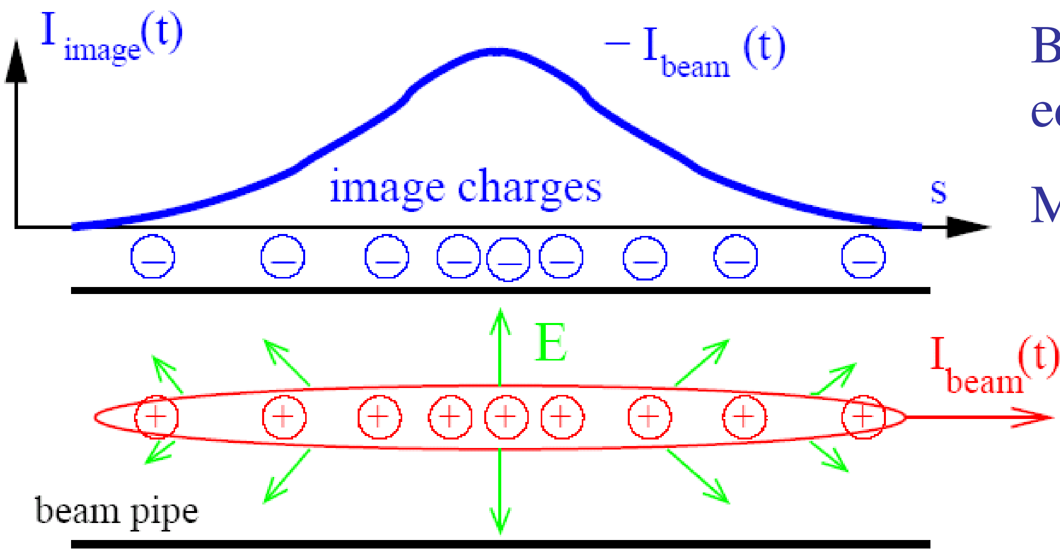
The image current at the beam pipe is monitored on a high frequency basis i.e. the ac-part given by the bunched beam.



Beam Position Monitor **BPM** equals Pick-Up **PU**

Most frequent used instrument!

For relativistic velocities, the electric field is transversal:

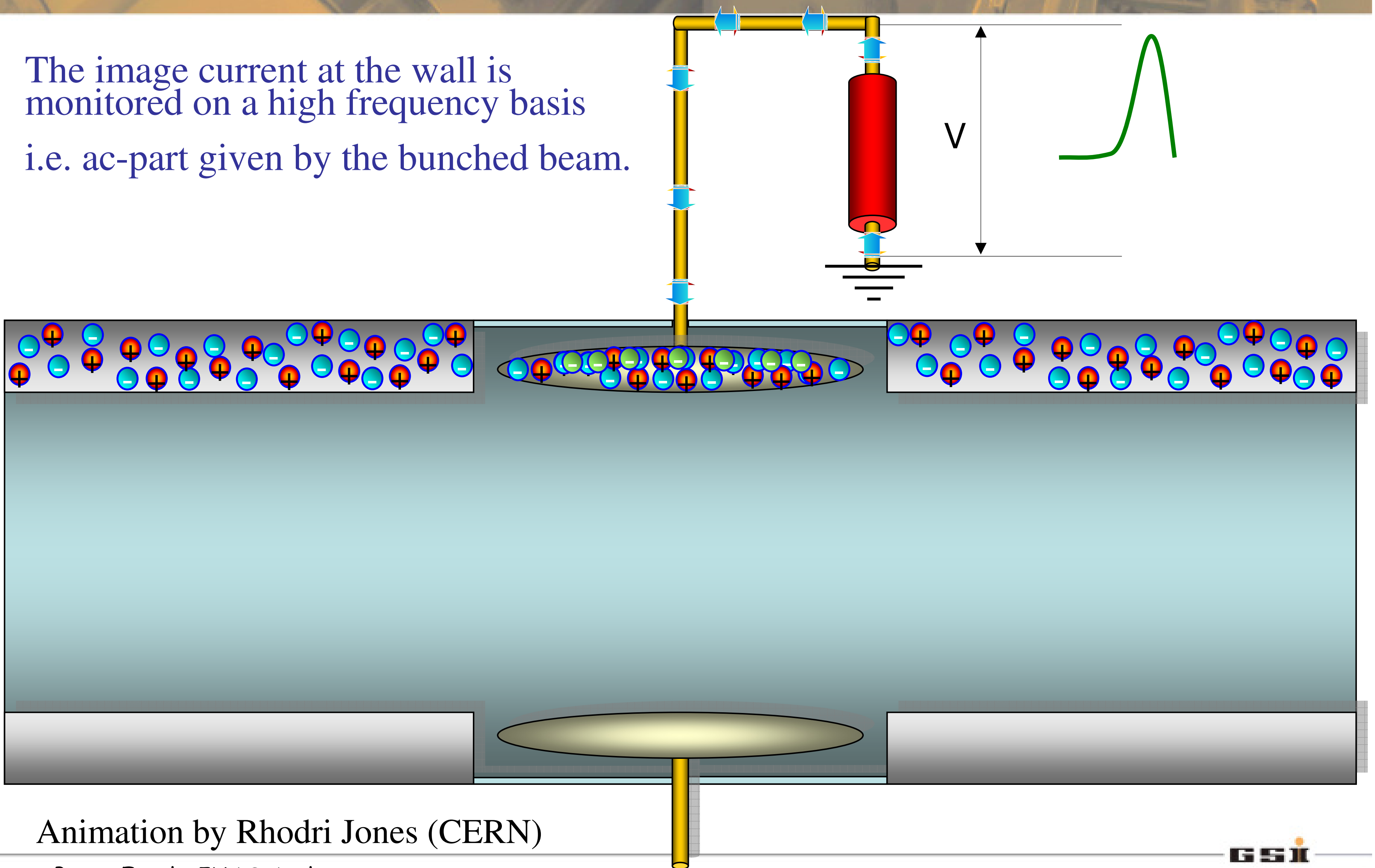
$$E_{\perp,lab}(t) = \gamma \cdot E_{\perp,rest}(t')$$

- **Signal treatment for capacitive pick-ups:**
  - Longitudinal bunch shape
  - Overview of processing electronics for Beam Position Monitor (BPM)
- **Measurements:**
  - Closed orbit determination
  - Tune and lattice function measurements (synchrotron only).

Principle of Signal Generation of capacitive BPMs

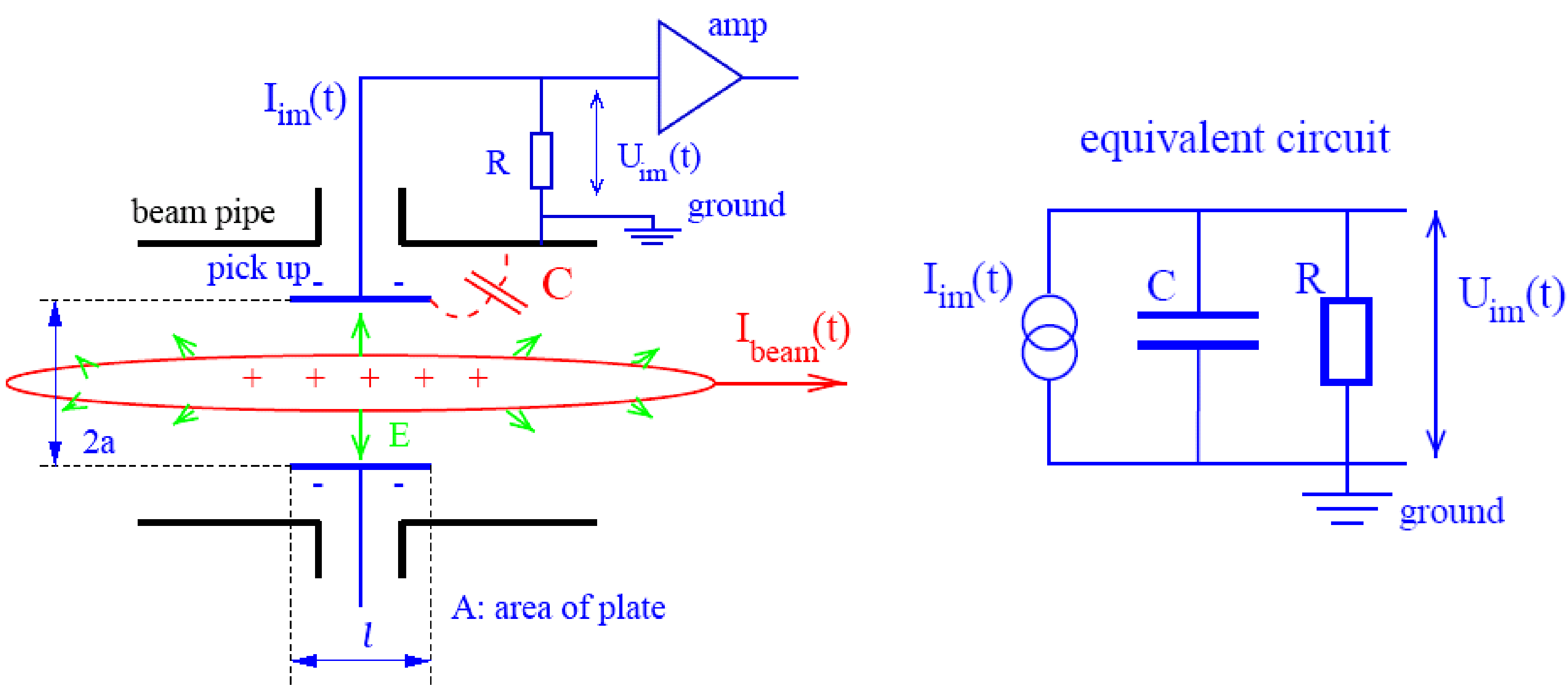
The image current at the wall is monitored on a high frequency basis

i.e. ac-part given by the bunched beam.



Animation by Rhodri Jones (CERN)

The wall current is monitored by a plate or ring inserted in the beam pipe:



The image current I_{im} at the plate is given by the beam current and geometry:

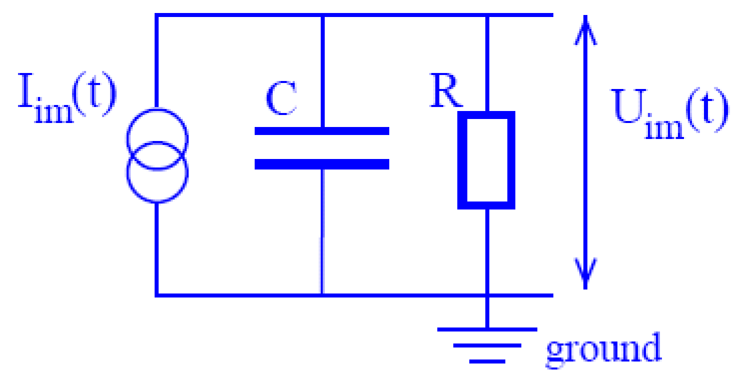
$$I_{im}(t) = -\frac{dQ_{im}(t)}{dt} = \frac{-A}{2\pi a l} \cdot \frac{dQ_{beam}(t)}{dt} = \frac{-A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{dI_{beam}(t)}{dt} = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot i\omega I_{beam}(\omega)$$

Using a relation for Fourier transformation: $I_{beam} = I_0 e^{-i\omega t} \Rightarrow dI_{beam}/dt = -i\omega I_{beam}$.

Transfer Impedance for a capacitive BPM

At a resistor R the voltage U_{im} from the image current is measured.
The transfer impedance Z_t is the ratio between voltage U_{im} and beam current I_{beam} in *frequency domain*: $U_{im}(\omega) = R \cdot I_{im}(\omega) = Z_t(\omega, \beta) \cdot I_{beam}(\omega)$.

equivalent circuit



$$\frac{1}{Z} = \frac{1}{R} + i\omega C \Leftrightarrow Z = \frac{R}{1+i\omega RC}$$

Capacitive BPM:

- The pick-up capacitance C:
  plate ↔ vacuum-pipe and cable.
- The amplifier with input resistor R.
- The beam is a high-impedance current source:

$$U_{im} = \frac{R}{1+i\omega RC} \cdot I_{im}$$

$$= \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{i\omega RC}{1+i\omega RC} \cdot I_{beam}$$

$$\equiv Z_t(\omega, \beta) \cdot I_{beam}$$

This is a high-pass characteristic with $\omega_{cut} = 1/RC$:

Amplitude: $|Z_t(\omega)| = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{\omega/\omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$ **Phase:** $\varphi(\omega) = \arctan(\omega_{cut}/\omega)$

Example of Transfer Impedance for Proton Synchrotron

The high-pass characteristic for typical synchrotron BPM:

$$U_{im}(\omega)=Z_t(\omega)\cdot I_{beam}(\omega)$$

$$|Z_t| = \frac{A}{2\pi a}\cdot\frac{1}{\beta c}\cdot\frac{1}{C}\cdot\frac{\omega/\omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$$

$$\varphi = \arctan(\omega_{cut}/\omega)$$

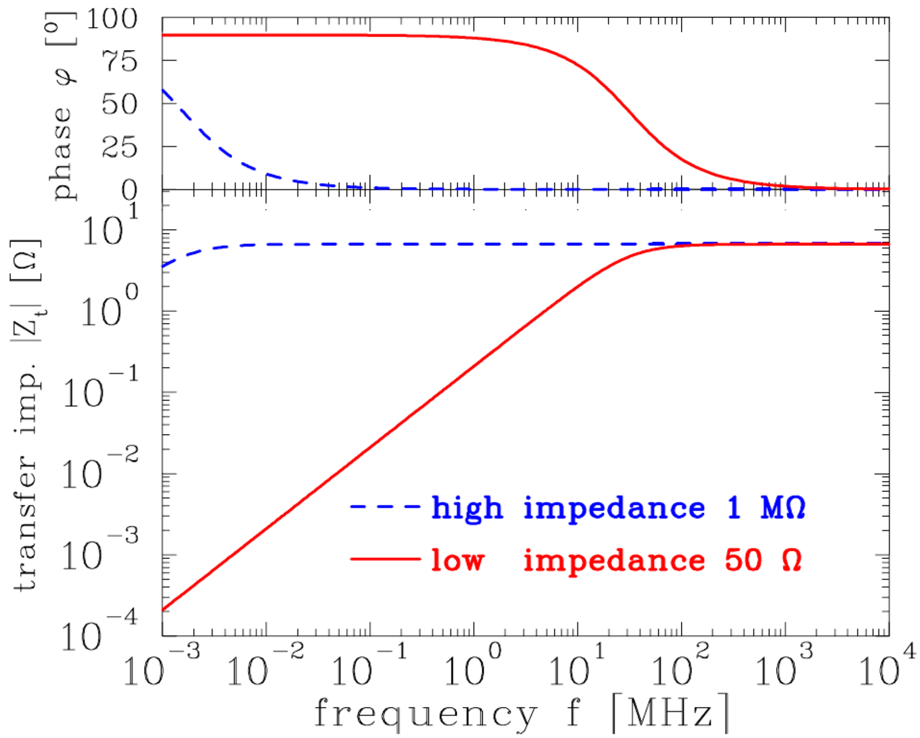
Parameter for shoe-box BPM:

C=100pF, l=10cm, β=50%

$f_{cut}=\omega/2\pi=(2\pi RC)^{-1}$

for R=50 Ω ⇒ f_{cut}= 32 MHz

for R=1 MΩ ⇒ f_{cut}= 1.6 kHz



Large signal strength → **high impedance**

Smooth signal transmission → **50 Ω**

Depending on the frequency range ***and*** termination the signal looks different:

- ***High frequency range $\omega >> \omega_{cut}$:***

$$Z_t \propto \frac{i\omega/\omega_{cut}}{1+i\omega/\omega_{cut}} \rightarrow 1 \Rightarrow U_{im}(t) = \frac{1}{C}\cdot\frac{1}{\beta c}\cdot\frac{A}{2\pi a}\cdot I_{beam}(t)$$

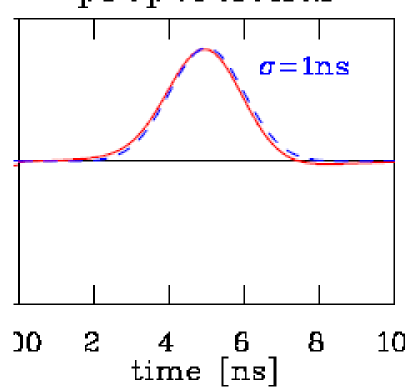
$\Rightarrow$ **direct image** of the bunch. Signal strength $\boldsymbol{Z_t \propto A/C}$ i.e. nearly independent on length

- ***Low frequency range $\omega << \omega_{cut}$:***

$$Z_t \propto \frac{i\omega/\omega_{cut}}{1+i\omega/\omega_{cut}} \rightarrow i\frac{\omega}{\omega_{cut}} \Rightarrow U_{im}(t) = R\cdot\frac{A}{\beta c\cdot 2\pi a}\cdot i\omega I_{beam}(t) = R\cdot\frac{A}{\beta c\cdot 2\pi a}\cdot\frac{dI_{beam}}{dt}$$

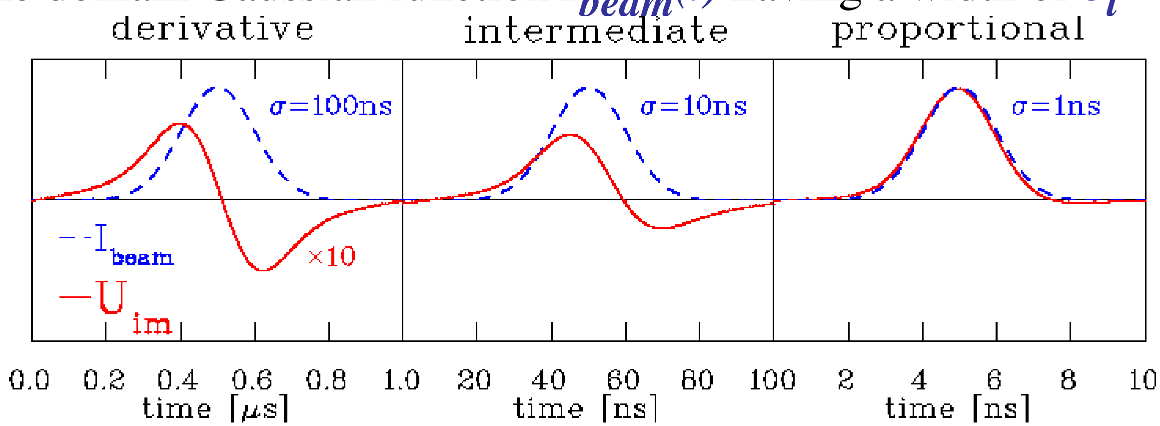
$\Rightarrow$ **derivative** of bunch, single strength $\boldsymbol{Z_t \propto A}$, i.e. (nearly) independent on $\boldsymbol{C}$

Example from synchrotron BPM with 50 Ω termination (reality at p-synchrotron : $\boldsymbol{\sigma}$>>1 ns):

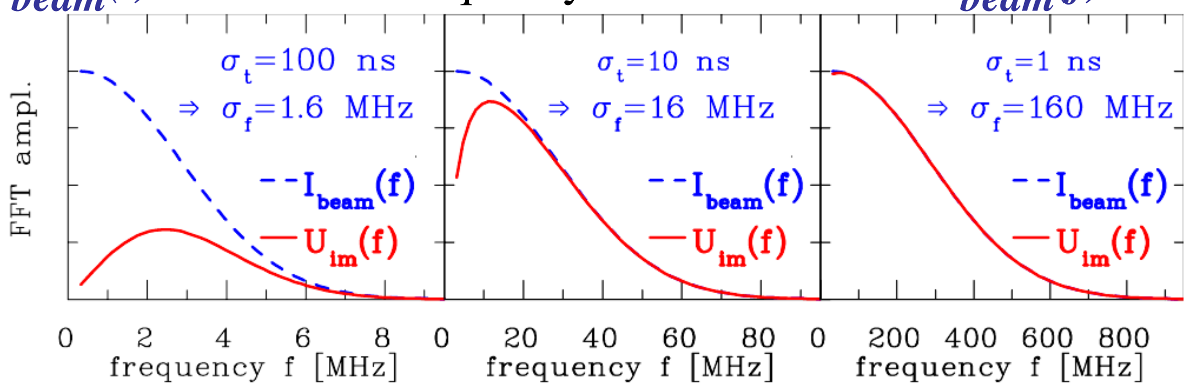


The transfer impedance is used in frequency domain! The following is performed:

1. Start: Time domain Gaussian function $I_{beam}(t)$ having a width of σ_t



2. FFT of $I_{beam}(t)$ leads to the frequency domain Gaussian $I_{beam}(f)$ with $\sigma_f=(2\pi\sigma_t)^{-1}$



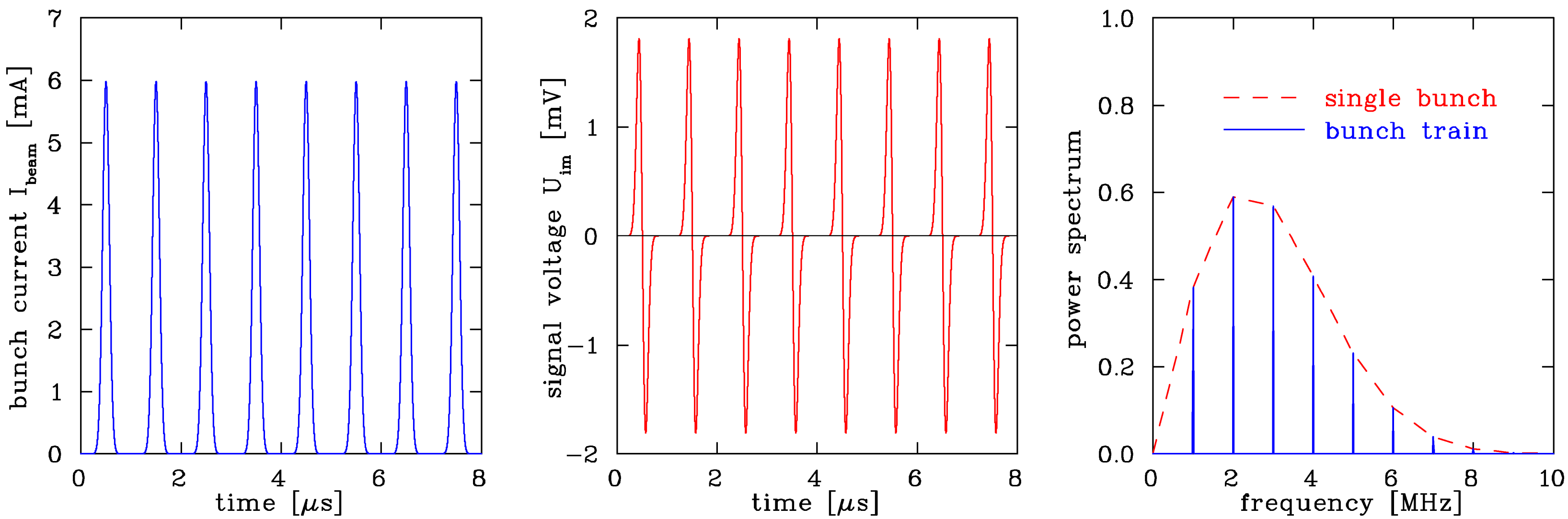
3. Multiplication with $Z_t(f)$ with f_{cut}=32 MHz leads to $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$

4. Inverse FFT leads to $U_{im}(t)$

Calculation of Signal Shape: repetitive Bunch in a Synchrotron

Synchrotron filled with 8 bunches accelerated with f_{acc}=1 MHz

BPM terminated with R=50 Ω $\Rightarrow f_{acc} << f_{cut}$:



Parameter: R=50 Ω $\Rightarrow f_{cut}$=32 MHz, all buckets filled

C=100pF, l=10cm, β=50%, σ_t=100 ns

- Fourier spectrum is concentrated at acceleration harmonics with single bunch spectrum as an envelope.
- Bandwidth up to typically 10*f_{acc}

Examples for differentiated & proportional Shape

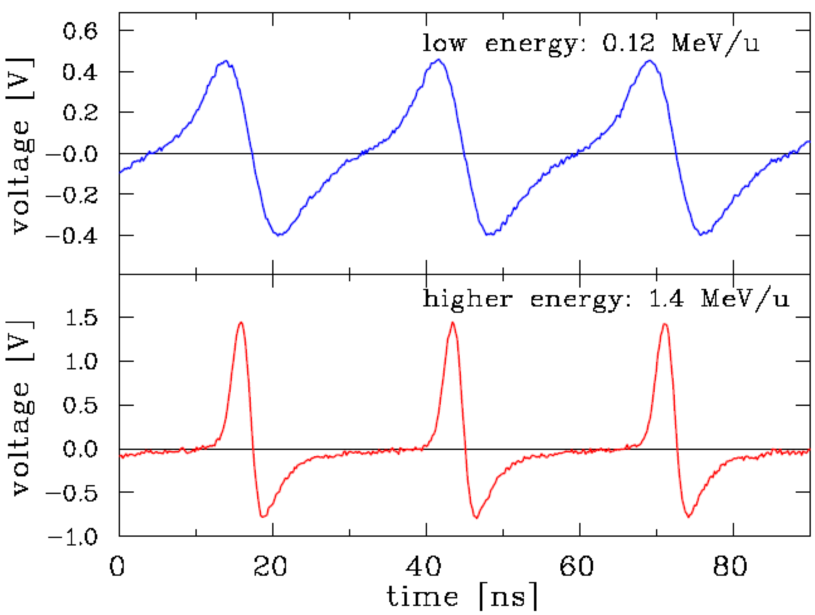
Proton LINAC, e⁻-LINAC&synchtrotron:

100 MHz < f_{rf} < 1 GHz typically

R=50 Ω processing to reach bandwidth

$C \approx 5$ pF ⇒ $f_{cut} = 1/(2\pi RC) \approx$ 700 MHz

Example: 36 MHz GSI ion LINAC



Proton synchtrotron:

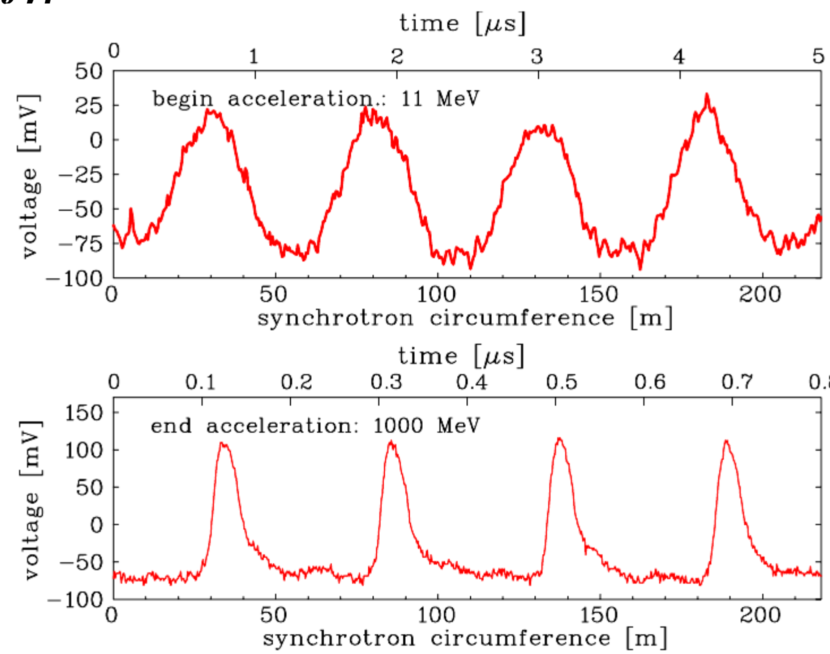
1 MHz < f_{rf} < 30 MHz typically

R=1 MΩ for large signal i.e. large Z_t

$C \approx 100$ pF ⇒ $f_{cut} = 1/(2\pi RC) \approx$10 kHz

Example: non-relativistic GSI synchrotron

f_{rf} : 0.8 MHz → 5 MHz



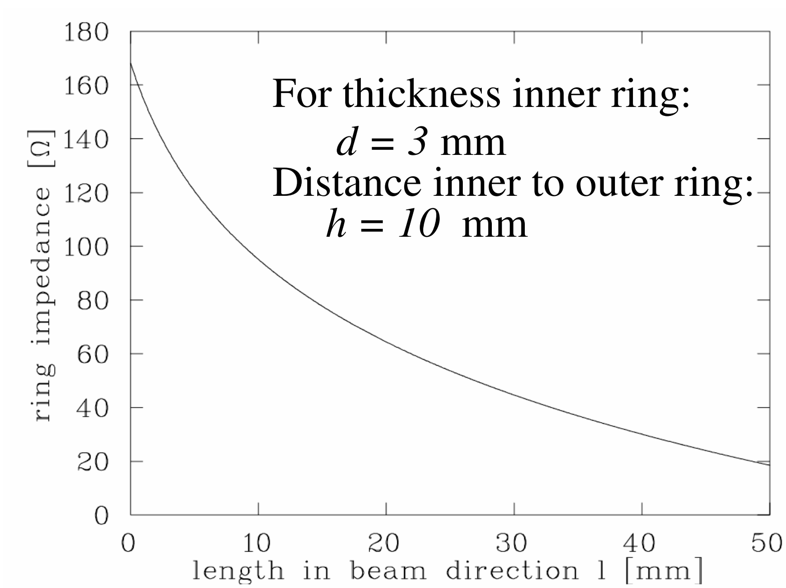
Remark: During acceleration the bunching-factor is increased: 'adiabatic damping'.

