## **RF Engineering Introduction** Uppsala University, December 2011



#### **Superconducting LEP cavity**

Fritz.Caspers@cern.ch Slides selected by Roger Ruber

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

### **RF Tutorial Contents**

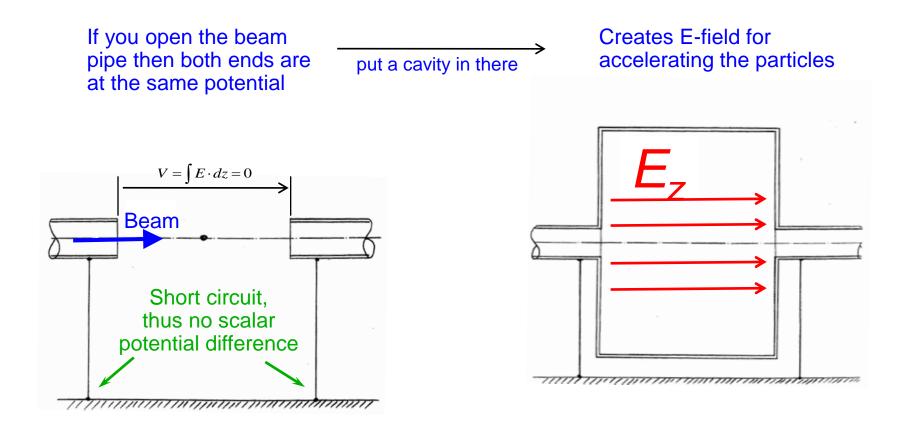
#### Part I

- Basics
- Cavity structures
- Equivalent circuit
- Characterisation in time and in frequency domain
- Beam-cavity interaction

#### Part II

- Diagnostics using RF instrumentation
- Wall current monitor
- Button pick-up
- Cavity type pick-up
- Travelling wave structures
- Possibilities and limitations of Schottky diagnostics

## From L and C to a cavity

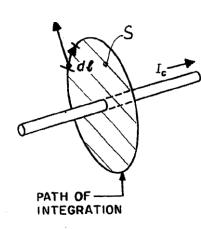


Capacitor at high frequencies, The Feynman Lectures on Physics Can the short-circuit be avoided?

Answer: No - but it doesn't bother us at high frequencies.

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## Maxwell's equations (1)



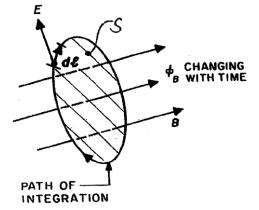
Ampere's Law :  $\oint H \cdot dl = I = I_{conduction} + I_{displacement}$   $\partial \Phi_{r}$ 

 $I_{displacement} = \frac{\partial \Phi_D}{\partial t}$ 

where the electric flux  $\Phi_D$ is given by  $\Phi_D = \int_{S} D \cdot dS = \mathcal{E} \int_{S} E \cdot dS$ ,

D designating the electric flux density.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$
  
with the current density *J*  
and the magnetic field *H*



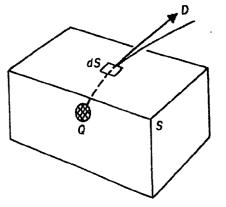
Faraday's Law :  $\oint E \cdot dl = -\frac{\partial \Phi_B}{\partial t}$ 

with the electric flux  $\Phi_B$  $\Phi_B = \int_S BdS = \mu \int_S HdS$ 

$$\nabla \times E = -\frac{\partial B}{\partial t}$$
  
with the electric field *E*  
and the magnetic field *B*

scalar vs. vector potential: path of integration makes a difference

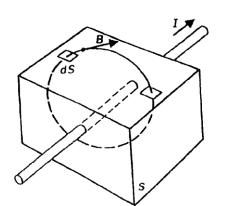
## Maxwell's equations (2)



S = TOTAL SURFACE Q = TOTAL CHARGE INSIDE S Gauss' Law (Electrici ty):  $\int_{S} D \cdot dS = Q$ 

with the electric displacement D

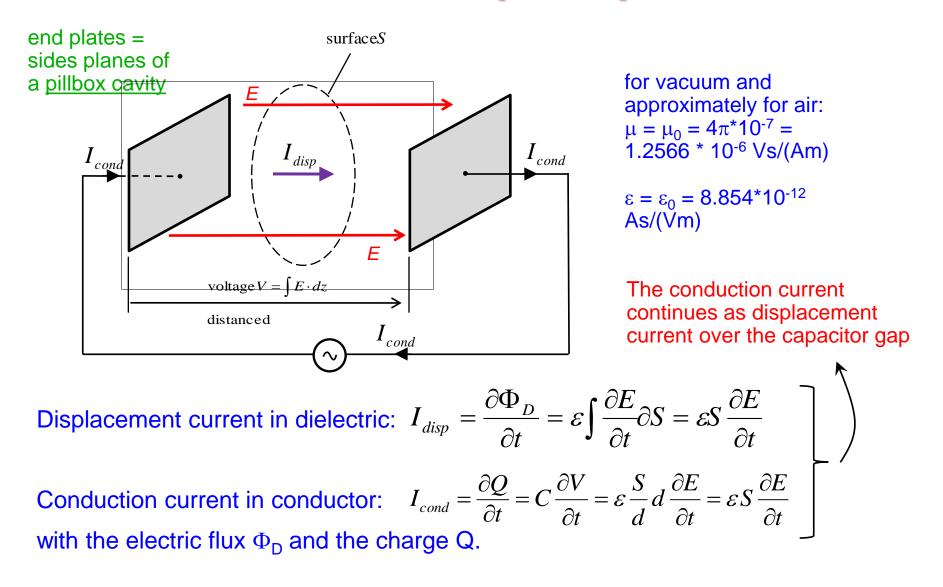
 $\nabla \cdot \mathbf{D} = \rho$ with the charge density  $\rho$ 



Gauss' Law (Magnetism):  $\int_{S} B \cdot dS = 0$   $\nabla \cdot \mathbf{B} = \mathbf{0}$ 

There are no magnetic charges

## Displacement and conduction currents in a simple capacitor



F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

### General Solution for a Rectangular (brick-type) Cavity

When describing field components in a Cartesian coordinates system (assuming a homogeneous and isotropic material in a space charge free volume) with harmonic functions (angular frequency  $\omega$ ) then each Cartesian component needs to fulfill Laplace's equation:

 $\Delta \Psi + k_0^2 \varepsilon_r \mu_r \Psi = 0 \qquad \begin{aligned} k_0^2 &= \omega^2 \varepsilon_0 \mu_0 & k_0 &\text{free space wavenumber} \\ k_0 &= 2\pi / \lambda_0 & \lambda_0 &\text{free space wavelength} \end{aligned}$ 

As a general solution we can use the product ansatz for  $\Psi$ 

$$\Psi = X(x)Y(y)Z(z)$$

From this one obtains the general solution for  $\Psi$  ( $\Psi$  may be a vector potential or field) standing waves

$$\Psi = \begin{cases} A \cdot \cos(k_x x) + B \cdot \sin(k_x x) \\ A' \cdot e^{-jk_x x} + B' \cdot e^{jk_x x} \end{cases} \begin{cases} C \cdot \cos(k_y y) + D \cdot \sin(k_y y) \\ C' \cdot e^{-jk_y y} + D' \cdot e^{jk_y y} \end{cases} \begin{cases} E \cdot \cos(k_z z) + F \cdot \sin(k_z z) \\ E' \cdot e^{-jk_z z} + F' \cdot e^{jk_z z} \end{cases} \checkmark$$

travelling waves

#### with the separation condition

$$(k_x)^2 + (k_y)^2 + (k_z)^2 = (k_0)^2 \varepsilon_r \mu_r$$

see also: G. Dome, RF Theory Proceeding Oxford CAS, April 91 CERN Yellow Report 92-03, Vol. I

## General Solution in Cylindrical Coordinates

As a general solution we can use the product ansatz for  $\Psi$ 

 $\Psi = R(\rho)F(\varphi)Z(z)$ 

From this one obtains the general solution for  $\Psi$  ( $\Psi$  may be a vector potential or field) standing waves

$$\Psi = \begin{cases} A \cdot J_{m}(k_{\rho}\rho) + B \cdot N_{m}(k_{\rho}\rho) \\ A' \cdot H_{m}^{(2)}(k_{\rho}\rho) + B' \cdot H_{m}^{(1)}(k_{\rho}\rho) \end{cases} \begin{cases} C \cdot \cos(m\varphi) + D \cdot \sin(m\varphi) \\ C' \cdot e^{-jm\varphi} + D' \cdot e^{jm\varphi} \end{cases} \begin{cases} E \cdot \cos(k_{z}z) + F \cdot \sin(k_{z}z) \\ E' \cdot e^{-jk_{z}z} + F' \cdot e^{jk_{z}z} \end{cases} \end{cases}$$

#### and the functions

travelling waves

 $J_m$  ... cylindrical harmonics of the Bessel function of order m

- $N_m$  ... cylindrical harmonics of the Neumann function of order m
- $H_m^{(1)}$ ...Hankel function of the first kind of order *m* (outwardtravelling wave)
- $H_m^{(2)}$ ...Hankel function of the second kind of order *m* (inward travelling wave)

$$H_{m}^{(1)} = J_{m} + jN_{m}$$
$$H_{m}^{(2)} = J_{m} - jN_{m}$$

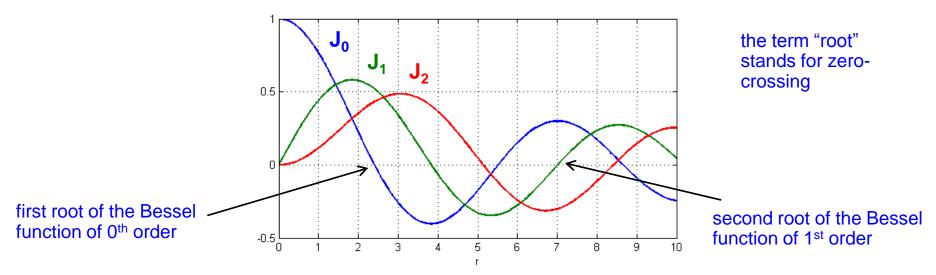
Here the separation condition is

$$(k_{\rho})^2 + (k_z)^2 = (k_0)^2 \varepsilon_r \mu_r$$

Hint: the index m indicating the order of the Bessel and Neumann function shows up again in the argument of the sine and cosine for the azimuthal dependency.

## **Bessel Functions (1)**

A nice example of the derivation of a Bessel function is the solution of the cylinder problem of the capacitor given in the Feynman reference (Bessel function via a series expansion).



Comment: For the generalized solution of cylinder symmetrical boundary value problems (e.g. higher order modes on a coaxial resonator) Neumann functions are required. Standing wave patterns are described by Bessel- and Neumann functions respectively, radially travelling waves in terms of Hankel functions. Hint: Sometimes a Bessel function is called Bessel function of first kind, a Neumann function is Bessel function of second kind, and a Hankel function=Bessel function of third kind.

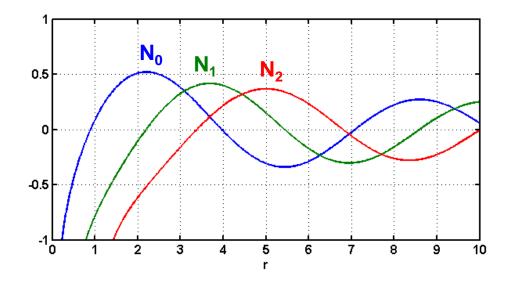
### **Bessel Functions (2)**

Some practical numerical values:

k	$J_0(x)$	$J_1(x)$	$J_{2}\left( x ight)$	$J_{3}\left( x ight)$	$J_4(x)$	$J_5(x)$
1	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715
2	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386
3	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002
4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801
5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178

See: http://mathworld.wolfram.com/BesselFunctionZeros.html

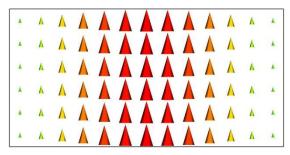
### **Neumann Functions**



Neumann functions are often also denoted as  $Y_m(r)$ .

