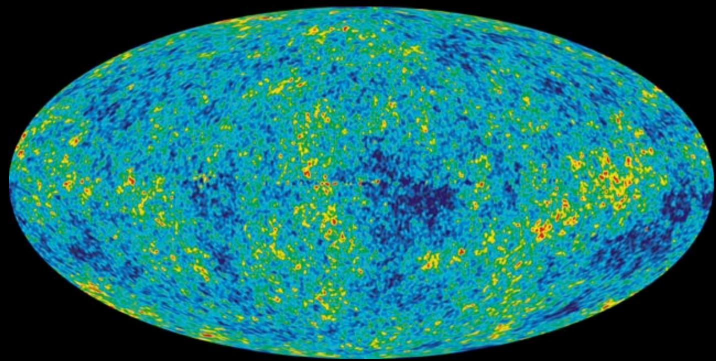
Loop corrections to cosmological correlations and experimental constraints on chaotic inflation



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Outline

- Inflation and the initial condition challenge
- Loop corrections to cosmological correlations
- Experimental constrains on chaotic inflation

Inflation

Inflation successful:

- solves th. problems of SBB model
- explains data very well

But, we know little about the theory of inflation itself

- the energy scale is unknown within 10 orders of magnitude

Future experiments may soon change that situation and let us rule out some of the popular models of inflation

→ **Exciting window to physics >> TeV**

What can we learn?

- From the power spectrum we can reconstruct a limited part of the inflaton potential
- From non-gaussianities we can constrain the complexity of inflation (single field vs. multifield)
- From B-mode polarization we can constrain the energy scale of inflation (primordial gavi. waves)

↓

These are all effects probing energy scales less or equal H_*, when observable modes exit the horizon - can we go beyond???

UV corrections to inflation

In an effective field theory of inflation with cutoff M and probing physics at scale H_*, we expect UV corrections of order $H_*^2/M^2 \approx 10^{-10}$ [Kaloper,Kleban,Lawrence,Shenker,Susskind]

$\Rightarrow$ Seems unobservable unless exotic physics is envoked (trans-planck effects)

- Modifications of dispersion relations or Heisenbergs uncertainty relations [Brandenberger et. al.]
- Modifications of the initial vacuum [Danielsson]

However, in models of very long inflation the initial Hubble rate H_i is a new UV scale close to the Planck scale

$\longrightarrow$ Loop corrections much larger than H_*^2/M^2 and sensitive to $H_i >> H_*$

[MSS, 2006]

Example: suppression of loops in $\lambda\varphi^4$

$H \approx$ const. approx:

$$\frac{H_*^2}{M_p^2}$$

Long Inflation:

$$\frac{H_i^4}{H_*^2 M_p^2}$$

[MSS, 2006]

Quantum corrections to cosmological correlations

Can quantum corrections to cosmological correlations depend on the whole history of the universe, not just on the behavior of fields at horizon exit ??? [S. Weinberg, 2005]

- If yes, may loop corrections be large?
- In any case important in order to understand what our theory entails...

The answer is yes, but not always !

Loop corrections in very long inflation

Loop corrections to classical background in chaotic inflation >> H_*^2/M_p^2 [Abramo,Brandenberger,Mukhanov, '97]

- Consider a scalar field φ in de Sitter with a potential $V(\varphi)$ and expand

$$\phi = \phi_c + \delta\phi \,, \qquad \langle \delta\phi \rangle = 0 \quad \longleftrightarrow \quad \text{Tadpole condition}$$

$$\downarrow$$

$$\left\langle \partial^2_{g+\delta g}\phi \right\rangle + V' + \frac{1}{2} V''' \left\langle \delta\phi^2 \right\rangle = 0$$

- Loop correction divergent in IR if spectral tilt negative

$$\left\langle \delta\phi^2 \right\rangle \propto \int_{a_i H_i}^{a\Lambda} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k) \,, \qquad \mathcal{P}_{\delta\phi}(k) \propto H^2 \left(\frac{k}{aH} \right)^{n-1}$$

- Note, the calculation requires E.O.M. $\mathcal{O}(\delta\phi^2)$ or action $\mathcal{O}(\delta\phi^3)$
- From S_3 and tadpole condition => eff. E.O.M.