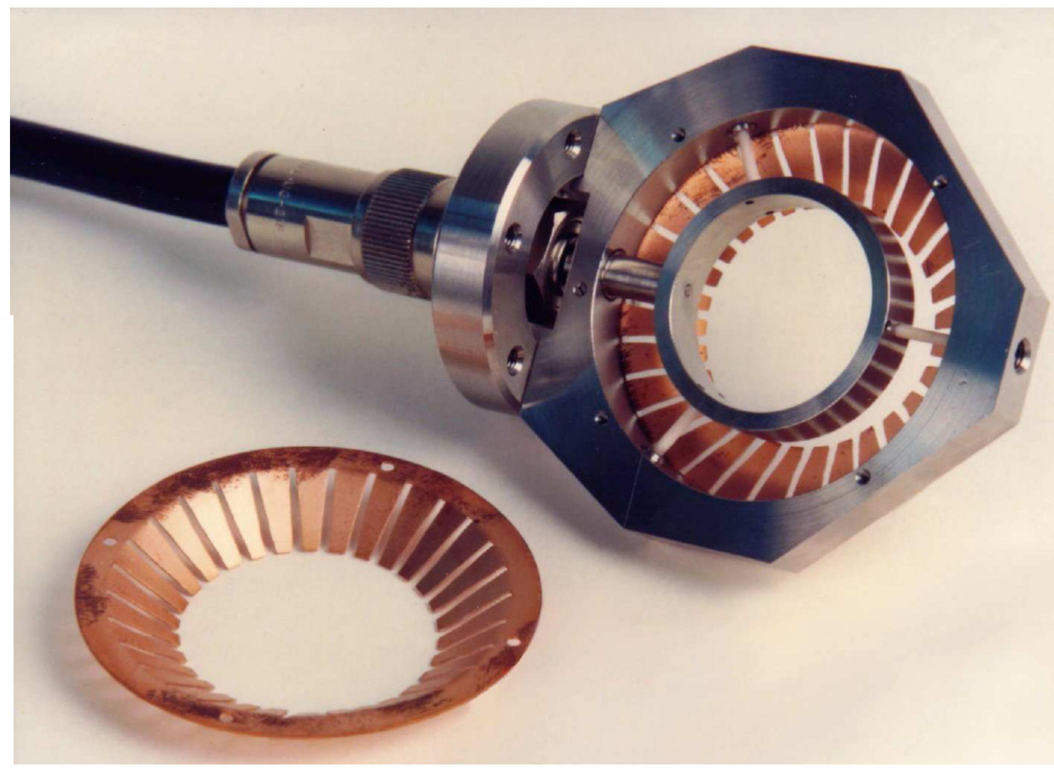
One ring in 50 Ω geometry to reach ≈1 GHz bandwidth.

The impedance is like a strip-line with 100 Ω due to the two passes of the signal:

$$Z_0(l) = \frac{87\ [\Omega]}{\sqrt{\varepsilon_r + 1.4}} \ln\left(\frac{5.98h}{0.8 \cdot l + d}\right)$$

⇒ Impedance depends strongly on geometry





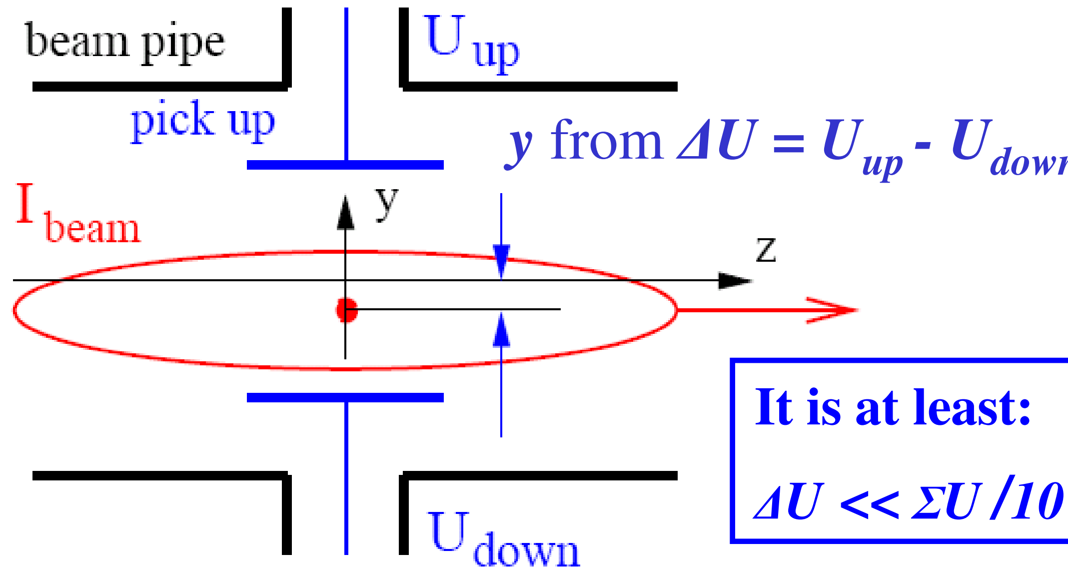
The difference voltage between plates gives the beam's center-of-mass
→**most frequent application**

'Proximity' effect leads to different voltages at the plates:

$$y = \frac{1}{S_y(\omega)} \cdot \frac{U_{up} - U_{down}}{U_{up} + U_{down}} + \delta_y(\omega)$$

$$\equiv \frac{1}{S_y} \cdot \frac{\Delta U_y}{\Sigma U_y} + \delta_y$$

$$x = \frac{1}{S_x(\omega)} \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} + \delta_x(\omega)$$

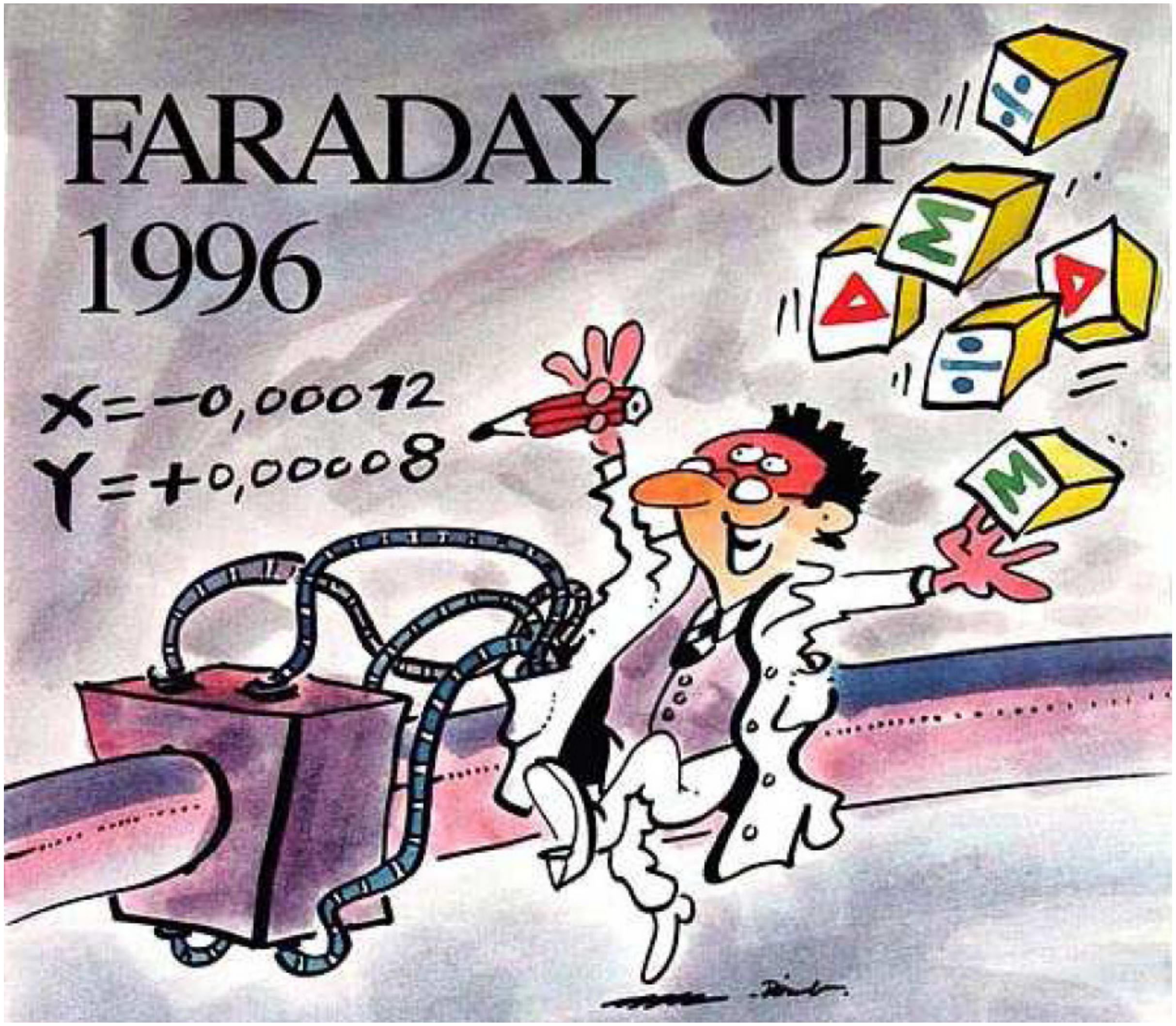


It is at least:

$\Delta U << \Sigma U / 10$

S(ω,x) is called **position sensitivity,** sometimes the inverse is used ***k(ω,x)=1/S(ω,x)***
S is a geometry dependent, non-linear function, which have to be optimized
Units: ***S***=[%/mm] and sometimes ***S***=[dB/mm] or ***k***=[mm].

The Artist View of a BPM

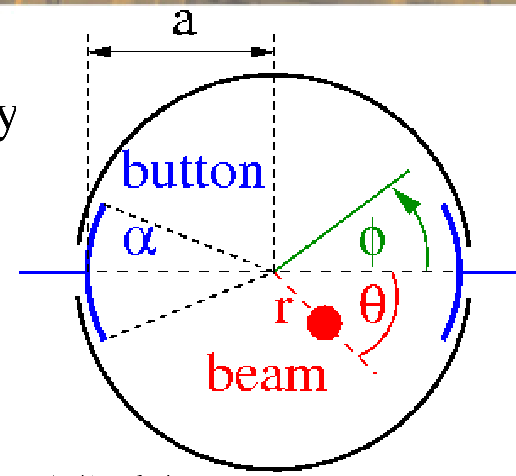


Outline:

- Signal generation → transfer impedance
- Capacitive ***button*** BPM for high frequencies

  used at most proton LINACs and electron accelerators
- Capacitive ***shoe-box*** BPM for low frequencies
- Electronics for position evaluation
- BPMs for measurement of closed orbit, tune and further lattice functions
- Summary

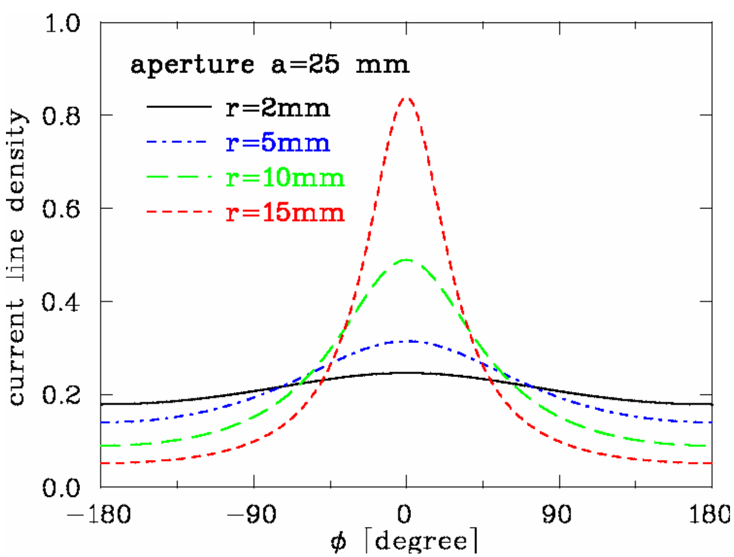
'Proximity effect': larger signal for closer plate

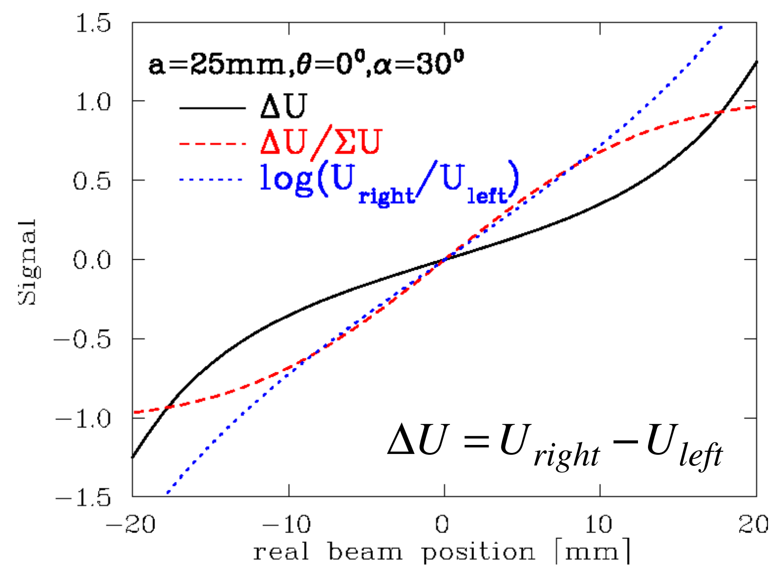


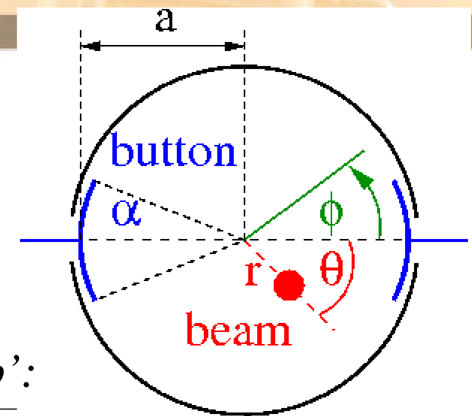
Ideal 2-dim model: Cylindrical pipe → image current density via 'image charge method' for 'pensile' beam:

$$j_{im}(\phi) = \frac{I_{beam}}{2\pi a} \cdot \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\phi - \theta)} \right)$$

Image current: Integration of finite BPM size: $I_{im} = a \cdot \int_{-\alpha/2}^{\alpha/2} j_{im}(\phi) d\phi$





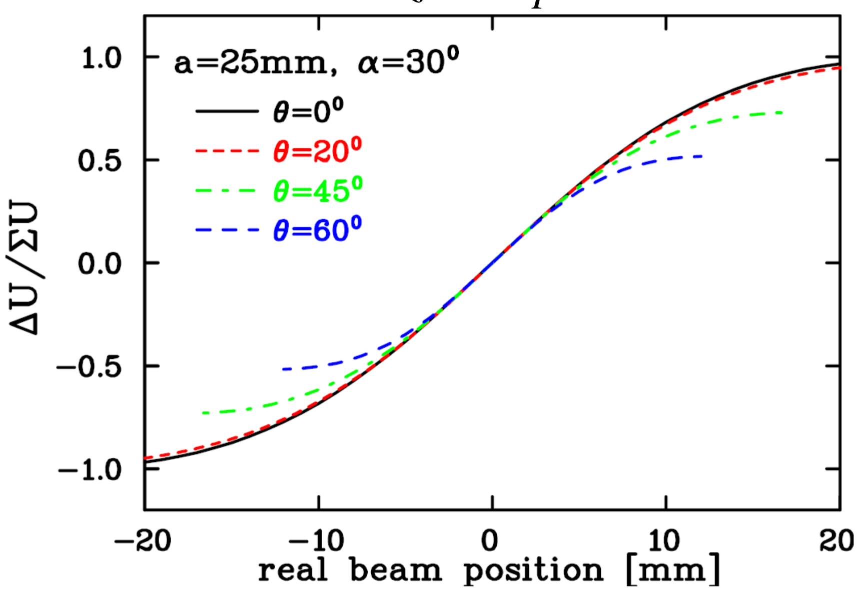


Ideal 2-dim model: Non-linear behavior and hor-vert coupling:

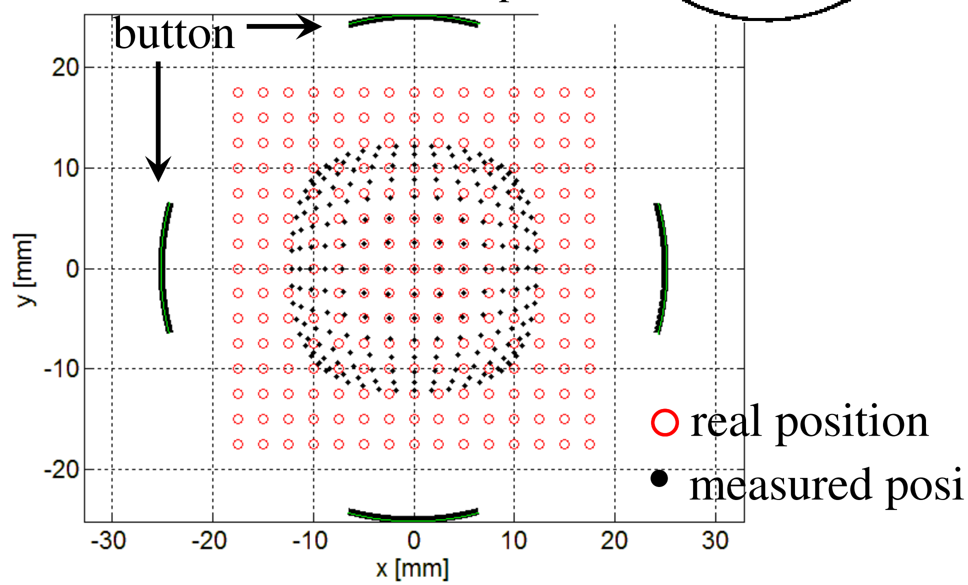
Sensitivity: $x=1/S\cdot\Delta U/\Sigma U$ with S [%/mm] or [dB/mm]

For this example: center part S=7.4%/mm ⇔ $k=1/S$=14mm

Horizontal plane:



'Position Map':



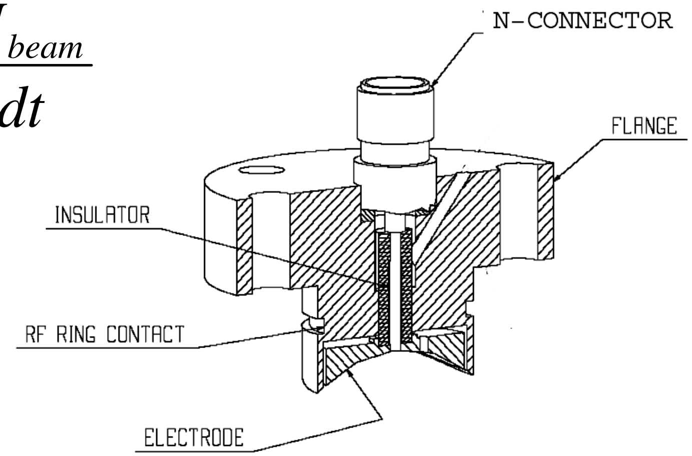
The measurement of U delivers: $x = \frac{1}{S_x} \cdot \frac{\Delta U}{\Sigma U}$ → here $S_x = S_x(x, y)$ i.e. non-linear.

Button BPM Realization

LINACs, e$^-$-synchrotrons: 100 MHz $< f_{rf} <$ 3 GHz → bunch length ≈ BPM length

→ 50 Ω signal path to prevent reflections

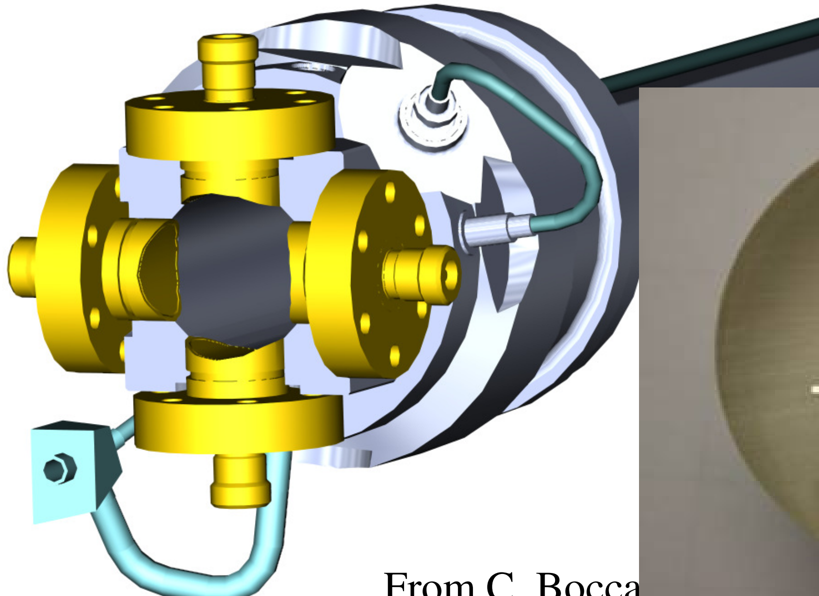
Button BPM with 50 Ω $\Rightarrow U_{im}(t) = R \cdot \frac{A}{\beta c \cdot 2\pi a} \cdot \frac{dI_{beam}}{dt}$



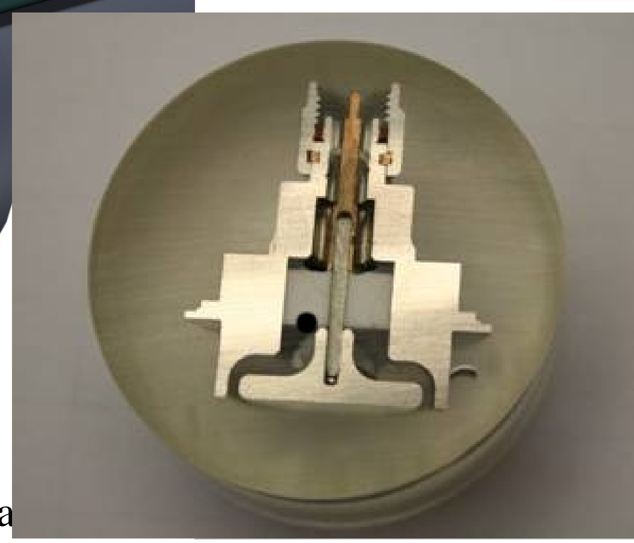
Example: LHC-type inside cryostat:

∅24 mm, half aperture a=25 mm, C=8 pF

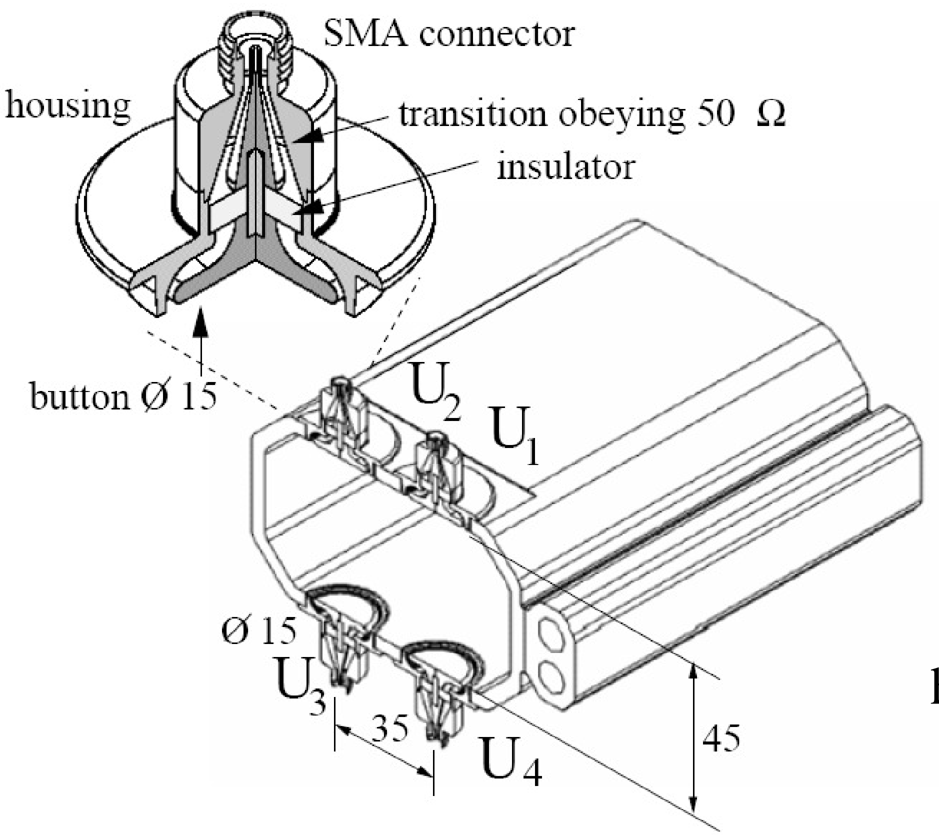
$\Rightarrow f_{cut}$=400 MHz, Z_t = 1.3 Ω above f_{cut}



From C. Bocca



Due to synchrotron radiation, the button insulation might be destroyed
⇒buttons only in vertical plane possible ⇒ increased non-linearity



PEP-realization

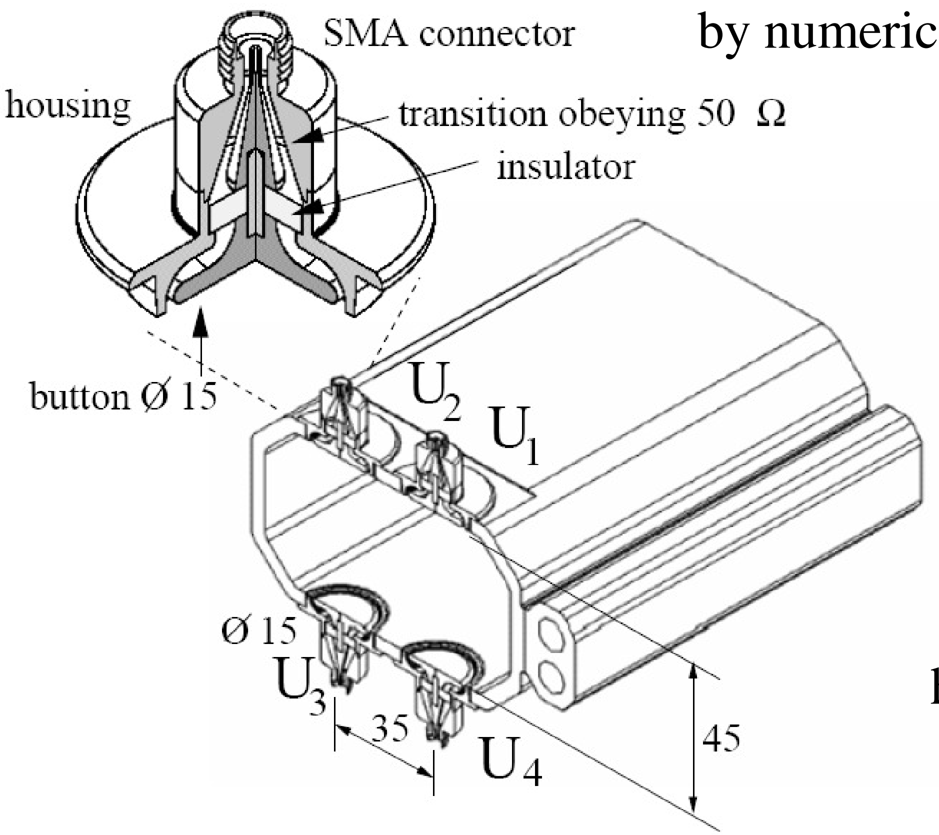
HERA-e realization

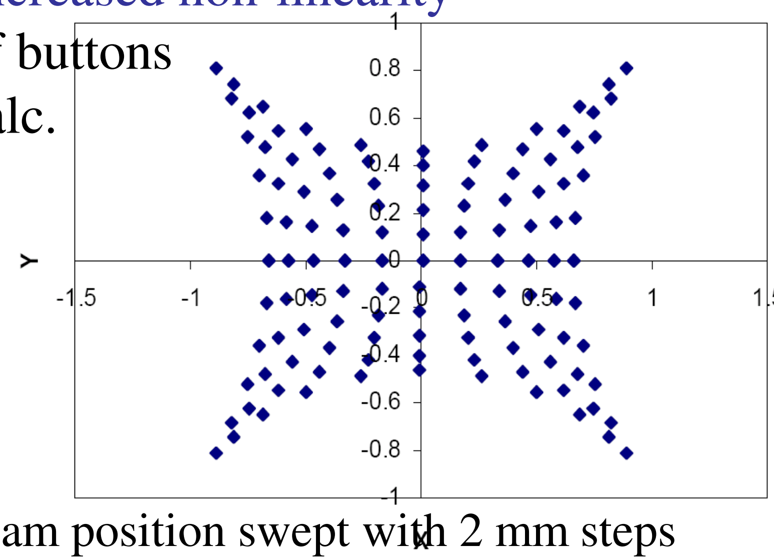
$$\text{horizontal}: x = \frac{1}{S_x} \cdot \frac{(U_2+U_4)-(U_1+U_3)}{U_1+U_2+U_3+U_4}$$

$$\text{vertical}: y = \frac{1}{S_y} \cdot \frac{(U_1+U_2)-(U_3+U_4)}{U_1+U_2+U_3+U_4}$$

Due to synchrotron radiation, the button insulation might be destroyed
⇒buttons only in vertical plane possible ⇒ increased non-linearity

Optimization: horizontal distance and size of buttons by numerical calc.





- Beam position swept with 2 mm steps
- Non-linear sensitivity and hor.-vert. coupling
- At center S_x=8.5%/mm in this example

$$\text{horizontal}: x = \frac{1}{S_x} \cdot \frac{(U_2+U_4)-(U_1+U_3)}{U_1+U_2+U_3+U_4}$$

$$\text{vertical}: y = \frac{1}{S_y} \cdot \frac{(U_1+U_2)-(U_3+U_4)}{U_1+U_2+U_3+U_4}$$

PEP-realization

From S. Varnasseri, SESAME, DIPAC 2005

Outline:

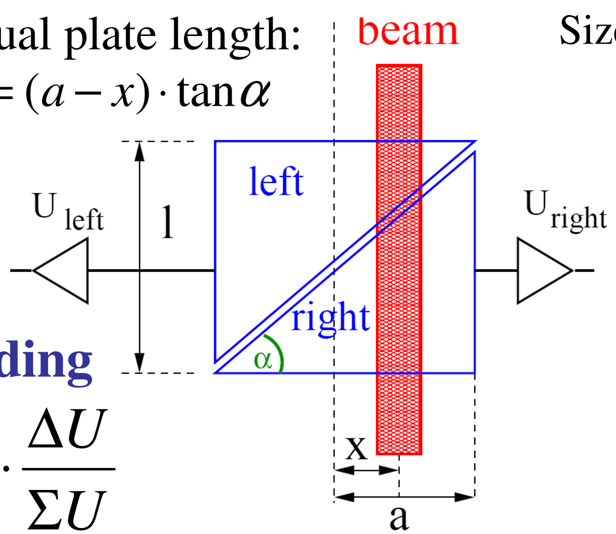
- Signal generation $\rightarrow$ transfer impedance
- Capacitive *button* BPM for high frequencies
  used at most proton LINACs and electron accelerators
- **Capacitive *shoe-box* BPM for low frequencies**
  used at most proton synchrotrons due to linear position reading
- **Electronics for position evaluation**
- **BPMs for measurement of closed orbit, tune and further lattice functions**
- **Summary**

Frequency range: 1 MHz < f_{rf} < 10 MHz ⇒ bunch-length >> BPM length.

