The Entropic Approach to Understanding the Cosmological Constant

Jim Cline, McGill University
with Andrew Frey and Gil Holder

CERN, 8 August 2007

Outline

- History of the cosmological constant problem
- The anthropic approach: successes and shortcomings
- A new approach: the causal entropic principle
- New results
- A connection to dark matter?

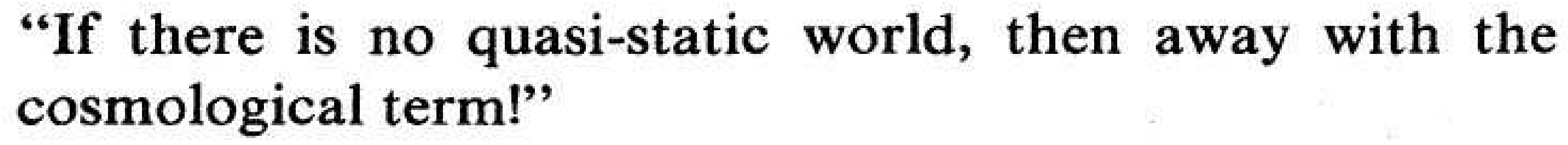
History of the Cosmological Constant

1917: Einstein introduces Λ to obtain a static universe

$$\mathcal{L} = \frac{1}{16\pi G}\left(R - 2\Lambda\right)$$

1912–1924: Slipher measures redshifts of galaxies,
suggesting expanding universe before Hubble (1929)

1923: Einstein writes to Weyl,



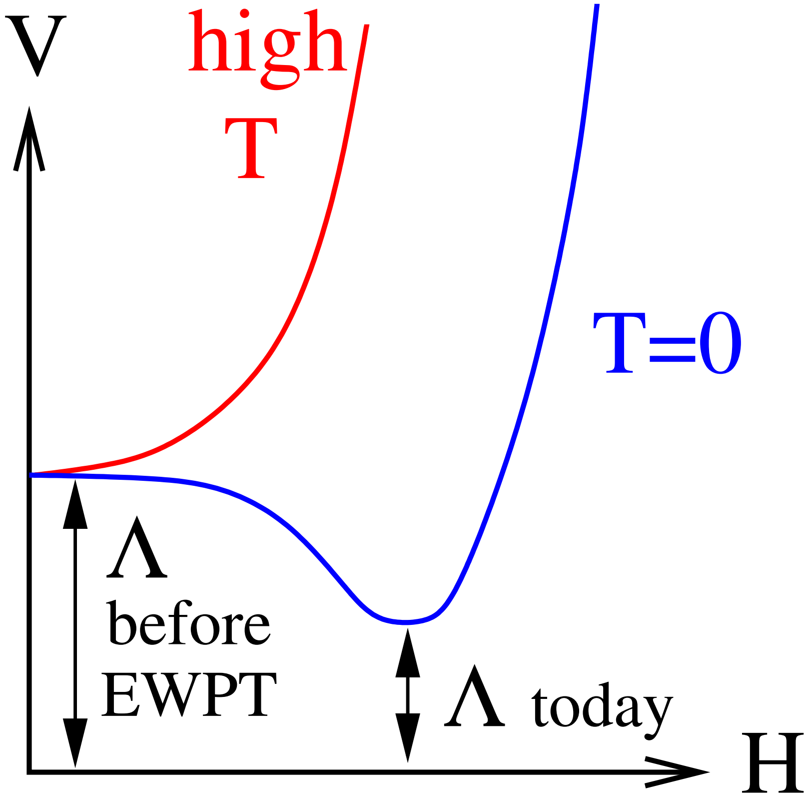
"If there is no quasi-static world, then away with the cosmological term!"

1967-1968: Λ invoked to get loitering universe, to explain
excess of quasars at $z = 1.95$

1967: Zeldovich proposes quantum fluctuations
as origin of Λ, but they give too big result!

History of the Cosmological Constant

1970's: spontaneous symmetry breaking in electroweak theory forces particle theorists to take vacuum energy more seriously



Early 1980's: first attempts to explain why Λ is small.
A. Zee refers to the Λ "problem"

History of the Cosmological Constant

1984: Hawking proposes probability distribution for Λ peaked at zero, from Euclidean quantum gravity,

$$P \sim \exp\left(M_p^2/\Lambda\right)$$

1986-1988: Linde's eternal inflation provides universes with different values of Λ

1987: Weinberg derives anthropic bound on Λ (but Hawking discussed idea at conference in 1981)

1988: Coleman's wormhole approach (> 500 citations) $\Rightarrow$

1987-1988: Brown and Teitelboim suggest relaxation of Λ through nucleation of bubbles with successively smaller values of Λ

1989: Weinberg's no-go theorem against smooth relaxation of $\Lambda \to 0$ (in review article, 1300 citations)