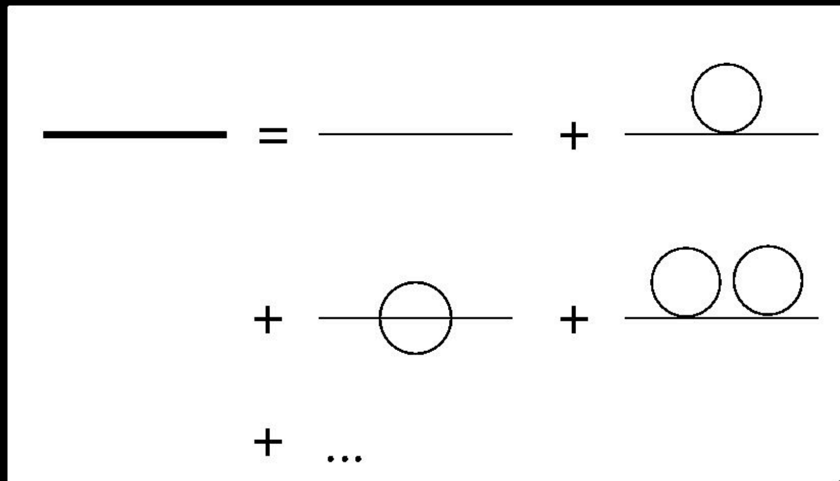
However it is not clear if the corrections has any physical significance!

- In single field inflation the effect can be gauged away by a time reparameterization in the background [Unruh, '98]

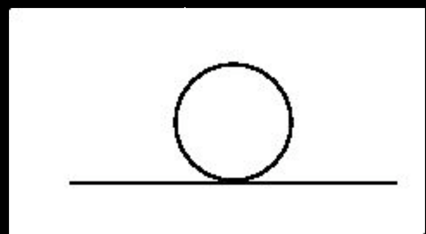
Now, look at loop correction to two-point function instead

Effective two-point function

Eff. two-point function:



Leading loop correction is the seagull:



Requires $\mathcal{O}(\delta\phi^4)$
Action: S_4

ADM formalism

Consider

$$S = \frac{1}{2} \int \sqrt{g} \left[R - (\partial\phi)^2 - 2V(\phi) \right],$$

in the ADM metric

$$ds^2 = -\mathcal{N}^2 dt^2 + h_{ij} \left(dx^i + \mathcal{N}^i dt \right) \left(dx^j + \mathcal{N}^j dt \right).$$

With the ADM metric one obtains

$$S = \frac{1}{2}\int \sqrt{h}[\mathcal{N}R^{(3)} - 2\mathcal{N}V + \mathcal{N}^{-1}(E_{ij}E^{ij} - E^2) + \mathcal{N}^{-1}(\dot{\phi} - \mathcal{N}^i\partial_i\phi)^2 - \mathcal{N}h^{ij}\partial_i\phi\partial_j\phi],$$

with

$$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i\mathcal{N}_j - \nabla_j\mathcal{N}_i).$$

[Arnowitt,Deser,Misner]

- Benefit: the GR constraint equations now follow from variation in *N*, *N*^{i}, which acts as Lagrange multipliers
- In uniform curvature gauge

$$\phi = \phi_c + \delta\phi, \qquad h_{ij} = a^2\delta_{ij}, \qquad \mathcal{N} = 1 + \alpha, \qquad \mathcal{N}^i = \partial_i\chi.$$

Varying the action iteratively, we can obtain constraint equations at each order and obtain $S(\delta\varphi)$

S_2

$$S_2 = \frac{1}{2}\int a^3\left[\delta\dot{\phi}^2 + \partial^i\delta\phi\partial_i\delta\phi - V''\delta\phi^2 + 2\frac{\dot{\phi}_c^2}{H^2}V\delta\phi^2\right].$$

[Mukhanov]

