Exercise 1

Compute the transverse space charge forces and the tune shifts for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, gaussian

parabolic
$$\lambda(z) = \frac{3Ne}{2l_o}\left[1 - \left(\frac{2z}{l_o}\right)^2\right]$$

sinusoidal modulation
$$\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z) \quad ; \quad k_z = 2\pi / \lambda_w$$

Gaussian
$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

Exercise 2

Compute the transverse space charge forces and the tune shifts for a cylindrical beam in a circular beam pipe, having a bi-gaussian longitudinal and transverse distribution.

$$bi-gaussian$$

$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$\rho(r,z) = \frac{\lambda(z)}{2\pi\sigma_r^2}\exp\left(\frac{-r^2}{2\sigma_r^2}\right)$$

Exercise 3

Compute the longitudinal space charge force of a transverse uniform cylindrical beam in a circular perfectly conducting beam pipe

$$E_z(r,z) = -\frac{1}{\gamma^2}\frac{\partial}{\partial z}\int_r^b E_r(r,z)dr \quad \Rightarrow \quad F_z(r,z) = -\frac{e}{\gamma^2}\frac{\partial}{\partial z}\int_r^b E_r(r',z)dr'$$

Exercise 4

Compute the longitudinal space charge forces for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian

parabolic $$\lambda(z) = \frac{3Ne}{2l_o}\left[1 - \left(\frac{2z}{l_o}\right)^2\right]$$

sinusoidal modulation $$\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z) \quad ; \quad k_z = 2\pi / l_w$$

Gaussian $$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$F_z(r,z) = -\frac{e}{\gamma^2}\frac{\partial}{\partial z}\int_r^b E_r(r',z)dr'$$

$$F_z(r,z) = -\frac{e}{4\pi\varepsilon_0\gamma^2}\left(1 - \frac{r^2}{a^2} + 2\ln\frac{b}{a}\right)\frac{\partial\lambda(z)}{\partial z}$$

Exercise 5

Compute the incoherent betatron tune shift of a uniform proton beam inside two parallel plates

Wake fields exercises

Calculate the amplitude of the resonator wake field given $R_s = 1\ k\Omega$, $\omega_r = 5\ GHz$, $Q = 10^4$

Calculate the ratio $Z(\omega_r)\ /\ Z(2\omega_r)$ for $Q = 1, 10^3, 10^5$

Show that the impedance of an RLC parallel circuit is that of the resonator one and relate R, L and C to Q, R_s and ω_r

BBU exercise

Consider a beam in a linac at 1 GeV without acceleration. Obtain the growth of the oscillation amplitude after 3 km if:

N = 5e10, $w_\perp$(-1 mm) = 63 V/(pC m), L_w = 3.5 cm, k_y = 0.06 1/m

BBU exercise (2)

Consider the same beam of the previous exercise being now accelerated from 1 GeV with a gradient g =16.7 MeV/m. Obtain the growth of the oscillation amplitude

$$E_f = E_0 + gL_L \approx gL_L = 50\,\text{GeV}$$

$$\left(\frac{\Delta\hat{y}_2}{\hat{y}_2}\right)_{\max} = \frac{cNew_{\perp}(z)L_L}{4\omega_y(E_f/e)L_w}\ln\frac{E_f}{E_0} = ?$$

Exercise: Haissinski equation with pure inductive impedance

Given the wake field in case of a pure inductive impedance, determine the longitudinal distribution

$$w_{\|}(z) = -c^2 L \delta'(z) \quad \longrightarrow \quad \Psi(z) = ?$$

Exercise: microwave instability

Calculate the threshold average current for the microwave instability with a bunch having the following parameters:

$|Z_{||} / n| = .5\ \Omega$, $\sigma_z = 1$ cm, $\sigma_\varepsilon = 10^{-3}$, $\alpha_c = 0.027$,
$E_0 = 510$ MeV, $L_0 = 97.69$ m