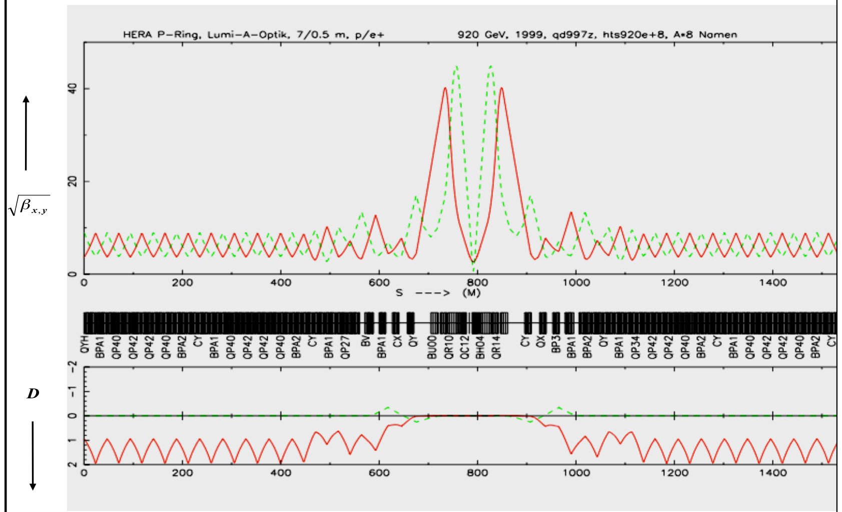
26.) Insertions



Insertions

... the most complicated one: the drift space

Question to the audience: what will happen to the beam parameters α, β, γ if we stop focusing for a while ...?

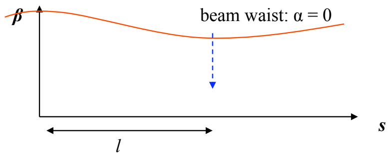
$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_S = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC'+S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} * \begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_0$$

transfer matrix for a drift:

$$M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \longrightarrow$$

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$
$$\alpha(s) = \alpha_0 - \gamma_0 s$$
$$\gamma(s) = \gamma_0$$

location of the waist:



given the initial conditions $\alpha_0, \beta_0, \gamma_0$: where is the point of smallest beam dimension in the drift … or at which location occurs the beam waist ?

beam waist:

$$\alpha(s) = 0 \quad \rightarrow \quad \alpha_0 = \gamma_0 * s$$

$$\ell = \frac{\alpha_0}{\gamma_0}$$

beam size at that position:

$$\left.\begin{array}{l} \gamma(\ell) = \gamma_0 \\ \alpha(\ell) = 0 \end{array}\right\} \rightarrow \gamma(l) = \frac{1+\alpha^2(\ell)}{\beta(\ell)} = \frac{1}{\beta(\ell)}$$

$$\beta(\ell) = 1/\gamma_0$$

β-Function in a Drift:

let's assume we are at a symmetry point in the center of a drift.

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$

as $\alpha_0 = 0, \quad \rightarrow \quad \gamma_0 = \frac{1+\alpha_0^2}{\beta_0} = \frac{1}{\beta_0}$

and we get for the β function in the neighborhood of the symmetry point

$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$ **! ! !**

Nota bene:

1.) this is very bad !!!

2.) this is a direct consequence of the conservation of phase space density (... in our words: ε = const) … and there is no way out.

3.) Thank you, Mr. Liouville !!!

Joseph Liouville, 1809-1882

β-Function in a Drift:

If we cannot fight against Liouvuille theorem ... at least we can optimise

Optimisation of the beam dimension:

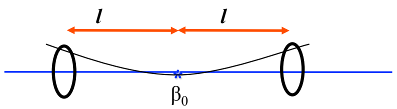
$$\beta(\ell) = \beta_0 + \frac{\ell^2}{\beta_0}$$