S_3

$$S_3 = \int a^3\Big[-\frac{1}{4}\frac{\dot{\phi}_c}{H}\dot{\delta\phi}^2\delta\phi - \frac{a^2}{4}\frac{\dot{\phi}_c}{H}\delta\phi(\partial\delta\phi)^2 - \dot{\delta\phi}\partial_i\chi_1\partial_i\delta\phi$$
$$+\frac{3}{8}\frac{\dot{\phi}_c^3}{H}\delta\phi^3 - \frac{1}{4}\frac{\dot{\phi}_c}{H}V_{,\phi\phi}\delta\phi^3 - \frac{1}{6}V_{,\phi\phi\phi}\delta\phi^3 + \frac{1}{4}\frac{\dot{\phi}_c^3}{H^2}\delta\phi^2\dot{\delta\phi} + \frac{1}{4}\frac{\dot{\phi}_c^2}{H}\delta\phi^2\partial^2\chi_1$$
$$+\frac{1}{4}\frac{\dot{\phi}_c}{H}\left(-\delta\phi\partial_i\partial_j\chi_1\partial_i\partial_j\chi_1 + \delta\phi\partial^2\chi_1\partial^2\chi_1\right)\Big],$$

$$\alpha_1 = \frac{1}{2}\frac{\dot{\phi}_c}{H}\delta\phi, \qquad \partial^2\chi_1 = -\frac{1}{2}\frac{\dot{\phi}_c}{H}\dot{\delta\phi} - \frac{1}{2}\dot{\phi}_c\frac{\dot{H}}{H}\delta\phi + \frac{1}{2}\frac{\ddot{\phi}}{H}\delta\phi.$$

[Maldacena]

S_4

$$S_4 = \int a^3\Big[-\frac{1}{24}V_{,\phi\phi\phi\phi}\delta\phi^4 + \frac{1}{2}\partial_j\chi_1\partial^j\delta\phi\partial_m\chi_1\partial^m\delta\phi - \delta\dot{\phi}\partial_j\chi_2\partial^j\delta\phi$$
$$+\left(\alpha_1^2\alpha_2 - \frac{1}{2}\alpha_2^2\right)(-6H^2+\dot{\phi}^2) + \frac{\alpha_1}{2}\Big\{-\frac{1}{3}V_{,\phi\phi\phi}\delta\phi^3 - 2\alpha_1^2 V_{,\phi}\delta\phi$$
$$+\alpha_1(-\partial^i\delta\phi\partial_i\delta\phi - V_{,\phi\phi}\delta\phi^2) - 2\partial_i\partial_j\chi_2\partial^i\partial^j\chi_1 + 2\partial^2\chi_2\partial^2\chi_1$$
$$+2\dot{\phi}\partial_j\chi_2\partial^j\delta\phi + 2\delta\dot{\phi}\partial_j\chi_1\partial^j\delta\phi\Big\}\Big].$$

$$\alpha_2 = \frac{\dot{\phi}_c^2}{8H^2}\delta\phi^2 + F(\delta\phi,\dot{\phi}),$$

[MSS]

[Seery,Lidsey,MSS]

$$\partial^2\chi_2 = \frac{3}{8}\frac{\dot{\phi}_c^2}{H}\delta\phi^2 + \frac{3}{4}\frac{\ddot{\phi}_c}{\dot{\phi}_c}\delta\phi^2 - \frac{a^2}{4H}(\partial\delta\phi)^2 - \frac{1}{4H}\dot{\delta\phi}^2 + \frac{\dot{\phi}_c}{2H}\partial_i\chi_1\partial_i\delta\phi$$
$$+\frac{1}{4H}\left((\partial^2\chi_1)^2 - (\partial_i\partial_j\chi_1)^2\right) - \frac{V}{H}F(\delta\phi,\dot{\delta\phi}),$$

$$F(\delta\phi,\delta\dot{\phi}) = \frac{1}{2H}\partial^{-2}\left[\partial^2\alpha_1\partial^2\chi_1 - \partial_i\partial_j\alpha_1\partial_i\partial_j\chi_1 + \partial_i\dot{\delta\phi}\partial_i\delta\phi + \dot{\delta\phi}\partial^2\delta\phi\right].$$

Tree - level stuff

- $S_2 \sim \mathcal{O}(1)$ => $\langle \delta\phi^2 \rangle \propto H^2$
  $\zeta = \epsilon^{-1/2}\delta\phi/M_p$ => $\langle \zeta^2 \rangle \propto \epsilon^{-1}(H/M_p)^2$