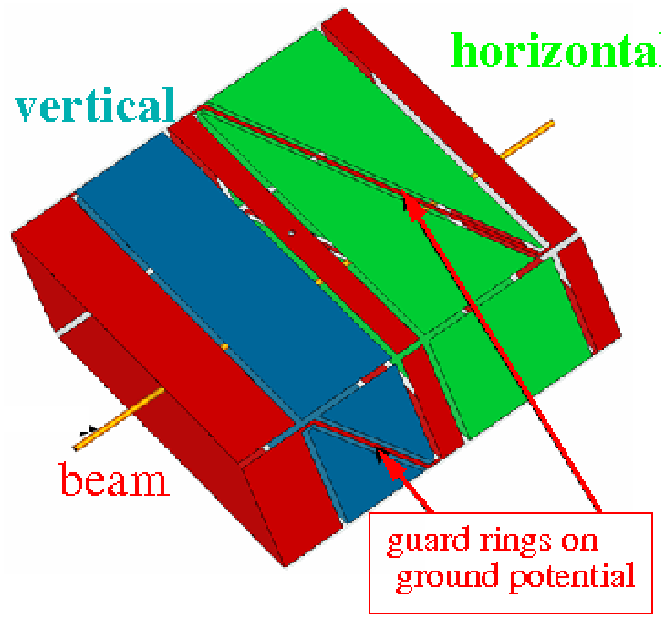
Signal is proportional to actual plate length:

$$l_{\text{right}} = (a+x)\cdot\tan\alpha, \quad l_{\text{left}} = (a-x)\cdot\tan\alpha$$

$$\Rightarrow \quad x = a\cdot\frac{l_{\text{right}} - l_{\text{left}}}{l_{\text{right}} + l_{\text{left}}}$$

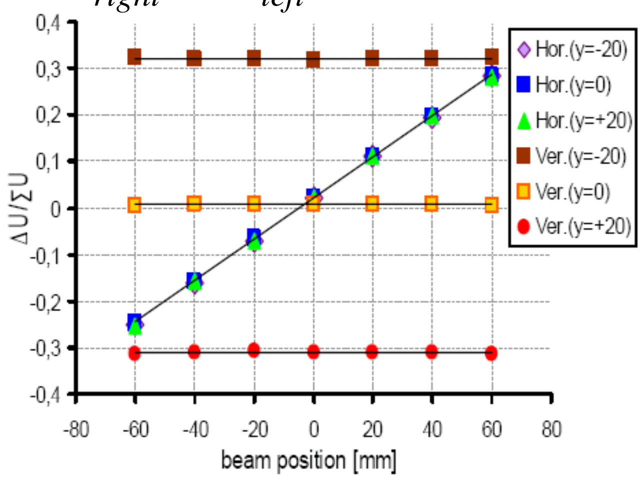


Size: 200x70 mm^2



In ideal case: linear reading

$$x = a\cdot\frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a\cdot\frac{\Delta U}{\Sigma U}$$

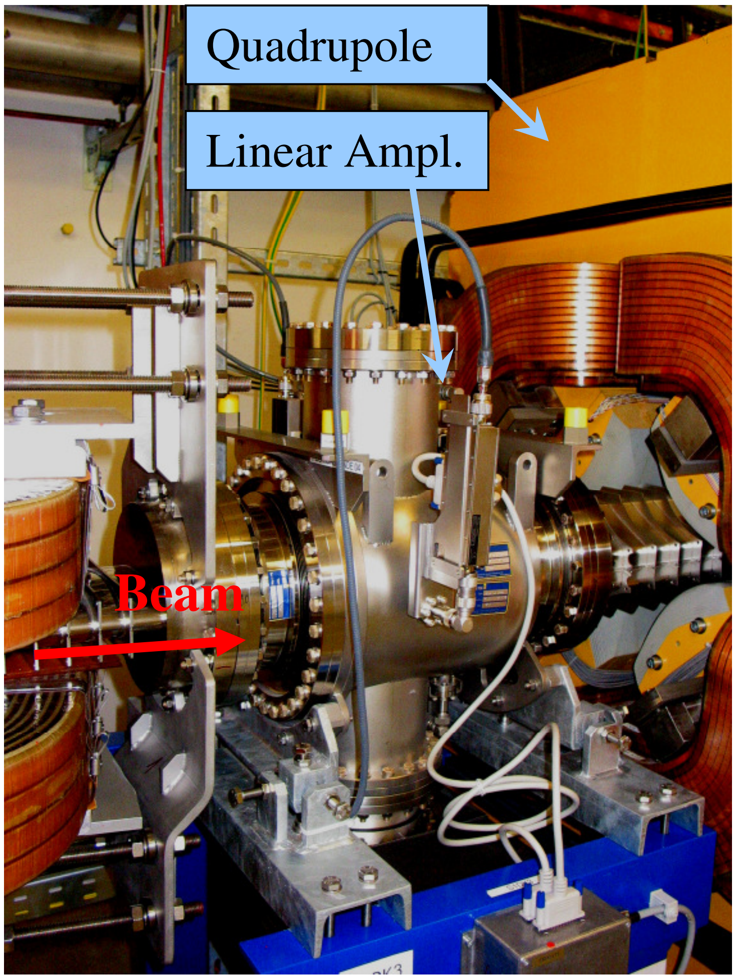


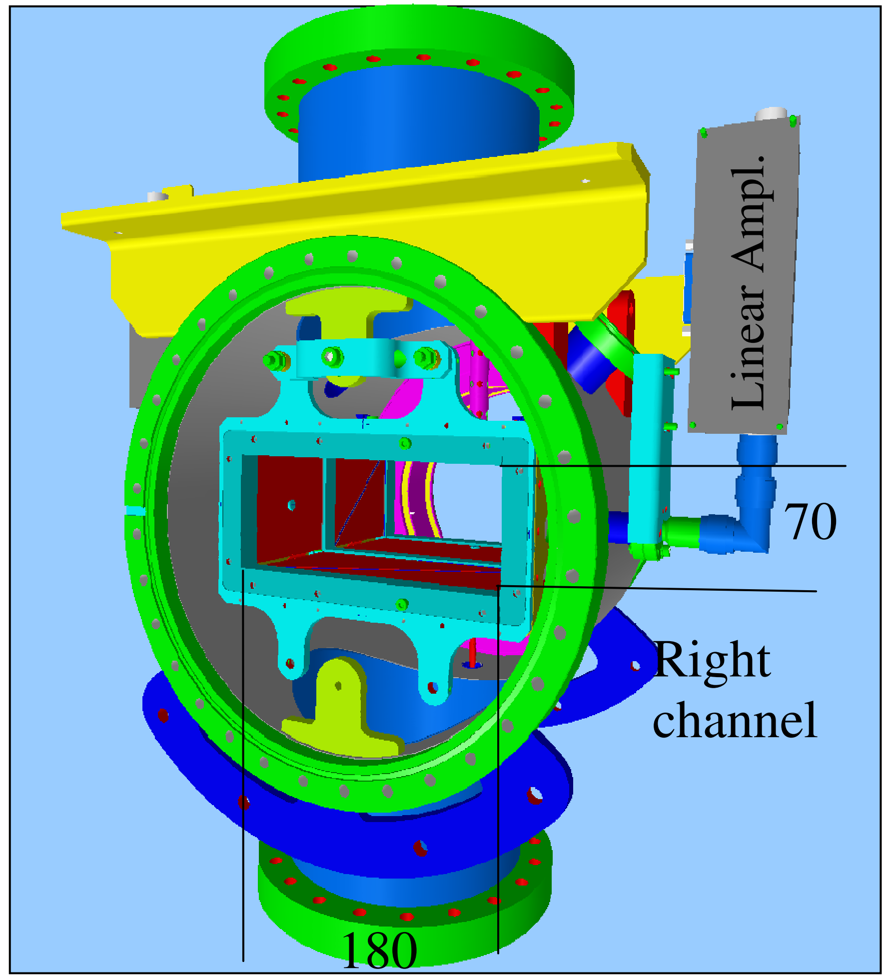
Shoe-box BPM:

Advantage: Very linear, low frequency dependence i.e. position sensitivity ***S*** is constant

Disadvantage: Large size, complex mechanics high capacitance

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u→ 440 MeV/u
BPM clearance: 180x70 mm^2, standard beam pipe diameter: 200 mm.

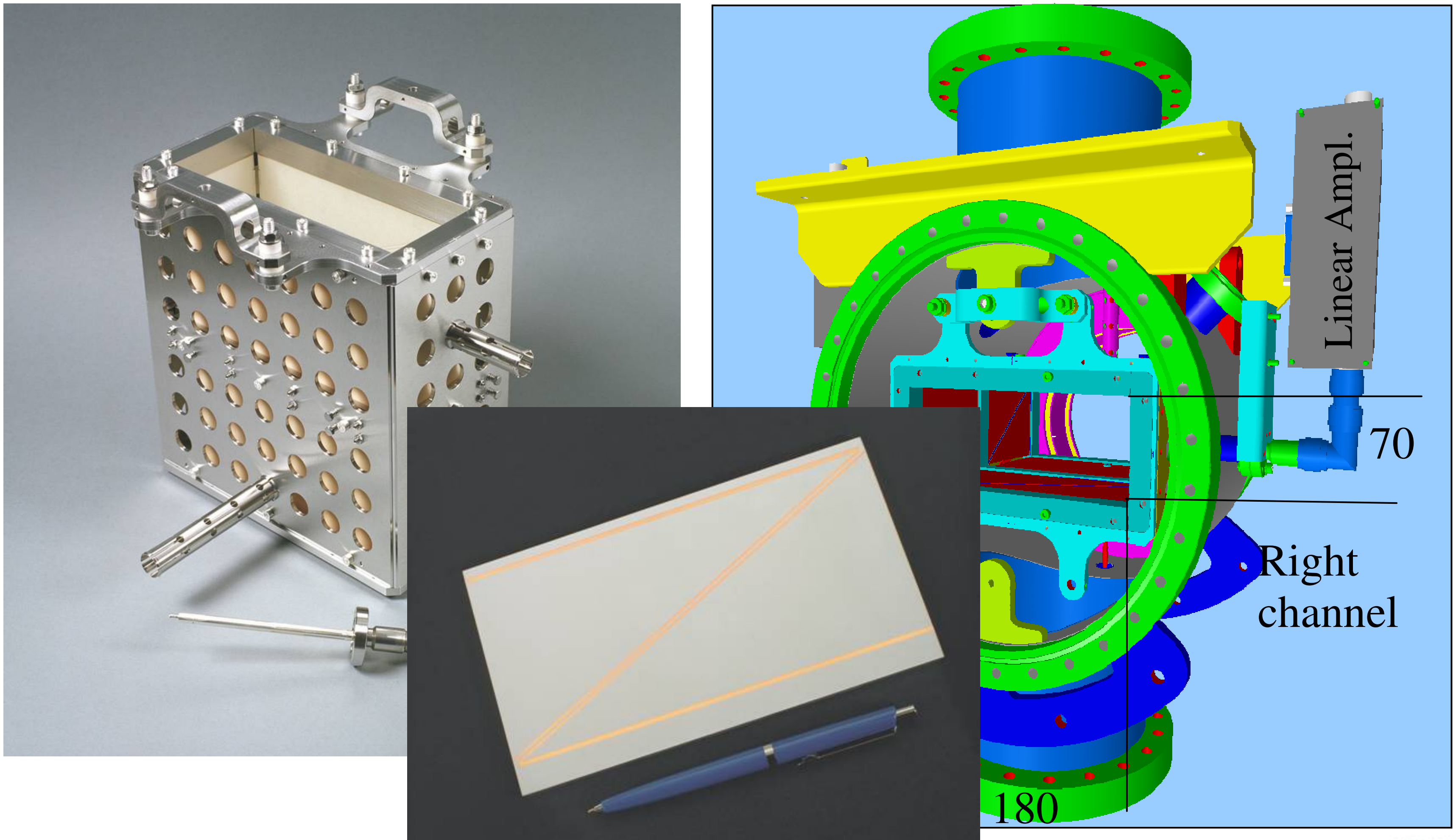




Technical Realization of a Shoe-Box BPM

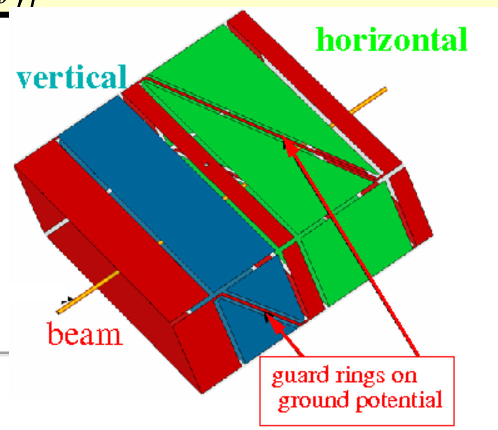
Technical realization at HIT synchrotron of 46 m length for 7 MeV/u→ 440 MeV/u

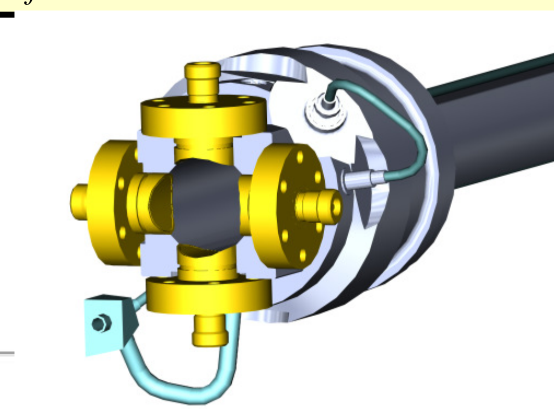
BPM clearance: 180x70 mm^2, standard beam pipe diameter: 200 mm.