## **Electromagnetic waves**

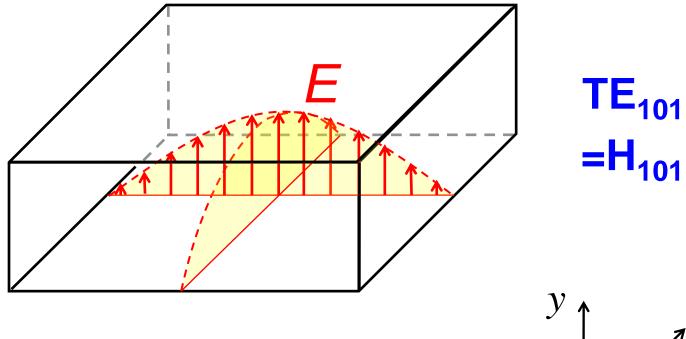
- Propagation of electromagnetic waves inside empty metallic channels is possible: there exist solutions of Maxwell's equations describing waves
- These waves are called waveguide modes
- There exist two types of waves,
  - Transverse electric (TE) modes:
    - $\rightarrow$  the electric field has only transverse components
  - Transverse magnetic (TM) modes:
    - $\rightarrow$  the magnetic field has only transverse components
- Propagate at above a characteristic cut-off frequency
- In a rectangular waveguide, the first mode that can propagate is the  $TE_{10}$  mode. The condition for propagation is that half of a wavelength can "fit" into the cross-section => cut-off wavelength  $\lambda_c = 2a$
- The modes are named according to the number of field maxima they have along each dimension. The E field of the TE<sub>10</sub> mode for instance has 1 maximum along x and 0 maxima along the y axis.
- For circular waveguides, the maxima are counted in the radial and azimuthal direction



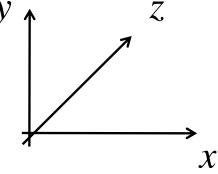
E field of the fundamental  $TE_{10}$  mode

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

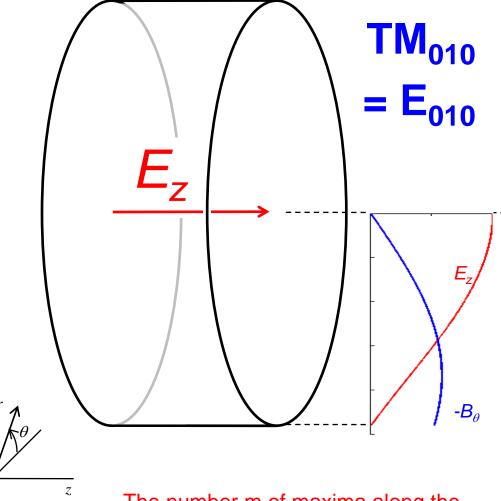
## Mode Indices in Resonators (1)



For a structure in <u>rectangular coordinates</u> the mode indices simply indicate the number of half waves (standing waves) along the respective axis. Here we have one maximum along the x-axis, no maximum in vertical dimension (y-axis), and one maximum along the z-axis.  $TE_{101}$  corresponds to  $TE_{xyz}$ 



## Mode Indices in Resonators (2)



For a structure in <u>cylindrical</u> <u>coordinates:</u>

The first index is the order of the Bessel function or in general cylindrical function.

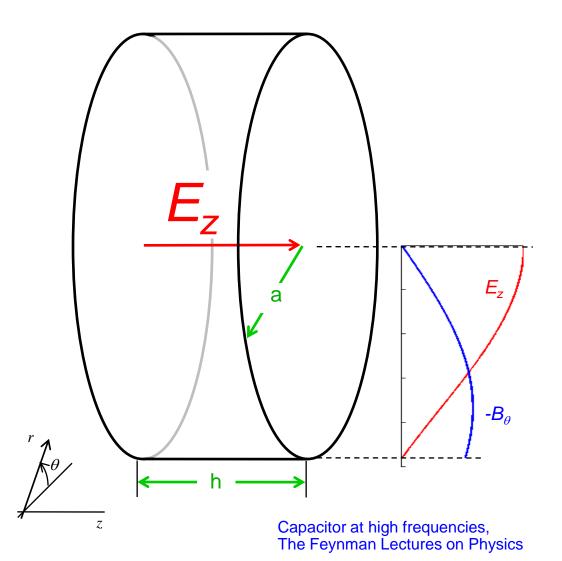
The second index indicates "the root" of the cylindrical function which is the number of zero-crossings.

The third index is the number of half waves (maxima) along the z-axis.

Hint: In an empty pillbox there will be <u>no</u> Neumann function as it has a pole in the center (conservation of energy). However we need Bessel and Neumann functions for higher order modes of coaxial structures.

The number m of maxima along the azimuth is coupled to the order of the Bessel function (see slide on theory).

## Fields in a pillbox cavity



Cavity height: h cavity radius: a

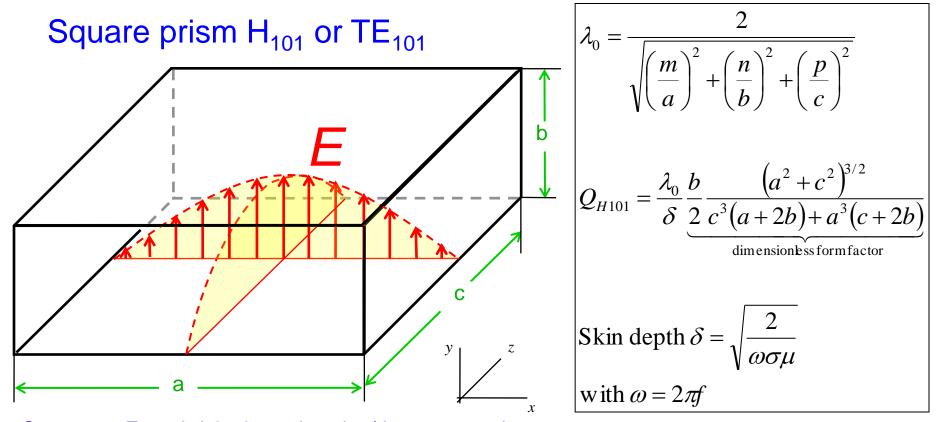
 $TM_{010}$  mode resonance =  $E_{010}$  mode resonance for

$$a = 0.383\lambda = 1.53\lambda/4$$

## TM<sub>010</sub> resonance frequency independent of h!!!

In the cylindrical geometry the E and H fields are proportional to Bessel functions for the radial dependency.

## **Common cavity geometries (1)**



Comment: For a brick-shaped cavity (the structure is described in Cartesian coordinates) the E and H fields would be described by sine and cosine distributions. The mode indices indicate the number of half waves along the x-,y-, and z-axis, respectively.

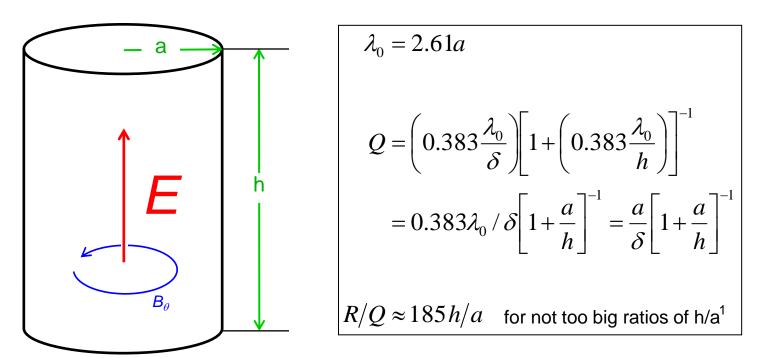
this simplifies in the case a=c:

$$\lambda_0 = \sqrt{2a}$$
$$Q = \frac{1}{\delta} \frac{ab}{a+2b}$$

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## **Common cavity geometries (2)**

Circular cylinder:  $E_{010}$ , =  $TM_{010}$ 



Note: h denotes the **full** height of the cavity In some cases and also in certain numerical codes, h stands for the half height

1: formula uses Linac definition and includes time transit factor F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## **R/Q for cavities**

## The full formula for calculating the R/Q value of a cavity is $\sin^2(\frac{\chi_{01}}{h})$

$$\frac{R}{Q} = \frac{4\eta}{\chi_{01}^{3} \pi J_{1}^{2}(\chi_{01})} \frac{\sin^{2}(\frac{\chi_{01}}{2} \frac{h}{a})}{\frac{h}{a}} s$$

see lecture: RF cavities, E. Iensen, Varna CAS 2010

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\mu_0^2 c_0^2} = 4\pi \times 10^{-7} \times 3 \times 10^8 = 377\Omega$$

 $\chi_{01} = 2.4048$  (First zero of the Bessel function of 0<sup>th</sup> order)  $J_1(\chi_{01}) = 0.5192$ 

h

This leads to

$$\frac{R}{Q} = 128 \frac{\sin^2(1.2024\frac{n}{a})}{\frac{h}{a}}$$

## The sinus can be approximated by sinx = x (for small values of x) leading to

$$\frac{R}{Q} \approx 128 \frac{\left(1.2024 \frac{h}{a}\right)^2}{\underline{h}} = 185 \frac{h}{a}$$

a

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## **Common cavity geometries (3)**

#### Circular cylinder:

**H**<sub>011</sub>

$$Q = 0.61 \frac{\lambda_0}{\delta} \frac{\left[1 + 0.17 \left(\frac{2a}{h}\right)^2\right]^{3/2}}{1 + 0.17 \left(\frac{2a}{h}\right)^3}$$

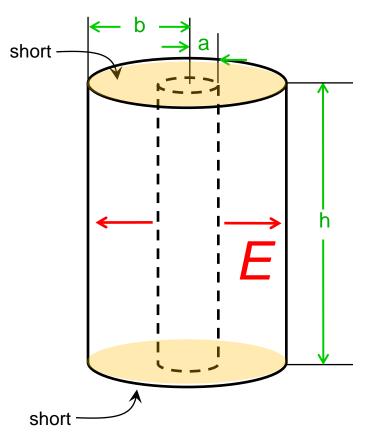
H<sub>111</sub>

$$Q = 0.206 \frac{\lambda_0}{\delta} \frac{\left[1 + 0.73 \left(\frac{2a}{h}\right)^2\right]^{3/2}}{1 + 0.22 \left(\frac{2a}{h}\right)^2 + 0.51 \left(\frac{2a}{h}\right)^3}$$

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## **Common cavity geometries (4)**

#### **Coaxial TEM**



$$\lambda_{0} = 2h \text{ or } h = \lambda_{0} / 2$$

$$Q = \frac{\lambda_{0}}{\delta} \frac{1}{4 + \frac{h}{b} \cdot \frac{1 + b / a}{\ln(b / a)}}$$
Optimum Q for  $(b/a) = 3.6 \quad (Z_{0} = 77\Omega)$ 

$$Q_{optimum} = \frac{\lambda_{0}}{\delta} \frac{1}{4 + 7.2 \frac{h}{b}}$$

Coaxial line with minimum loss
→ slide TEM transmission lines (3)

Taken from S. Saad et.al., Microwave Engineers' Handbook, Volume I, p.180

## **Common cavity geometries (5)**

#### Sphere

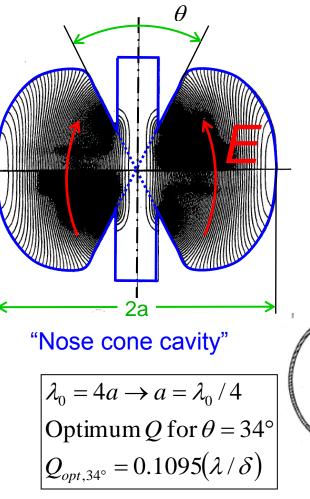
#### Sphere with cones



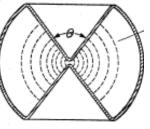
"Energy storage in LEP"

$$\lambda_0 = 2.28a$$
$$Q = 0.318(\lambda / \delta)$$

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011



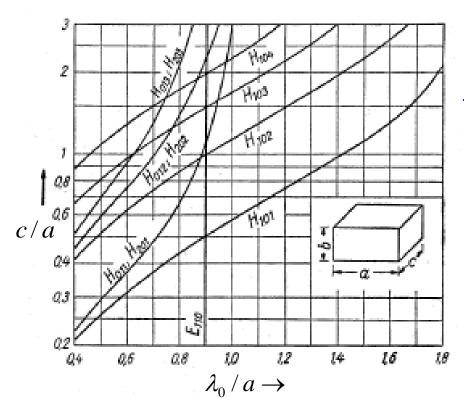
a spherical "λ/4-resonator" Cavity structures



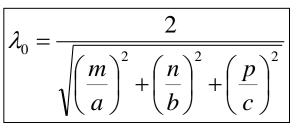
the tips of the cone don't touch

## Mode chart of a brick-shaped cavity

Carol G. Montgomery, 1st ed., 1947; by permission, McGraw-Hill Book Co., N. Y. and Reprinted from Meinke, H. and Gundlach, F. W., Taschenbuch der Hochfrequenztechnik,S.471 1968) Microwave Measurements by Springer-Verlag, Berlin Erste Auflage Techniques of

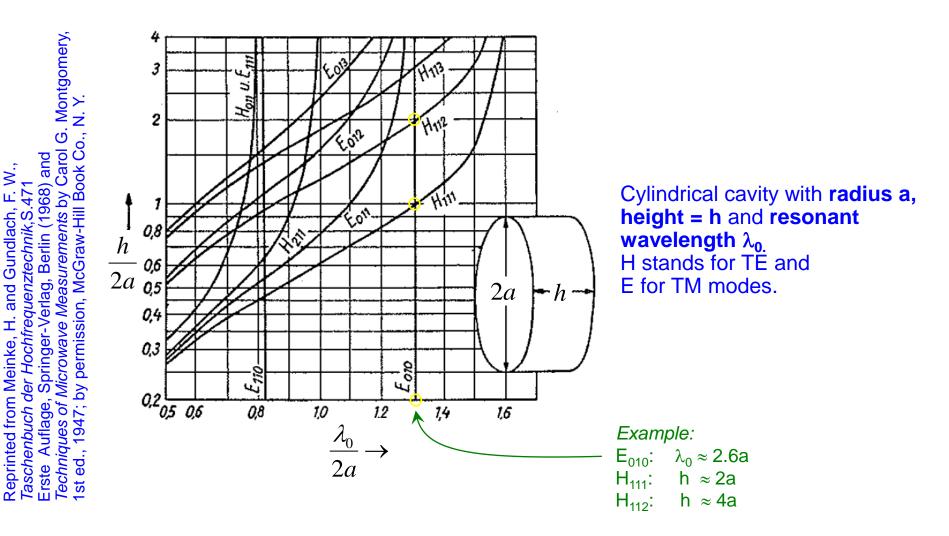


The resonant wavelength of the  $H_{mnp}$  resonance calculates as

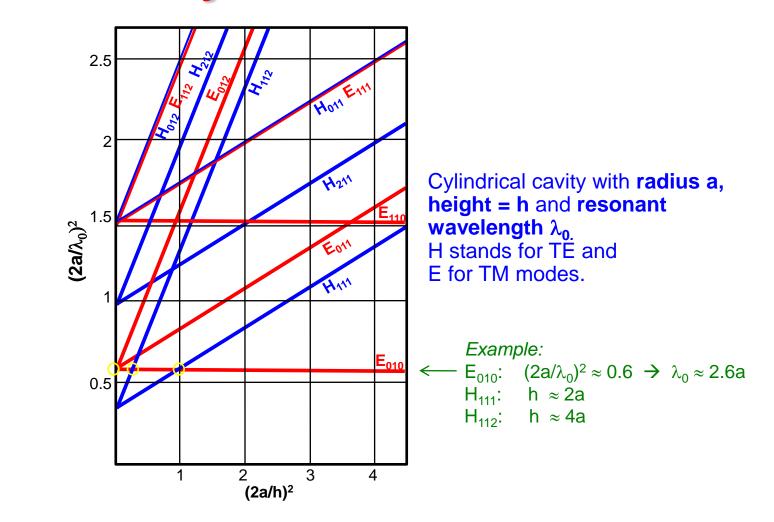


for a  $E_{mn}$  or a  $H_{mn}$  wave with p half waves along the c-direction.