- $S_3 \sim \mathcal{O}(\epsilon^{1/2})$ => $\langle \delta\phi^3 \rangle \propto \epsilon^{1/2} H^4$
  => $\langle \zeta^3 \rangle \propto \epsilon^{-1}(H/M_p)^4 \propto \epsilon \mathcal{P}_\zeta^2$ => $f_{NL} \approx \varepsilon$

[Maldacena]

- $S_4 \sim \mathcal{O}(1)$ => $\langle \delta\phi^4 \rangle \propto H^6$
  => $\langle \zeta^4 \rangle \propto \epsilon^{-2}(H/M_p)^6 \propto \epsilon \mathcal{P}_\zeta^3$ => $\tau_{NL} \approx \varepsilon$

[Seery,Lidsey,MSS]

Observational bounds

Present:

$$|f_{NL}| \leq 100 \qquad |\tau_{NL}| \leq 10^8$$

In foreseeable future we can probe:

$$|f_{NL}| \sim 3 \qquad |\tau_{NL}| \sim 560$$

[Komatsu,Spergel] [Kogo,Komatsu]

Loop corrections quantitatively

From S_4 we can calculate the loop corrected power-spectrum of inflaton perturbations in the "in-in" formalism

$$\langle \zeta^4(t) \rangle = \langle U_{int}^{-1} \zeta^4(t) U_{int}(t,t_0) \rangle \ , \qquad U_{int} = T\ e^{-i\int_{t_0}^{t} H_{int}(t')dt'}$$

[Schwinger-Keldysh]

↓

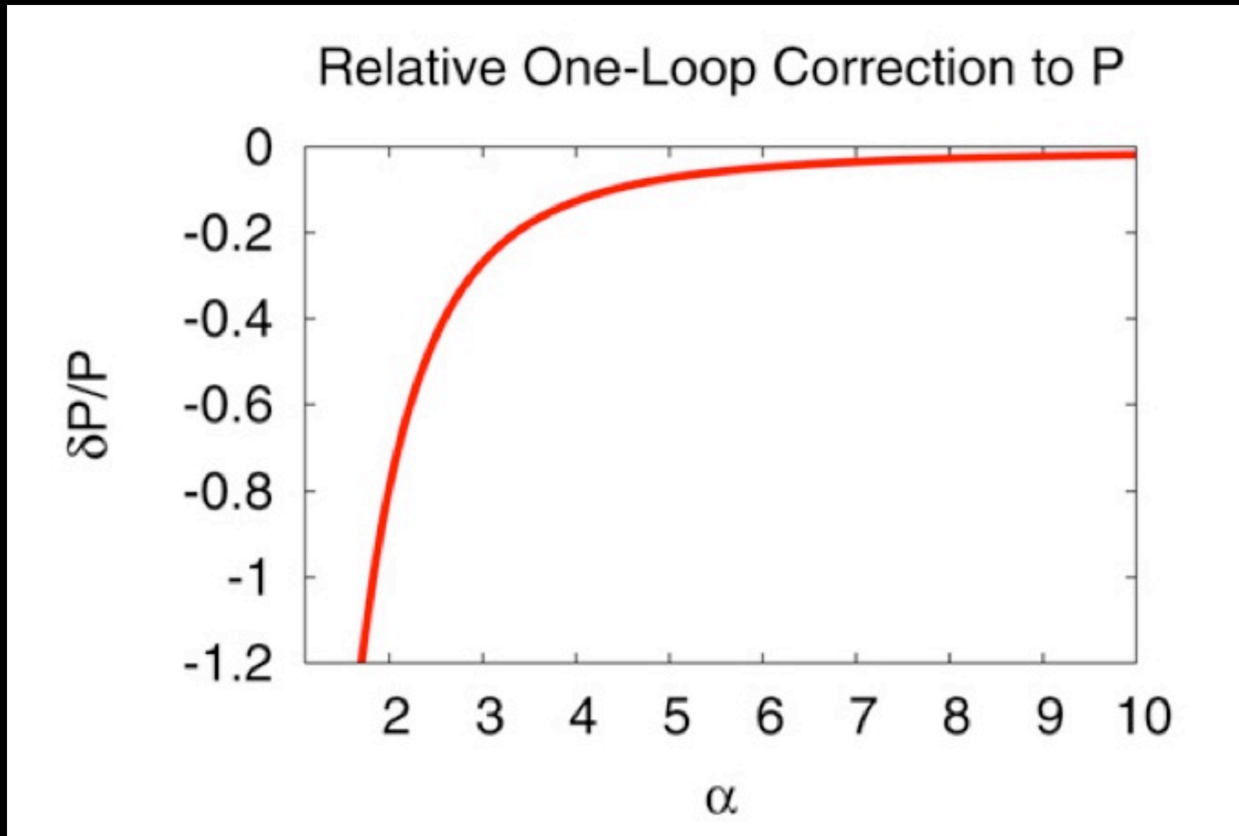
$$\mathcal{P}(\eta_0, k) \approx \frac{H^2}{4\pi^2}\left[1 - \left(\frac{1}{16}\epsilon + \frac{1}{2}(2\epsilon - \eta) - \frac{3}{8}(2\epsilon - \eta)\,\mathrm{Ci}(-2k\eta_0)\right)\langle \delta\phi^2 \rangle\right],$$