Coleman’s wormhole mechanism

S Coleman / Cosmological constant

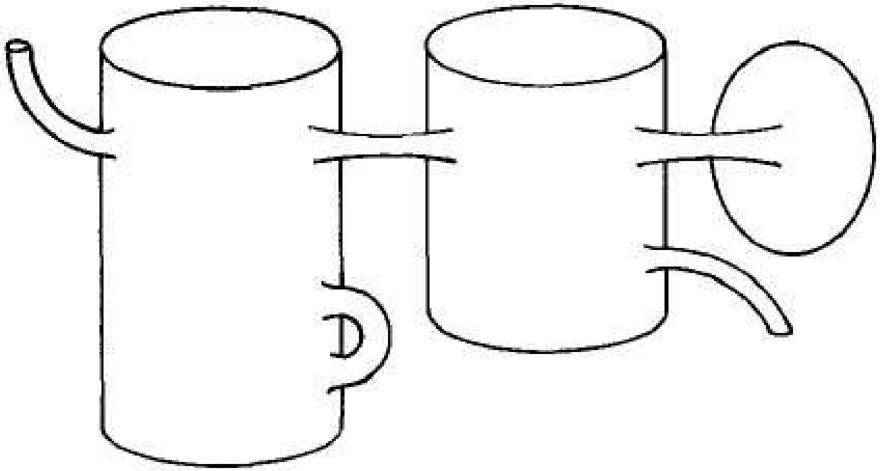


Fig 3 A manifold with wormholes and baby universes

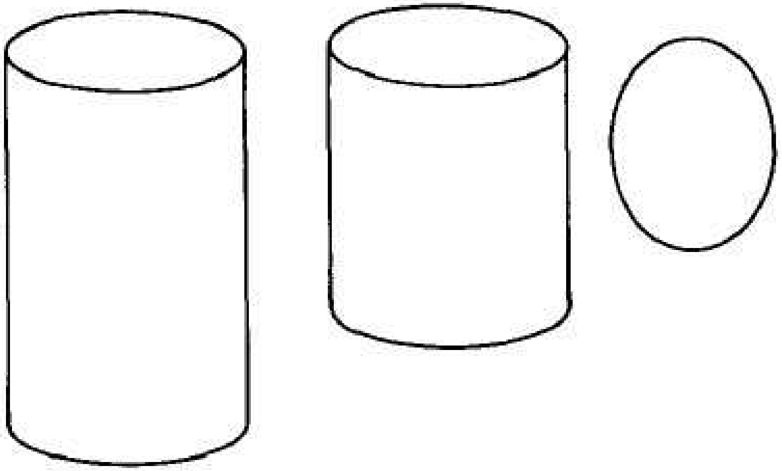


Fig 4 The manifold of fig 3, with wormholes and baby universes stripped away

Wormholes → superselection sectors with different values of Λ

Probability distribution for Λ:

$$P \sim e^{e^{1/\Lambda}}$$

Other constants of nature also predicted, *e.g.* $\theta_{\text{QCD}} = \pi$

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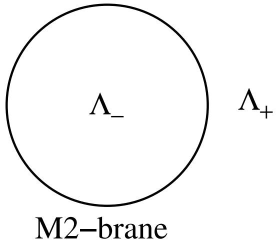
History of the Cosmological Constant

1990-91: Efstathiou, Turner propose ΛCDM universe to make $\Omega_{\rm CDM} \sim 0.2$ consistent with $\Omega = 1$, as already indicated by CMB. Astronomers don't like it!

1998: High-z type Ia supernovae establish $\Lambda > 0$ (> 3000 citations); also CMB and galaxy clusters $\Rightarrow$

1999: Huterer and Turner coin the term "Dark Energy"

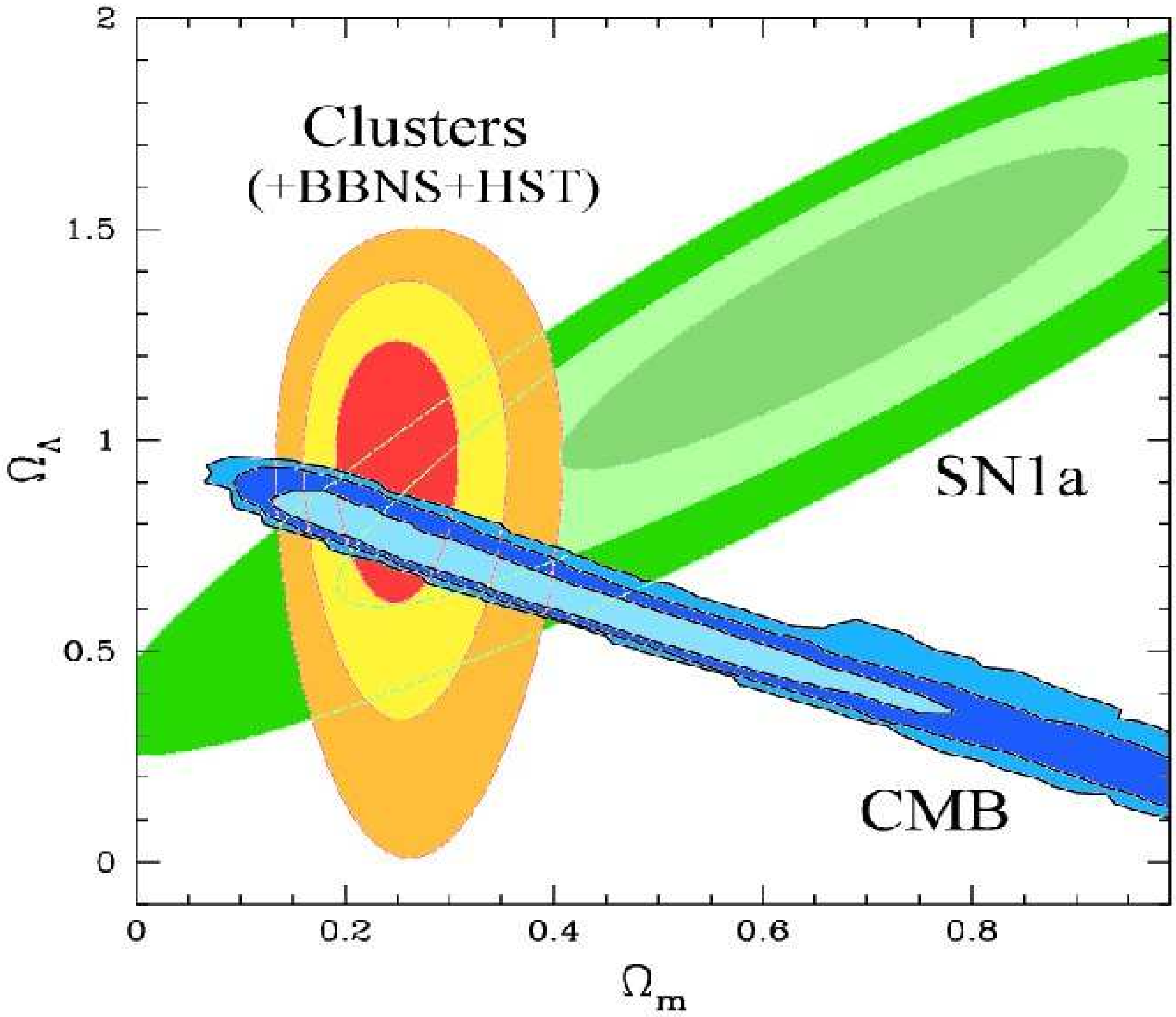
2000: Bousso and Polchinski realize the mechanism of Brown and Teitelboim within string theory



2000 – present: many authors try to use braneworlds to self-tune $\Lambda \to 0$. But this violates Weinberg's no-go theorem.

