteristic impedance of the coaxial line.



What about all these rulers below the Smith chart (5)

Third ruler / right, marked as TRANSM.COEFF.P refers to the transmitted power as a function of mismatch and displays essentially the relation $P_t = 1 - |\Gamma|^2$. Thus, in the center of the SC full match, all the power is transmitted. At the boundary we have total reflection and e.g. for a Γ value of 0.5 we see that 75% of the incident power is transmitted.



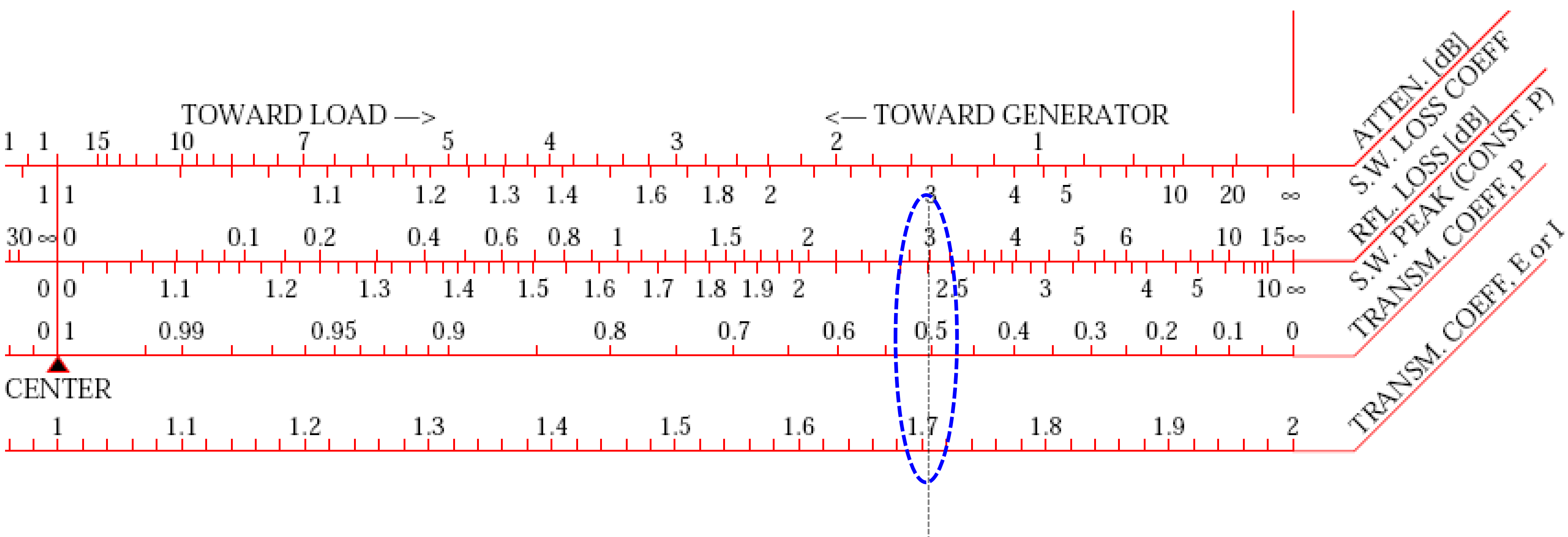
Note that the voltage of the transmitted wave in this case is 1.5 x the incident wave (Z_{load} = real)

What about all these rulers below the Smith chart (6)

Second ruler / right / upper part, denoted as RFL.LOSS in dB = reflection loss. This ruler refers to the loss in the **transmitted** wave, not to be confounded with the return loss referring to the **reflected** wave. It displays the relation $P_t = 1 - |\Gamma|^2$ in dB.

Example: $|\Gamma| = 1/\sqrt{2} = 0.707$, transmitted power = 50% thus loss = 50% = 3dB.

Note that in the lowest ruler the voltage of the transmitted wave (Z_{load} = real) would be $V_t = 1.707 = 1 + 1/\sqrt{2}$ if referring to the voltage.