[MSS]

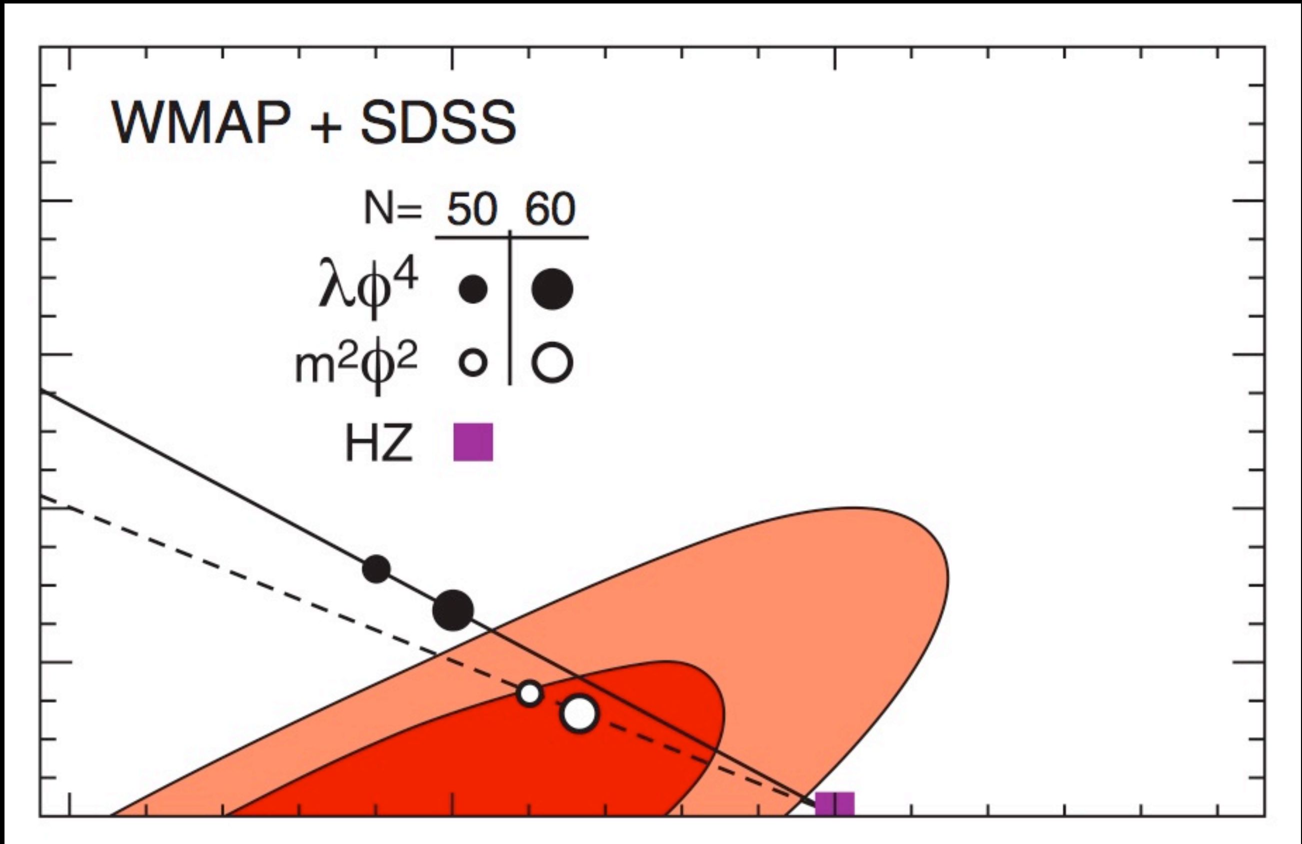
$$\langle \delta\phi^2 \rangle = \int_{a_i H_i}^{a\Lambda} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k) \propto N^{(4+\alpha)/2} \ , \qquad N \approx \frac{1}{2\alpha M_p^2}\phi_i^2$$