Observational evidence for Λ

Turner and Huterer, 2007



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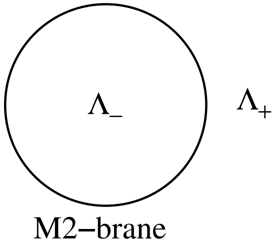
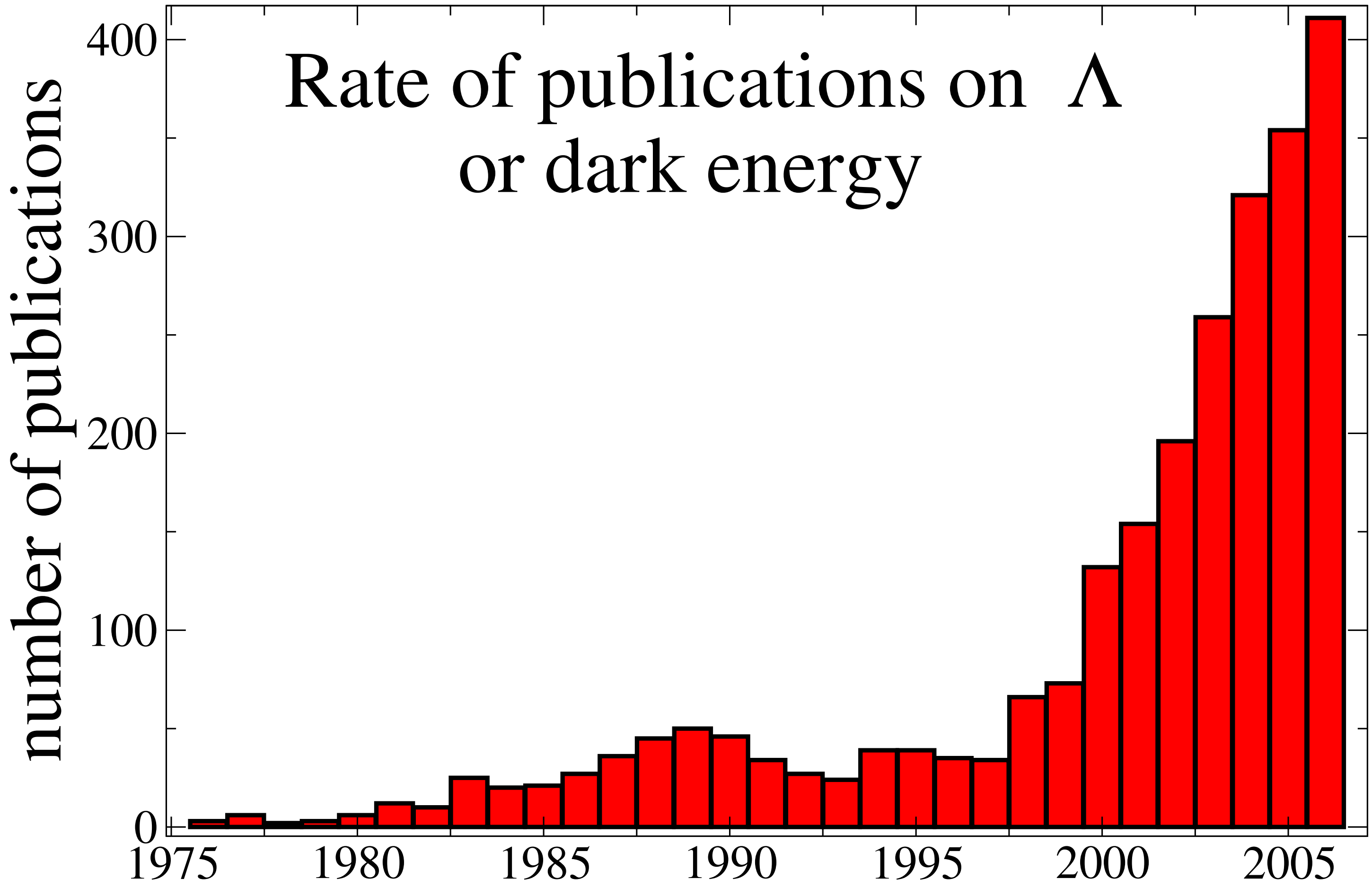


2000 – present: many authors try to use braneworlds to self-tune $\Lambda \to 0$. But this violates Weinberg's no-go theorem.

History of the Cosmological Constant

2003: Susskind notes that string theory landscape justifies existence of universes with many possible values of Λ.

Douglas counts $\sim 10^{500}$ string theory vacua—can explain existence of universes with $\Lambda \sim 10^{-123}\, M_p^2$



The lesson of history

What do we learn from these recollections?

- Much effort (and $$$) is being spent on understanding Λ
- Self-tuning where $\Lambda \to 0$ continuously is ruled out
- The most promising ideas involve ensemble of universes with different values of Λ, and probability distribution $P(\Lambda)$.
- The probabalistic view cannot be be divorced from anthropic considerations since

$$P_{\rm obs}(\Lambda) = P_{\rm prior}(\Lambda) \times (\text{Probability to have observers})$$

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The Anthropic Approach

There is no agreed-upon explanation of the hierarchy

$$\frac{\rho_\Lambda}{M_p^4} \cong 10^{-123}$$

but one idea predicted such a value before its measurement: the anthropic principle (Weinberg, 1987). Too-fast expansion (or collapse) of universe prevents formation of structure at redshift z_c unless

$$-\rho_{\mathrm{m},0} \;<\; \rho_\Lambda \;<\; 3(1+z_c)^3 \rho_{\mathrm{m},0} \;\sim\; 400\,\rho_{\mathrm{m},0}$$

for $z_c = 4$. If prior probability distribution for Λ is flat, then probability of our universe is

$$P(\rho_\Lambda \lesssim \rho_{\mathrm{m},0}) \cong \frac{1}{400} = 0.25\%$$

Not as unlikely as $\frac{1}{10^{123}}$!

