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Tutorial
Particle sources

1 – Electron diode : Child-Langmuir law

Let's consider two metallic plates of infinite dimensions. Plates are separated by a distance ***d***. The axis perpendicular to planes is called ***x***. The plate 1 located in the plane ***x=0*** is heated and biased to 0 Volt, while the second plate at ***x=d*** is biased to ***+V*** Volts. Thermionic electrons are generated on plate 1 and accelerated toward plate 2. We are going to study the space charge effect of the electrons in the gap ***d***.

Due to the symmetry of the system, we can consider that all the physical parameters are only depending on ***x***. We also assume that a stationary solution is reached. The following conventions are proposed to describe the problem :

- ***n(x)*** electronic density,
- ***-ρ(x)*** electronic charge density, we assume that $\rho>0$
- ***v(x)*** velocity,
- ***-e*** electron charge,
- ***V(x)*** electric potential,
- ***E(x)*** Electric field,
- ***-J(x)*** current density, ***J(x)***>0

1) Plot ***V(x)*** and ***E(x)*** when there are no electrons in the gap. (case of ideal condensator)
2) Express ***v(x)*** as a function of ***V(x)*** in the gap.
3) Show that J is a constant. Express ***ρ(x)*** as a function of ***J*** and ***V(x)***.
4) Write Poisson law and find the differential equation that should satisfy ***V(x)***.
5) Show that the differential equation can be written :

$$EdE = k\frac{dV}{V^{1/2}} \text{ with } k = \frac{J}{\varepsilon_0}\sqrt{\frac{m}{2e}}$$

6) Express ***E(x)*** as a function of ***E(0)***
7) When ***E(0)***-> 0, electrons emitted by the plate 1 cannot be accelerated anymore and we say that the diode as reached its space charge limit. Solve the equation as a function of ***V(x)*** when E(0)=0. Find that in x=d, equation reduces to the relation :

$$J_{max} = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{U^{3/2}}{d^2}$$

2- Calculation of the Plasma frequency

Let's consider a plasma with a density n of electron and ions. The mass of ions M being higher than the one of electrons, we assume that ions are frozen and that only electrons move (no contribution of ions to Current).

The plasma is baking in a sinusoidal electric field with pulsation ω // y axis and propagating toward x>0.

We will use the complex notation for convenience. $\tilde{E} = E e^{i(\omega t \;\; ky)}$

1) write the second Newton law for electrons. Express complex velocity v as a function of $\tilde{E}$ and derive the complex electrical conductivity of electrons as a function of n, e ,m_e, ω.

2) Helped with Maxwell equations, show that the dispersion wave equation can be written as follow: $k^2 = \frac{\omega^2 - \omega_p{}^2}{c^2}$,

   where ω_p is the plasma frequency we will express as a function of the parameters of the problem.