Comparison Shoe-Box and Button BPM

	Shoe-Box BPM	Button BPM
Precaution	Bunches longer than BPM	Bunch length comparable to BPM
BPM length (typical)	10 to 20 cm length per plane	∅1 to 5 cm per button
Shape	Rectangular or cut cylinder	Orthogonal or planar orientation
Bandwidth (typical)	0.1 to 100 MHz	100 MHz to 5 GHz
Coupling	1 MΩ or ≈1 kΩ (transformer)	50 Ω
Cutoff frequency (typical)	0.01… 10 MHz (C=30…100pF)	0.3… 1 GHz (C=2…10pF)
Linearity	Very good, no x-y coupling	Non-linear, x-y coupling
Sensitivity	Good, care: plate cross talk	Good, care: signal matching
Usage	At proton synchrotrons, f_{rf}< 10 MHz	All electron acc., proton Linacs, f_{rf}> 100 MHz





Outline:

- Signal generation $\rightarrow$ transfer impedance
- Capacitive *button* BPM for high frequencies
  used at most proton LINACs and electron accelerators
- Capacitive *shoe-box* BPM for low frequencies
  used at most proton synchrotrons due to linear position reading
- **Electronics for position evaluation**
  analog signal conditioning to achieve small signal processing
  (today's technology based on *digital* signal processing)
- **BPMs for measurement of closed orbit, tune and further lattice functions**
- **Summary**

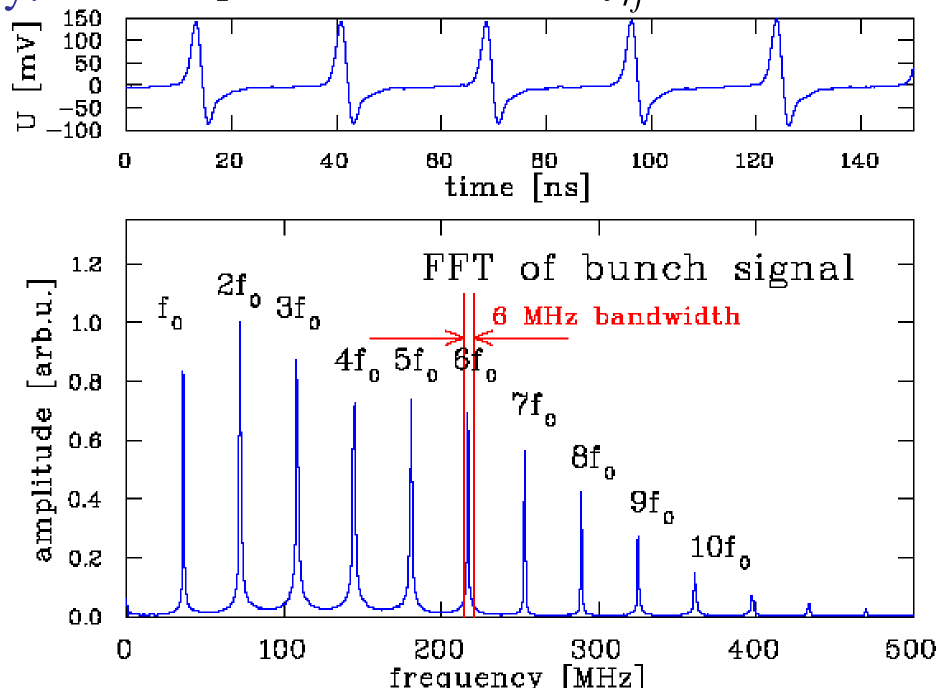
General: Noise Consideration

1. Signal voltage given by: $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$
2. Position information from voltage difference: $x = 1/S \cdot \Delta U / \Sigma U$
3. Thermal noise voltage given by: $U_{eff}(R, \Delta f) = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$

⇒ Signal-to-noise $\Delta U_{im}/U_{eff}$ is influenced by:

- Input signal amplitude
  → large or matched Z_t
- Thermal noise at R=50Ω for T=300K
  (for shoe box R=1kΩ …1MΩ)
- Bandwidth Δf
  ⇒ Restriction of frequency width because the power is concentrated on the harmonics of f_{rf}

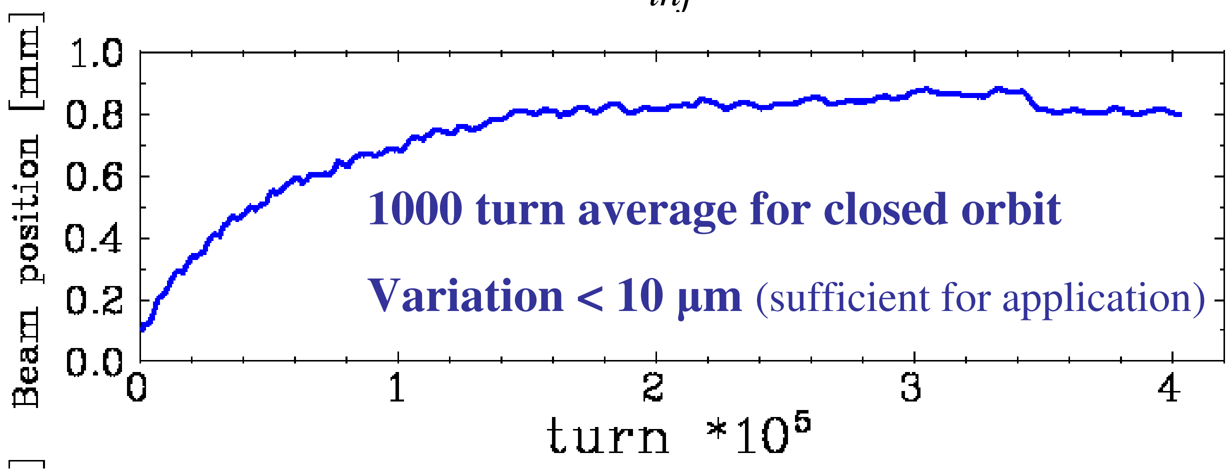
Example: GSI-LINAC with f_{rf}=36 MHz



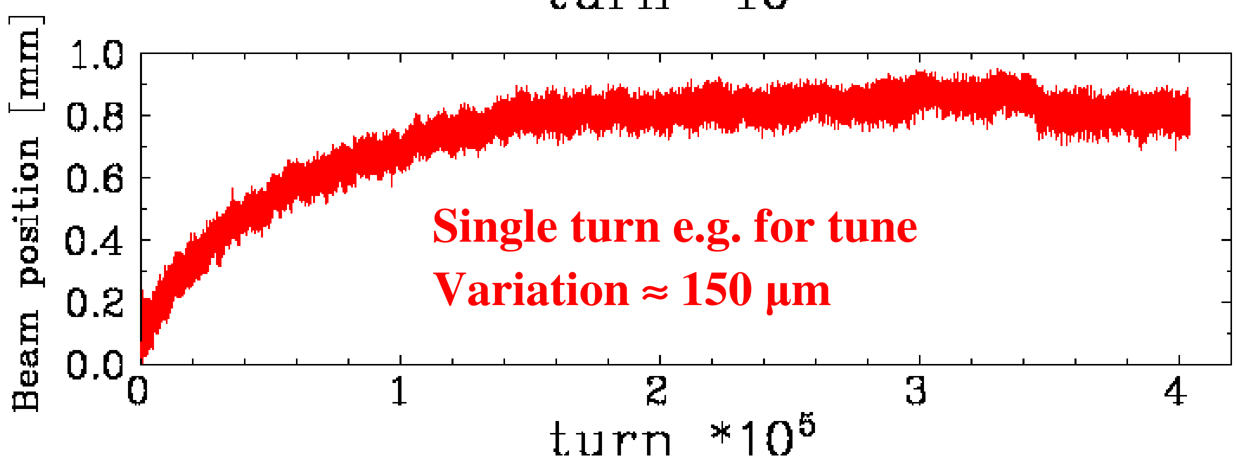
Remark: Additional contribution by non-perfect electronics typically a factor 2
Moreover, pick-up by electro-magnetic interference can contribute ⇒ good shielding required

Comparison: Filtered Signal ↔ Single Turn

Example: GSI Synchr.: U^{73+}, E_{inj}=11.5 MeV/u→ 250 MeV/u within 0.5 s, 10^9 ions



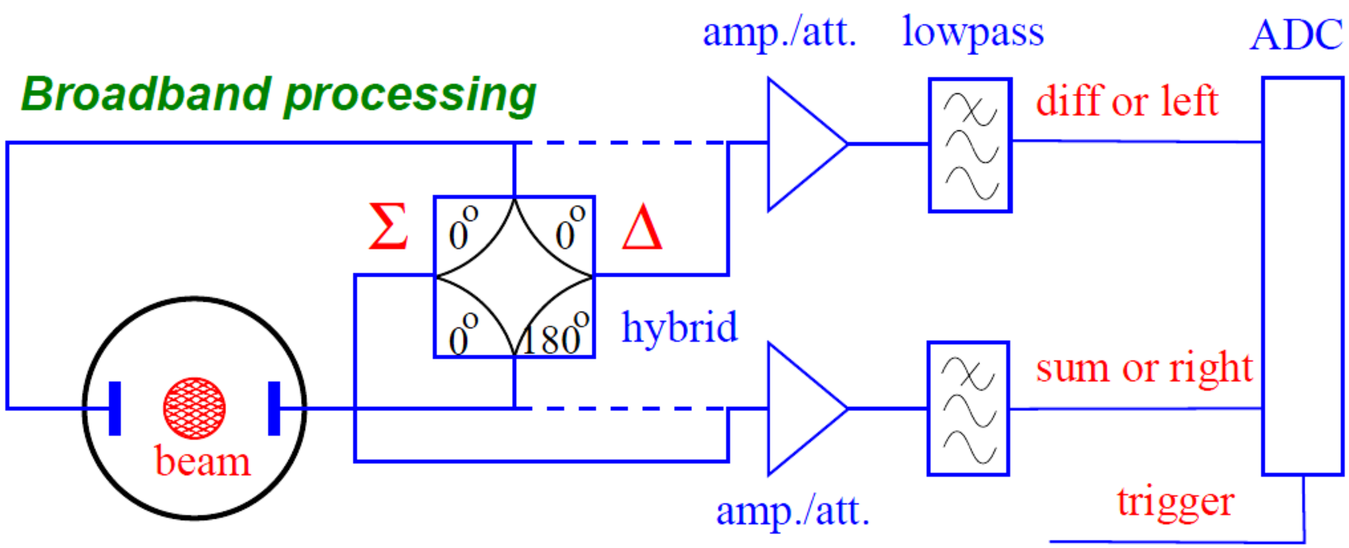
- Position resolution < 30 µm (BPM half aperture *a*=90 mm)
- average over 1000 turns corresponding to ≈1 ms or ≈1 kHz bandwidth



- Turn-by-turn data have much larger variation

However: not only noise contributes but additionally **beam movement** by betatron oscillation
⇒ broadband processing i.e. turn-by-turn readout for tune determination.

Broadband Signal Processing

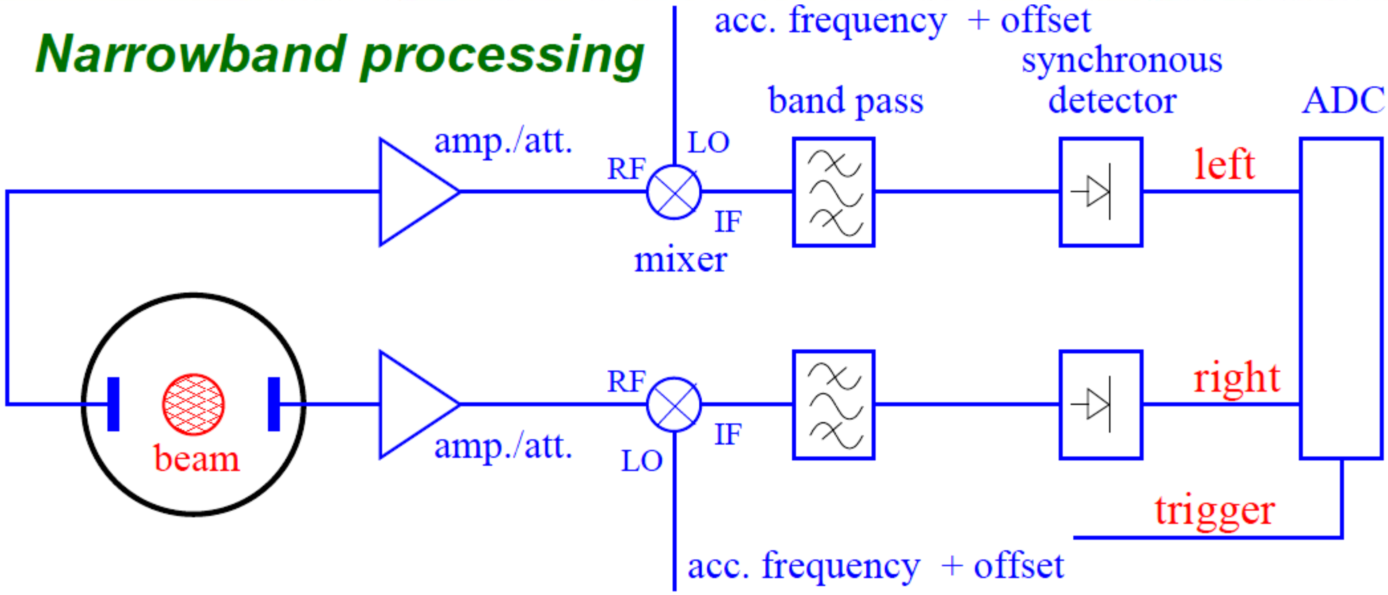


- Hybrid or transformer close to beam pipe for analog ***ΔU*** & ***ΣU*** generation or U_{left} & U_{right}
- Attenuator/amplifier
- Filter to get the wanted harmonics and to suppress stray signals
- ADC: digitalization ⟶ followed by calculation of of ***ΔU /ΣU***

Advantage: Bunch-by-bunch possible, versatile post-processing possible

Disadvantage: Resolution down to ≈ 100 μm for shoe box type , i.e. ≈0.1% of aperture,
resolution is worse than narrowband processing

Narrowband Processing for improved Signal-to-Noise



Narrowband processing equals heterodyne receiver (e.g. AM-radio or spectrum analyzer)

- Attenuator/amplifier
- Mixing with accelerating frequency $f_{rf} \Rightarrow$ signal with sum and difference frequency
- Bandpass filter of the mixed signal (e.g at 10.7 MHz)
- Rectifier: synchronous detector
- ADC: digitalization ⟶ followed calculation of $\Delta U/\Sigma U$

Advantage: spatial resolution about 100 time better than broadband processing

Disadvantage: No turn-by-turn diagnosis, due to mixing = 'long averaging time'

For non-relativistic p-synchrotron: → variable f_{rf} leads via mixing to constant intermediate freq.

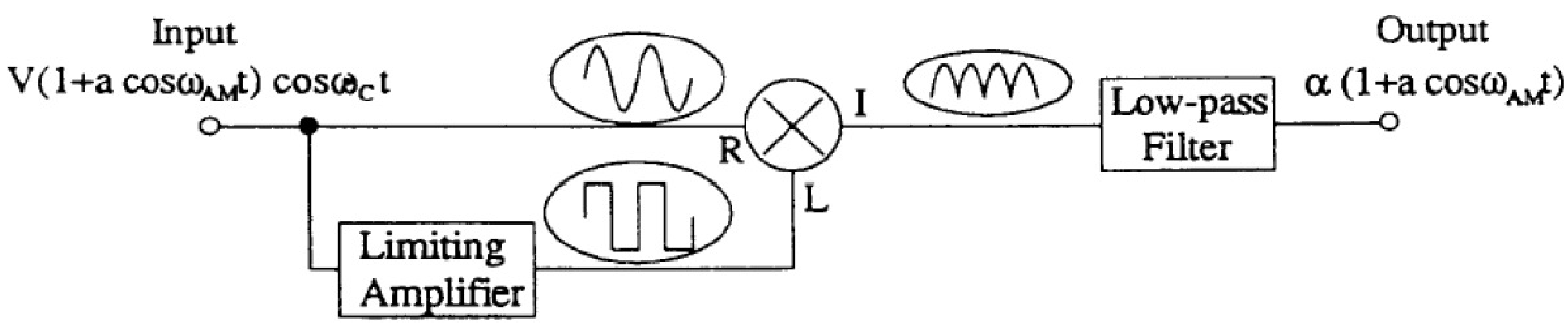
Mixer: A passive rf device with

- Input RF (radio frequency): Signal of investigation $A_{RF}(t) = A_{RF}\cos\omega_{RF}t$
- Input LO (local oscillator): Fixed frequency $A_{LO}(t) = A_{LO}\cos\omega_{LO}t$
- Output IF (intermediate frequency)

$$A_{IF}(t) = A_{RF}\cdot A_{LO}\cos\omega_{RF}t\cdot\cos\omega_{LO}t$$
$$= A_{RF}\cdot A_{LO}\left[\cos(\omega_{RF}-\omega_{LO})t+\cos(\omega_{RF}+\omega_{LO})t\right]$$

$\Rightarrow$ Multiplication of both input signals, containing the sum and difference frequency.

Synchronous detector: A phase sensitive rectifier



Outline:

- Signal generation $\rightarrow$ transfer impedance
- Capacitive *button* BPM for high frequencies
  used at most proton LINACs and electron accelerators
- Capacitive *shoe-box* BPM for low frequencies
  used at most proton synchrotrons due to linear position reading
- Electronics for position evaluation
  analog signal conditioning to achieve small signal processing
  (today's technology based on *digital* signal processing)
- **BPMs for measurement of closed orbit, tune and further lattice functions**
  frequent application of BPMs
- **Summary**

Detected position on a analog narrowband basis → closed orbit with ms time steps.