Problems with Anthropic Approach

We now know of dwarf galaxies which formed at $z = 10$.
This weakens anthropic constraint to

$$\rho_\Lambda \; < \; 5000\,\rho_{\mathrm{m},0}$$

Our universe looks less likely.
What if other quantities are allowed to vary, like density contrast

$$Q = \frac{\delta\rho}{\rho} \text{ from inflation } \; \cong \; 2 \times 10^{-5} \;?$$

Structure can form more easily, despite fast expansion.
Tegmark and Rees (1997): bound on Λ is further weakened,

$$\rho_\Lambda \; < \; 5000 \left(\frac{Q}{Q_{\mathrm{obs}}} \right)^3 \rho_{\mathrm{m},0}$$

If $Q \rightarrow 0.1$, then [bound on ρ_Λ] $\rightarrow 10^{14} \rho_{\mathrm{m},0}$

Problems with Anthropic Approach

To save anthropic explanation, need further assumptions, *e.g.,* life requires galaxies of some minimum mass (heavy elements, needed for life, not retained if $M_{\rm galaxy} < 10^9 M_\odot$).

Such assumptions seem arbitrary and hard to justify. Perhaps life can exist without carbon or water? Perhaps “observers” completely unlike us can exist?

Argument in favor of anthropics

If Λ_{obs} were 0, a dynamical mechanism for explaining it would be likely.

But why should a physics mechanism coincidentally predict Λ in the bizarre anthropic range?

That sounds more anthropocentric than admitting our existence is a data point!

Perhaps $\Lambda = 0$ and dark energy is quintessence, but I think quintessence creates more problems than Λ by itself.

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The Causal Entropic Principle

Idea suggested by

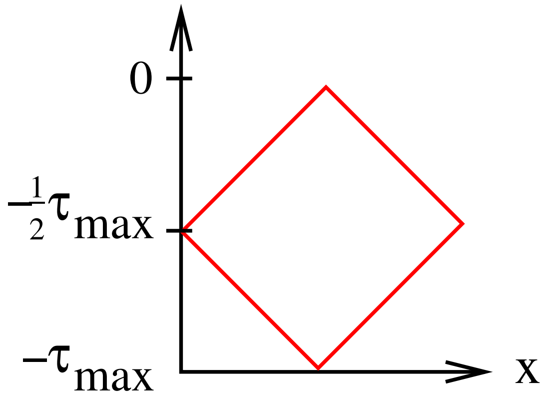
R. Bousso, R. Harnik, G. D. Kribs and G. Perez,
“Predicting the Cosmological Constant from the Causal
Entropic Principle,” arXiv:hep-th/0702115.

A replacement for the anthropic principle:

$\big[$ Probability of finding observers $\big] \propto$
$\big[$ entropy produced in causally connected region of universe $\big]$

- Simple, quantitative, makes minimal assumptions about nature of observers;
- Circumvents measure problem of eternal inflation

The Causal Diamond



- Conformal time $ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^{\,2})$
- Start at some early time, *e.g.*, reheating
- End at $t = \infty$, but τ is finite, due to de Sitter horizon,

In Planck units,

$$\tau_{\max} = \int \frac{dt}{a(t)} \sim 2.8\, t_\Lambda^{1/3} \equiv 2.8 \left(\frac{3}{\Lambda}\right)^{1/6}$$

assuming universe is first matter-, then Λ-dominated,

$$a(t) = \left[t_\Lambda \sinh\left(\frac{3t}{2\Lambda}\right)\right]^{2/3} .$$

Comoving volume of diamond $V_c = \frac{4\pi}{3} r^3(\tau)$ is finite at all times

Entropic Principle

Amount of entropy created inside the diamond determines probability for evolving complexity, structure, observers:

$$\frac{dP}{d\Lambda} = \Delta S(\Lambda)$$

ΔS indicates free energy was available, needed for making measurements. Minimal assumption: observers must obey laws of thermodynamics!

Oppposite situation: universe which remains in thermal equilibrium. No complexity can arise.

Same approach might be used to predict other variables besides Λ, if they vary over landscape.

How to compute ΔS

Product of two factors:

$$\Delta S = \int dt\, V_c(t)\, \frac{dS}{dV_c dt}$$

$V_c =$ comoving volume of causal diamond; determined by cosmology;

$\frac{dS}{dV_c dt}$ = entropy production rate per unit volume; depends on astrophysics. Sources of entropy:

Stars:	$\Delta S_* \cong 10^6$ per baryon
Active Galactic Nuclei:	$< \frac{1}{6}\, \Delta S_*$
Supernovae:	$< \frac{1}{10}\, \Delta S_*$
Galaxy cooling:	$< \frac{1}{100}\, \Delta S_*$

Entropy production by stars

Starlight is the biggest source of entropy:

1 UV γ scattered by dust $\Longrightarrow\ > 100$ IR (20 meV) γ's.

Rate of entropy production per comoving volume:

$$\frac{dS}{dV_c\,dt}(t) = \int_0^t dt'\,\frac{d^2S}{dM_* dt}(t-t')\,\dot\rho_*(t')$$

$\frac{d^2S}{dM_* dt}(t-t')$ = rate of entropy production per stellar mass, at time t, from stars born at time t',

$$\frac{d^2S}{dM_* dt}(t-t') = \frac{1}{\langle M\rangle}\int_{0.08M_\odot}^{M_{\max}(t-t')} dM\,\xi_{\rm IMF}(M)\,\frac{d^2s}{dN_* dt}$$