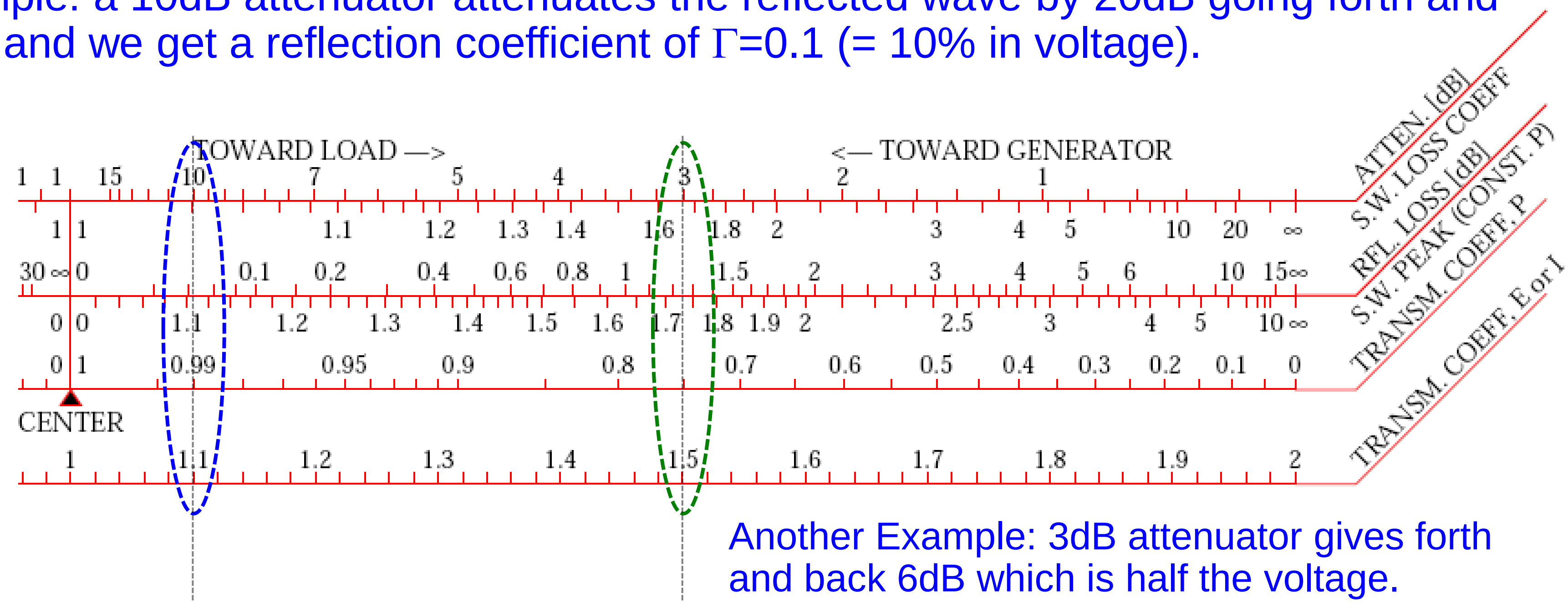


What about all these rulers below the Smith chart (7)

First ruler / right / upper part, denoted as ATTEN. in dB assumes that we are measuring an attenuator (that may be a lossy line) which itself is terminated by an open or short circuit (full reflection). Thus the wave is travelling twice through the attenuator (forward and backward). The value of this attenuator can be between zero and some very high number corresponding to the matched case.

The lower scale of ruler #1 displays the same situation just in terms of VSWR.

Example: a 10dB attenuator attenuates the reflected wave by 20dB going forth and back and we get a reflection coefficient of Γ=0.1 (= 10% in voltage).



Another Example: 3dB attenuator gives forth and back 6dB which is half the voltage.