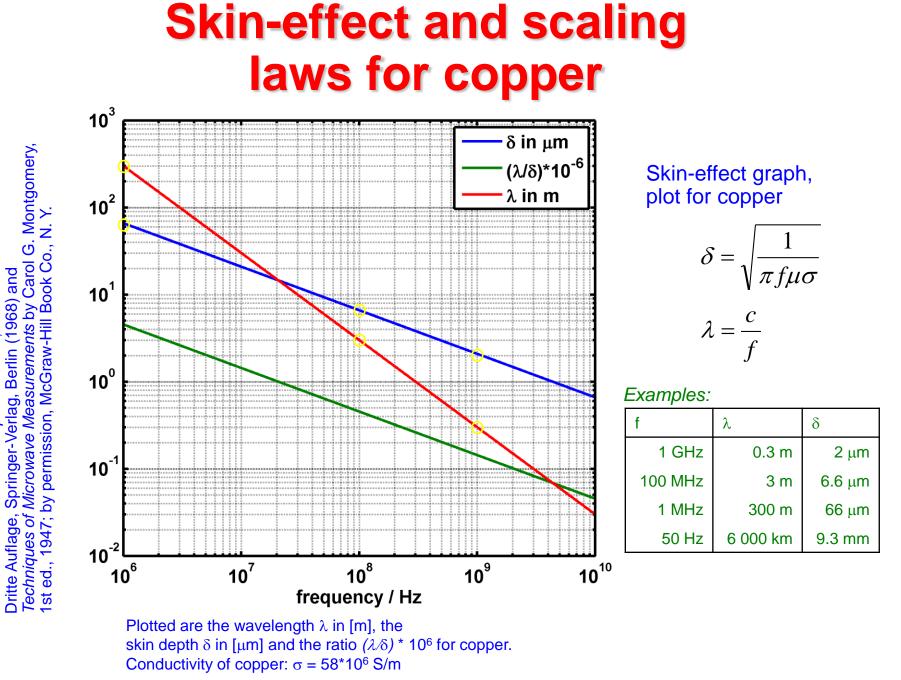
## Mode chart of a Pillbox cavity – Version 1



## Mode chart of a Pillbox cavity – Version 2



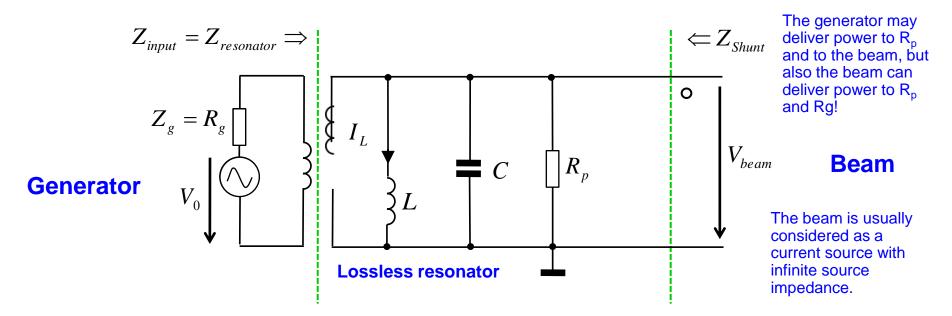
Carol G. Montgomery, 1st ed., 1947; by permission, McGraw-Hill Book Co., N. Y. and Reprinted from Meinke, H. and Gundlach, F. W., Taschenbuch der Hochfrequenztechnik,S.471 1968) Microwave Measurements by Springer-Verlag, Berlin Erste Auflage, Techniques of



F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

Reprinted from Meinke, H. and Gundlach, F. W., Taschenbuch der Hochfrequenztechnik,

## Equivalent circuit (1)



 $R_p$  = resistor representing the losses of the parallel RLC equivalent circuit

We have Resonance condition, when 
$$\omega L = \frac{1}{\omega C}$$
  
 $\Rightarrow$  Resonance frequency:  $\omega_{res} = 2\pi f_{res} = \frac{1}{\sqrt{LC}} \Rightarrow \int f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$ 

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

Equivalent circuit

## Equivalent circuit (2)

 $X = \frac{R}{Q} = \omega_{res}L = \frac{1}{\omega_{res}C} = \sqrt{L/C}$ Characteristic impedance "R upon Q" (R/Q) is independent of Q and a pure geometry factor for any cavity or resonator! This formula assumes a HOMOGENEOUS field in the capacitor !  $W = \frac{CV^2}{2} = \frac{LI_L^2}{2}$ Stored energy at resonance  $P = \frac{V^2}{2R}$ Dissipated power  $Q = \frac{R}{X} = \frac{\omega_{res}W}{P}$ Q-factor  $R = \frac{V^2}{2P}$ Shunt impedance (circuit definition)  $\frac{\Delta f}{f} = -\frac{1}{2} \frac{\Delta C}{C}$ Tuning sensitivity

 Coupling parameter (shunt impedance over generator or feeder impedance Z)

 $k^2 = \frac{R}{R_{innut}}$ 

## The Quality Factor (1)

 The quality (Q) factor of a resonant circuit is defined as the ratio of the stored energy W over the energy dissipated P in one cycle.

$$Q = \frac{\omega_{res}W}{P}$$

- The Q factor can be given as
  - Q<sub>0</sub>: Unloaded Q factor of the unperturbed system, e.g. a closed cavity
  - Q<sub>L</sub>: Loaded Q factor with measurement circuits etc connected
  - Q<sub>ext</sub>: External Q factor of the measurement circuits etc
- These Q factors are related by

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

# The Quality Factor (2) Q as defined in a Circuit Theory Textbook:

$$Q = \frac{\omega_{res}L}{R}$$

#### Q as defined in a Field Theory Textbook:

 $Q = 2\pi \frac{\text{energy stored in the resonator}}{\text{energy dissipated per cycle}}$ 

Q as defined in an optoelectronics Textbook:

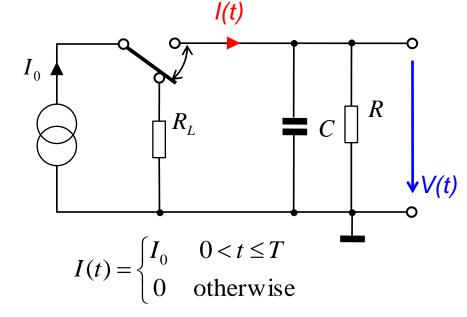
$$Q = \frac{v_0}{v_{1/2}}$$

 $v_0$  = the resonant frequency

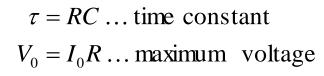
 $v_{1/2}$  = "full - width at half power maximum" (FWHM)

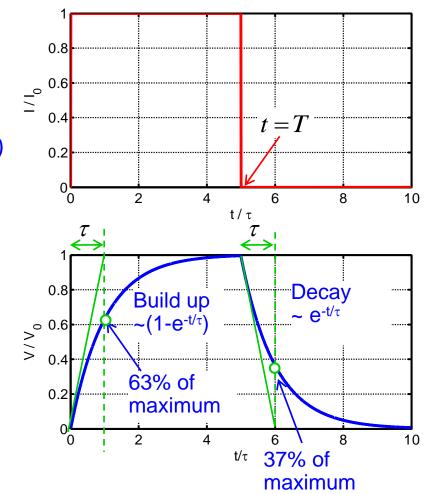
Equivalent circuit

## **Transients on an RC-Element (1)**

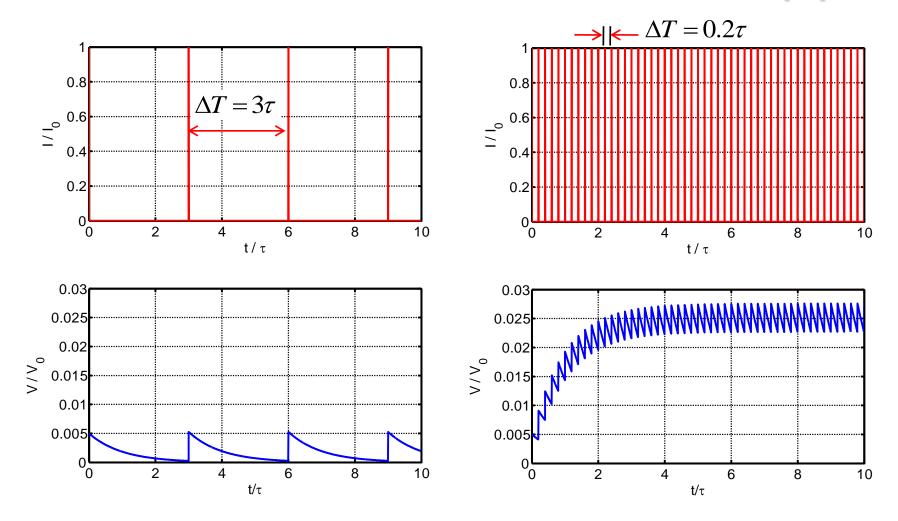


A voltage source would not work here! Explain why.



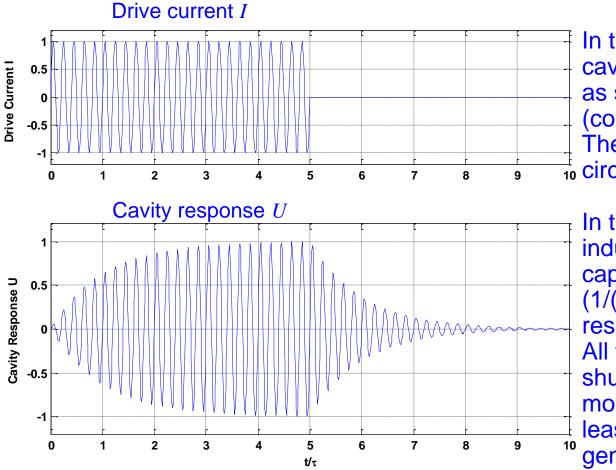


## **Transients on an RC-Element (2)**



F. Caspers, RF Engineering Introduction, Uppsala, Dec. 20 Behaviour in time and in frequency domain

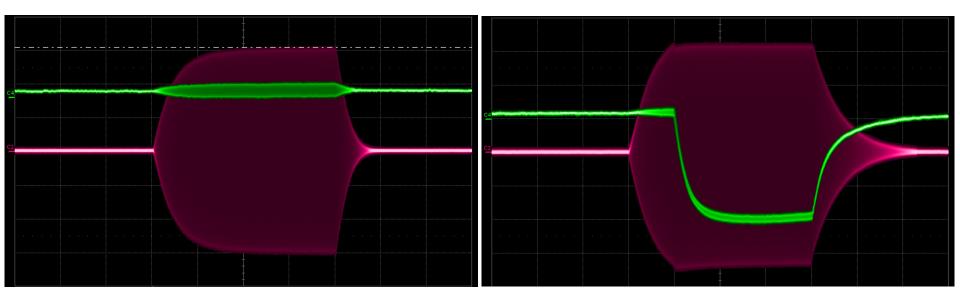
## Response of a tuned cavity to sinusoidal drive current (1)



In the first moment, the
 cavity acts like a capacitor,
 as seen from the generator
 (compare equivalent circuit).
 The RF is therefore short circuited

In the stationary regime, the inductive ( $\omega$ L) and capacitive reactances (1/( $\omega$ C)) cancel (operation at resonance frequency!). All the power goes into the shunt impedance *R* => no more power reflected, at least for a matched generator...

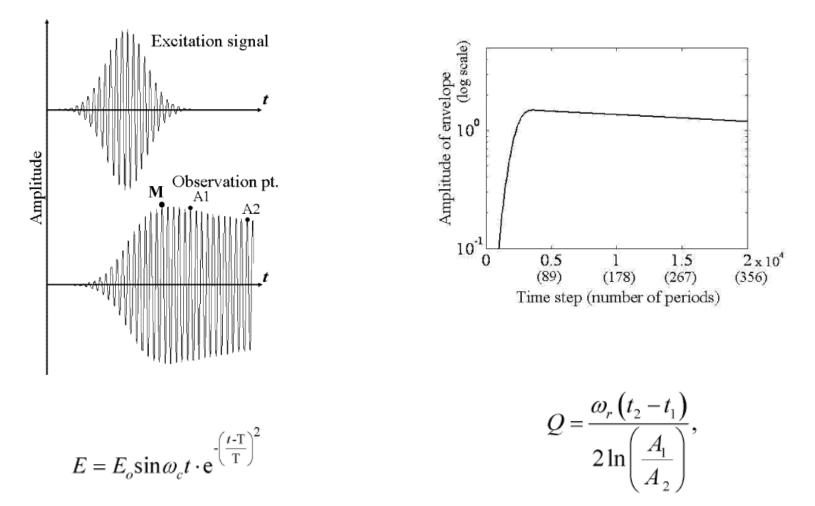
## Measured time domain response of a cavity



#### Cavity E field (red trace) and electron probe signal (green trace) with and without multipacting. 200 µs RF burst duration.

see: O. Heid, T Hughes, COMPACT SOLID STATE DIRECT DRIVE RF LINAC EXPERIMENTAL PROGRAM, IPAC Kyoto, 2010

## Numerically calculated response of a cavity in the time domain



see: I. Awai, Y. Zhang, T. Ishida, Unified calculation of microwave resonator parameters, IEEE 2007 F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## Response of a tuned cavity to sinusoidal drive current (2)

#### Differential equation of the envelope

(shown without derivation)

$$\dot{V} = \frac{1}{2C} = (I - \frac{V}{Z}) = \frac{1}{2ZC}(IZ - V)$$

V, V, I, Z are complex quantities, evaluated at the stimulus (drive) frequency.