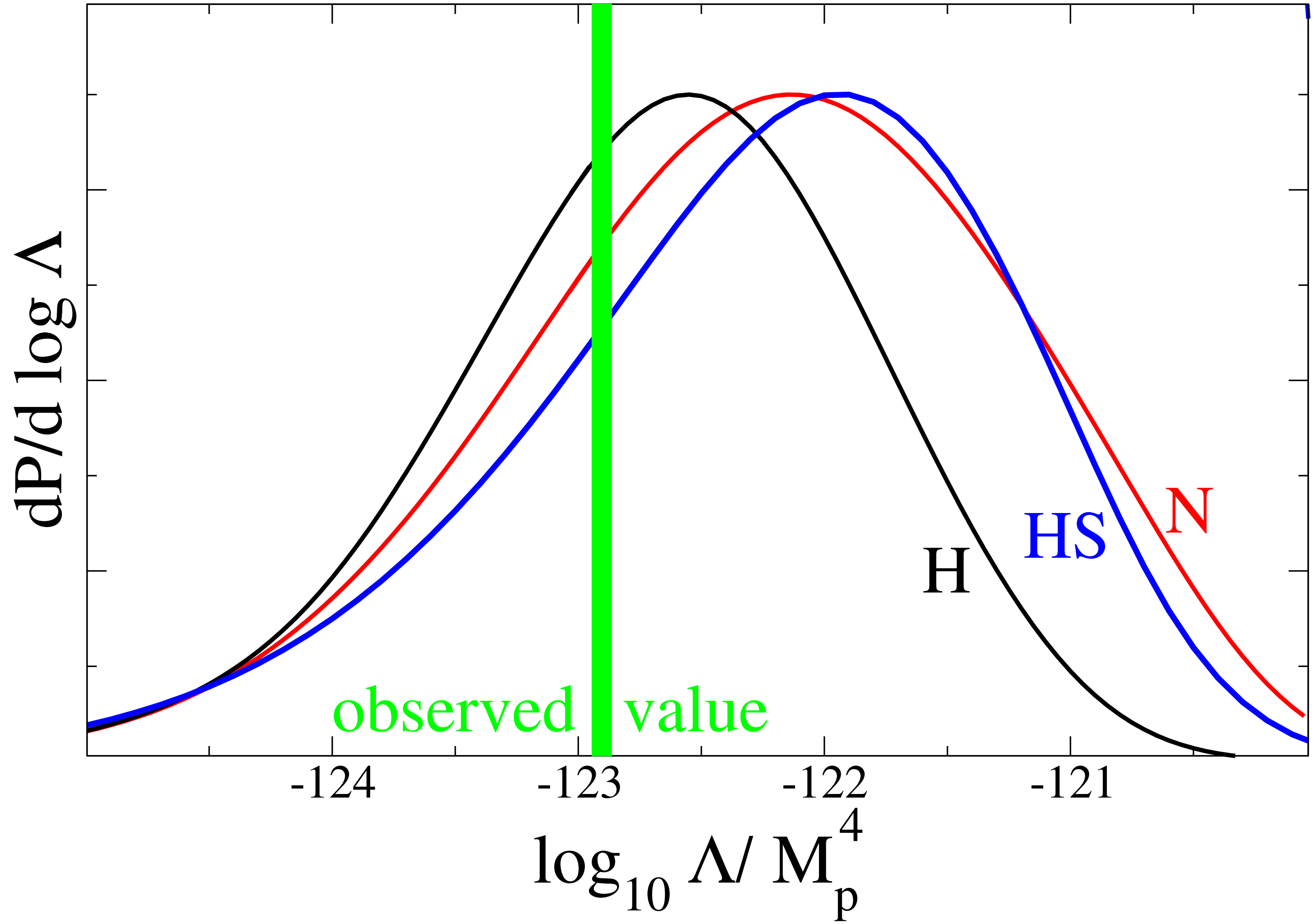
$\dot\rho_* =$ stellar mass formation rate

$\xi_{\rm IMF} =$ “initial mass function” $\sim M^{-2.35}$

$\frac{d^2s}{dN_* dt} =$ rate per star $= L_*/T_{\rm eff} \sim (M/M_\odot)^{3.5}\times 10^{54}$ yr^{-1}

Probability Distributions for Λ

For different published star formation rates $\dot{\rho}_*$
(Hopkins-Beacom, Nagamine *et al.*, Hernquist-Springel)



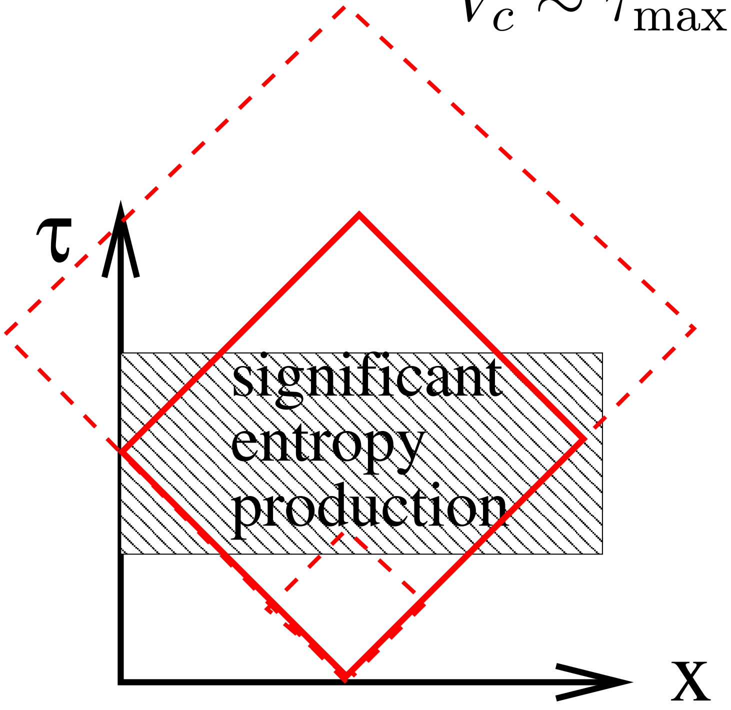
Probability to observe $\Lambda = 10^{-123} M_p^4$ is $O(1)$!

Basic predictions are robust

The basic prediction remains unaffected by details of $\dot{\rho}_*$: $dP/d\log\Lambda$ peaks near Λ_{obs}.

Easy to understand: Main Λ-dependence is through volume of causal diamond,

$$V_c \sim \tau_{\max} \sim \frac{1}{\sqrt{\Lambda}}, \quad \int dt\, V_c \sim \frac{1}{\Lambda}$$



Preferred Λ has time of maximum diamond volume coinciding with time of maximum entropy production by stars.

Contrast with anthropic prediction

Suppose Λ_a is anthropic cutoff on Λ:

$$\frac{dP}{d\Lambda} = \begin{cases} \text{const.}, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases}$$

Then $$\frac{dP}{d\log\Lambda} = \begin{cases} c\Lambda, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases}$$

is sharply peaked at Λ_a.