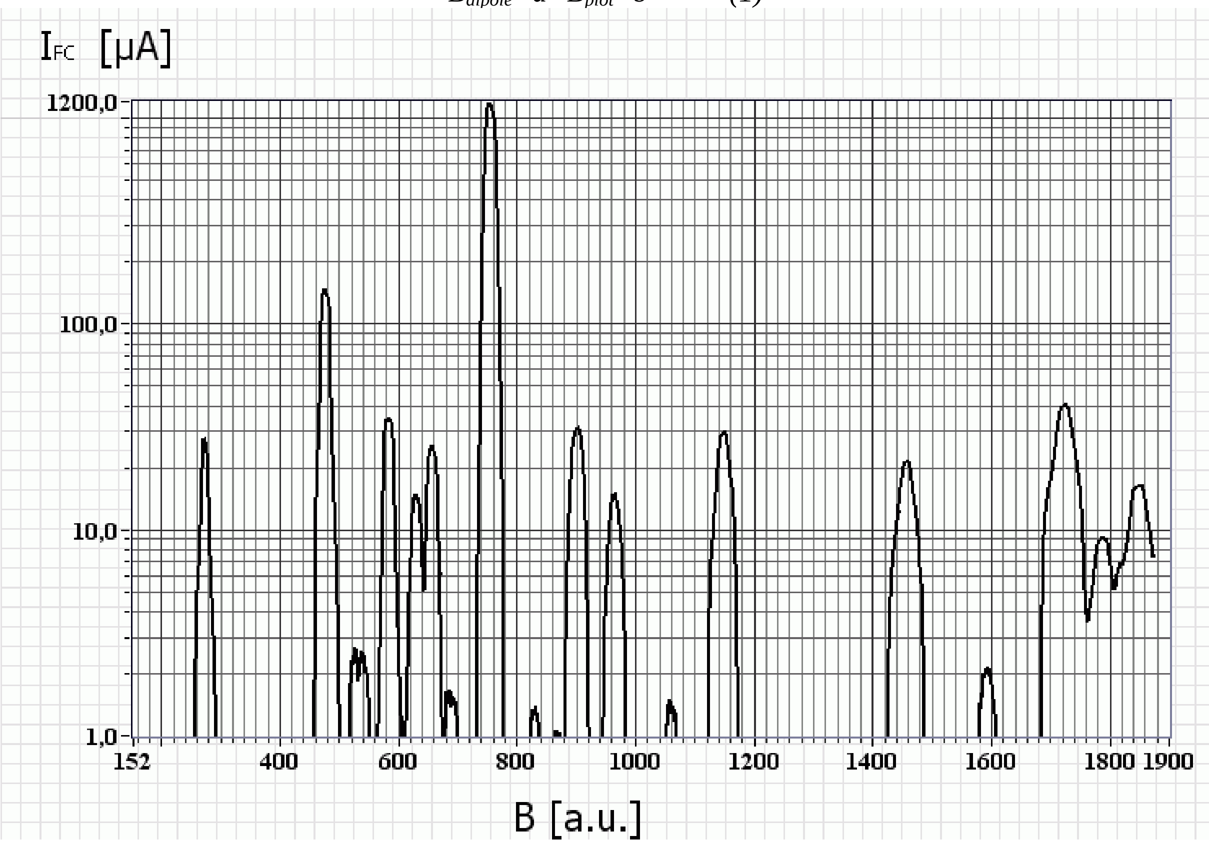
1. We observe a stop of propagation of electromagnetic waves for a frequency ν < 3 MHz (case of ionosphere). Derive density N of ionosphere.

N.A. : $m = 9.1 \times 10^{-31}$ kg
$e = 1.6 \times 10^{-19}$ C
$1/(4\pi\varepsilon_0) = 9 \times 10^{9}$ SI

3- Ionic Spectrum

An ion source is producing beam with an electrostatic extraction voltage ***V*** of 26 kV. The extracted beam is composed of several chemical elements (C,N,O,H,He) with several charge states for each elements. The different species are separated through a magnetic bending magnet (dipole) with a radius of curvature ρ and magnetic field induction ***B***. The beams are measured after the dipole in a Faraday cup. When the magnetic field is ramped in the magnet, one obtains the ionic spectrum of Figure 1 below. The axis of the plot is a measurement of the magnetic field intensity in the fringe field of the magnet. The strong hysteresis of the dipole makes the real magnetic field value $\boldsymbol{B_{dipole}}$ in the dipole gap being:

$$B_{dipole} = a \times B_{plot} + b \qquad (1)$$



3.1) Show that the dipole enables to separate chemical species according to the following formula:

$$B = \frac{1}{\rho}\sqrt{\frac{2m_a V}{e}}\sqrt{\frac{A}{Z}} \qquad (2)$$

where:

-A is the atomic number, m_A is atomic mass unit (931.46 MeV/c^2)

- e is electric charge and Z is the charge state of the ion

3.2) The peak on the right is H_2O^+ molecule, the peak on the left is H^+. Helped with (1) and (2), find the He^+ peak and the O^{7+} peak. Then identify the other chemical species with their charge states.

3.3) Assuming that the arbitrary unit of the figure 1 is Gauss, estimate the radius of curvature of the bending magnet.