For a tuned cavity all quantities become real. In particular Z = R, therefore

$$\dot{V} = \frac{1}{2RC} (IR - V)$$

 $\rightarrow$  time constant becomes

$$\underline{\underline{\tau}} = 2RC = 2\frac{R}{Q}QC = \frac{2Q}{\omega_0} = \frac{Q}{\frac{\pi f}{\omega_0}} = \frac{QT}{\frac{\pi}{\omega_0}} = \frac{QT}{\frac{\pi}{\omega_0}}$$
"Q over  $\pi$  periods"

V... envelope amplitude C... cavity capacitance I... drive current Z... cavity impedance R... real part of cavity impedance

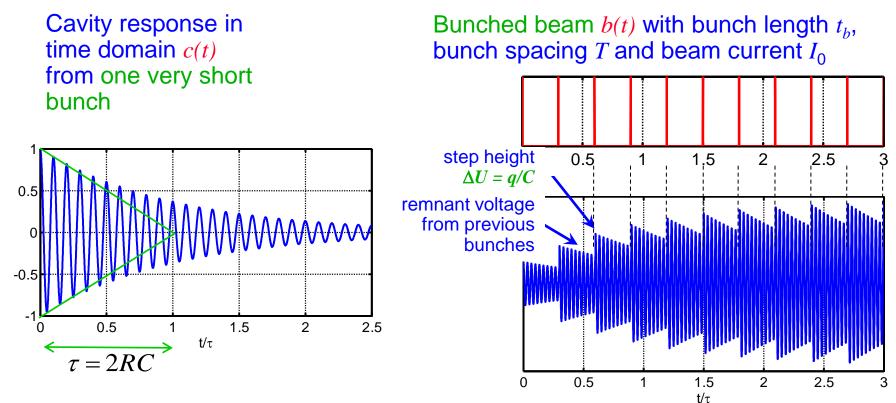
This  $\tau$  value refers to the 1/e decay of the <u>field</u> in the cavity. Sometimes one finds  $\tau_w$  referring to the energy with  $2\tau_w = \tau$ .

The voltage (or current) decreases to 1/e of the initial value within the time  $\tau$ .

see also: H. Klein, Basic concepts I Proceeding Oxford CAS, April 91 CERN Yellow Report 92-03, Vol. I

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 20 Behaviour in time and in frequency domain

## **Beam-cavity interaction (1)**



Resulting response for bunched beam obtained by convolution of the bunch sequence with the cavity response  $r(t) = b(t) \otimes c(t)$ Condition that the induced signals in the cavity add up: cavity resonant frequency  $f_{res}$  must be an integer multiple of bunch frequency 1/T

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

Beam-cavity interaction

## **Beam-cavity interaction (2)**

For a quantitative evaluation the worst case is considered with the induced signals adding up in phase.

Two approaches:

• Equilibrium condition: Voltage drop between two bunch passages compensated by newly induced voltage

$$U_{end}e^{-T/\tau} = U_{step} = U_{end} - \frac{q}{C} \implies U_{end} = \frac{q}{C}\frac{1}{1 - e^{-T/\tau}}$$

Summing up individual stimuli

$$U_{end} = \frac{q}{C} (1 + e^{-T/\tau} + e^{-2T/\tau} + \dots) = \frac{q}{C} \frac{1}{1 - e^{-T/\tau}}$$

Approximation for  $T/\tau \ll 1$ :

$$1 - e^{-T/\tau} = 1 - (1 - T/\tau + ...) \approx T/\tau$$

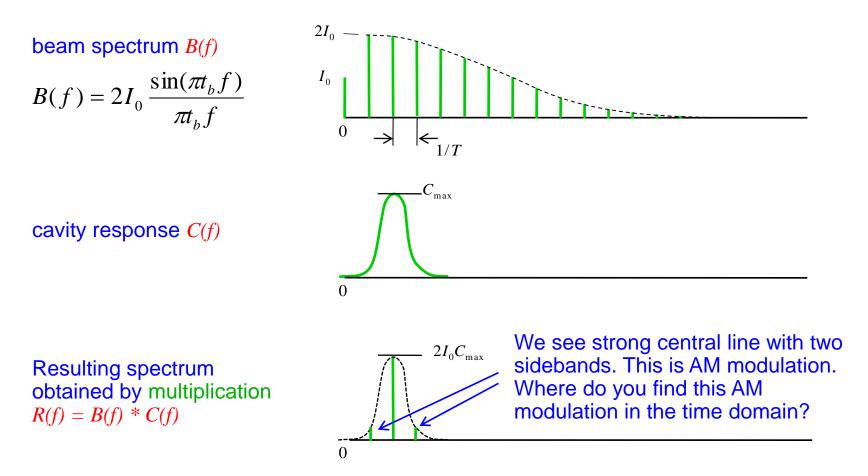
$$\underline{U_{end}} = \frac{q}{C} \frac{1}{T/\tau} = \frac{q}{C} \frac{2RC}{T} = 2R\frac{q}{T} = \frac{2RI_0}{T}$$

where  $I_0$  is the mean beam current.

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## Beam-cavity interaction in Frequency domain

#### • Frequency domain



## Typical parameters for different cavity technologies

Cavity type	<i>R/Q</i>	Q	R
Ferrite loaded cavity (low frequency, rapid cycling)	4 kΩ	50	200 kΩ
Room temperature copper cavity (type 1 with nose cone)	<b>192</b> Ω	<b>30 * 10</b> <sup>3</sup>	5.75 <b>M</b> Ω
Superconducting cavity (type 2 with large iris)	50 Ω	$1 * 10^{10}$	500 GΩ

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## **Electromagnetic scaling laws**

A cavity of a given geometry can be scaled using three rules:

- The ratio of any cavity dimension to λ is constant. To put it another way, all cavity dimensions are inversely proportional to frequency
- Characteristic impedance R/Q = const.

$$Q * \delta / \lambda = \text{const.}$$

#### The skin depth $\delta$ is given by

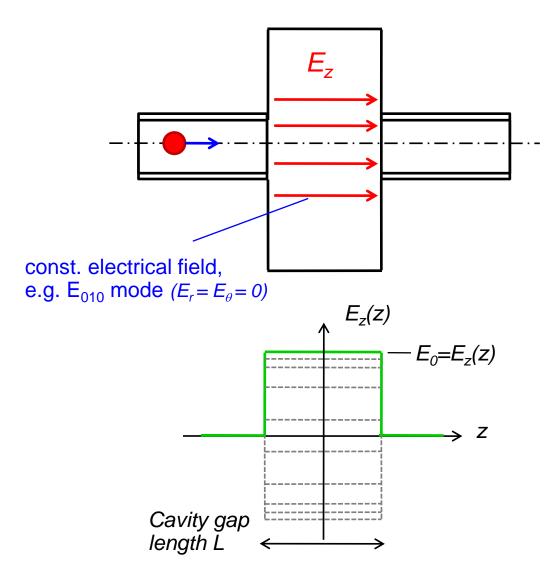
$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}}$$

with the conductivity  $\sigma$ , the permeability  $\mu$ , and the angular frequency  $\omega = 2\pi f$ .

Note that it is proportional to  $\frac{1}{\sqrt{f\sigma}}$ For instance, in copper ( $\sigma_{copper} = 5.8*10^7$  S/m) the skin depth is ≈9 mm at 50 Hz, while it decreases to ≈ 2 µm at 1 GHz.

Scaling laws

## **Transit time factor (1)**



F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

The "voltage" in a cavity along the particle trajectory (which coincides with the axis of the cavity) is given by the integral along this path for a fixed moment in time:

$$V = \int_{L} E_{z}(z) \, dz$$

But: the field in the cavity is varying in time:

$$E_{z}(z,t) = E_{z}(z)f(t)$$
$$= E_{z}(z)\cos(\omega t + \varphi)$$

Thus, the field seen by the particle is

$$V = E_0 \int_{-L/2}^{L/2} \cos(\omega t + \varphi) dz$$

## **Transit time factor (2)**

The transit time factor describes the amount of the supplied RFenergy that is effectively used to accelerate the traversing particle.

Usually, as a reference the moment of time is taken when the longitudinal field strength of the cavity is at its maximum, i.e.  $\cos(\varphi)=1$ . A particle with infinite velocity passing through the cavity at this moment would see

$$\hat{V} = E_0 L$$

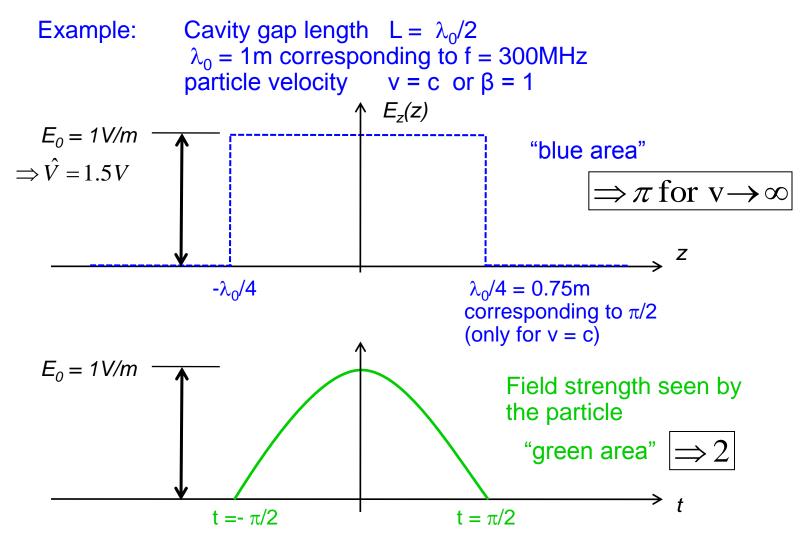
Now the particle is sampling this field with a <u>finite velocity</u>. This velocity is given by  $v = \beta c$ . The resulting transit time factor returns therefore as

$$T = \sin\left(\frac{L}{2}\frac{\omega}{\beta c}\right) / \left(\frac{L}{2}\frac{\omega}{\beta c}\right)$$

Transit time factor, p.565f. ,Alexander Wu Chao, Handbook of Accelerator Physics and Engineering

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## **Transit time factor (3)**



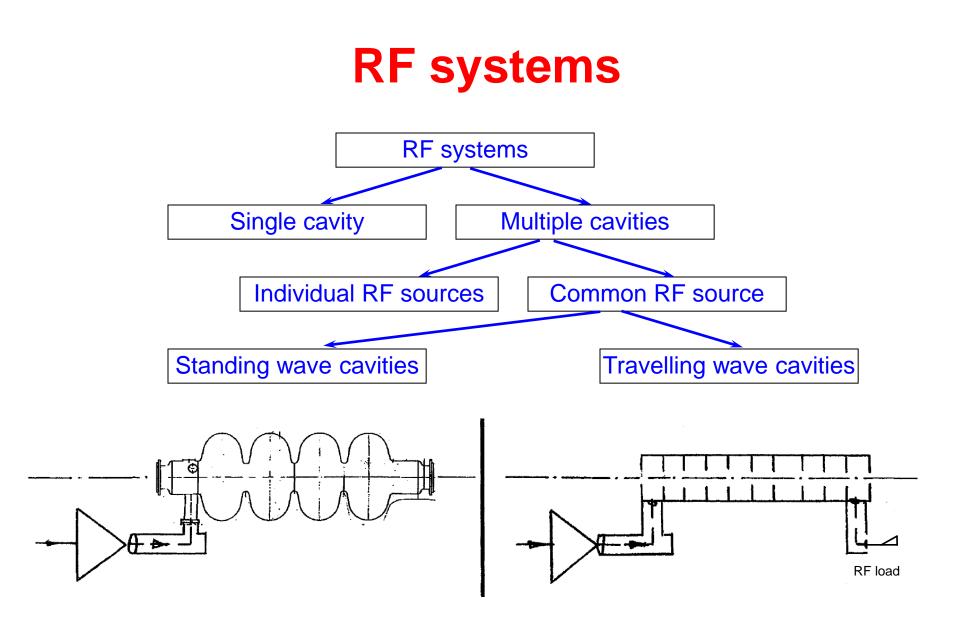
F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011

## Acceleration

We have "slow" particles with  $\beta$  significantly below 1. They become faster when they gain energy and in a circular accelerator with fixed radius we must tune the cavity (increase its resonance frequency).

When already highly relativistic particles become accelerated (gaining momentum) they cannot become significantly faster as they are already very close to c, but they become heavier. Here we can see very nicely the conversion of energy into mass. In this case no or little tuning of the resonance frequency of the cavity is required. It is sufficient to move the frequency of the RF generator within the 3dB bandwidth of the cavity.

Fast tuning (fast cycling machines) can only be done electronically and is implemented in most cases by varying the inductance via the effective  $\mu$  of a ferrite.