It differs from ideal orbit by misalignments of the beam or components.

Example from GSI-Synchrotron:

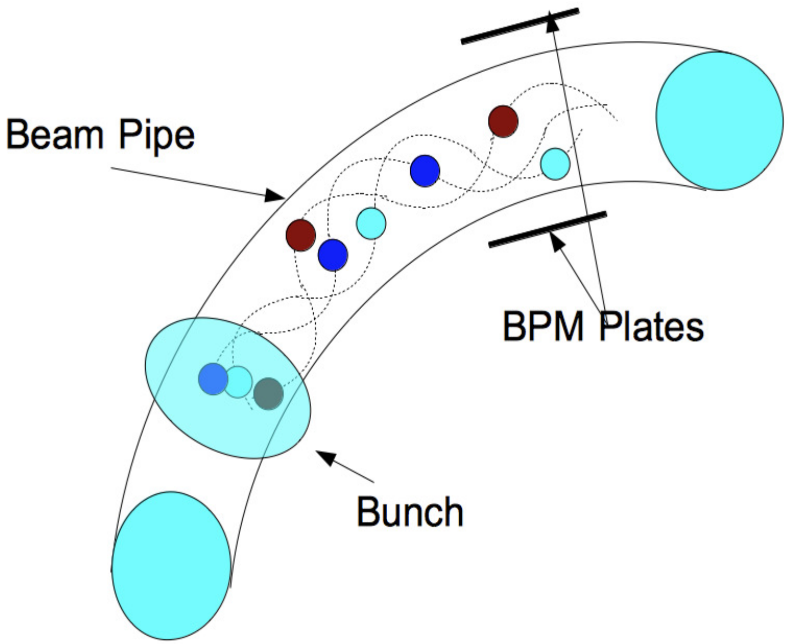
Closed orbit:

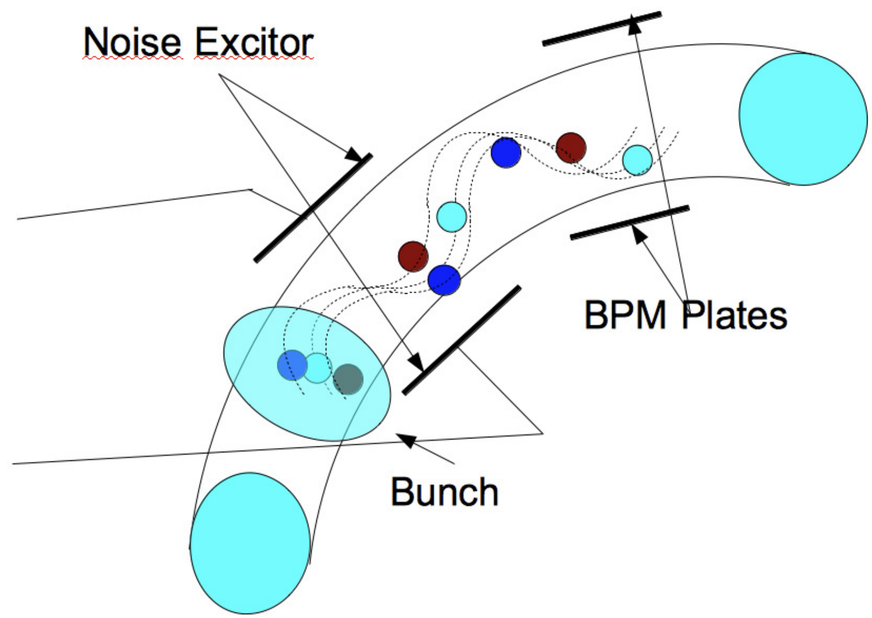
Beam position
averaged over many
betatron oscillations.

Coherent excitations are required for the detection by a BPM

Beam particle's in-coherent motion
⇒ center-of-mass stays constant

Excitation of all particles by rf
⇒ Coherent motion
⇒ center-of-mass variation turn-by-turn





Graphics by R. Singh, GSI

The tune Q is the number of betatron oscillations per turn.

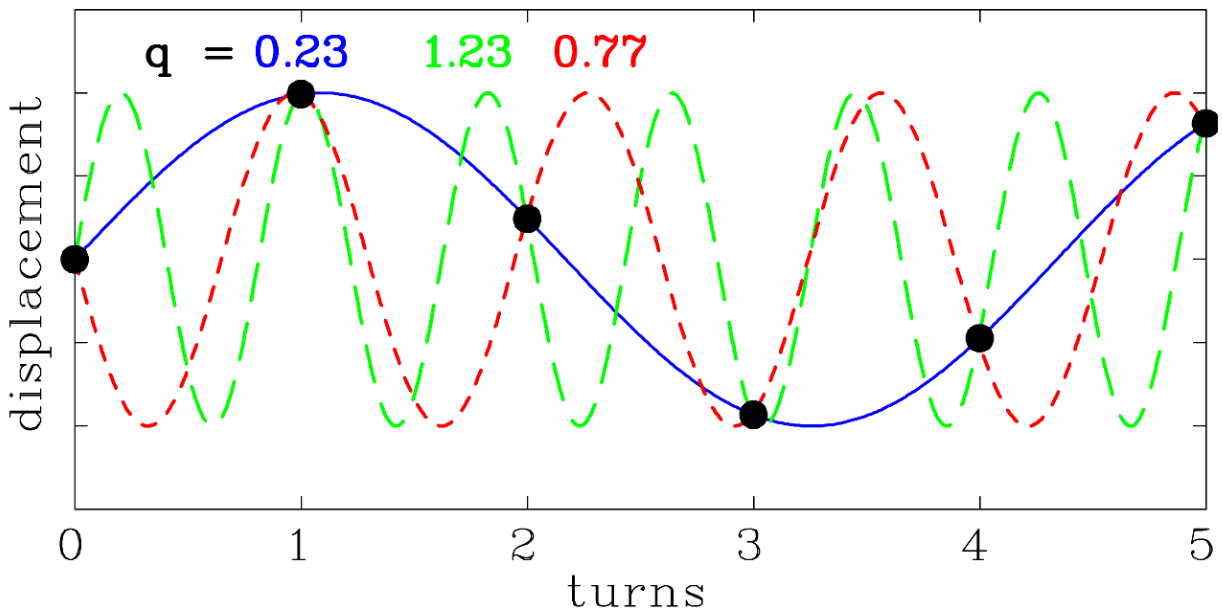
The betatron frequency is $f_\beta = Qf_0$.

Measurement: excitation of ***coherent*** betatron oscillations + position from one BPM.

From a measurement one gets only the non-integer part q of Q with $Q=n\pm q$.

Moreover, only $0 < q < 0.5$ is the unique result.

Example: Tune measurement for six turns with the three lowest frequency fits:

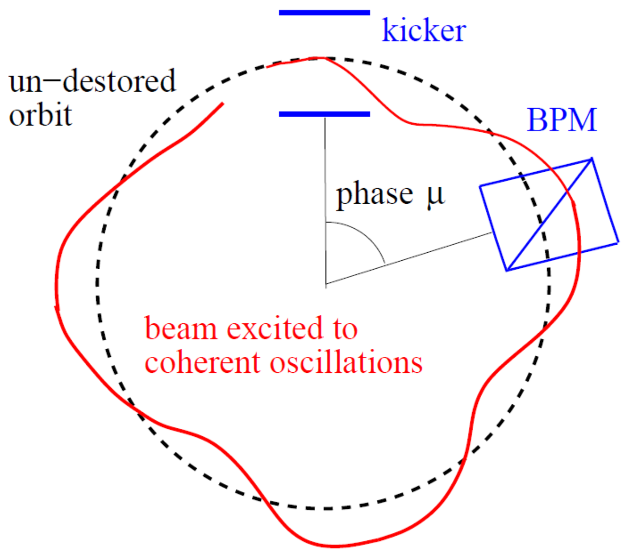


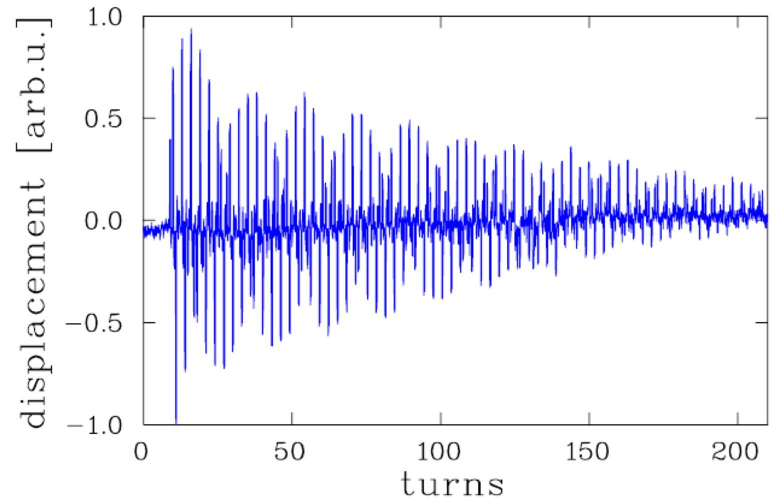
To distinguish for $q < 0.5$ or $q > 0.5$:

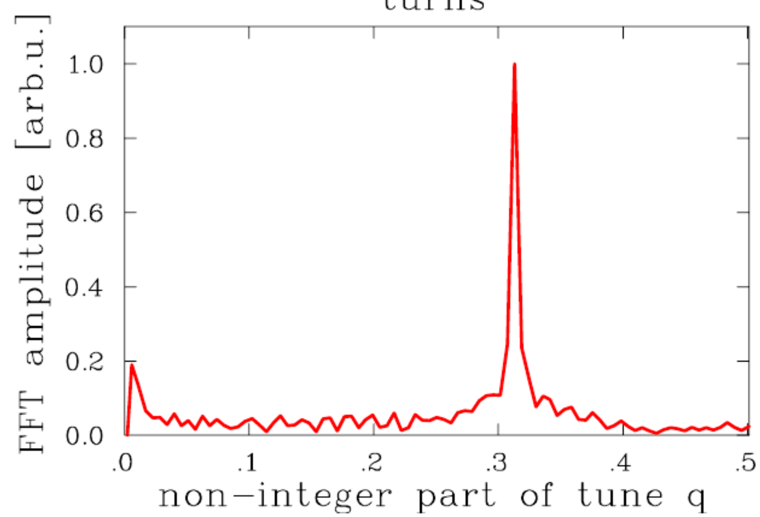
Changing the tune slightly, the direction of q shift differs.

The beam is excited to coherent betatron oscillation
→ the beam position measured each revolution ('turn-by-turn')
→ Fourier Trans. gives the non-integer tune ***q***.

Short kick compared to revolution.







The de-coherence time limits the **resolution**:

N non-zero samples

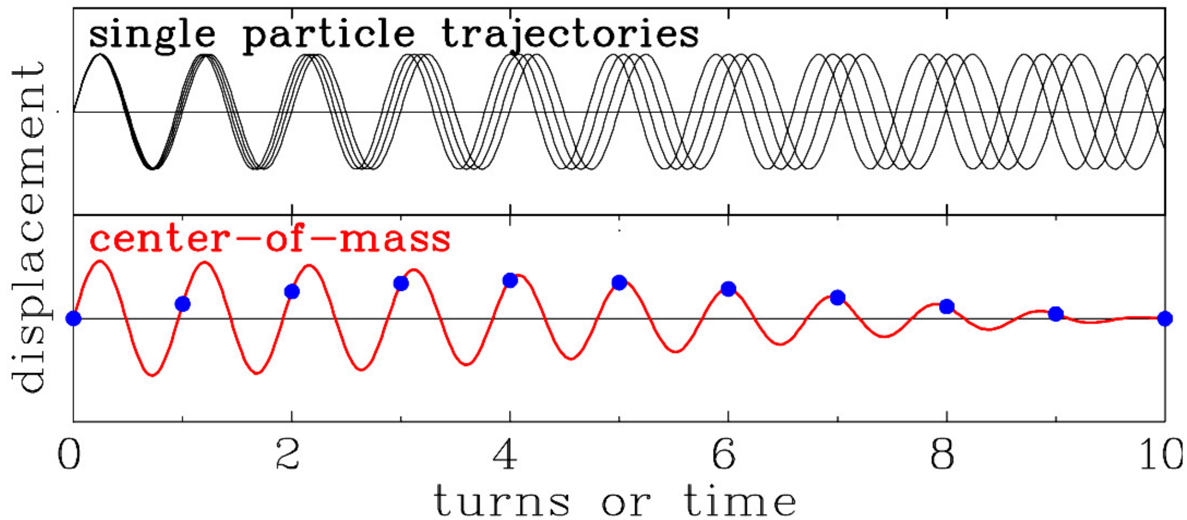
⇒ General limit of discrete FFT: $\Delta q > \frac{1}{2N}$

N = 200 turn ⇒ $\Delta q > 0.003$ as resolution

(tune spreads are typically $\Delta q \approx 0.001$!)

Tune Measurement: De-Coherence Time

The particles are excited to betatron oscillations, but due to the spread in the betatron frequency, they getting out of phase ('Landau damping'):



Scheme of the individual trajectories of four particles after a kick (top) and the resulting ***coherent*** signal as measured by a pick-up (bottom).

$\Rightarrow$ Kick excitation leads to limited resolution

Remark: The tune spread is much lower for a real machine.

Tune Measurement: Beam Transfer Function in Frequency Domain

Instead of one kick, the beam can be excited by a sweep of a sine wave, called 'chirp'

→ **Beam Transfer Function (BTF) Measurement**

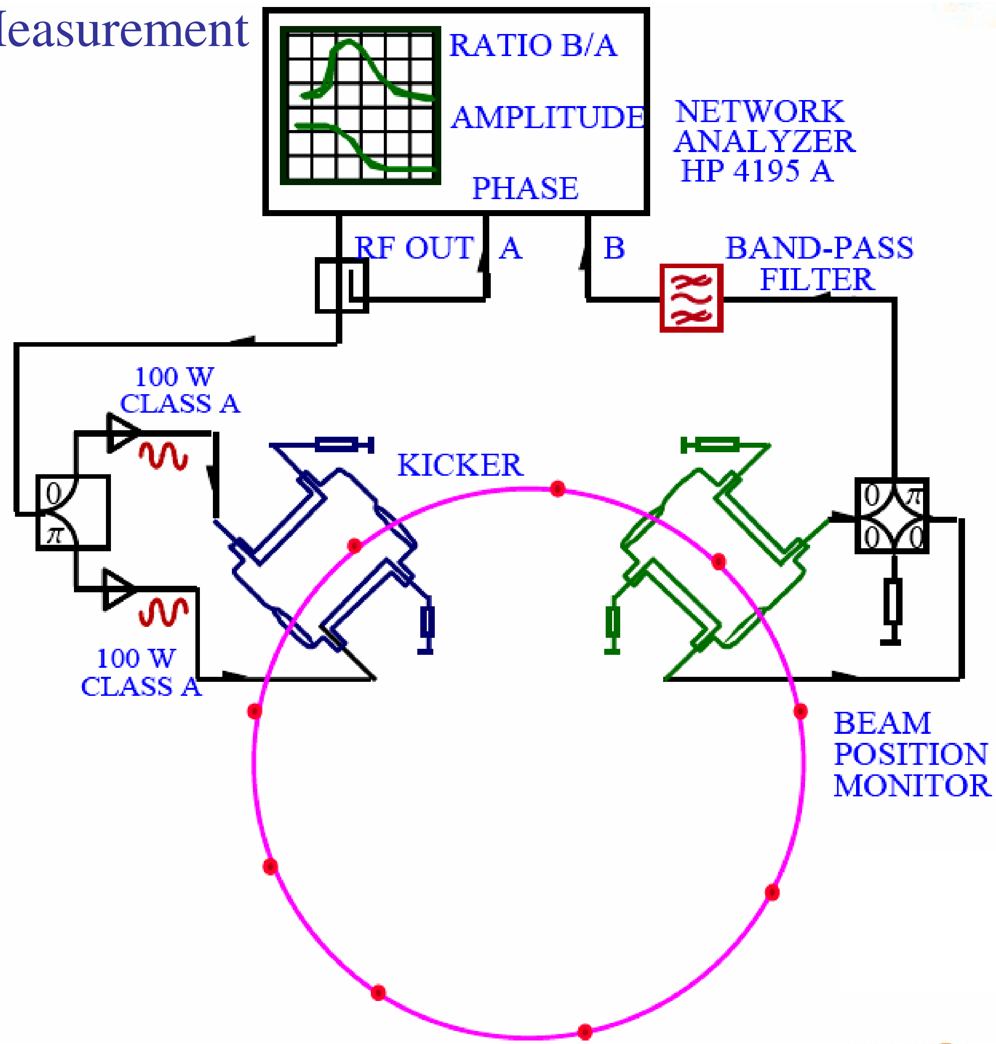
as the velocity response to a kick

Prinziple:

Beam acts like a driven oscillator!