Find the β at the center of the drift that leads to the lowest maximum β at the end:

$$\overset{!}{\frac{d\hat{\beta}}{d\beta_0} = 1 - \frac{\ell^2}{\beta_0^2} = 0}$$

$$\rightarrow \beta_0 = \ell$$

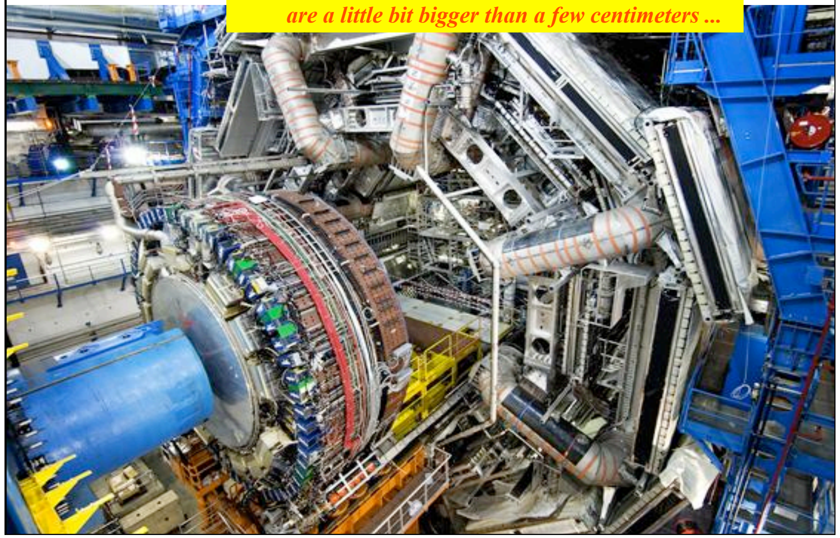
$$\rightarrow \hat{\beta} = 2\beta_0$$



If we choose $\beta_0 = \ell$ we get the smallest β at the end of the drift and the maximum β is just twice the distance ℓ

... clearly there is an

But: ... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

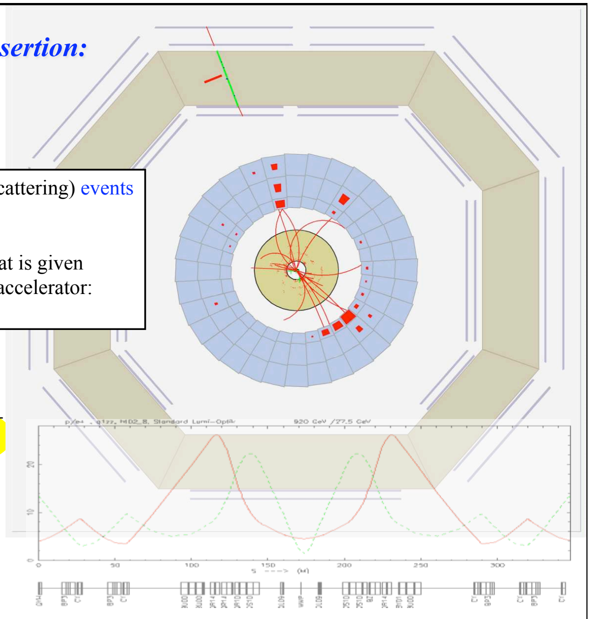


27.) The Mini-β Insertion:

$$R = L * \Sigma_{react}$$

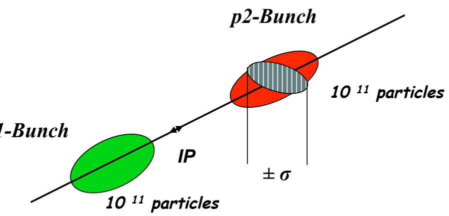
production rate of (scattering) events
is determined by the
cross section Σ_{react}
and a parameter L that is given
by the design of the accelerator:
… the luminosity

$$L = \frac{1}{4\pi e^2 f_0 b} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$



ZEUS detector: inelastic
scattering event of e+/p

Luminosity



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$

$\sigma_{x,y} = 17\ \mu m$

$f_0 = 11.245\ kHz$

$n_b = 2808$

$I_p = 584\ mA$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

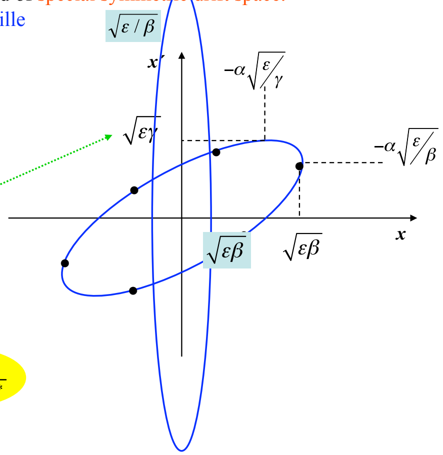
Mini-β Insertions: Betafunctions

A mini-β insertion is always a kind of special symmetric drift space.
→greetings from Liouville