Further reading

Introductory literature

- A very good general introduction in the context of accelerator physics: CERN Accelerator School: RF Engineering for Particle Accelerators, Geneva
- The basics of the two most important RF measurement devices: Byrd, J M, Caspers, F, Spectrum and Network Analysers, CERN-PS-99-003-RF; Geneva

General RF Theory

- RF theory in a very reliable compilation: Zinke, O. and Brunswig H., Lehrbuch der Hochfrequenztechnik, Springer
- Rather theoretical approach to guided waves: Collin, R E, Field Theory of Guided Waves, IEEE Press
- Another very good one, more oriented towards application in telecommunications: Fontolliet, P.-G.. Systemes de Telecommunications, Traite d'Electricite, Vol. 17, Lausanne
- And of course the classic theoretical treatise: Jackson, J D, Classical Electrodynamics, Wiley

For the RF Engineer

- All you need to know in practice: Meinke, Gundlach, Taschenbuch der Hochfrequenztechnik, Springer
- Very useful as well: Matthaei, G, Young, L and Jones, E M T, Microwave Filters, Impedance-Matching Networks, and Coupling Structures, Artech House

Appendix

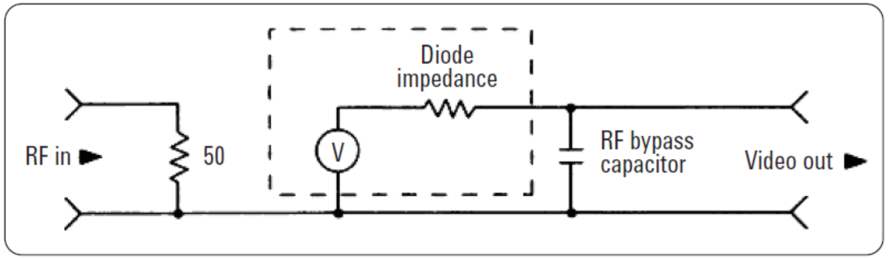
- **The RF diode**
- **The RF mixer**

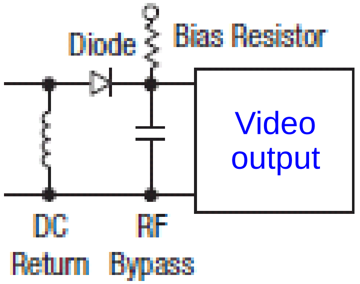
The RF diode (1)

- **We are not discussing the generation of RF signals here, just the detection**
- **Basic tool: fast RF* diode (= Schottky diode)**
- **In general, Schottky diodes are fast but still have a voltage dependent junction capacity (metal – semi-conductor junction)**

- A typical RF detector diode
  - Try to guess from the type of the connector which side is the RF input and which is the output

- **Equivalent circuit:**





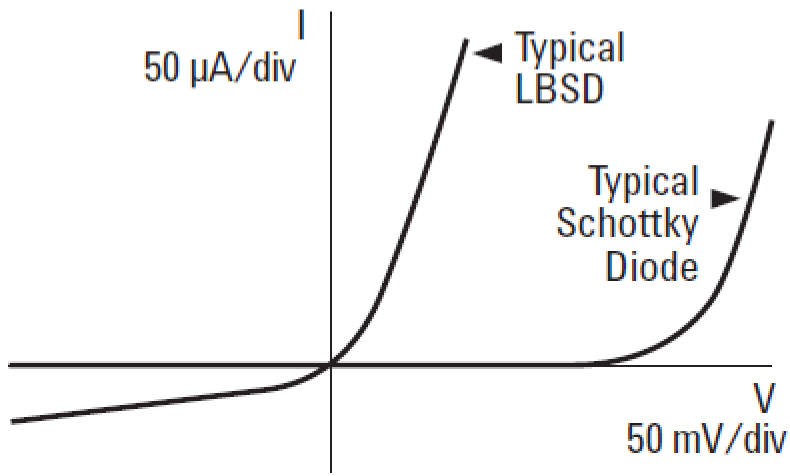
- *Please note, that in this lecture we will use RF for both the RF and micro wave (MW) range, since the borderline between RF and MW is not defined unambiguously

