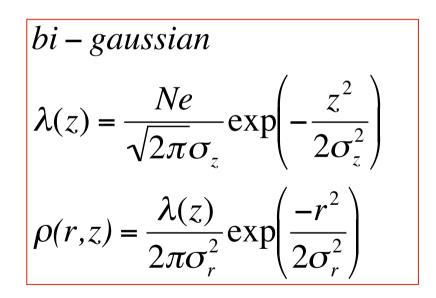
#### **Exercise 1**

Compute the transverse space charge forces and the tune shifts for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, gaussian

$$parabolic \qquad \lambda(z) = \frac{3Ne}{2l_o} \left[ 1 - \left(\frac{2z}{l_o}\right)^2 \right]$$
  
sinusoidal modulation  $\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z)$ ;  $k_z = 2\pi/\lambda_w$   
Gaussian  $\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$ 

#### **Exercise 2**

Compute the transverse space charge forces and the tune shifts for a cylindrical beam in a circular beam pipe, having a bi-gaussian longitudinal and transverse distribution.



### **Exercise 3** Compute the longitudinal space charge force of a transverse uniform cylindrical beam in a circular perfectly conducting beam pipe

$$E_{z}(r,z) = -\frac{1}{\gamma^{2}} \frac{\partial}{\partial z} \int_{r}^{b} E_{r}(r,z) dr \implies F_{z}(r,z) = -\frac{e}{\gamma^{2}} \frac{\partial}{\partial z} \int_{r}^{b} E_{r}(r',z) dr'$$

#### **Exercise 4**

Compute the longitudinal space charge forces for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian

$$parabolic \qquad \lambda(z) = \frac{3Ne}{2l_o} \left[ 1 - \left(\frac{2z}{l_o}\right)^2 \right]$$
  
sinusoidal modulation  $\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z)$ ;  $k_z = 2\pi/l_w$   
Gaussian  $\lambda(z) = \frac{Ne}{\sqrt{2\pi\sigma_z}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$ 

$$F_{z}(r,z) = -\frac{e}{\gamma^{2}}\frac{\partial}{\partial z}\int_{r}^{b}E_{r}(r',z)dr' \qquad F_{z}(r,z) = -\frac{e}{4\pi\varepsilon_{0}\gamma^{2}}(1-\frac{r^{2}}{a^{2}}+2\ln\frac{b}{a})\frac{\partial\lambda(z)}{\partial z}$$

**Exercise 5** *Compute the incoherent betatron tune shift of a uniform proton beam inside two parallel plates* 

## Wake fields exercises

Calculate the amplitude of the resonator wake field given  $R_s = 1 \ k\Omega$ ,  $\omega_r = 5 \ GHz$ ,  $Q = 10^4$ 

Calculate the ratio  $Z(\omega_r) / Z(2\omega_r)$  for  $Q = 1, 10^3, 10^5$ 

Show that the impedance of an RLC parallel circuit is that of the resonator one and relate R, L and C to Q,  $R_s$  and  $\omega_r$ 

## **BBU exercise**

Consider a beam in a linac at 1 GeV without acceleration. Obtain the growth of the oscillation amplitude after 3 km if:

N = 5e10,  $w_{\perp}(-1 \text{ mm}) = 63 \text{ V/(pC m)}, L_w = 3.5 \text{ cm}, k_y = 0.06 \text{ 1/m}$ 

# **BBU exercise (2)**

Consider the same beam of the previous exercise being now accelerated from 1 GeV with a gradient g = 16.7 MeV/m. Obtain the growth of the oscillation amplitude

 $E_f = E_0 + gL_L \approx gL_L = 50 \text{ GeV}$ 

$$\left(\frac{\Delta \hat{y}_2}{\hat{y}_2}\right)_{\max} = \frac{cNew_{\perp}(z)L_L}{4\omega_y(E_f/e)L_w}\ln\frac{E_f}{E_0} = ?$$

### **Exercise: Haissinski equation with pure inductive impedance**

Given the wake field in case of a pure inductive impedance, determine the longitudinal distribution

$$w_{\parallel}(z) = -c^2 L \delta'(z) \longrightarrow \Psi(z) = ?$$

### **Exercise: microwave instability**

Calculate the threshold average current for the microwave instability with a bunch having the following parameters:

 $|Z_{\parallel} / n| = .5 \Omega$ ,  $\sigma_z = 1 \text{ cm}$ ,  $\sigma_{\varepsilon} = 10^{-3}$ ,  $\alpha_c = 0.027$ ,  $E_0 = 510 \text{ MeV}$ ,  $L_0 = 97.69 \text{ m}$