- JUAS 2012 -

GUIDED STUDY AND TUTORIAL ON RF LINEAR ACCELERATORS

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Problem 1: *Pillbox cavity*

Suppose that you have to design a TM_{010} mode pillbox cavity (see Fig. 1) with a square-wave electric field distribution and you are free to choose the length. If the length is too short, the voltage gain across the cavity is small; if it is too long, the transit-time factor is small.

- **a.** Plot the transit-time factor versus the ratio of length to $\beta\lambda$ (for 0< $\beta\lambda$ <2).
- **b.** Find the ratio of the length to $\beta\lambda$ that maximizes the value of the energy gain for the cavity.
- **c.** Calculate the transit-time factor at this length.
- **d.** What should be the length of the cavity to maximize the energy gain per unit length? Is it a practical solution?

Problem 2: *Drift-tube linac (DTL)*

Suppose that we want to design a CW room-temperature drift-tube linac to accelerate a 100 mA proton beam from 3 to 20 MeV. Assume for the RF power we can purchase 350 MHz klystrons of 1 MW capacity each. Suppose we run the SUPERFISH electromagnetic field-solver code and obtain for all β values the following results: transit-time factor $T=0.8$, effective shunt impedance $ZT^2=50\ M\Omega/m$ and the ratio of the peak surface electric field to the average axial electric field $E_s/E_0=6$. We restrict the peak surface electric field at a bravery factor $b=E_s/E_K=1.2$ (see the Kilpatrick limit criterion plot). For adequate longitudinal acceptance we choose the synchronous phase $\phi_s = -30^\circ$.

- **a.** Calculate the average axial electric field E_0.
- **b.** Calculate the length of the linac assuming it consists of a single tank.
- **c.** Calculate the structure power P_S (power dissipated in the cavity), beam power P_B and the total RF power required.
- **d.** Calculate the structure efficiency ε_s (ratio of beam power to total RF power).
- **e.** How many klystrons do we need for our structure?

Problem 3: *Longitudinal phase advance*

For the DTL of Problem 2 calculate the zero current longitudinal phase advance per focusing period σ_{0l} in degrees at the injection energy for a FODO focusing lattice (period $P = 2\beta\lambda$) and assuming that β does not change through one period.

Problem 4: *Quadrupole gradient*

For stability reasons we would like to have the zero current transverse phase advance per focusing period σ_{0t} =70°. For the DTL of Problem 2 calculate the quadrupole gradient G (T/m) necessary to provide such phase advance at the injection energy. All the quadrupoles in the tank have the same length l_q =45 mm.

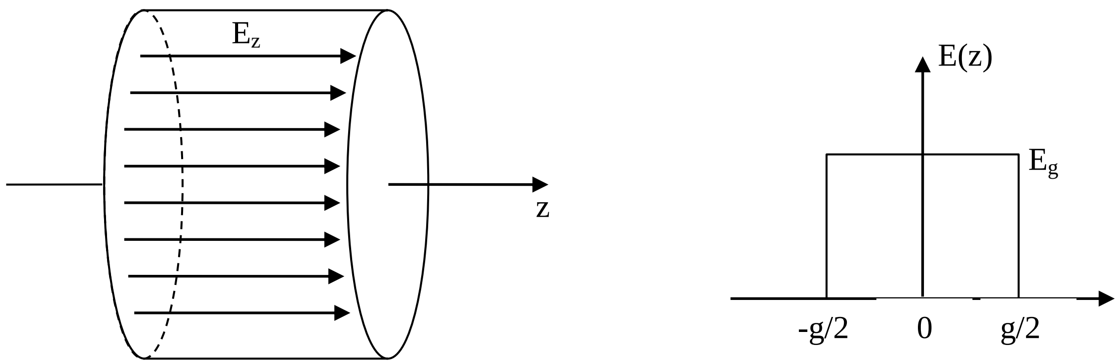
Problem 5: *Energy acceptance*

For the DTL of Problem 2 derive and calculate the maximum energy acceptance $\Delta W_{\max}$ (see Fig.3) for the synchronous phase ϕ_s = -30° at the injection energy from the equation of the separatrix in the longitudinal phase-space:

$$\frac{\omega}{2m_0c^3\beta_s^3\gamma_s^3}\Delta W^2 + qE_0T[\sin(\phi_s+\Delta\phi)+\sin\phi_s-(2\phi_s+\Delta\phi)\cos\phi_s]=0\,.$$

Problem 6: *Longitudinal acceptance*

Suppose that for the designed DTL the longitudinal acceptance is an upright ellipse in the longitudinal phase-space and can be expressed as $A=\Delta\phi\Delta W$. Suppose it is smaller than the emittance of the beam. Which parameter and how should be changed to increase the longitudinal acceptance of the accelerator?