Groups of cavities

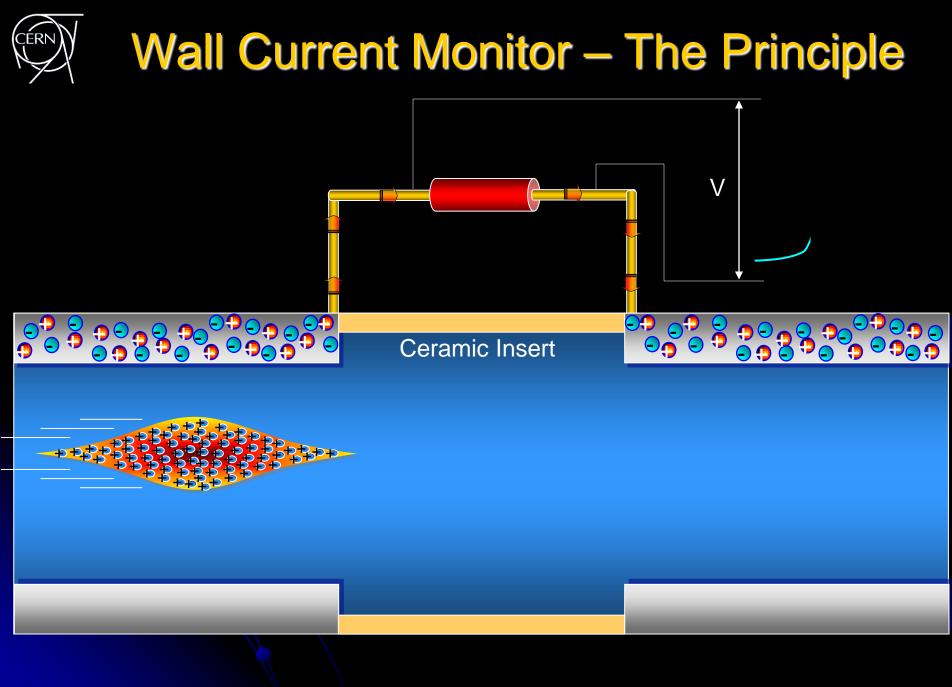
#### Diagnostics Using RF Instrumentation F. Caspers CERN

- An overview of classical non-intercepting electromagnetic sensors used for charged particle accelerators
- Examples of Schottky mass spectroscopy of single circulating ions
- Stochastic beam cooling, a feedback process based on Schottky signals in the microwave range
- Synchrotron light in the microwave range



# 

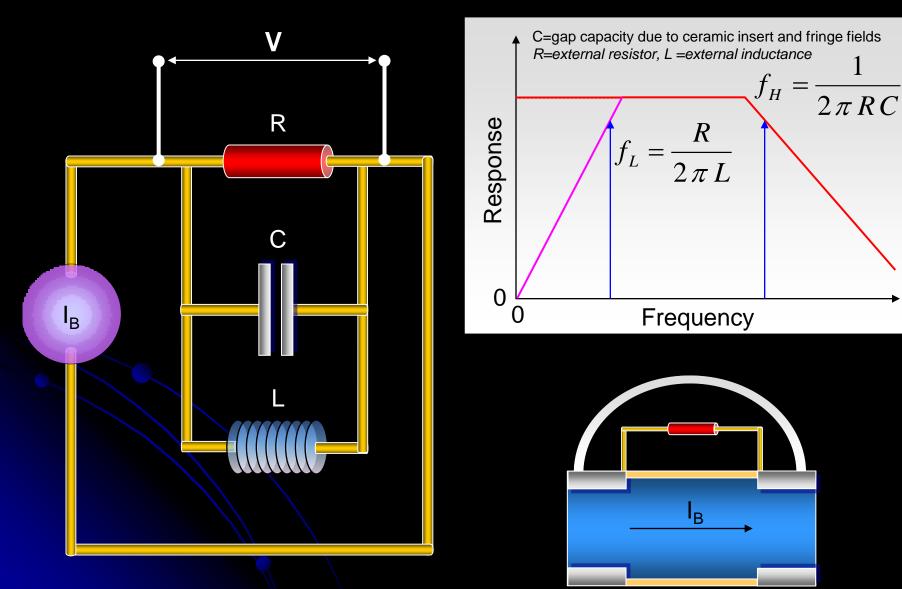
Courtesy R. Jones



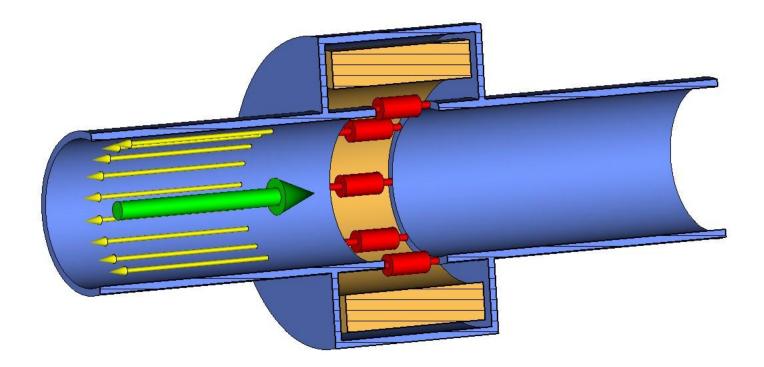
Courtesy R. Jones



## Wall Current Monitor – Beam Response





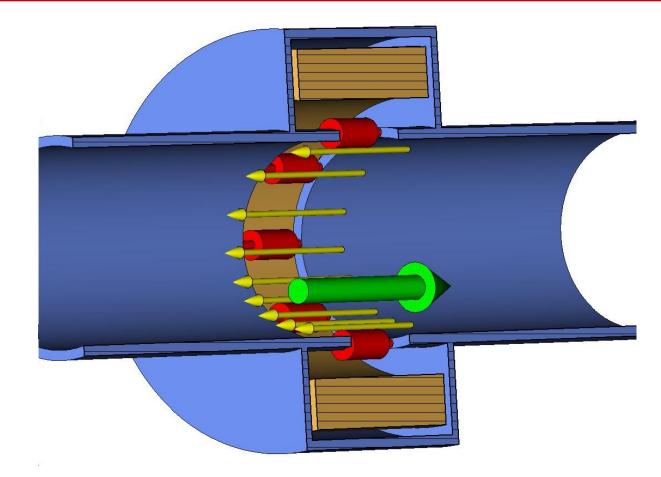


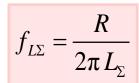
- The BEAM current is accompanied by its IMAGE
- A voltage proportional to the beam current develops on the **RESISTORS** in the beam pipe gap
- The gap must be closed by a box to avoid floating sections of the beam pipe
- The box is filled with the FERRITE to force the image current to go over the resistors
- The ferrite works up to a given frequency and lower frequency components flow over the box wall



#### WCM as a Beam Position Monitor







$$f_{L\Delta} = \frac{R}{2\pi L_{\Delta}}$$

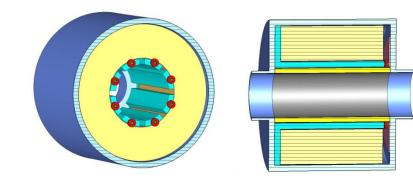
- For a centered **BEAM** the **IMAGE** current is evenly distributed on the circumference
- The image current distribution on the circumference changes with the beam position
- Intensity signal (Σ) = resistor voltages summed
- Position dependent signal ( $\Delta$ ) = voltages from opposite resistors subtracted
- The  $\Delta$  signal is also proportional to the intensity, so the position is calculated according to  $\Delta/\Sigma$
- Low cut-offs depend on the gap resistance and box wall (for  $\Sigma$ ) and the pipe wall (for  $\Delta$ ) inductances



#### **Inductive Pick-Up New Design**



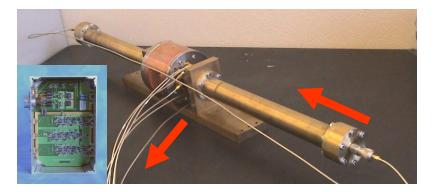
- The ceramic tube is coated with low resistance titanium layer, resistance: end-to-end ≈10 Ω, i.e. ≈ 15 Ω/□
- Primary circuit has to have small parasitic resistances (Cu pieces, CuBe screws, gold plating)
- Tight design, potential cavities damped with the ferrite
- The transformers are mounted on a PCB and connected by pieces of microstrip lines (minimizing series inductances)



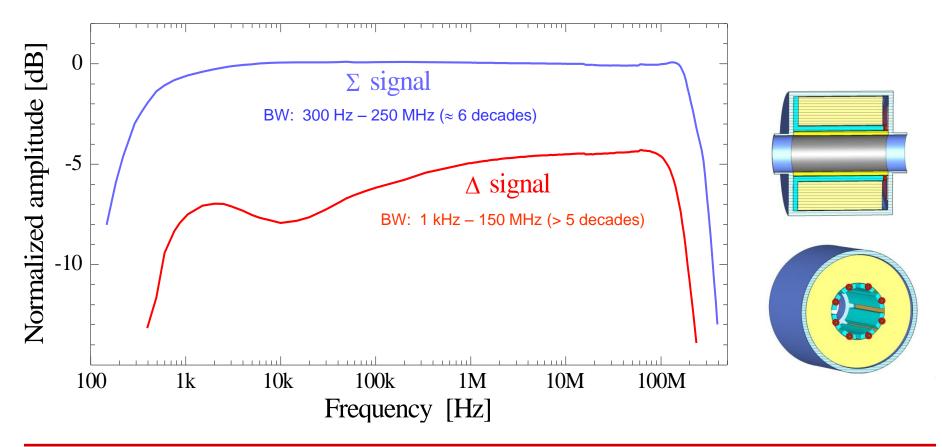


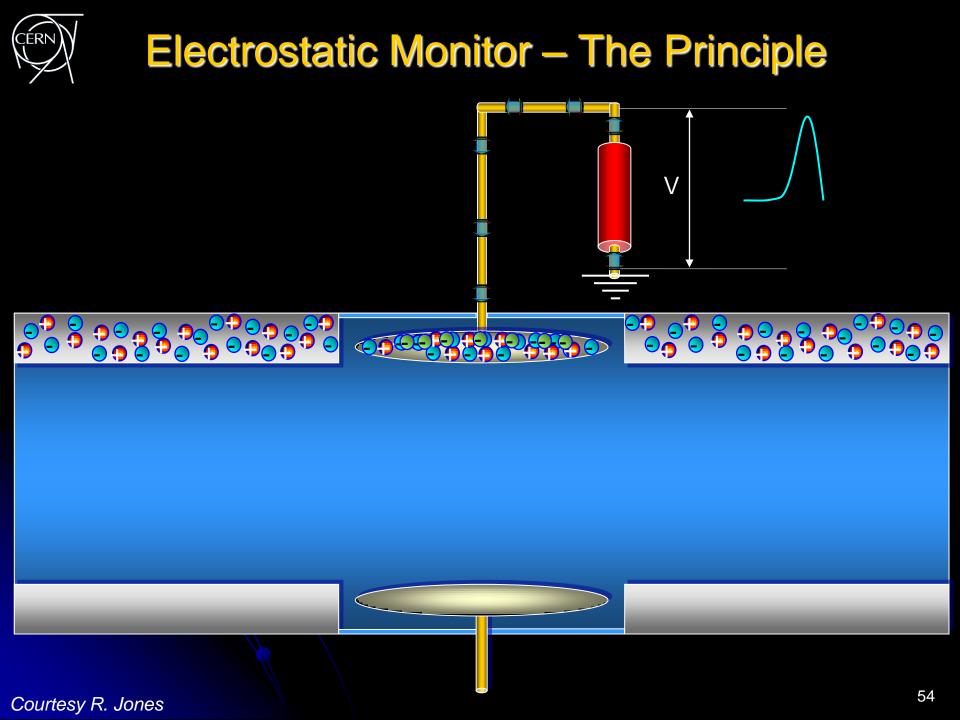




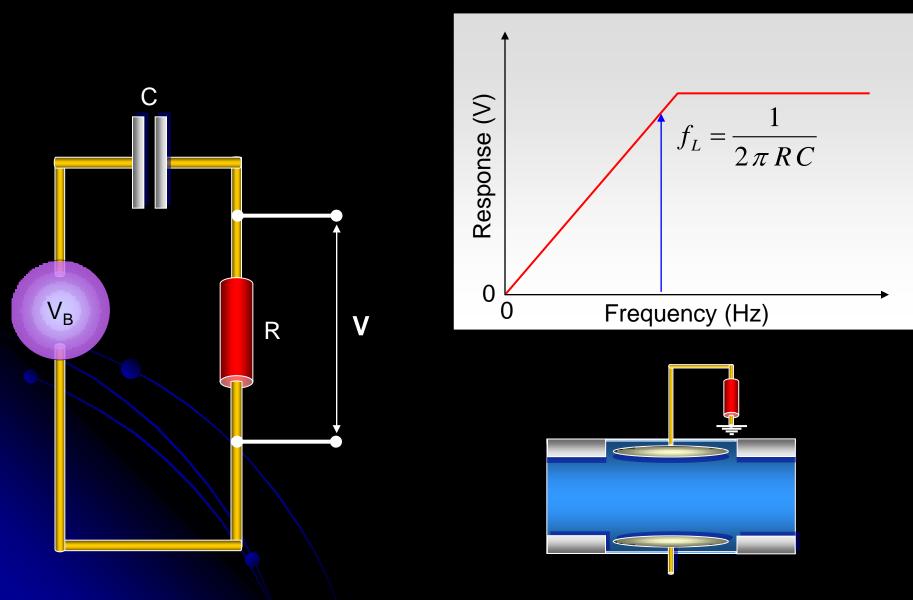


- A wire method with a 50 Ω coaxial setup which the IPU is a part
- Σ signal flat to 0.5 dB within 5 decades, almost
   6 decades of 3 dB bandwidth (no compensation)
- $\Delta$  signal 5 decades (four decades + one with an extra gain for low frequencies)