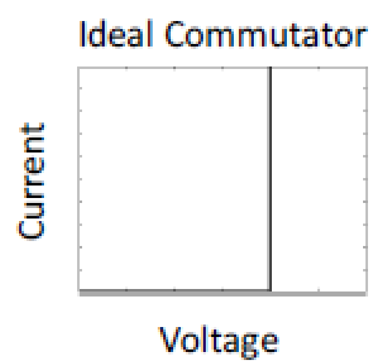
The RF diode (2)

- **Characteristics of a diode:**
- **The current as a function of the voltage for a barrier diode can be described by the Richardson equation:**

$$I = AA^{**}{}_2 \exp\left(-\frac{q\phi B}{kT}\right)\left[\exp\left(\frac{qv}{NkT}\right) - 1\right]$$

where
A = area (cm²)
A** = modified Richardson constant (amp/oK)²/cm²)
k = Boltzman's Constant
T = absolute temperature (°K)
ϕB = barrier heights in volts
V = external voltage across the depletion layer
(positive for forward voltage) - V - IR_S
R_S = series resistance
I = diode current in amps (positive forward current)
n = ideality factor





- The RF diode is NOT an ideal commutator for small signals! We cannot apply big signals otherwise burnout

The RF diode (3)

- **In a highly simplified manner, one can approximate this expression as:**

$$I = I_S \left[\exp\left(\frac{V_J}{0.028} \right) - 1 \right]$$

- V_J … junction voltage

- **and show as sketched in the following, that the RF rectification is linked to the second derivation (curvature) of the diode characteristics:**

If the DC current is held constant by a current regulator or a large resistor, then the total junction current, including RF, is

$$I = I_0 = i \cos \omega t$$

and the I–V relationship can be written

$$V_J = 0.028 Ln \left(\frac{I_S + I_0 + i\cos \omega t}{I_S} \right)$$

$$= 0.028 Ln \left(\frac{I_0 + I_S}{I_S} \right) + 0.028 Ln \left(\frac{i\cos \omega t}{I_0 + I_S} \right)$$

If the RF current, i, is small enough, the IN-term can be approximated in a Taylor series:

$$V_J \approx 0.028 Ln \left(\frac{I_0 + I_S}{I_S} \right) + 0.028 \left[\frac{i \cos \omega t}{I_0 + I_S} - \frac{i^2 \cos^2 \omega t}{2(I_0 + I_S)^2} + \ldots \right]$$

$$= V_{DC} + V_J \cos \omega t + \textit{higher frequency terms}$$

If you use the fact that the average value of $\cos^2$ is 0.50, then the RF and DC voltages are given by the following equations:

$$V_J = \frac{0.028}{I_0 + I_S} \quad i = R_S i$$

$$V_{DC} = 0.028/n \left(1 + \frac{I_0}{I_S} \right) - \frac{0.028^2}{4(I_0 + I_S)^2} = V_0 - \frac{V_J{}^2}{0.112}$$