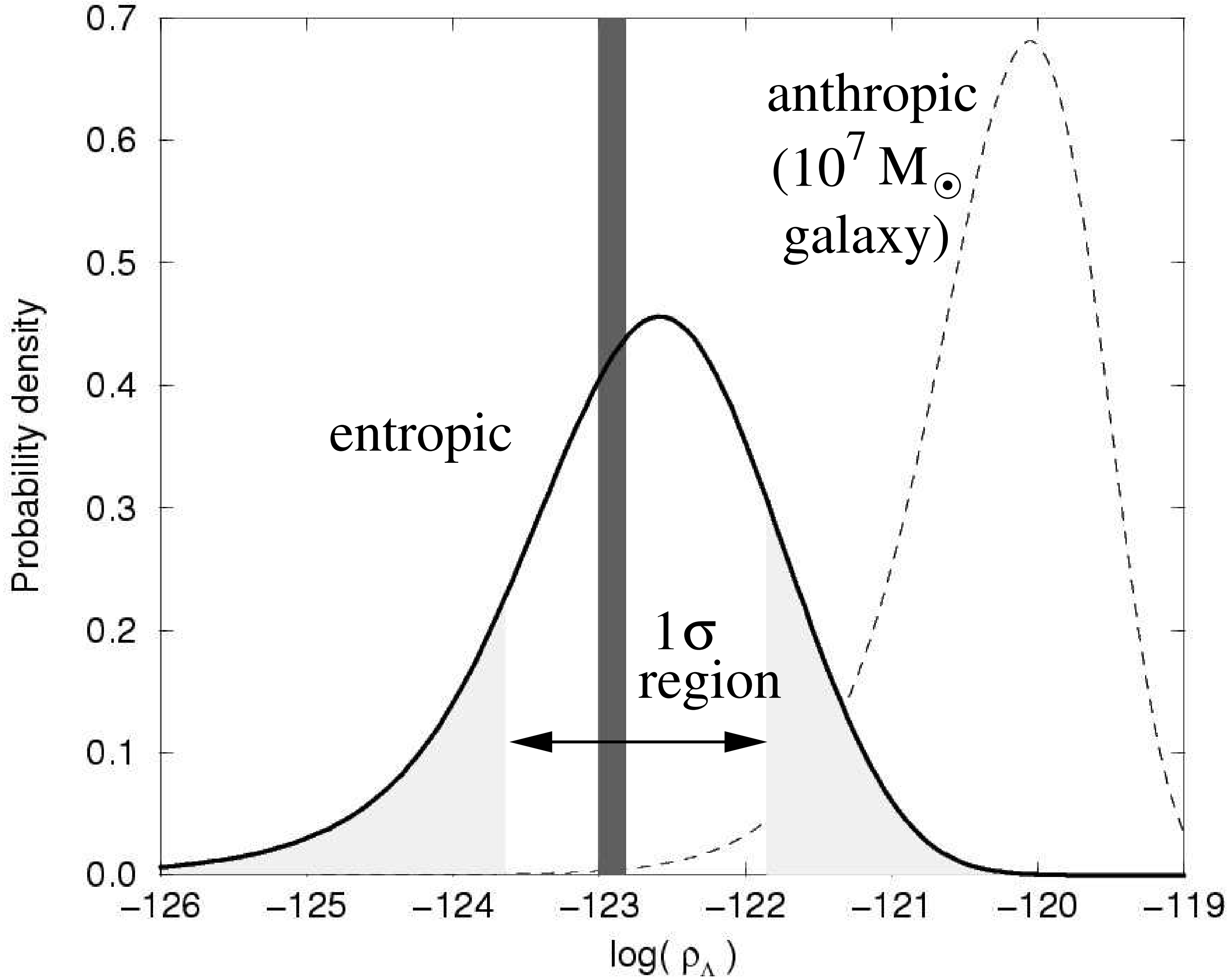
Causal diamond volume's Λ-dependence changes this to

$$\frac{dP}{d\Lambda} = \begin{cases} \frac{c}{\Lambda}, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases} \quad \Longrightarrow \quad \frac{dP}{d\log\Lambda} = \begin{cases} c, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases}$$

ignoring time-dependence of $\frac{dS}{dV_c dt}$. Including $\frac{dS}{dV_c dt}$, $c \to c(\Lambda)$, $\frac{dP}{d\log\Lambda}$ is no longer peaked at Λ_a

Contrast with anthropic prediction

from Bousso *et al.*



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How can this idea be further tested?

Other quantities besides Λ should be scanned by the landscape. Can we predict their values? *E.g.,*

Q (density contrast)

k (spatial curvature)

$\Omega_b/\Omega_{\rm DM}$ (baryon to dark matter ratio)