Using a network analyzer:

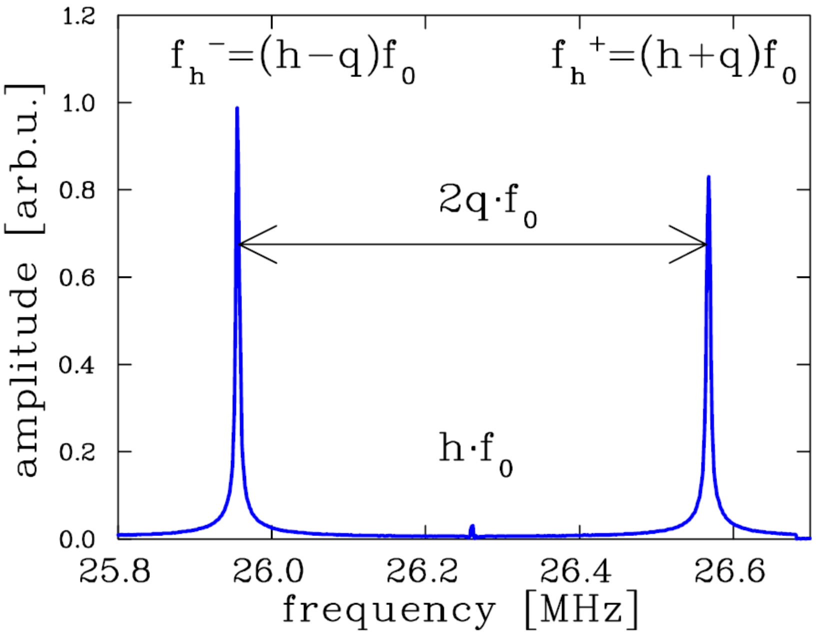
- RF OUT is feed to the beam by a kicker (reversed powered as a BPM)
- The position is measured at one BPM
- Network analyzer: amplitude and phase of the response
- Sweep time up to seconds due to de-coherence time per band
- resolution in tune: up to 10^{-4}



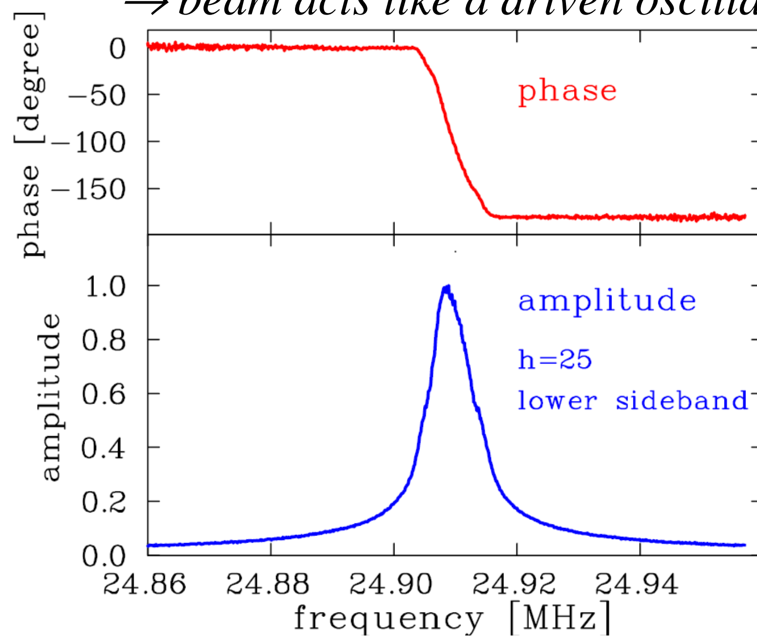
Tune Measurement: Result for BTF Measurement

BTF measurement at the GSI synchrotron, recorded at the 25th harmonics.

A wide scan with both sidebands at $h=25^{th}$-harmonics:



*A detailed scan for the **lower** sideband → beam acts like a driven oscillator:*



From the position of the sidebands $\boldsymbol{q} = 0.306$ is determined. From the width $\Delta f/f \approx 5 \cdot 10^{-4}$ the tune spread can be calculated via $\Delta f_h^- = \eta \frac{\Delta p}{p} \cdot h f_0 \left(h - q + \frac{\xi}{\eta} Q \right)$

Advantage: High resolution for tune and tune spread (also for de-bunched beams)

Disadvantage: Long sweep time (up to several seconds).

Tune Measurement: *Gentle* Excitation with Wideband Noise

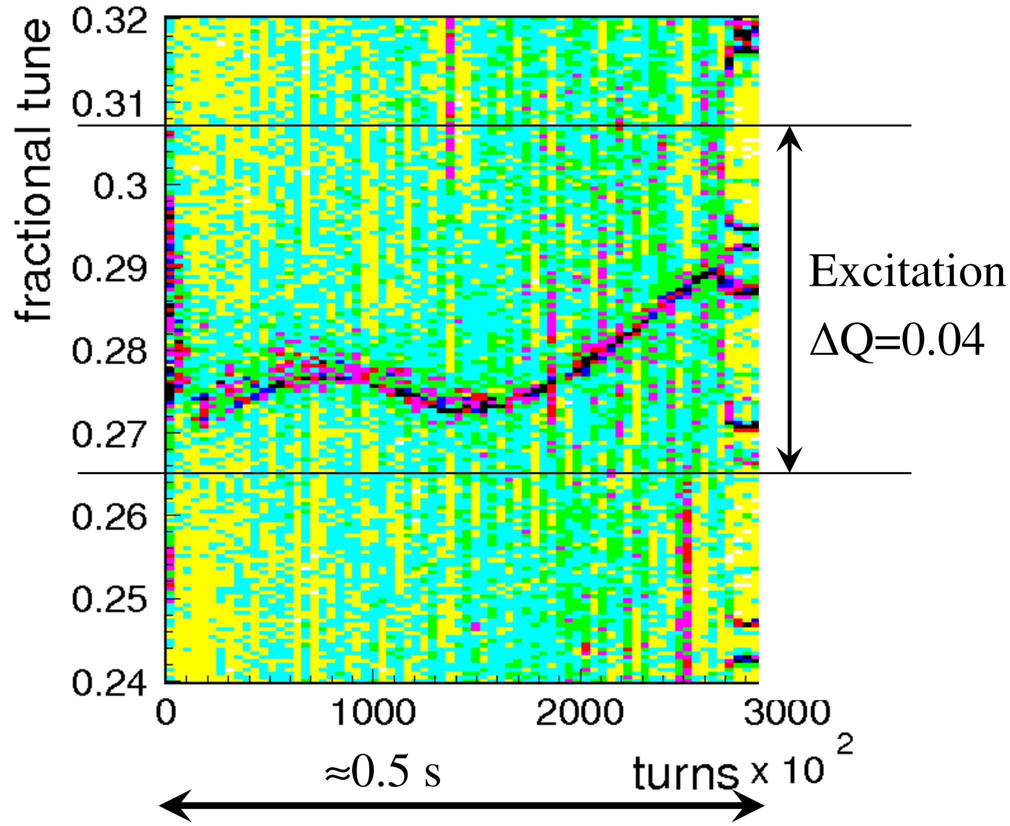
Instead of a sine wave, noise with adequate bandwidth can be applied

→ beam picks out its resonance frequency:

- broadband excitation with white noise of ≈ 10 kHz bandwidth
- turn-by-turn position measurement by fast ADC
- Fourier transformation of the recorded data

⇒ Continues monitoring with low disturbance

Example: *Vertical tune within 2048 turn at GSI synchrotron 11 → 250 MeV/u 2048 turn FFT equals ≃ 5 ms.*



Advantage:
Fast scan with good time resolution
Disadvantage: Lower precision

β-Function Measurement from Bunch-by-Bunch BPM Data

Excitation of coherent betatron oscillations: From the position deviation x_{ik} at the BPM i and turn k the β-function $\beta(s_i)$ can be evaluated.

The position reading is: ($\hat{x}_i$ amplitude, μ_i phase at i, Q tune, s_0 reference location)

$$x_{ik} = \hat{x}_i \cdot \cos(2\pi Qk + \mu_i) = \hat{x}_0 \cdot \sqrt{\beta(s_i)/\beta(s_0)} \cdot \cos(2\pi Qk + \mu_i)$$

→ a turn-by-turn position reading at many location (4 per unit of tune) is required.

The ratio of β-functions at different location:

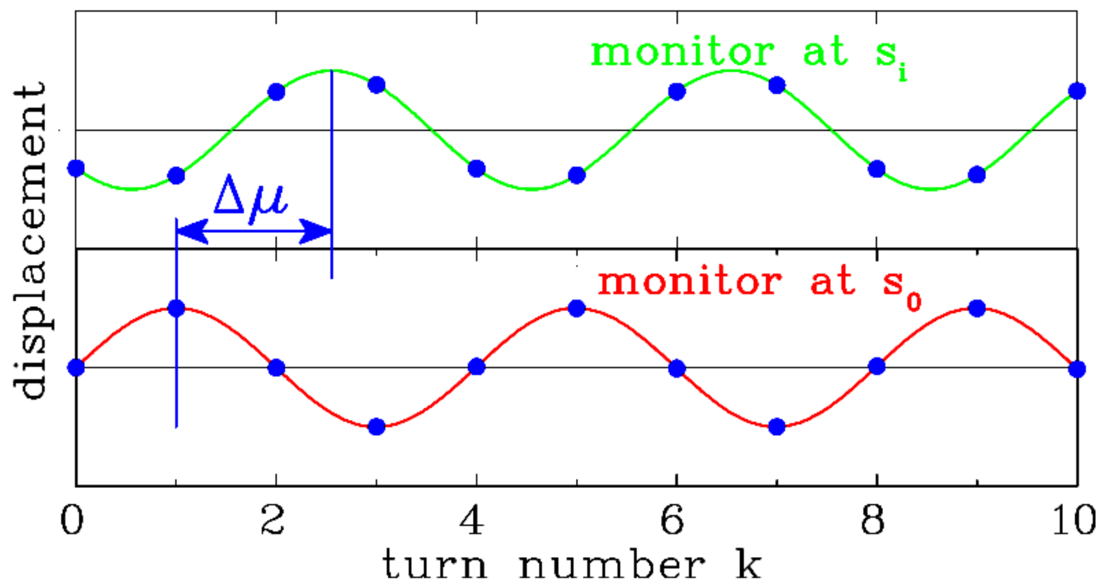
$$\frac{\beta(s_i)}{\beta(s_0)} = \left(\frac{\hat{x}_i}{\hat{x}_0}\right)^2$$

The phase advance is:

$$\Delta\mu = \mu_i - \mu_0$$

Without absolute calibration, β-function is more precise:

$$\Delta\mu = \int_{S0}^{Si} \frac{ds}{\beta(s)}$$



Dispersion D(s_i): Excitation of coherent betatron oscillations and change of momentum ***p*** by detuned rf-cavity:

→ Position reading at one location: $x_i = D(s_i) \cdot \frac{\Delta p}{p}$

→ Result from plot of $\boldsymbol{x_i}$ as a function of ***Δp/p*** ⇒ slope is local dispersion ***D(s_i).***

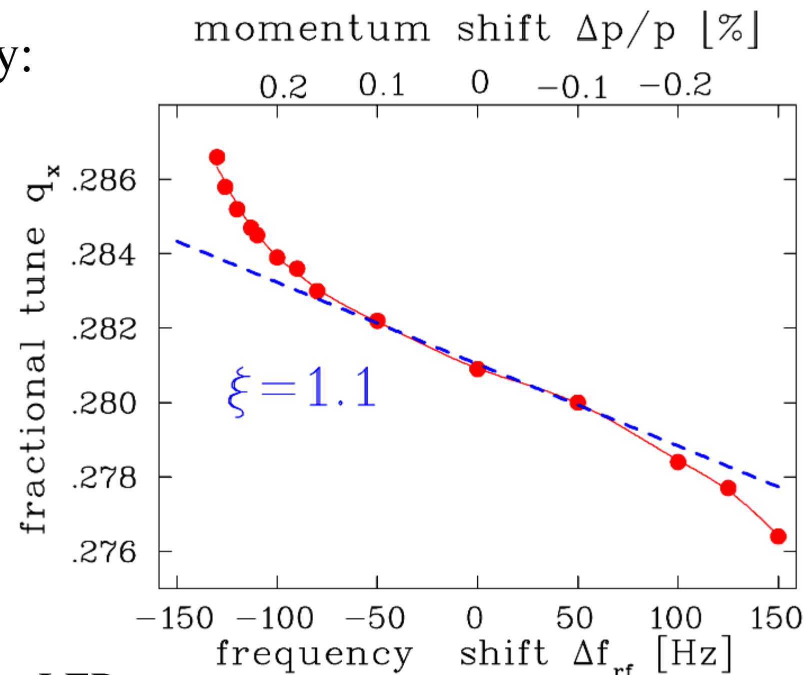
Chromaticity ξ: Excitation of coherent betatron oscillations and change of momentum ***p*** by detuned rf-cavity:

→ Tune measurement (kick-method, BTF, noise excitation):

$$\frac{\Delta Q}{Q} = \xi \cdot \frac{\Delta p}{p}$$

Plot of ***ΔQ/Q*** as a function of ***Δp/p***

⇒ slope is dispersion ξ.



Measurement at LEP

Summary Pick-Ups for bunched Beams

The electric field is monitored for bunched beams using rf-technologies ('frequency domain'). Beside transfromers they are the most often used instruments!

Differentiated or proportional signal: rf-bandwidth ↔ beam parameters

Proton synchrotron: 1 to 100 MHz, mostly 1 MΩ → proportional shape

LINAC, e–-synchrotron: 0.1 to 3 GHz, 50 Ω → differentiated shape

Important quantity: transfer impedance $Z_t(\omega, \beta)$.

Types of capacitive pick-ups:

Shoe-box (p-synch.), button (p-LINAC, e–-LINAC and synch.)

Remark: Stripline BPM as traveling wave devices are frequently used

Position reading: difference signal of four pick-up plates (BPM):

- **Non-intercepting** reading of center-of-mass → online measurement and control
  slow reading → closed orbit, ***fast bunch-by-bunch***→ trajectory
- Excitation of ***coherent betatron oscillations*** and response measurement
  excitation by short kick, white noise or sine-wave (BTF)
  → tune q, chromaticity ξ, dispersion D etc.