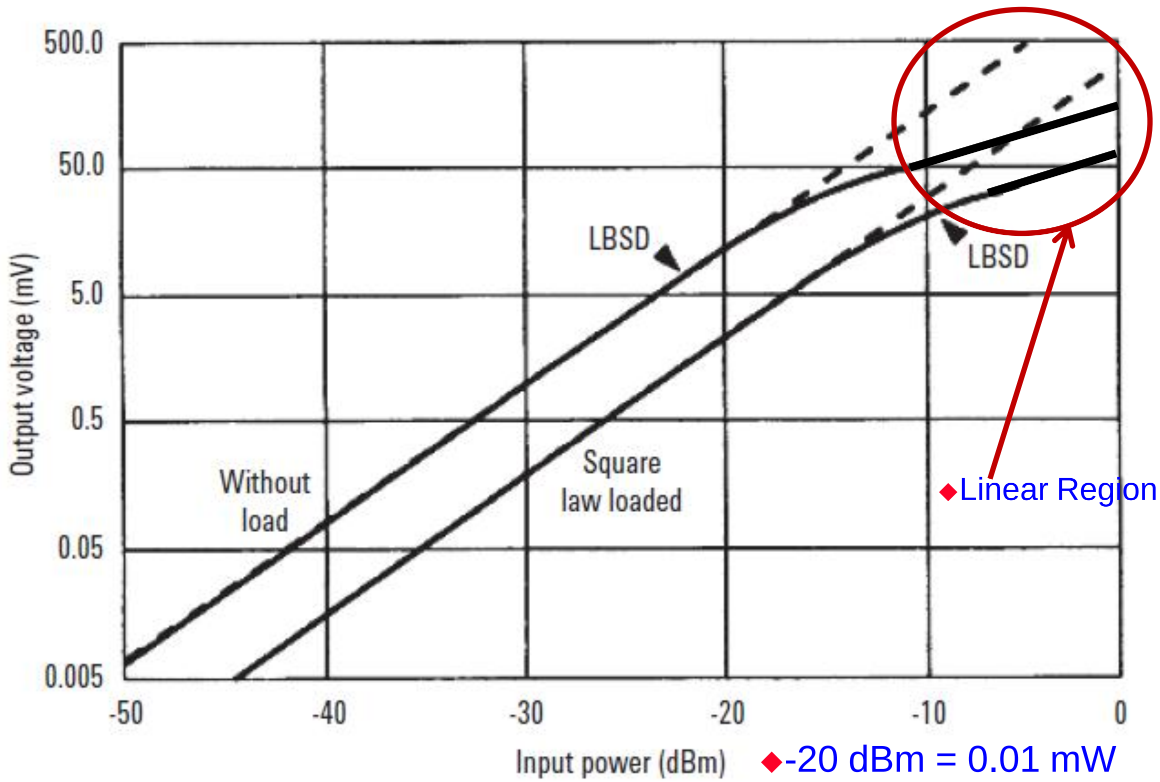
The RF diode (4)

- **This diagram depicts the so called square-law region where the output voltage (V_{Video}) is proportional to the input power**

Since the input power is proportional to the square of the input voltage (V_{RF}^2) and the output signal is proportional to the input power, this region is called square- law region.

In other words:

$V_{Video} \sim V_{RF}^2$



-20 dBm = 0.01 mW

- **The transition between the linear region and the square-law region is typically between -10 and -20 dBm RF power (see diagram)**

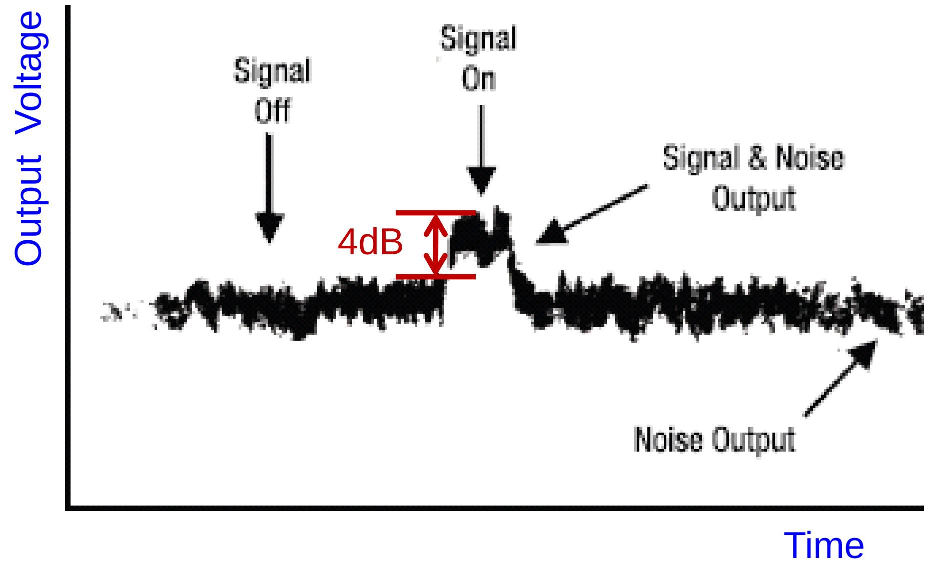
The RF diode (5)

- Due to the square-law characteristic we arrive at the thermal noise region already for moderate power levels (-50 to -60 dBm) and hence the V_{Video} disappears in the thermal noise

- This is described by the term *tangential signal sensitivity* (TSS)

  where the detected signal

  (Observation BW, usually 10 MHz) is 4 dB over the thermal noise floor



If we apply an RF-signal to the detector diode with the same power as its TSS, its output voltage will be 4 dB over the thermal noise floor.