TM$_{010}$ mode in a pillbox cavity

Square-wave electric field distribution

Fig. 1 *Pillbox cavity*

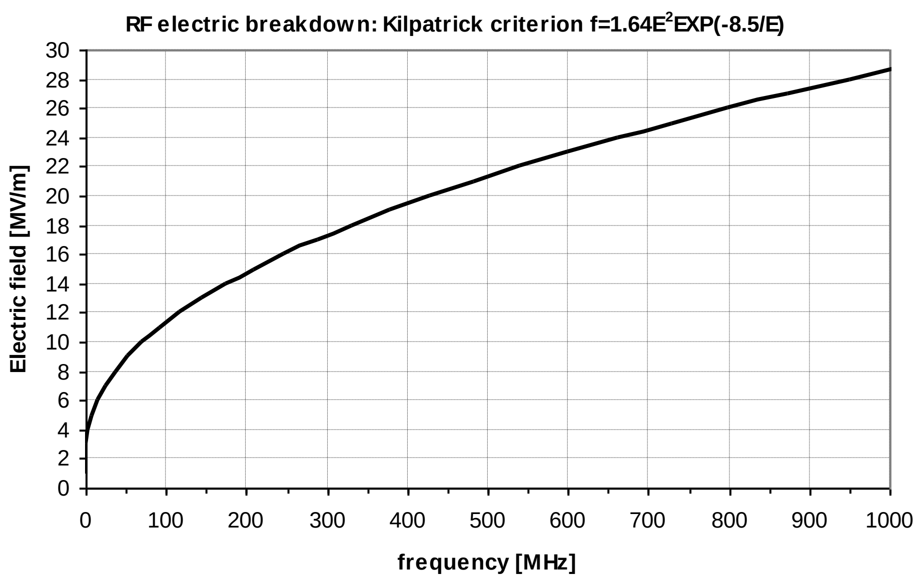


Fig. 2 *Kilpatrick limit for RF electric breakdown*

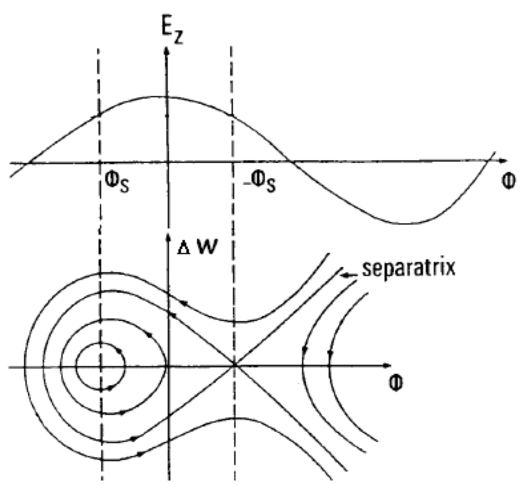


Fig. 3 *Separatrix in the longitudinal phase-space*

Constants

Proton rest mass $E_r = m_0 c^2$ *=938.27 MeV*

c *=299792458 m/s*

Useful expressions

$$f_{RF} = \frac{\omega}{2\pi} \quad \textit{(RF frequency)}$$

$$\gamma = 1 + \frac{W}{E_r},\ \beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad \textit{(relativistic factors)}$$

Transit-time factor approximation for a square-wave electric field distribution: $T = \dfrac{\sin(\pi g / \beta\lambda)}{(\pi g / \beta\lambda)}$

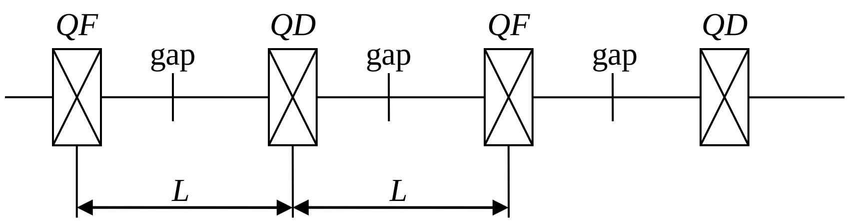
Energy gain in a cavity: $\Delta W = qE_0 TL \cos\phi_s$

Effective shunt impedance: $ZT^2 = \dfrac{(E_0 T)^2}{P_S / L}$

Beam power (CW): $P_B = I\Delta W / q$

Transverse phase advance per period (FODO lattice):

$$\sigma_{0t} \approx \sqrt{\left(\frac{qGl_q L}{m_0 c\gamma\beta}\right)^2 - \frac{\pi q E_0 T \sin(-\phi)(2L)^2}{m_0 c^2 \lambda (\beta\gamma)^3}}$$



FODO period P=2L

Longitudinal phase advance per unit length: $k_{0l} = \sqrt{\dfrac{2\pi q E_0 T \sin(-\phi_s)}{m_0 c^2 \beta_s^3 \gamma_s^3 \lambda}}$