n-DBI gravity

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Outline

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Scale Invariance and Inflation

- Observations suggest that the Universe is nearly scale invariant at early and late times, when it is believed to be approximately de Sitter space.
- At the present epoch, the accelerating expansion is thought to be driven by a nearly constant vacuum energy.
- To explain the inflation phase after the Big Bang, most current models involve a scalar field, the *inflaton*, which acts as the agent of the nearly exponential expansion of the Universe.
- However, the nature of such a field is far from clear from the Particle Physics point of view.

Outline

A new model of Gravity

Action for n-DBI Gravity

$$S = -\frac{3\lambda}{4\pi G_N^2}\int d^4x\sqrt{-g}\left\{\sqrt{1+\frac{G_N}{6\lambda}\left({}^{(4)}R+\mathcal{K}\right)}-q\right\}$$

- λ, q are dimensionless constants that set the scale of inflation and the cosmological constant.
- $\mathcal{K} = -2(K^2 + n^\alpha \partial_\alpha K)$, where K is the extrinsic curvature of hypersurfaces orthogonal to n^α.

A new model of Gravity

This model:

- yields the Dirac-Born-Infeld type conformal scalar theory when the Universe is conformally flat.
- reduces to Einstein's gravity with the Gibbons-Hawking-York boundary term in weakly curved spacetimes.

Moreover,

- it breaks Lorentz invariance by introducing a preferred time-like vector field (albeit recovering it the weak curvature limit).
- resembles Hořava-Lifshitz gravity in the sense that we treat time as distinct from space.

Outline

The conformally flat Universe

If we consider conformally flat universes, with metric

Friedmann-Robertson-Walker Ansatz

$$ds^2 = \ell_P^2 \phi(\tau, x)^2(-d\tau^2 + dx^2),$$

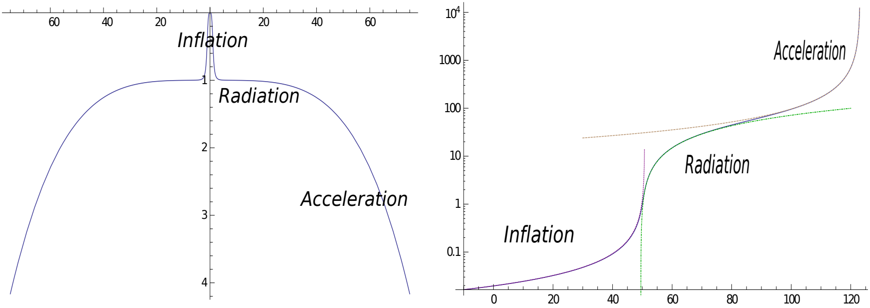
we get a conformal scalar theory with equation of motion

Effective Potential for the Scalar Field

$$\frac{1}{2}\dot{\phi}^2 + V(\phi) = 0, \; V(\phi) = -\frac{1}{2}\lambda\phi^4 \left[1 - \left(q + \frac{\epsilon}{\lambda\phi^4}\right)^{-2}\right].$$

- ℓ_P is the Planck lenght; ϵ is the radiation energy.

The conformally flat Universe



- The model naturally results in inflation at early times, followed by radiation- (and matter-) dominated epochs and subsequent acceleration at late times.

The Universe with a Perfect Fluid

Energy-Momentum Conservation

$$\dot{\rho} + 3H(\rho + p) = 0.$$

Friedmann Equation

$$H^2 = \frac{\lambda}{G_N}\left[1 - \left(q + \frac{4\pi G_N^2}{3\lambda}\rho\right)^{-2}\right].$$

Raychaudhury Equation

$$\partial_t\left[H\left(q + \frac{4\pi G_N^2}{3\lambda}\rho\right)\right] = -4\pi G_N(\rho + p).$$

Outline

Inflation and the Cosmological Constant Problem

- Taking the energy scale of inflation to be $E_{inf} = \sqrt{\lambda}\ell_P^{-1} \sim 10^{15}$ GeV,
- and the current CC $E_\Lambda = \sqrt{\lambda(1-q^{-2})}\ell_P^{-1} \sim 10^{-12}$ GeV,
- to generate such a large hierarchy, we need $\lambda \sim 10^{-8}$ and $q \sim 1 + 10^{-54}$.

However, at the level of the action the naive CC is

$$[\lambda(q-1)]^{1/4}\ell_P^{-1} \sim \text{TeV},$$

suggesting that SUSY breaking might be sufficient to solve this apparent fine-tuning problem.

Outline

The vector field n^α

- The introduction of the everywhere time-like vector field n^α yields equations which are at most second order in time, albeit higher order in spatial derivatives.
- This is a desirable property to avoid *ghosts* in the quantum theory.
- n^α defines a natural foliation of space-time by (constant time) hypersurfaces orthogonal to n^α.

ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \qquad n = -N dt.$$

The gauge group of n-DBI gravity

The general diffeomorphism group of General Relativity is broken down into:

Foliation-Preserving Diffeomorphisms

$$t \rightarrow t + \xi^0(t), \qquad x^i \rightarrow x^i + \xi^i(t, x^j).$$

- Quantities such as the shear σ_{ij} and the expansion θ of a congruence of time-like curves with tangent n^α acquire an invariant geometric meaning:

$$\sigma_{ij} = K_{ij} - \frac{1}{3} K h_{ij}, \qquad \theta = K.$$

Einstein's Gravity limit

Einstein's equations can be recovered by taking the limit

$$\lambda \to \infty, \qquad q \to 1,$$

with the product $\lambda(q-1)$ kept fixed.

- We get a CC term
$$\Lambda = \frac{6\lambda(q-1)}{G_N^2}.$$
- Full Lorentz invariance is restored.

Outline

Solutions with constant $\mathcal{R}$

The study of solutions with constant $\mathcal{R} = {}^{(4)}R + \mathcal{K}$ establishes the following theorem

Theorem

Any solution of Einstein's gravity with cosmological constant plus matter, admitting a foliation with constant $\mathcal{R}$, is a solution of n-DBI gravity.

Corollary

Any Einstein space admitting a foliation with constant $R - N^{-1}\Delta N$ (hypersurface metric and Ricci scalar), is a solution of n-DBI gravity.

Black Hole Solutions

If we require spherical symmetry and include a Maxwell field, we get the RN-(A)dS black hole metric, albeit in an unusual set of coordinates. The cosmological constant is an integration constant

$$\Lambda_C = \frac{3\lambda}{G_N^2}(2qC - 1 - C^2), \qquad C \equiv \sqrt{1 + \frac{G_N}{6\lambda}\mathcal{R}}.$$

Thus, asymptotically ($r \to \infty$), we can get either de Sitter, Anti-de Sitter or Minkowski space, depending on the value of C.

Outline

What are the degrees of freedom of n-DBI gravity?

- The breaking of Lorentz invariance down to the subgroup of FPD's apparently introduces an extra (scalar) degree of freedom, in addition to the two graviton polarizations of General Relativity.
- What is the dynamics of the scalar graviton? Are there any pathologies (like in Hořava-Lifshitz gravity)?
- Is there any time-dependent, spherically-symmetric, vacuum solution? (prohibited by Birkhoff's theorem in GR)

For Further Reading

- C. Herdeiro, S. Hirano, "Scale invariance and a gravitational model with non-eternal inflation," [arXiv:1109.1468 [hep-th]].
- C. Herdeiro, S. Hirano, Y. Sato, "n-DBI gravity," Phys. Rev. D 84 124048 (2011).
- F. Coelho, C. Herdeiro, S. Hirano, Y. Sato, *Work In Progress*.