The RF mixer (1)

- For the detection of very small RF signals we prefer a device that has a linear response over the full range (from 0 dBm (= 1mW) down to thermal noise (= -174 dBm/Hz = $4 \cdot 10^{-21}$ W/Hz)
- This is the RF mixer which is using 1, 2 or 4 diodes in different configurations (see next slide)
- Together with a so called LO (local oscillator) signal, the mixer works as a signal multiplier with a very high dynamic range since the output signal is always in the "linear range" provided, that the mixer is not in saturation with respect to the RF input signal (For the LO signal the mixer should always be in saturation!)
- The RF mixer is essentially a multiplier implementing the function

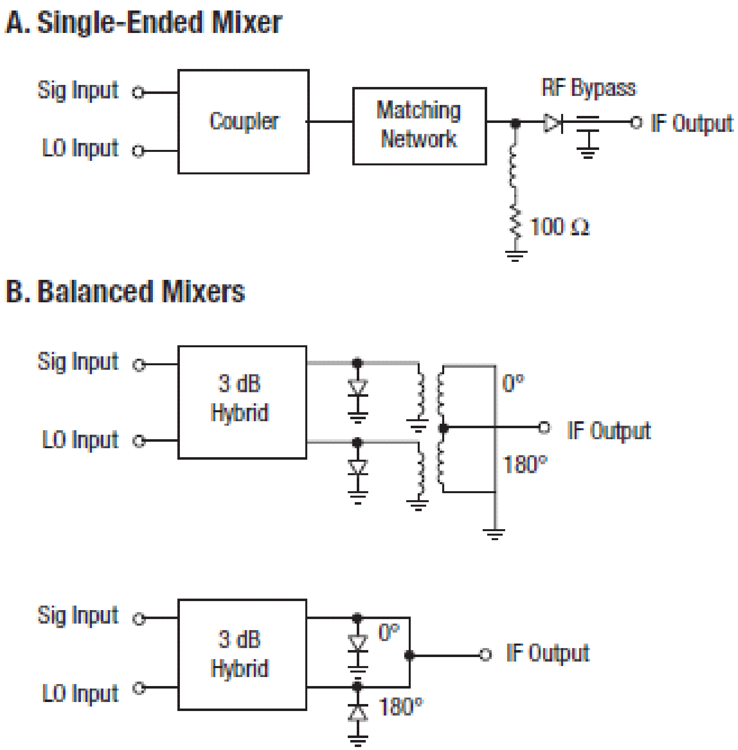
$f_1(t) \cdot f_2(t)$ with $f_1(t)$ = RF signal and $f_2(t)$ = LO signal

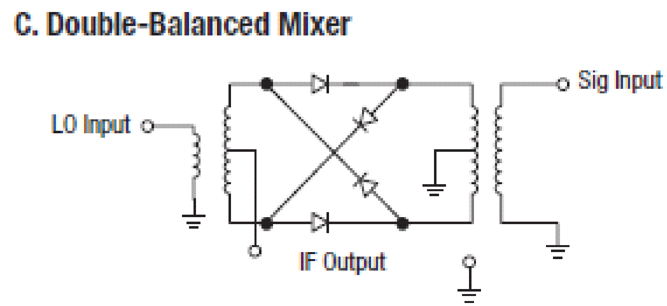
$$a_1 \cos(2\pi f_1 t + \varphi) \cdot a_2 \cos(2\pi f_2 t) = \frac{1}{2} a_1 a_2 [\cos((f_1 + f_2)t + \varphi) + \cos((f_1 - f_2)t + \varphi)]$$

- Thus we obtain a response at the IF (intermediate frequency) port that is at the sum and difference frequency of the LO and RF signals

The RF mixer (2)

- **Examples of different mixer configurations**





A typical coaxial mixer (SMA connector)