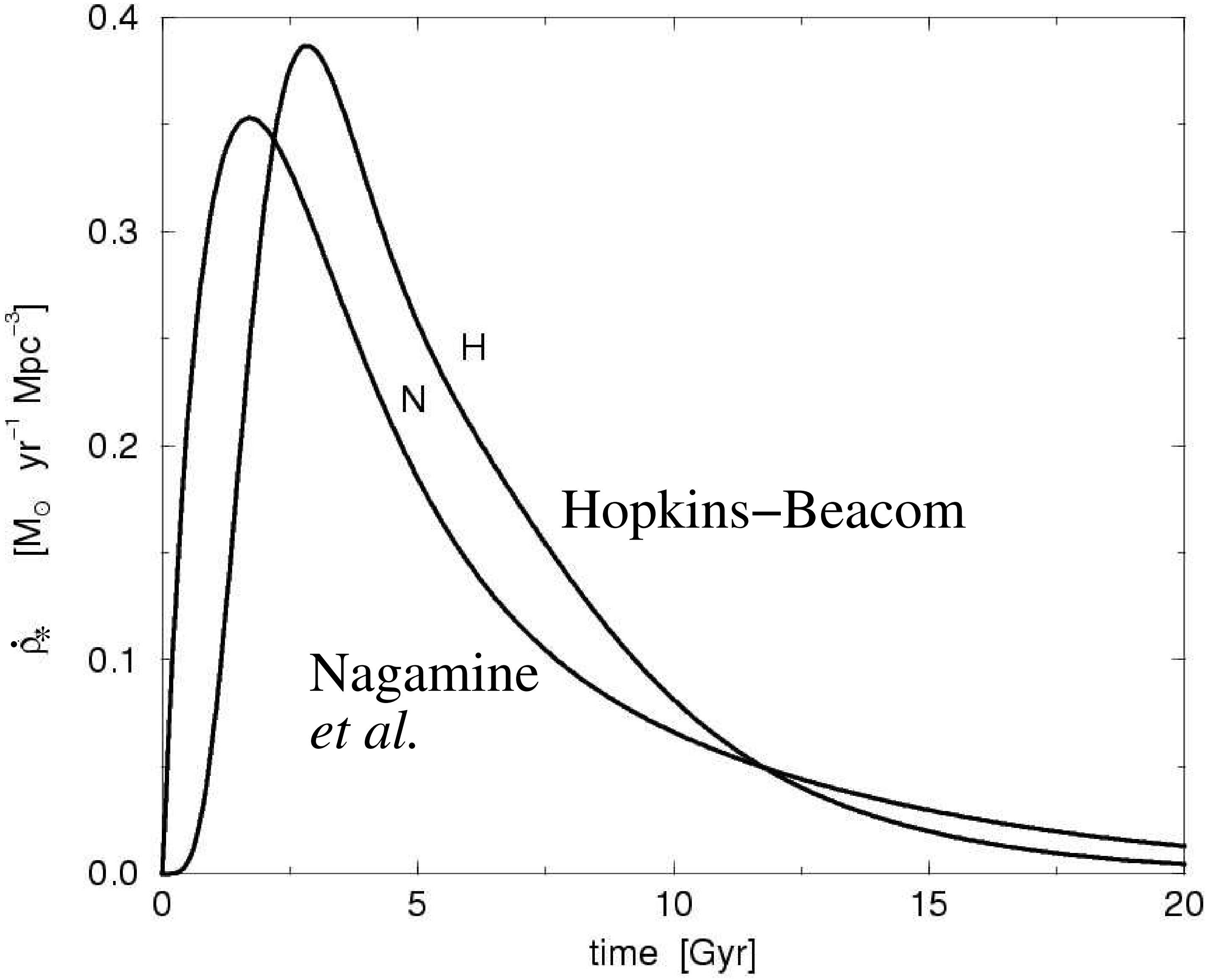
n_m/n_γ (matter to photon ratio)

Start small: explore region of landscape containing universes like ours—ΔS dominated by starlight

Challenge: how does star formation rate $\dot{\rho}_*$ depend on these quantities?

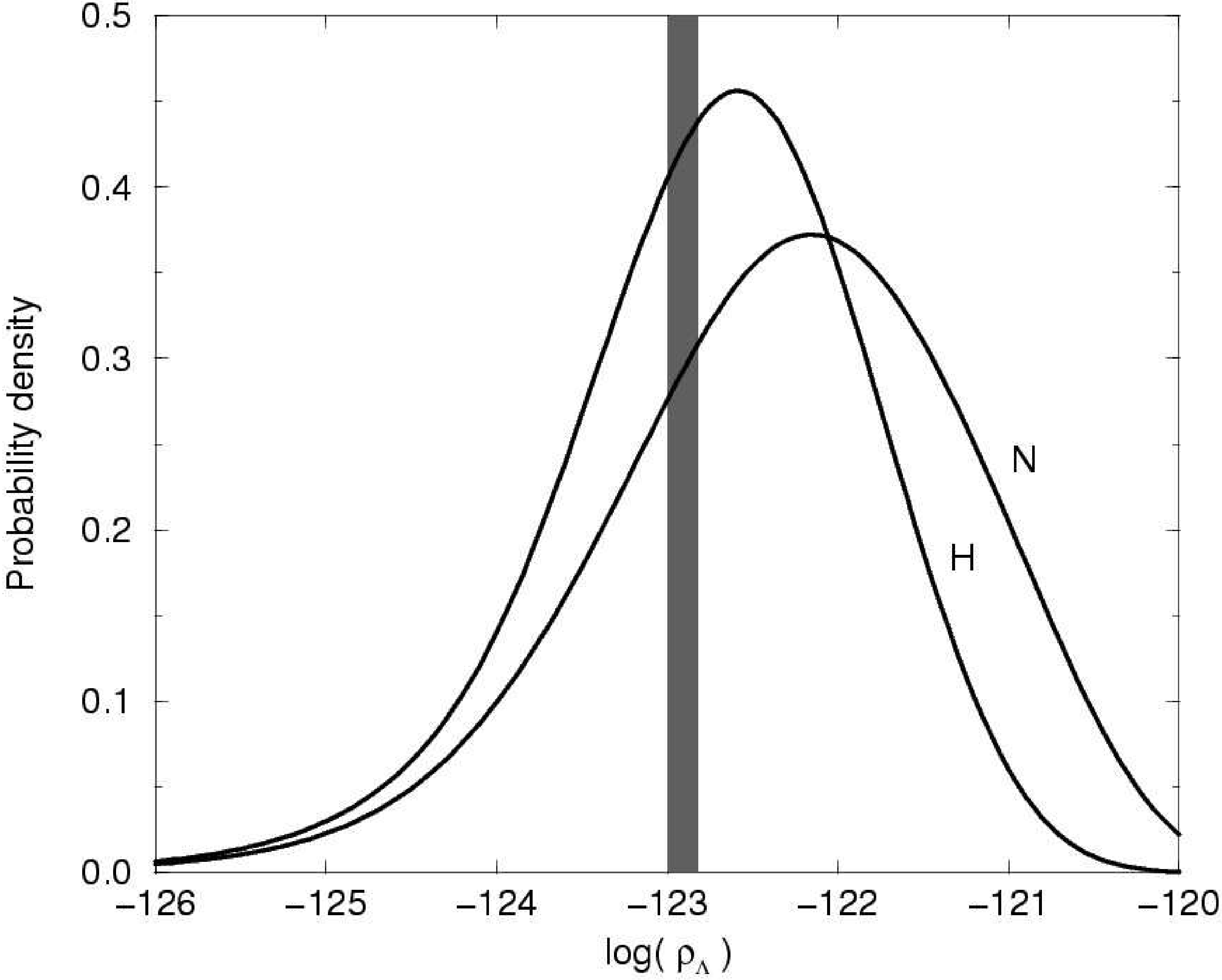
Star Formation History, $\dot{\rho}_*$

Bousso *et al.* tried two published versions of $\dot{\rho}_*$, based on fits to data/simulations,



Star Formation History, $\dot{\rho}_*$

leading to somewhat different predictions for $dP/d\log\Lambda$,



Star Formation History, $\dot{\rho}_*$

However, published SFR's are only valid for value of Λ in *our* universe. How to generalize to other values of Λ?