$$\Delta\phi = \dot{\phi}\Delta t < \delta\phi \Rightarrow \phi_i < \lambda^{-1/6} M_p \ , \qquad \Delta t = 1/H$$

Loop corrections quantitatively



Experimental constraints on chaotic inflation



"Vanilla model"
Ω_b
Ω_m
$\Omega_\Lambda = 1 - \Omega_m$
H_0
n_s
A_s
τ

Figure from WMAP III data paper [Spergel et. al.]

Effect of neutrinos

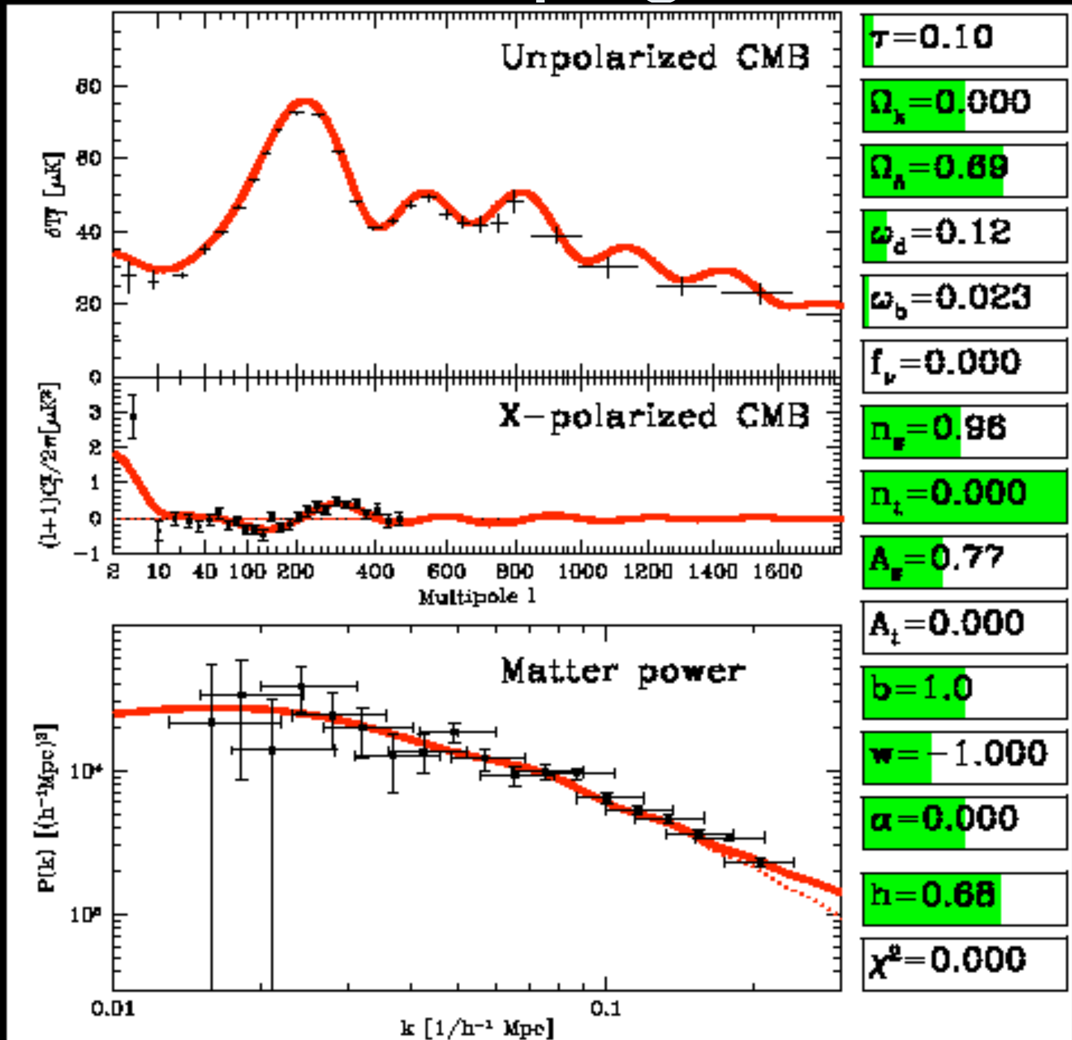
- Neutrinos contribute to DM
- However, neutrinos free-stream until they become non-rel. and get trapped in pot. wells
- Thus, on scales smaller than the horizon, when they become non-rel., they suppress structure formation