The RF mixer (3)

- **Response of a mixer in time and frequency domain:**

Input signals here:

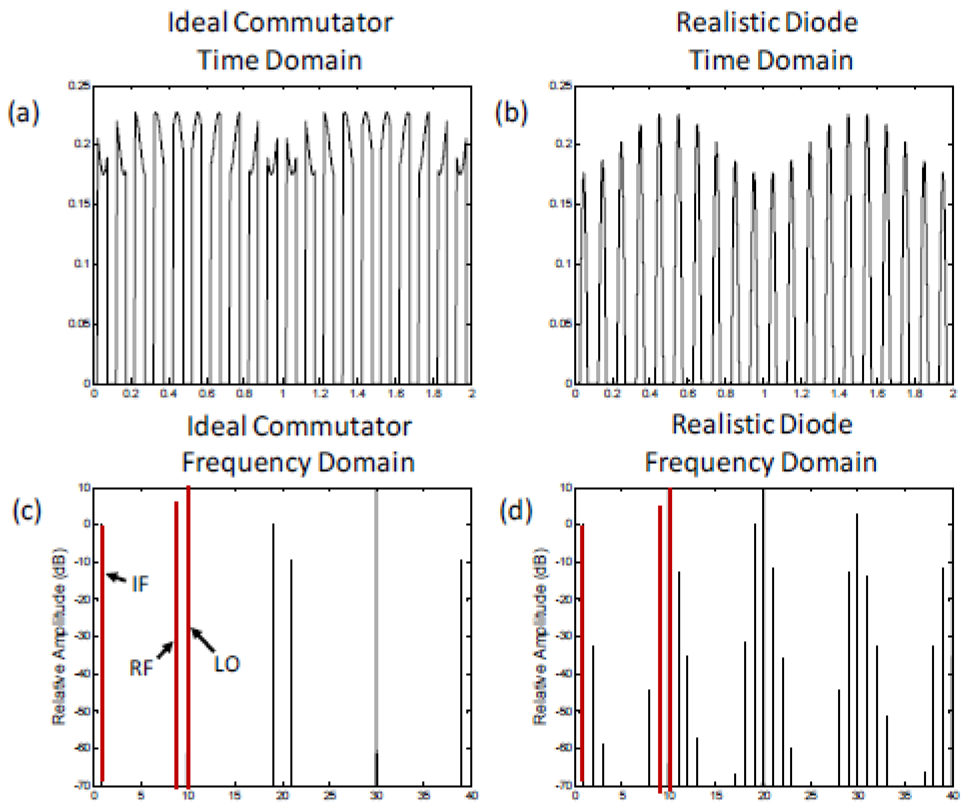
LO = 10 MHz

RF = 8 MHz

Mixing products at

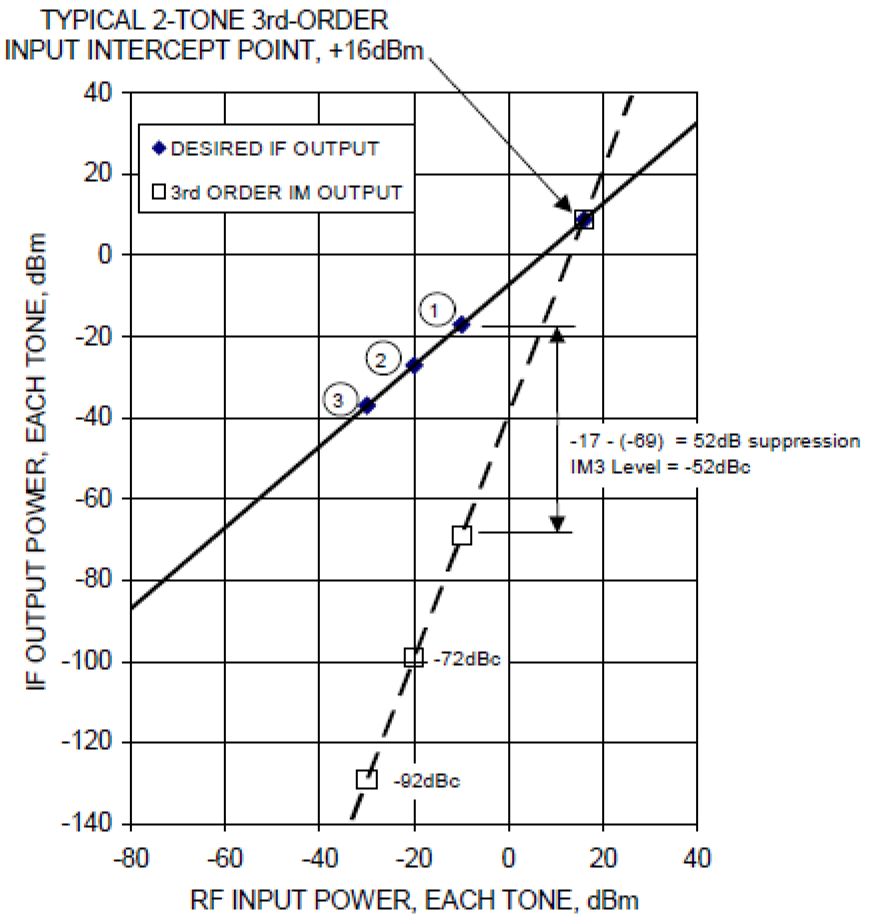
2 and 18 MHz

plus higher order terms at higher frequencies



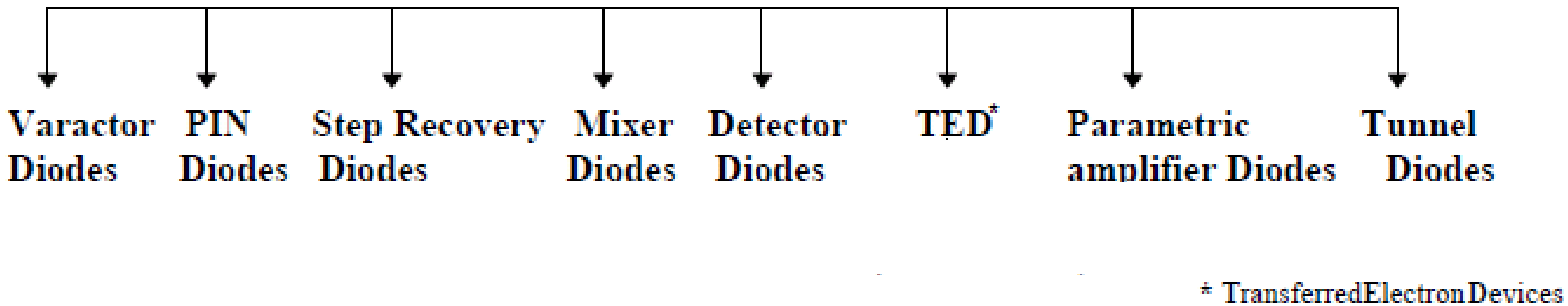
Dynamic range and IP3 of an RF mixer

- The abbreviation IP3 stands for the third order intermodulation point where the two lines shown in the right diagram intersect.
- Two signals (f_1,$f_2 > f_1$) which are closely spaced by Δf in frequency are simultaneously applied to the DUT.
- The intermodulation products appear at $+\Delta f$ above f_2 and at $-\Delta f$ below f_1
- This intersection point is usually not measured directly, but extrapolated from measurement data at much smaller power levels in order to avoid overload and damage of the DUT



Solid state diodes used for RF applications

- There are many other diodes which are used for different applications in the RF domain



- Varactor diodes: for tuning applications
- PIN diodes: for electronically variable RF attenuators
- Step Recovery diodes: for frequency multiplication and pulse sharpening
- Mixer diodes, detector diodes: usually Schottky diodes
- TED (GUNN, IMPATT, TRAPATT etc.): for oscillator
- Parametric amplifier Diodes: usually variable capacitors (vari caps)
- Tunnel diodes: rarely used these days, they have negative impedance and are usually used for very fast switching and certain low noise amplifiers