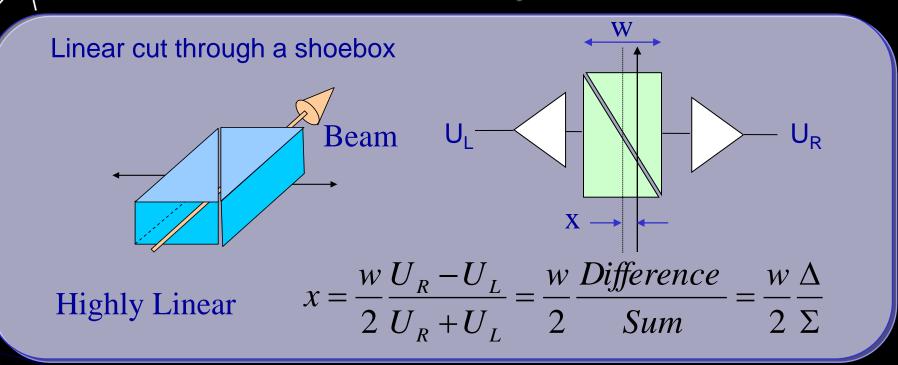


# Electrostatic Monitor – Beam Response



Courtesy R. Jones

## Electrostatic Pick-up – Shoebox



#### Measurement:

 Induced charges carried away by low-impedance circuit or sensed on a high impedance as a voltage





## **Electrostatic Pick-up – Button**

- Variant of electrostatic PU
- ✓ Low cost  $\Rightarrow$  most popular
- $\times$  Non-linear
  - requires correction algorithm when beam is off-centre

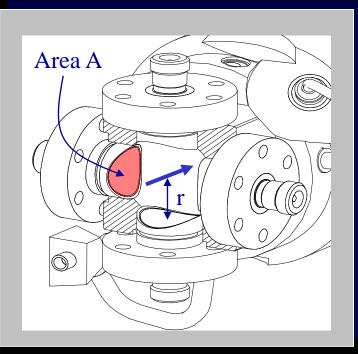
Transfer Impedance:

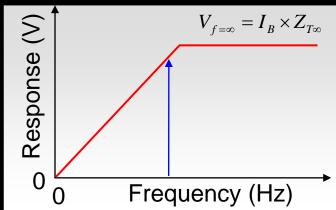
$$T_{\infty} = \frac{A}{(2\pi r) \times c \times C_e}$$

Low frequency<sub>1</sub>cut-off:

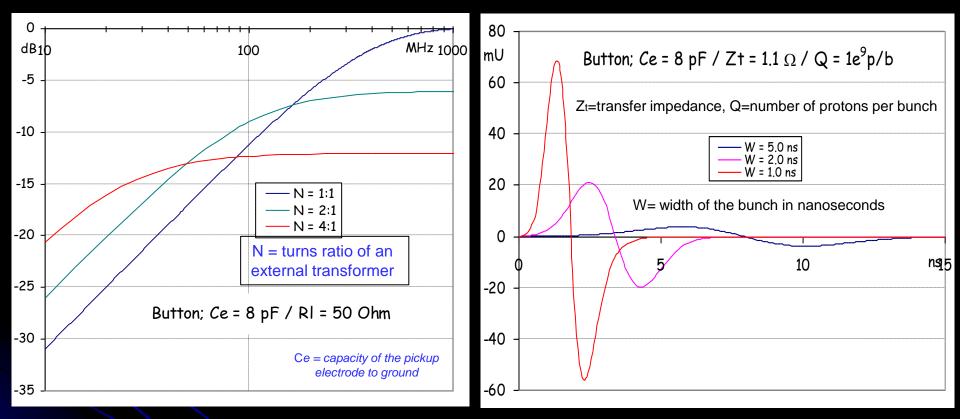
$$f_L = \frac{1}{2\pi RC}$$

Ce = capacity of the pickup electrode to ground





# Button Frequency & Time Response



### Frequency domain:

 Impedance transformers improve the low frequency levels at the expense of the high frequency

### • Time domain:

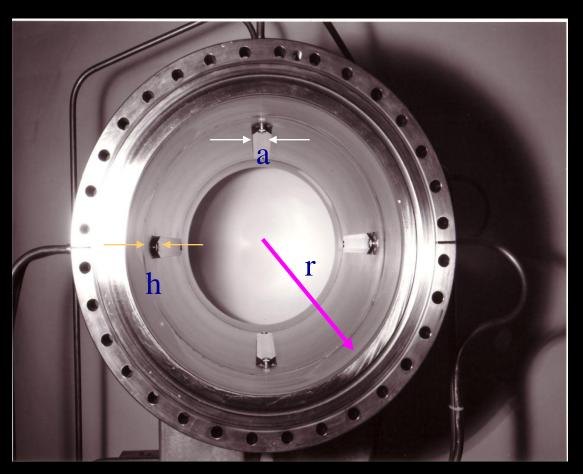
- Differentiated pulse
- Exponential dependence of amplitude on bunch length

# Electromagnetic (Directional) coupler

• A transmission line (stripline) which couples to the transverse electromagnetic (TEM) beam field

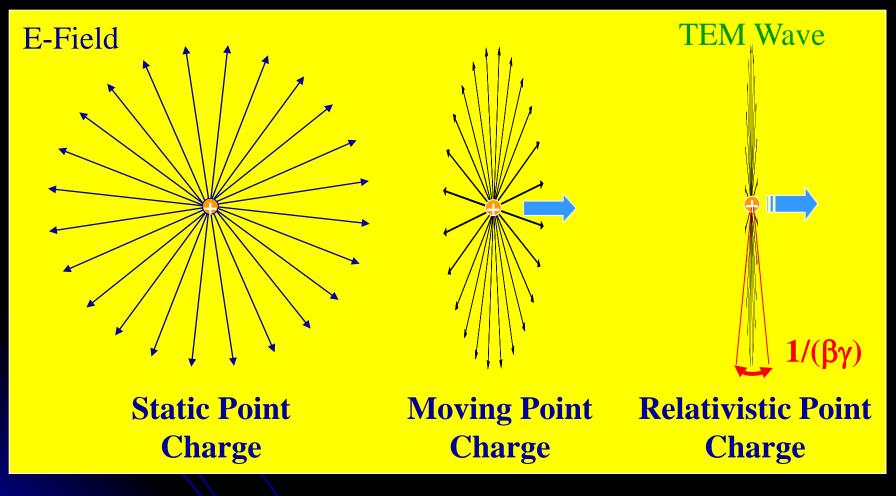
 $Z_{t \infty} = 60 \ln[(r+h)/r]$  $\equiv Z_0^*[a/2\pi(r+h)]$ 

- Z<sub>0</sub> is the characteristic impedance
- a, r, h, I are the mechanical dimensions
- t = I/c is the propagation time in the coupler



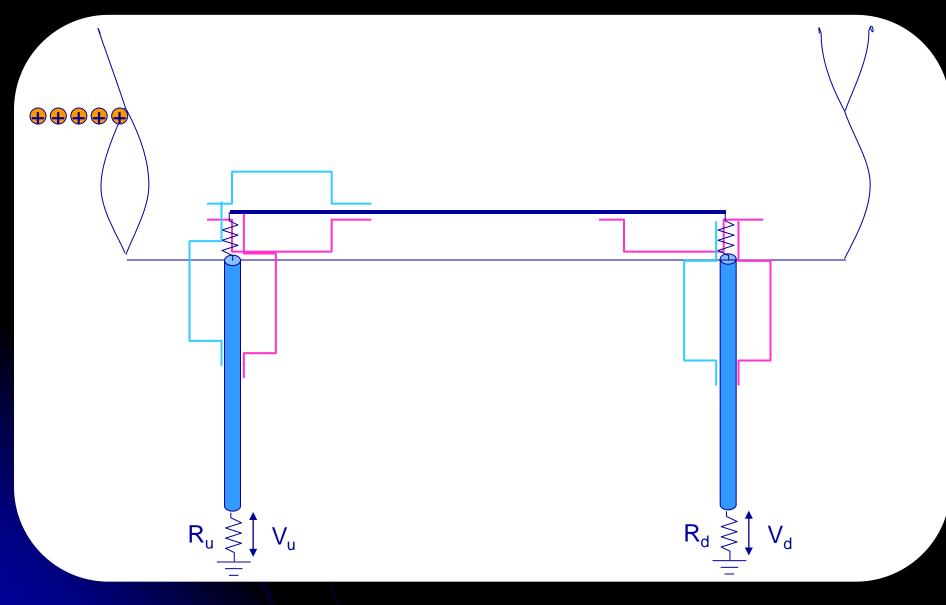


## **EM Fields & Relativity**



Relativistic case: Electric & magnetic fields become transverse to the direction of motion (TEM).

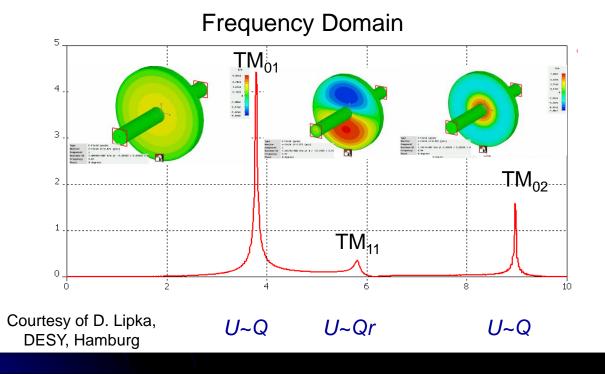
# Electromagnetic Stripline Coupler - Principle





## Improving the Precision for Next Generation Accelerators

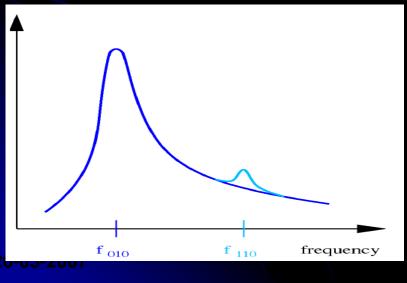
- Standard BPMs give intensity signals which need to be subtracted to obtain a difference which is then proportional to position
  - Difficult to do electronically without some of the intensity information leaking through
    - When looking for small differences this leakage can dominate the measurement
    - Typically 40-80dB (100 to 10000 in V) rejection  $\Rightarrow$  tens micron resolution for typical apertures
- Solution cavity BPMs allowing sub micron resolution
  - Design the detector to collect only the difference signal
    - Dipole Mode TM<sub>11</sub> proportional to position & shifted in frequency with respect to monopole mode

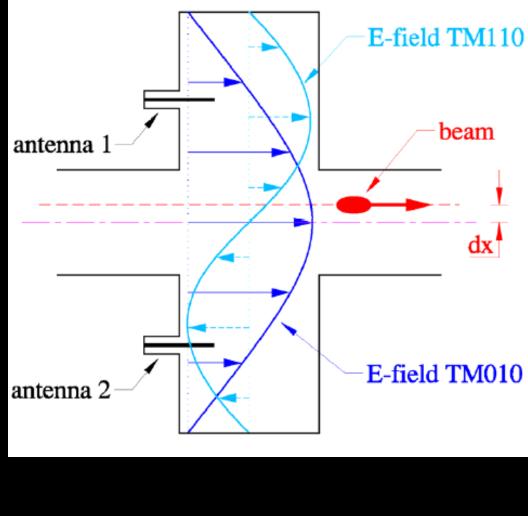


## **Cavity BPMs**



- BPM resolution typically limited by problem of taking a difference between large numbers (2 opposing electrodes)
- Cavity BPMs have different frequency response for fundamental and difference mode
  - Aids in fundamental rejection
  - Can give sub-micron resolution. BUT:
  - Damping time quite high due to intrinsic high Q >>1000
  - Poor time resolution (~100ns)

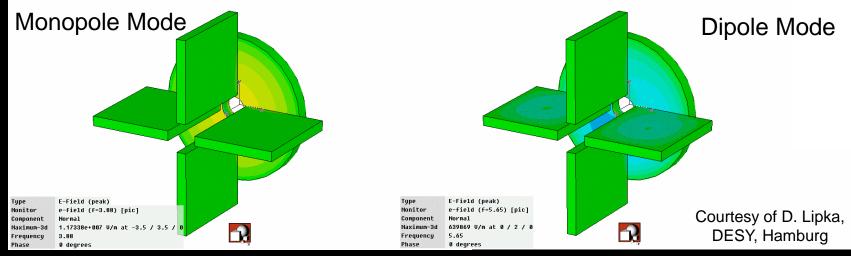




# CERN

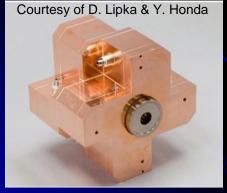
## Today's State of the Art BPMs

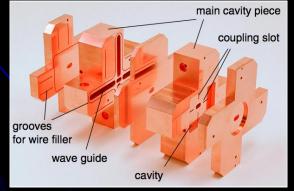
- Obtain signal using waveguides that only couple to dipole mode
  - Further suppression of monopole mode

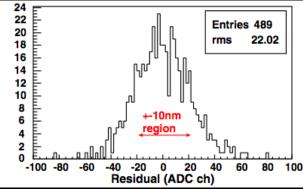


#### Prototype BPM for ILC Final Focus

Required resolution of 2nm (yes nano!) in a 6×12mm diameter beam pipe
 Achieved World Record (so far!) resolution of 8.7nm at ATF2 (KEK, Japan)

