Conformal Regge Theory

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- Notions on conformal symmetry

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- Notions on Regge theory

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- Results in $\mathcal{N} = 4$ SYM

Conformal Symmetry - Definition

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Let $\tilde{x}^{\mu} = \tilde{x}^{\mu}(x)$,

$$\frac{\partial \tilde{x}^{\mu}}{\partial x^{\theta}} \frac{\partial \tilde{x}^{\nu}}{\partial x^{\lambda}} g^{\theta\lambda} = f(x)\, g^{\mu\nu}$$

The generators of the theory can be derived from this equation.

Conformal Symmetry - Primaries

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with

$$a+b+c = \frac{\Delta_1+\Delta_2+\Delta_3}{2}.$$

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Once all Δ_i and C_{ij}^{k} are known, the conformal field theory is completely resolved.

Notions on Regge theory

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$$s = (p_a + p_b)^2$$

$$t = (p_a - p_c)^2$$

$$u = (p_a - p_d)^2 ,$$

then the amplitude $\mathcal{A}$ should be function of only two of these as $s + t + u = 4m^2$. It is assumed that the amplitude $\mathcal{A}$ is lorentz invariant, unitary property and analytic function.

Notions on Regge theory - Partial wave expansion

It is possible to write the amplitude using partial wave expansion in the Regge limit, i.e., s much larger than t.

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$$\mathcal{A}_{ab\to cd}(s,t) = \frac{1}{2i}\int_{\gamma} dl\,(2l+1)\,\frac{a(l,t)}{\sin \pi l} P\left(l, 1+\frac{2s}{t}\right)$$

Notions on Regge theory - Contour deformation

We deform the counter γ and pick a possible pole and then the amplitude is expressed as,

$$\mathcal{A}(s,t) \to \beta(t)\, s^{\alpha(t)}$$

This theory with simple assumptions has some agreement with experiment.

N=4 SYM

$\mathcal{N} = 4$ SYM is a supersymmetric gauge theory which has the property of being conformal.
We consider the scattering of two operators, $\mathcal{O}_1$ and $\mathcal{O}_2$.

$$\mathcal{A} = \frac{\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_1(x_3)\,\mathcal{O}_2(x_4)\rangle}{\langle \mathcal{O}_1(x_1)\,\mathcal{O}_1(x_3)\rangle\,\langle \mathcal{O}_2(x_4)\,\mathcal{O}_2(x_2)\rangle}.$$

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with

$$\mathcal{O}_1 = \mathrm{Tr}\left(Z^2\right) = \mathcal{O}_2$$

Results

We have obtained

$$C^i_{jk} \text{ at any order in } \left(\frac{\lambda}{J-1}\right)$$

for λ small and $\left(\frac{\lambda}{J-1}\right)$ fixed and for twist two operators. We already have a prediction for the fusion rules and at the moment we are checking the first terms.