$\alpha^* = 0$

$\gamma^* = \frac{1+\alpha^2}{\beta} = \frac{1}{\beta^*}$

$\sigma'^* = \sqrt{\frac{\varepsilon}{\beta^*}}$



$\beta^* = \frac{\sigma^*}{\sigma'^*}$

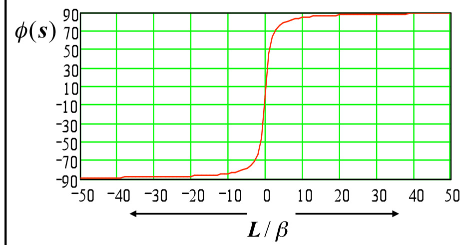
at a symmetry point β is just the ratio of beam dimension and beam divergence.

Mini-β Insertions: Phase advance

By definition the phase advance is given by: $\Phi(s) = \int \frac{1}{\beta(s)} ds$

Now in a mini β insertion: $\beta(s) = \beta_0 \; (1 + \frac{s^2}{\beta_0^2})$

$$\rightarrow \Phi(s) = \frac{1}{\beta_0} \int_0^L \frac{1}{1 + s^2/\beta_0^2} ds = \arctan \frac{L}{\beta_0}$$



Consider the drift spaces on both sides of the IP: the phase advance of a mini β insertion is approximately π, in other words: the tune will increase by half an integer.

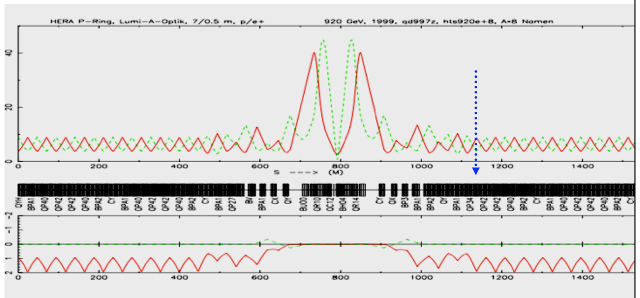
Mini-β Insertions: some guide lines

* *calculate the periodic solution in the arc*
* *introduce the drift space needed for the insertion device (detector ...)*
* *put a quadrupole doublet (triplet ?) as close as possible*
* *introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure*

parameters to be optimised & matched to the periodic solution:

$$\alpha_x,\ \beta_x \qquad D_x,\ D_x'$$
$$\alpha_y,\ \beta_y \qquad Q_x,\ Q_y$$

8 individually powered quad magnets are needed to match the insertion
(... at least)

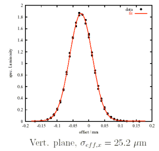


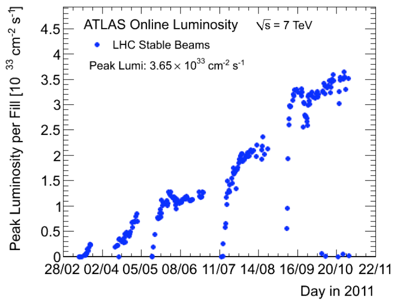
LHC Luminosity Parameters

$$L = \frac{n_B * N_p * N_e * f_0}{2\pi * \sqrt{(\sigma_{x,p}^2 + \sigma_{x,e}^2)} * \sqrt{(\sigma_{y,p}^2 + \sigma_{y,e}^2)}}$$

	LHC Parameters
Energy	*3500 / 7000 GeV*
I	*2 * 300 mA / 2*584 mA*
N pro Bunch	$1.4 * 10^{11}$
n_coll	*1380 / 2808*
β_x β_y ε_x	*1 m / 0.55 m* *1 m / 0.55 m* $5.6*10^{-10}$ *m* / $5*10^{-10}$ *m*
σ_x σ_y	*23 μm /16 μm* *23 μm /16 μm*
L_0	$3.6*10^{33}$ / $1*10^{34}$

Luminosity Scan of the two beams
-> effective cross section





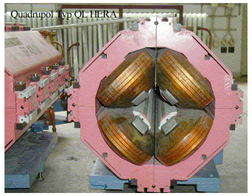
Are there any problems ??