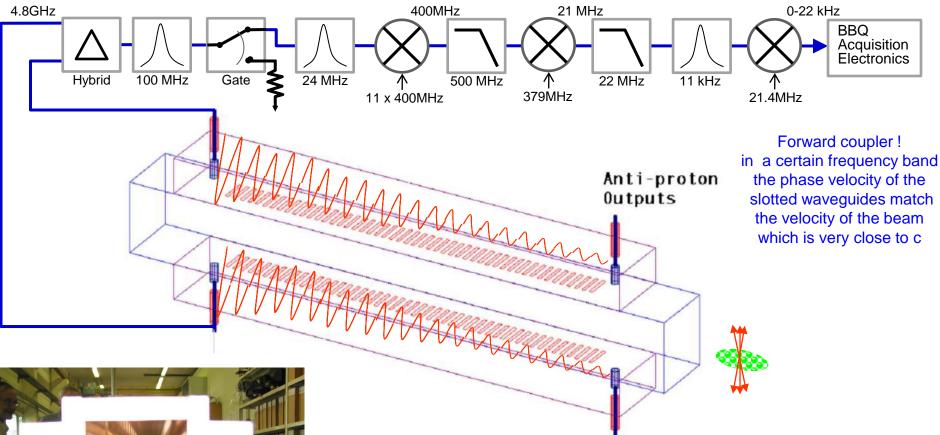


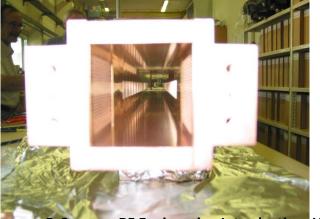






#### Schottky Measurements



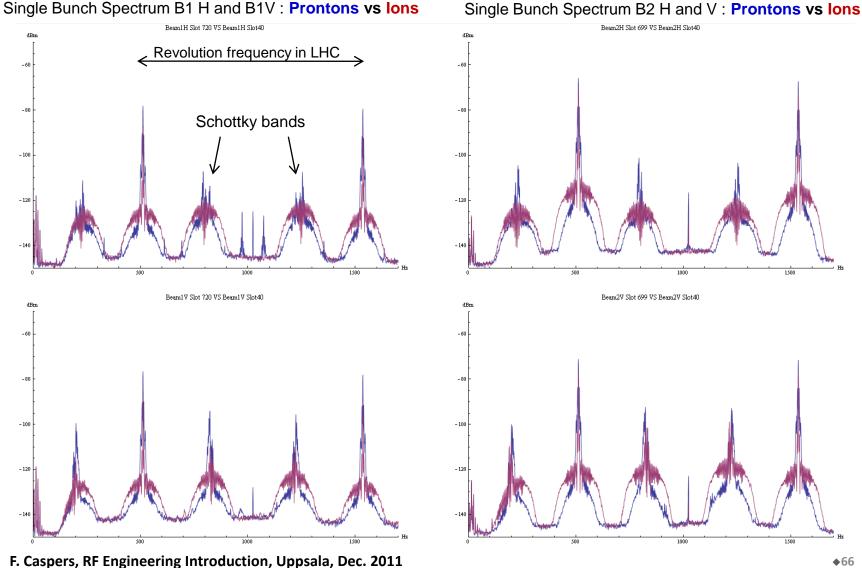


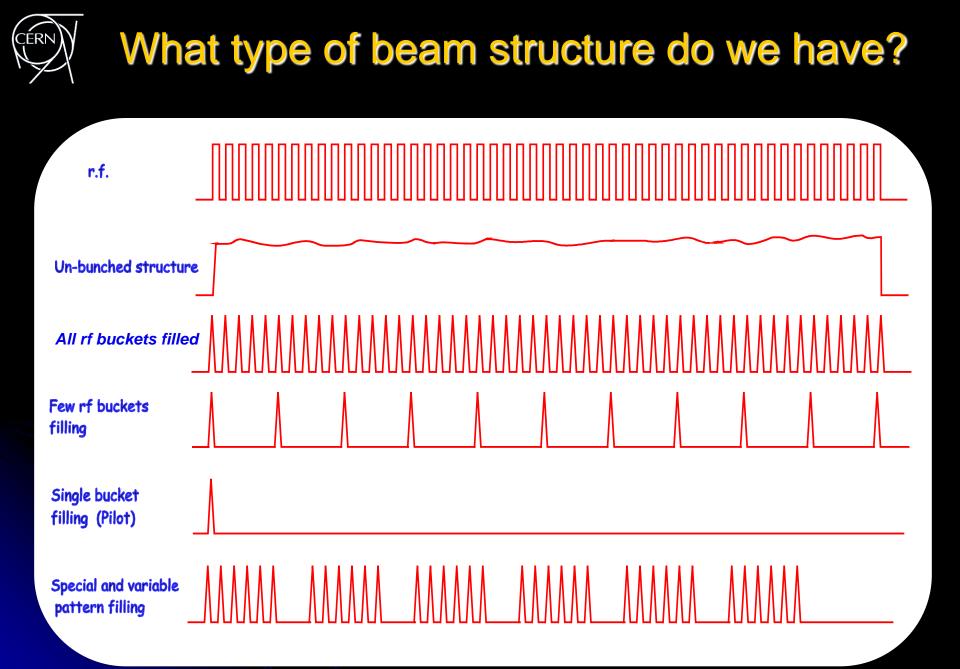
4.8 GHz Slotted Waveguide Structure
60 x 60 mm aperture x 1.5 meters long
Gated, triple down-mixing scheme to baseband
Successive filtering from bandwidth of 100MHz to 11kHz
Capable of Bunch by Bunch Measurement

F. Caspers, RF Engineering Introduction, Uppsala, Dec. 2011



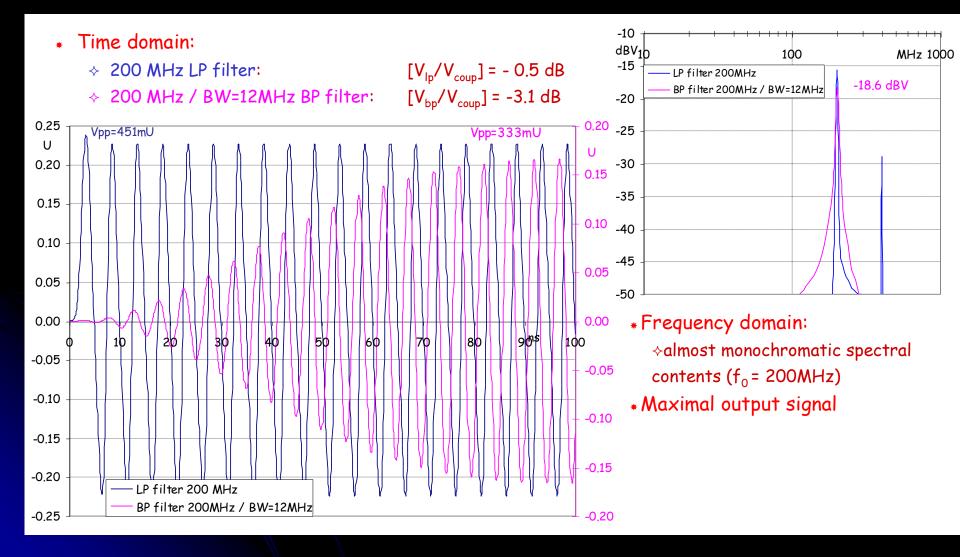
#### **Comparison of Ions and Protons Bunch to Bunch Spectra**







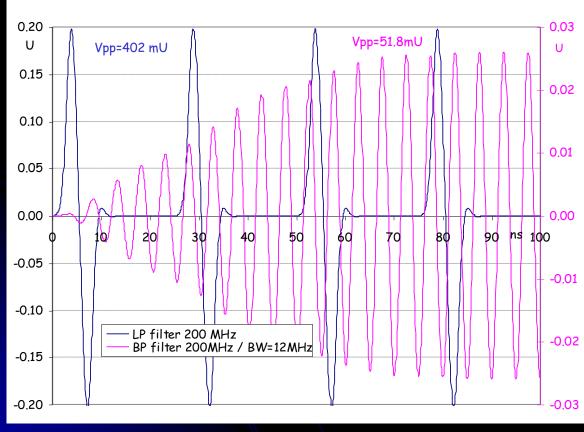
## All RF Buckets Filled (200MHz)

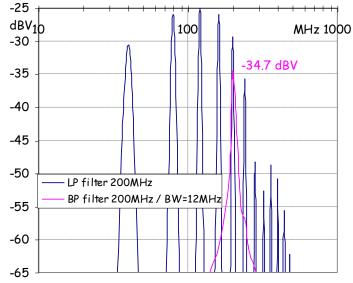




## Few RF Buckets Filled (40MHz)

- Time domain:
  - ♦ 200 MHz LP filter:  $[V_{lp}/V_{coup}] = -0.35 \text{ dB}$
  - $\diamond$  200MHz / 12MHz BP filter: [V<sub>lp</sub>/V<sub>coup</sub>] = -18.0 dB





#### \* Frequency domain:

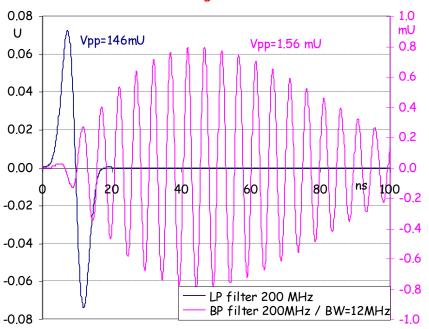
 Spectral contents shows all harmonics of the 40 MHz (1/25 ns)

♦BP filter selects only the 200 MHz line



## Single Bunch Response

Bunch length = 4.8 ns



#### \*Time domain:

#### $\diamond$ LP filter

- $\oplus$  Bunch length = 4.8 ns  $[V_{lp}/V_{coup}] = -0.0 \text{ dB}$
- $\oplus$  Bunch length = 2.1 ns  $[V_{lp}/V_{coup}]$  = -0.35 dB

#### $\diamond$ BP filter

- $\oplus$  Bunch length = 4.8 Ns  $[V_{lp}/V_{coup}]$  = -39.6 dB
- $\oplus$  Bunch length = 2.1 Ns  $[V_{lp}/V_{coup}]$  = -24.1 dB

15 0.20 υ mU Vpp= 402mU Vpp= 25.1mU 0.15 10 0.10 5 0.05 0.00 0 20 40 60 80 ns 100 -0.05 -5 -0.10 -10 -0.15 LP filter 200 MHz BP filter 200MHz / BW=12MHz -0.20 -15

Bunch length = 2.1 ns

- \* Frequency domain:
  - Quasi continuous spectrum
  - BP filter uses the fraction of the signal power that corresponds to its BW

#### Examples of Schottky mass spectroscopy

this and the following 4 slides (S.Litvinov) were provided by P. Kowina (GSI)

#### Schottky Mass Spectrometry (SMS) - Collaboration

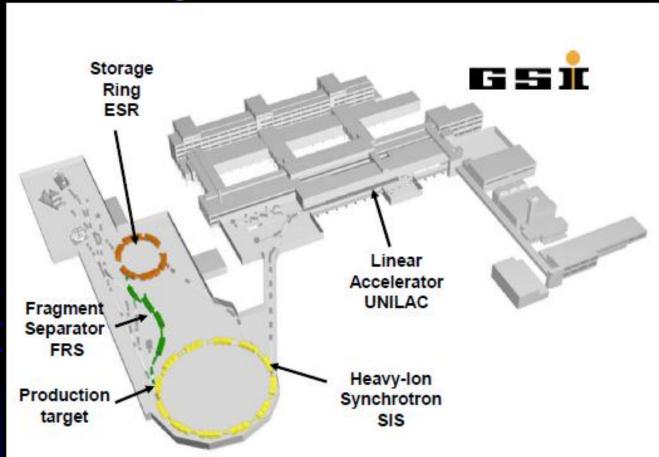
F. Attallah, G. Audi, K. Beckert, P. Beller<sup>†</sup>, F. Bosch, D. Boutin, C. Brandau, Th. Bürvenich, L. Chen, I. Cullen, Ch. Dimopoulou, H. Essel, B. Fabian, Th. Faestermann, M. Falch, A. Fragner, B. Franczak, B. Franzke, H. Geissel, E. Haettner, M. Hausmann, M. Hellström, S. Hess, G. Jones, E. Kaza, Th. Kerscher, P. Kienle, O. Klepper, H.-J. Kluge, Ch. Kozhuharov, K.-L. Kratz, R. Knöbel, J. Kurcewicz, S.A. Litvinov, Yu.A. Litvinov, Z. Liu, K.E.G. Löbner<sup>†</sup>, L. Maier, M. Mazzocco, F. Montes, A. Musumarra, G. Münzenberg, S. Nakajima, C. Nociforo, F. Nolden, Yu.N. Novikov, T. Ohtsubo, A. Ozawa, Z. Patyk, B. Pfeiffer, W.R. Plass, Z. Podolyak, M. Portillo, A. Prochazka, T. Radon, R. Reda, R. Reuschl, H. Schatz, Ch. Scheidenberger, M. Shindo, V. Shishkin, J. Stadlmann, M. Steck, Th. Stöhlker, K. Sümmerer, B. Sun, T. Suzuki, K. Takahashi, S. Torilov, M.B.Trzhaskovskaya, S.Typel, D.J. Vieira, G. Vorobjev, P.M. Walker, H. Weick, S. Williams, M. Winkler, N. Winckler, H. Wollnik, T. Yamaguchi



### Examples of Schottky mass spectroscopy (2)

production storage and cooling of short lived nuclei (slide by S. Litvinov)