=> Suppression of of matter power-spectrum at small scales

$$d_{\mathrm{FS}} \sim 1\ \mathrm{Gpc}\ m_{\mathrm{eV}}^{-1}$$

Shape of angular and matter power spectrum - movie from Tegmark's homepage

Indirect effect on CMB opposite of grav. waves which increase power at large scales



Degeneracy

Effect of neutrinos

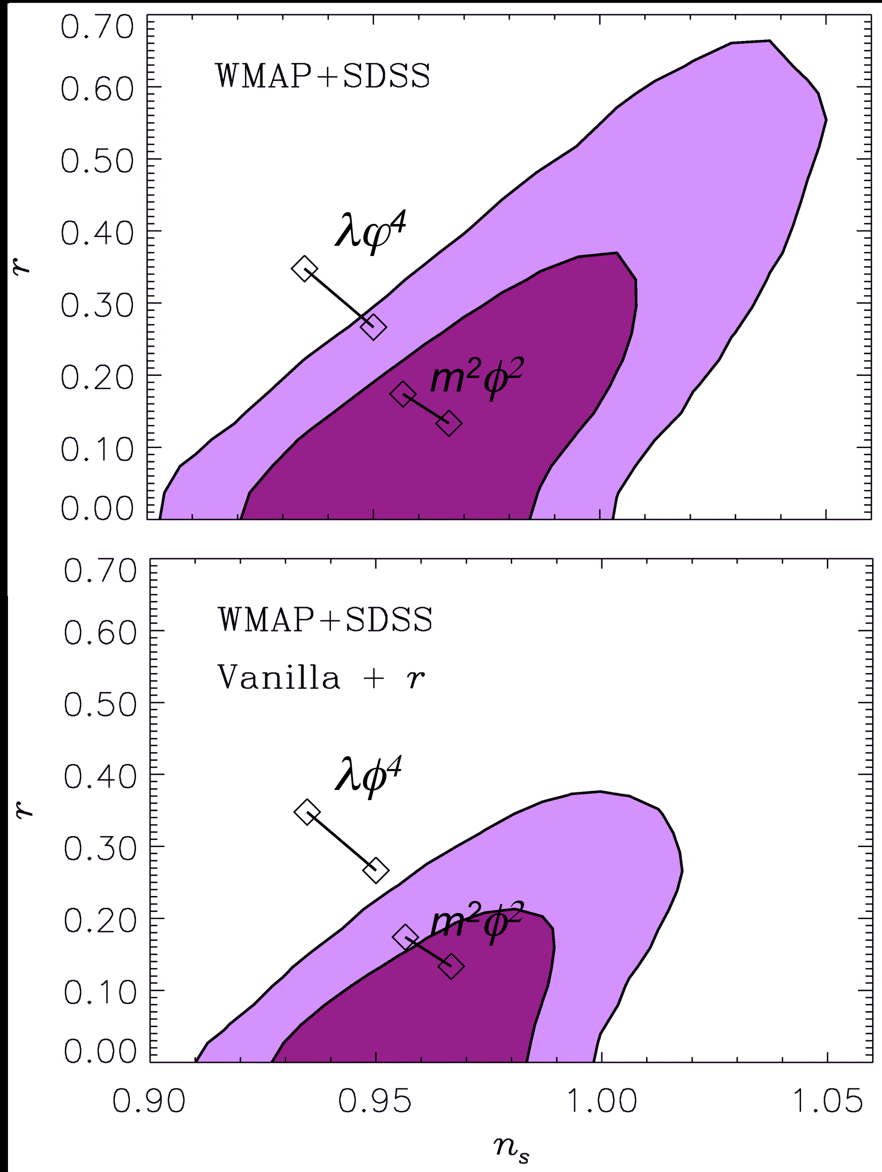
UNDERSTANDING NEUTRINO MASSES IS IMPORTANT FOR CONSTRAINING OTHER COSMOLOGICAL PARAMETERS!