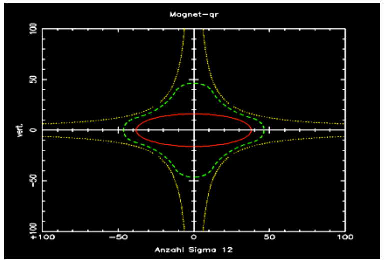
sure there are ...

* *aperture of mini β quadrupoles* limit the luminosity

* remember: *large quads are weak quads*

gradient of a quadrupole magnet: $g = \frac{2\mu_0 n I}{r^2}$





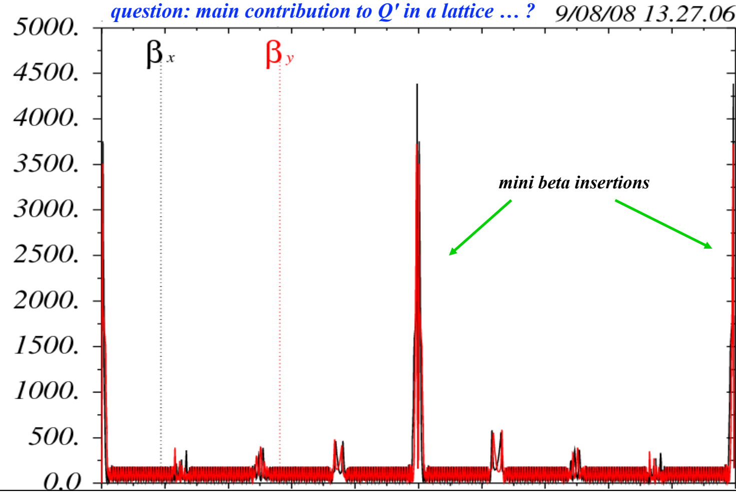
beam envelope at the first mini β quadrupole lens in the HERA proton storage ring

→ *keep distance „s“ to the first mini β quadrupole as small as possible*

… and now back to the Chromaticity

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

question: main contribution to Q' in a lattice … ?

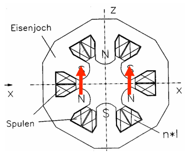


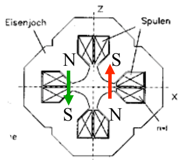
Correction of Q':

k_1 normalised quadrupole strength

k_2 normalised sextupole strength

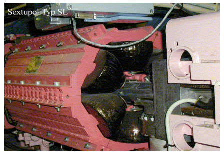
Sextupole Magnets:





$$k_1(sext) = \frac{\tilde{g}\, x}{p/e} = k_2 * x$$

$$k_1(sext) = k_2 * D * \frac{\Delta p}{p}$$



corrected chromaticity

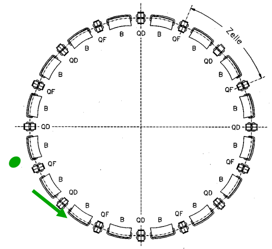
considering a single cell:

$$Q'_{cell\_x} = -\frac{1}{4\pi}\left\{k_{qf}\hat{\beta}_x l_{qf} - k_{qd}\breve{\beta}_x l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

$$Q'_{cell\_y} = -\frac{1}{4\pi}\left\{-k_{qf}\breve{\beta}_y l_{qf} + k_{qd}\hat{\beta}_y l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

Clearly there is another problem ...

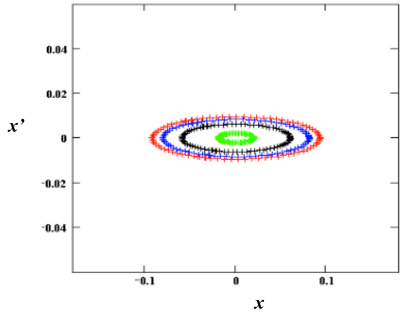
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



A beam of 4 particles

– each having a slightly different emittance:

25.) Particle Tracking Calculations

particle vector: $\begin{pmatrix} x \\ x' \end{pmatrix}$

Idea: *calculate the particle coordiantes x, x´ through the linear lattice*
… using the matrix formalism.
if you encounter a nonlinear element (e.g. sextupole): stop
calculate explicitly the magnetic field at the particles coordinate

$$B = \begin{pmatrix} g'xz \\ \frac{1}{2} g'(x^2 - z^2) \end{pmatrix}$$

calculate kick on the particle

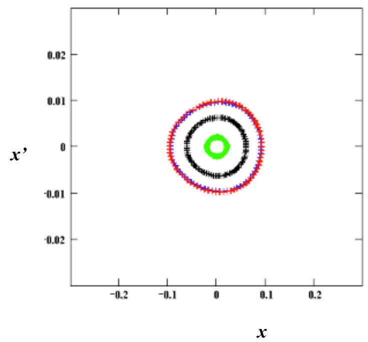
$$\Delta x_1' = \frac{B_z l}{p/e} = \frac{1}{2}\frac{g'}{p/e} l(x_1^2 - z_1^2) = \frac{1}{2} m_{sext} l(x_1^2 - z_1^2) \qquad \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \rightarrow \begin{pmatrix} x_1 \\ x_1' + \Delta x' \end{pmatrix}$$

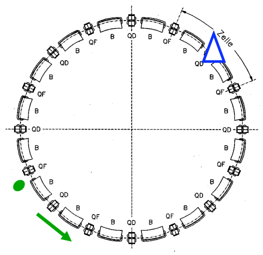
$$\Delta z_1' = \frac{B_x l}{p/e} = \frac{g' x_1 z_1}{p/e} l = m_{sext} l x_1 z_1 \qquad \begin{pmatrix} z_1 \\ z_1' \end{pmatrix} \rightarrow \begin{pmatrix} z_1 \\ z_1' + \Delta z_1' \end{pmatrix}$$

and continue with the linear matrix transformations

Installation of a weak (!!!) sextupole magnet

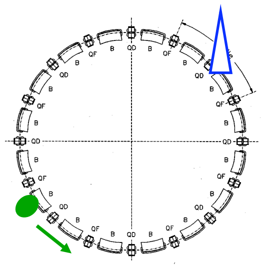
The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equatiuons; instead: Computer simulation
„ particle tracking "

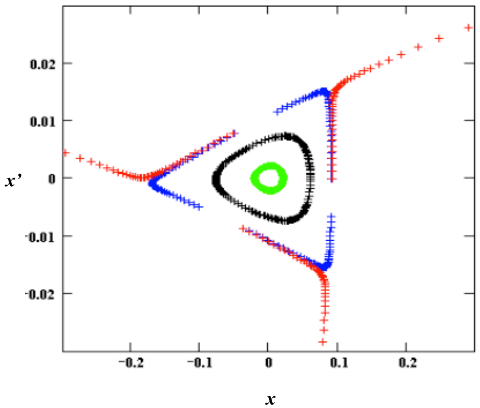




Effect of a strong (!!!) Sextupole …

→ Catastrophy !



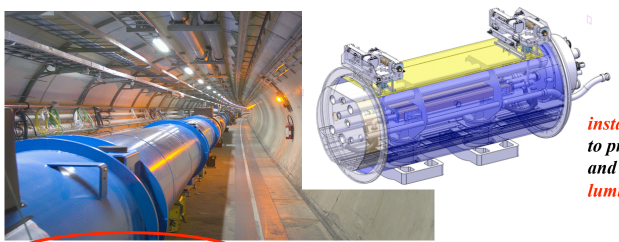


„dynamic aperture"

Talking about Reality

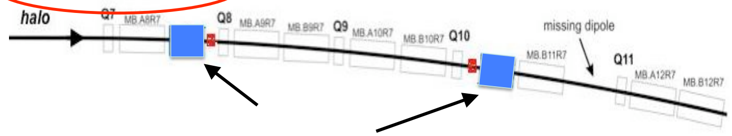
My daily problems:

LHC Upgrade Plans: install shorter / stronger dipoles



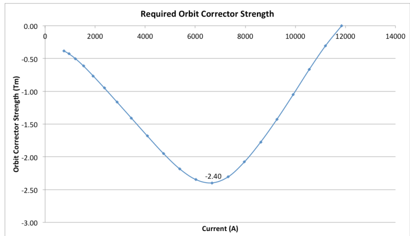
install additional collimator to protect machine and experiments from luminosity debris

New ~3..3.5 m shorter Nb_3Sn Dipoles (2 per DS)



Talking about Reality

Two main „daily" problems:



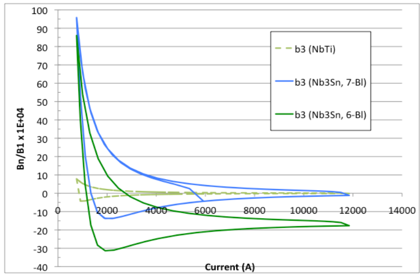
Below I_{nom} 11 T Dipole is stronger than MB

$$\int Bdl(I) \neq const$$

2.7% difference to ideal main field

strong (!) non-linear field errors

-> particle stability



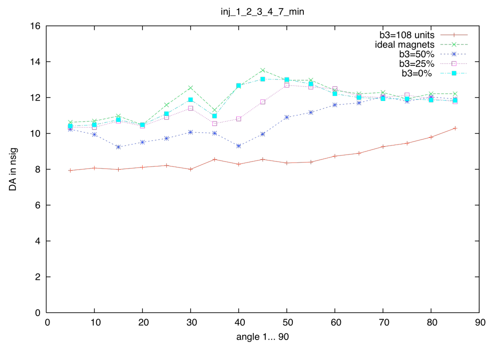
Talking about Reality

My daily problems:

Scanning magnet multipole coefficients to find the limit for individual magnet errors

... depending on optics, energy and magnet location

! an in collision

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:CD_col := 0.0000 ; a2R_MQXCD_col := 0.0000 ;
:CD_col := 0.8900 ; a3R_MQXCD_col := 0.8900 ;
:CD_col := 0.6400 ; a4R_MQXCD_col := 0.6400 ;
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:CD_col := 1.2700 ; a6R_MQXCD_col := 0.3300 ;
:CD_col := 0.2100 ; a7R_MQXCD_col := 0.2100 ;
:CD_col := 0.1600 ; a8R_MQXCD_col := 0.1600 ;
:CD_col := 0.0800 ; a9R_MQXCD_col := 0.0800 ;
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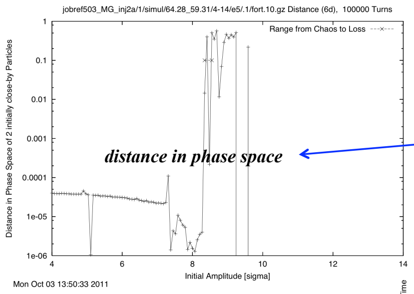
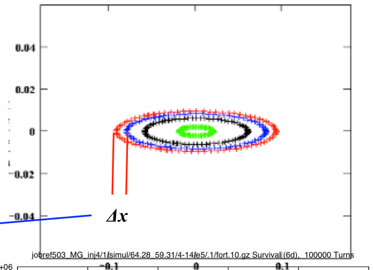
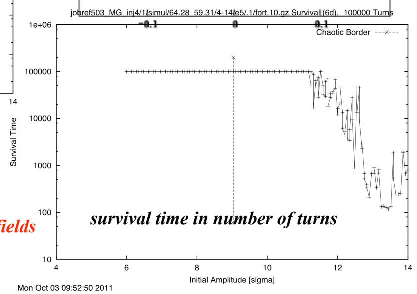
)XCD_col := 0.0000 ; a15R_MQXCD_col := 0.0000 ;
```

Talking about Reality

stability in phase space



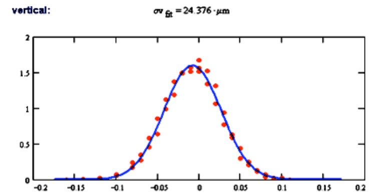




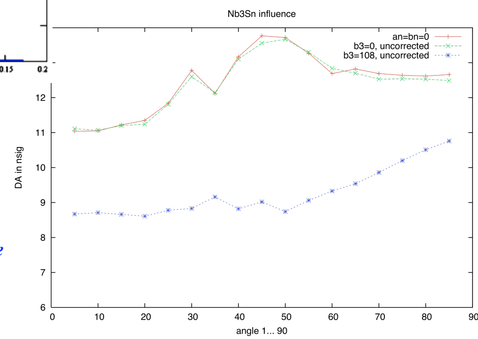
observe the distance of a pair of nearby particles
in phase space under the influence of non-linear fields

Talking about Reality

My daily problems: „Dynamic Aperture"



Gaussian beam distribution



Stability limit in sigma
for a good field quality and a
bad one.
We require at least 12 sigma on average

Than'x a lot

Bernhard, Reyes and Guido