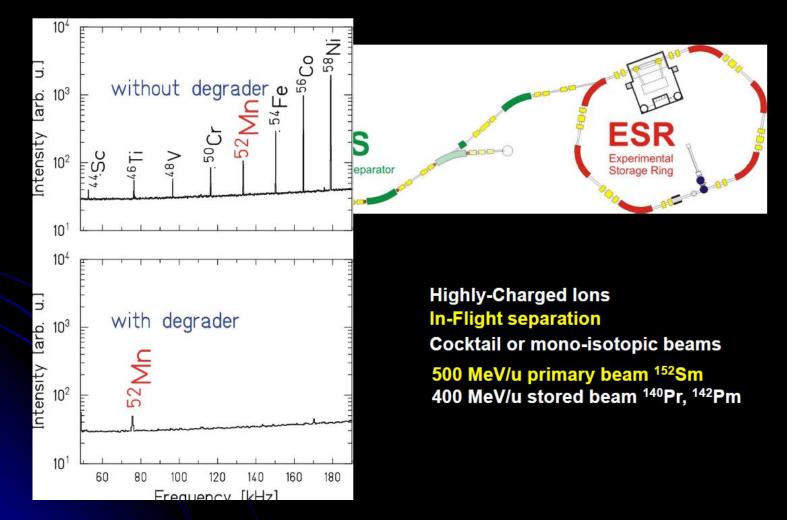
#### Secondary Beams of Short-Lived Nuclei



## Examples of Schottky mass spectroscopy (3)

production storage and cooling of short lived nuclei (slide by S. Litvinov)

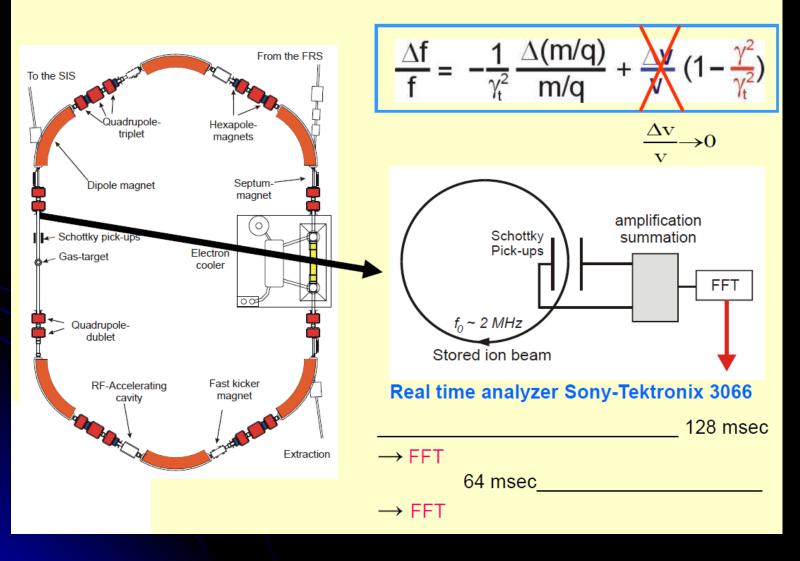
### **Production & Separation of Exotic Nuclei**



### Examples of Schottky mass spectroscopy (4)

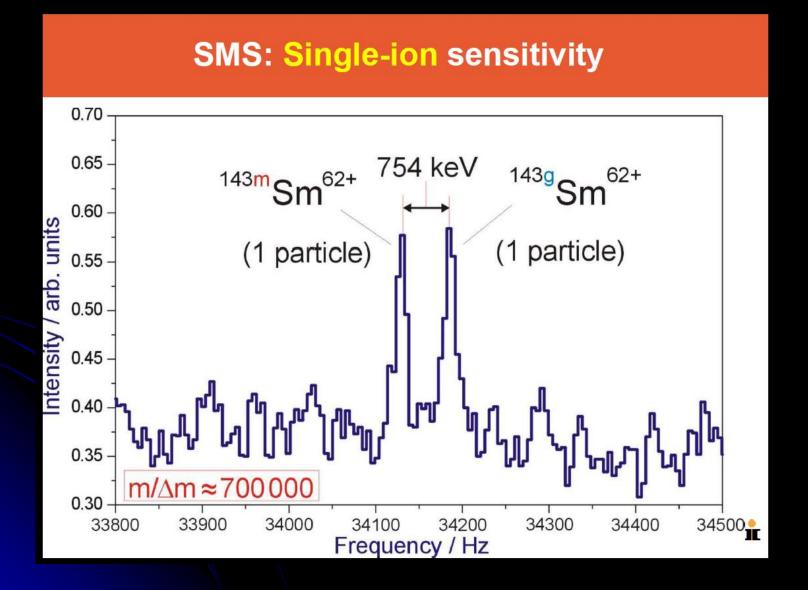
production storage and cooling of short lived nuclei (slide by S. Litvinov)

### **Recording the Schottky-noise**



### Examples of Schottky mass spectroscopy (4)

production storage and cooling of short lived nuclei (slide by S. Litvinov)





### Stochastic beam cooling (1)

invented by Simon van der Meer at CERN in 1967, Nobel prize in 1984

#### STOCHASTIC COOLING AND THE ACCUMULATION OF ANTIPROTONS

Nobel lecture, 8 December, 1984

by

SIMON VAN DER MEER CERN, CH- 1211 Geneva 23, Swi The Nobel prize was awarded to Carlo Rubbia and Simon van der Meer for "their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of the weak interaction".(quote) and the phrase was coined: Van der Meer made it possible, Rubbia made it happen.

#### 1. A general outline of the pp project

The large project mentioned in the motivation of this year's Nobel award in physics includes in addition to the experiments proper described by C. Rubbia, the complex machinery for colliding high-energy protons and antiprotons (Fig. 1). Protons are accelerated to 26 GeV/c in the PS machine and are used to produce p's in a copper target. An accumulator ring (AA) accepts a batch of these with momenta around 3.5 GeV/c every 2.4 s. After typically a day of accumulation, a large number of the accumulated p's (~10") are extracted from the AA, reinjected into the PS, accelerated to 26 Gev/c and transferred to the large (2.2 km diameter) SPS ring. Just before, 26 Gev/c protons, also from the PS, have been injected in the opposite direction. Protons and antiprotons are then accelerated to high energy (270 or 310 Gev) and remain stored for



### Stochastic beam cooling (2)

the text shown below is part of the Nobel lecture by S. v.d. Meer

2. Cooling, why and how?

A central notion in accelerator physics is phase space, well-known from other areas of physics. An accelerator or storage ring has an acceptance that is defined in terms of phase volume. The antiproton accumulator must catch many antiprotons coming from the target and therefore has a large acceptance; much larger than the SPS ring where the p's are finally stored. The phase volume must therefore be reduced and the particle density in phase space increased. On top of this, a large density increase is needed because of the requirement to accumulate many batches. In fact, the density in 6-dimensional phase space is boosted by a factor 10<sup>°</sup> in the AA machine.

This seems to violate Liouville's theorem that forbids any compression of phase volume by conservative forces such as the electromagnetic fields that are used by accelerator builders. In fact, all that can be done in treating particle beams is to distort the phase volume without changing the density anywhere.

Fortunately, there is a trick - and it consists of using the fact that particles are points in phase space with empty space in between. We may push each particle towards the centre of the distribution, squeezing the empty space outwards. The small-scale density is strictly conserved, but in a macroscopic sense the particle density increases. This process is called cooling because it reduces the movements of the particles with respect to each other.



#### **Stochastic beam cooling (3)** the text shown below is part of the Nobel lecture by S. v.d. Meer

#### 3. Qualitative description of betatron cooling

The cooling of a single particle circulating in a ring is particularly simple. Fig. 2 shows how it is done in the horizontal plane. (Horizontal, vertical and longitudinal cooling are usually decoupled.)

Under the influence of the focusing fields the particle executes betatron oscillations around its central orbit. At each passage of the particle a so-called differential pick-up provides a short pulse signal that is proportional to the distance of the particle from the central orbit. This is amplified and applied to the kicker, which will deflect the particle. If the distance between pick-up and kicker contains an odd number of quarter betatron wavelengths and if the gain is chosen correctly, any oscillation will be cancelled. The signal should arrive at

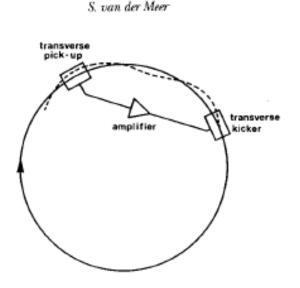


Fig. 2. Cooling of the horizontal betatron oscillation of a single particle

293



## Stochastic beam cooling (4)

- Today stochastic cooling is an important tool for charged particle beam conditioning, its used in all 3 planes (horizontal, vertical and longitudinal
- Stochastic cooling is applied on coasting (non-bunched) and bunched HADRON beams, where for bunched beam stochastic cooling large difficulties related to inter-modulation of the front end amplifier had to be mastered
- Stochastic cooling systems are in operation at CERN, GSI, FZJ, BNL and Fermilab and further systems are planned for NICA and in the frame of the FAIR project
- Stochastic cooling is very suitable to make HOT beam tempered, and electron cooling is very well suited to make tempered beam really cold..approaching the state of beam crystallization...
- There exist a considerable number of other particle cooling methods, such as ionization cooling (proposed for muons with very short lifetime), laser cooling, radiation cooling (leptons) resistive cooling (applied in traps) ...just to give a few examples
- Stochastic cooling has permitted to increase the "6 D phase space density" of antiprotons by more than 10 orders of magnitude...(these days)



## Synchrotron radiation (1)

- Synchrotron radiation occurs in any charged particle accelerator where highly relativistic particles are deflected by some bending magnet. It is nothing else than the radiation emitted by a electric charge which forced to travel on a curved trajectory due to external (usually) magnetic fields.
- Particle accelerators are used for leptons (electrons, positrons) and hadrons like protons, antiprotons and all kind of ions from negatively charged hydrogen H<sup>-</sup> to fully stripped uranium ions.
- Leptons radiate very easily (in contrast to hadrons) and this radiation is used in all kind of synchrotron light sources for the generation very monochromatic electromagnetic radiation pulses down to a small fraction of a pico-second in length.
- The spectral range of synchrotron radiation used for research (e.g. biological and chemical processes) as well as technical applications extents from far infrared to hard  $\gamma$ -rays.
- Synchrotron radiation is in may cases a desired effect and also used for beam cooling (radiation cooling) in lepton damping rings. But it may be also undesired like in the CERN-LEP machine, where synchrotron radiation of the electrons and positrons at 100 GeV/c generated 2 kW average power of X-ray radiation per meter which had to be removed by water cooling of the vacuum chamber



## Synchrotron radiation (2)

- Synchrotron radiation has a lower frequency bound, namely the cutoff frequency of the first waveguide mode of the beam-pipe. Below this frequency which is typically in the GHz range the mechanism of radiation cannot become effective (no propagating waveguide modes)
- We discriminate between coherent and incoherent synchrotron radiation
- Incoherent radiation is emitted by individual particles without having a defined phase relation to other particles in the bunch. In this case the total emitted power is proportional to the number of particles

• Coherent synchrotron radiation is related to a defined phase relation e.g. for the case that the bunch is very short compared to the emitted wavelength. Here the complete bunch acts as a single macro-particle and radiated power is proportional to the SQUARE of the number of particles. Thus for short bunches we often have coherent synchrotron radiation in the microwave range



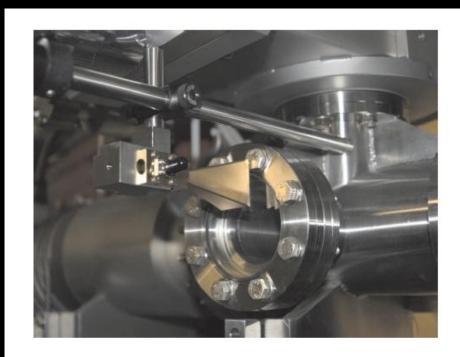
## Synchrotron radiation (3)

- Synchrotron radiation from infra-red to  $\gamma$  -rays has many diagnostic applications for particle accelerators such as measurement of the time structure of the beam with sub-femto second resolution as well as measurement of the transverse and longitudinal emittance.
- But also at the low end (microwave) synchrotron radiation is applied now as a diagnostic tool..e.g. for measuring the time structure.

Example of a 60-90 GHz detector with horn antenna mounted next to a visible light extraction port

From: G. Rehm et al. ULTRA-FAST MM-WAVE DETECTORS FOR OBSERVATION OF MICROBUNCHING INSTABILITIES IN THE DIAMOND STORAGE RING, Proceedings DIPAC 2009 Basel Abstract:

The operation of the Diamond storage ring with short electron bunches using low alpha optics for generation of Coherent THz radiation and short X-ray pulses for time-resolved experiments is limited by the onset of microbunching instabilities. We have installed two ultrafast (time response is about 250 ps) Schottky Barrier Diode Detectors sensitive to radiation within the 3.33-5 mm and 6-9 mm wavelength ranges. Bursts of synchrotron radiation at these wavelengths have been observed to appear periodically above certain thresholds of stored current per bunch.....



#### Conclusions

- For beam diagnostics in particle accelerators electromagnetic sensors operating from DC to well beyond the microwave range are and indispensible tool and modern accelerators cannot run without this kind of diagnostic.
- Stochastic beam cooling, a mixture of microwave based beam diagnostic and correction, has made important contributions to physics. This technique corrects the movements of individual particles. On average each simply charged particle (e.g. proton) passing through a stochastic cooling pick-up just gives off a single microwave photon per passage
- Microwave diagnostic can see a single charged particle circulating in a storage ring and also a single particle (e.g. antiproton) oscillating in a trap.
- And last not least: RF and microwave power systems are the indispensible working horse for virtually ALL particle accelerators used these days.