ALLOWING FOR A NON-ZERO NEUTRINO MASS MAKES THE SIMPLEST $\lambda\phi^4$ MODEL COMPATIBLE WITH DATA!

↕

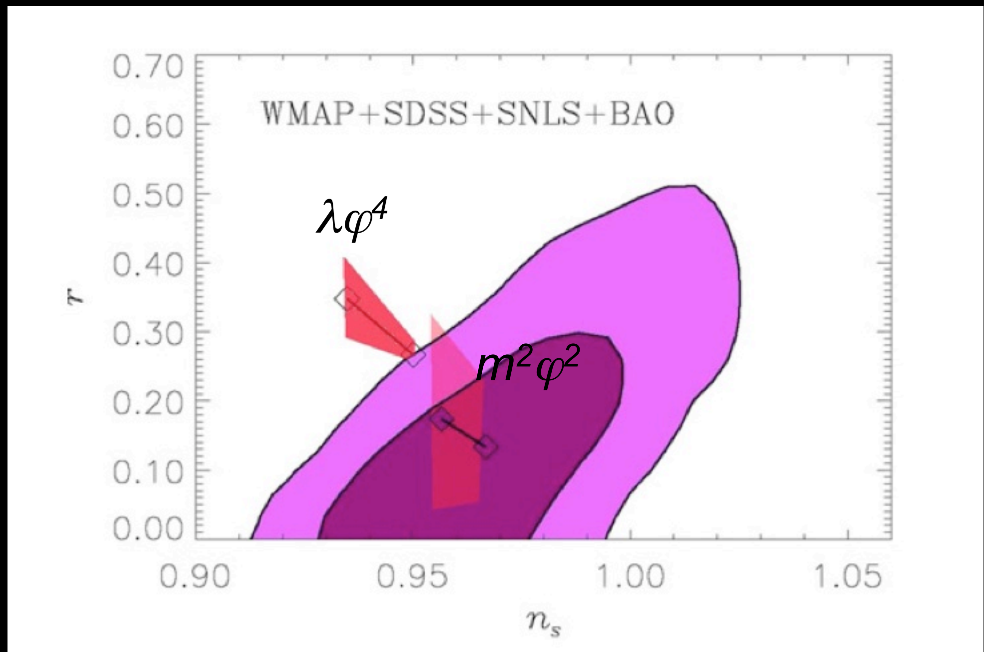
$\lambda\varphi^4$ predicts neutrino masses ~ 0.3 - 0.5 eV

[Hamann, Hannestad, MSS, Wong]



Observational implications of loop corrections

- Loop corrections shift the overall amplitude of power-spectrum
- Not observable from scalar spectrum alone
- May affect tensor-to-scalar relation



[MSS]

Conclusions

- Quantum contributions to cosmological correlations can probe the full history of the universe (not just at horizon crossing)
- In some models of inflation this may indirectly give us information about the initial conditions for inflation
- $\lambda\varphi^4$ is not excluded by data - yet!
- Loop corrections may make it less trivial to exclude chaotic inflation