Bousso *et al.* apply Press-Schechter formalism to $\dot{\rho}_*$ as an overall multiplicative factor,

$$\dot{\rho}_* \longrightarrow F(\Lambda)\dot{\rho}_*$$

F quantifies how efficiently matter collapses as structure forms

Press-Schechter Formalism

Using: statistics of Gaussian distribution of $\delta\rho$ fluctuations,
how fluctuations grow in spherical collapse model;
Obtain: fraction of matter collapsed into DM halos $\geq M$,

$$F(M,t) = \mathrm{erfc}\left[\frac{\delta_c}{\sqrt{2}\,\sigma(M,t)}\right]$$

$\sigma =$ r.m.s. fluctuations of mass M
$\delta_c = 1.686$ from studying growth of perturbations

To model Λ-dependence of $\dot{\rho}_*$, Bousso *et al.* rescale

$$\dot{\rho}_* \;\to\; F(10^7 M_\odot,\; t_{\mathrm{max}})\,\dot{\rho}_*$$

$t_{\mathrm{max}} =$ time when $\dot{\rho}_*$ is maximized, $10^7 M_\odot =$ typical M_{galaxy}

But this is a crude approximation $\ldots$

Hernquist-Springel SFR

Hernquist and Springel (2003) derived star formation rate from first principles + fits to simulations,

$$\dot{\rho}_* \;\propto\; \left[\frac{\chi\bar{\chi}}{(\chi^m + \bar{\chi}^m)^{1/m}}\right]^{9/2\eta} \frac{1}{1 + \frac{5}{2}\sqrt{\pi}u\exp(u^2)}$$

where
$$\chi = \left(\frac{H(t)}{H_0}\right)^{2/3}, \qquad u \;\propto\; \sigma_4(t)$$

and $\sigma_4 =$ r.m.s. density fluctuation at $M = M_4$ where $T_{\rm virial} = 10^4$ K.

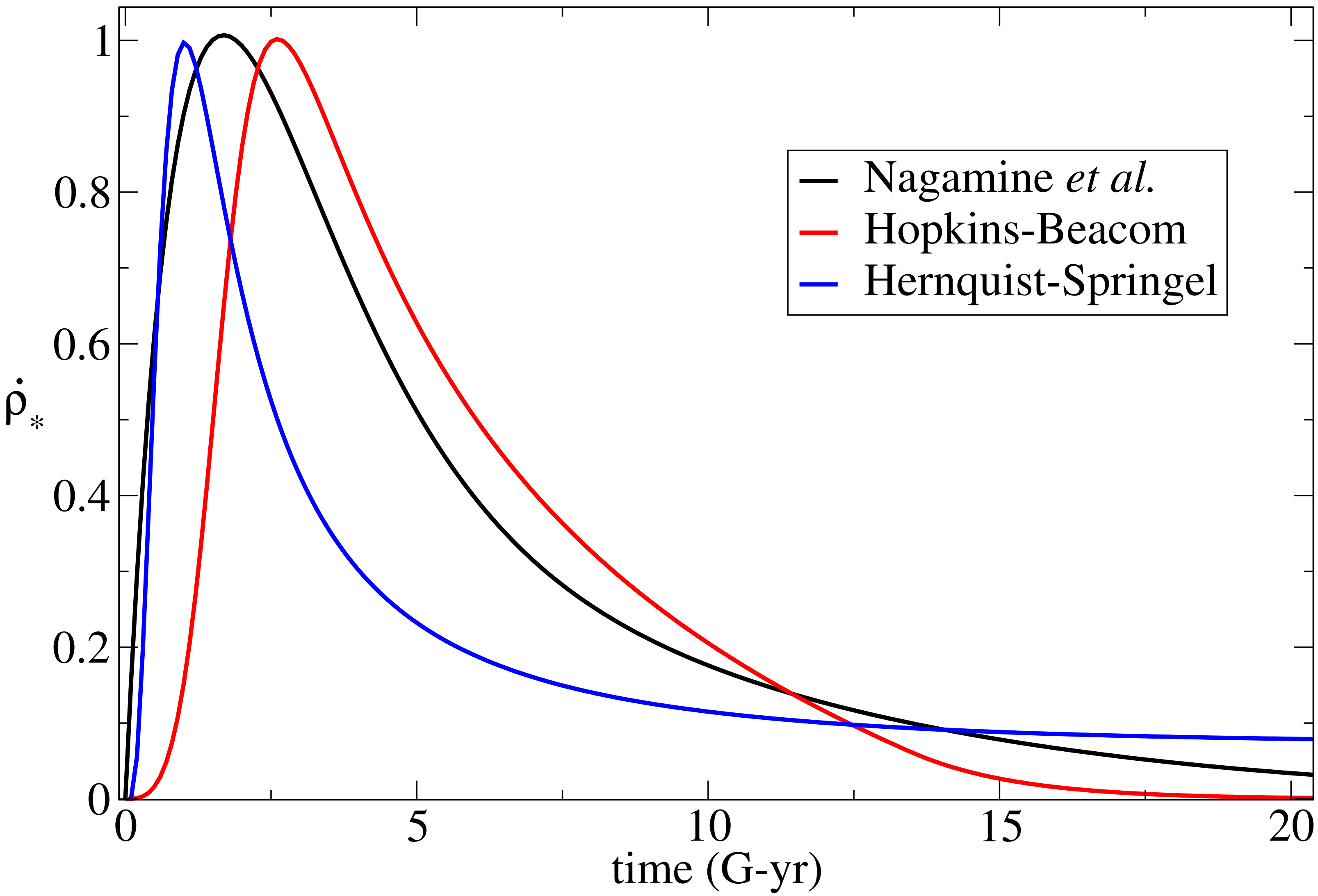
Tegmark, Aguirre, Rees, Wilczek (2005) show that

$$\sigma(M,t) \;\propto\; Q\rho_\Lambda^{-1/3} f(x(t))\, g(M), \quad x(t) \equiv \frac{\rho_\Lambda}{\rho_m(t)}$$

Note $\Lambda \sim Q^3$ to keep σ constant—large Q greatly weakens anthropic bound

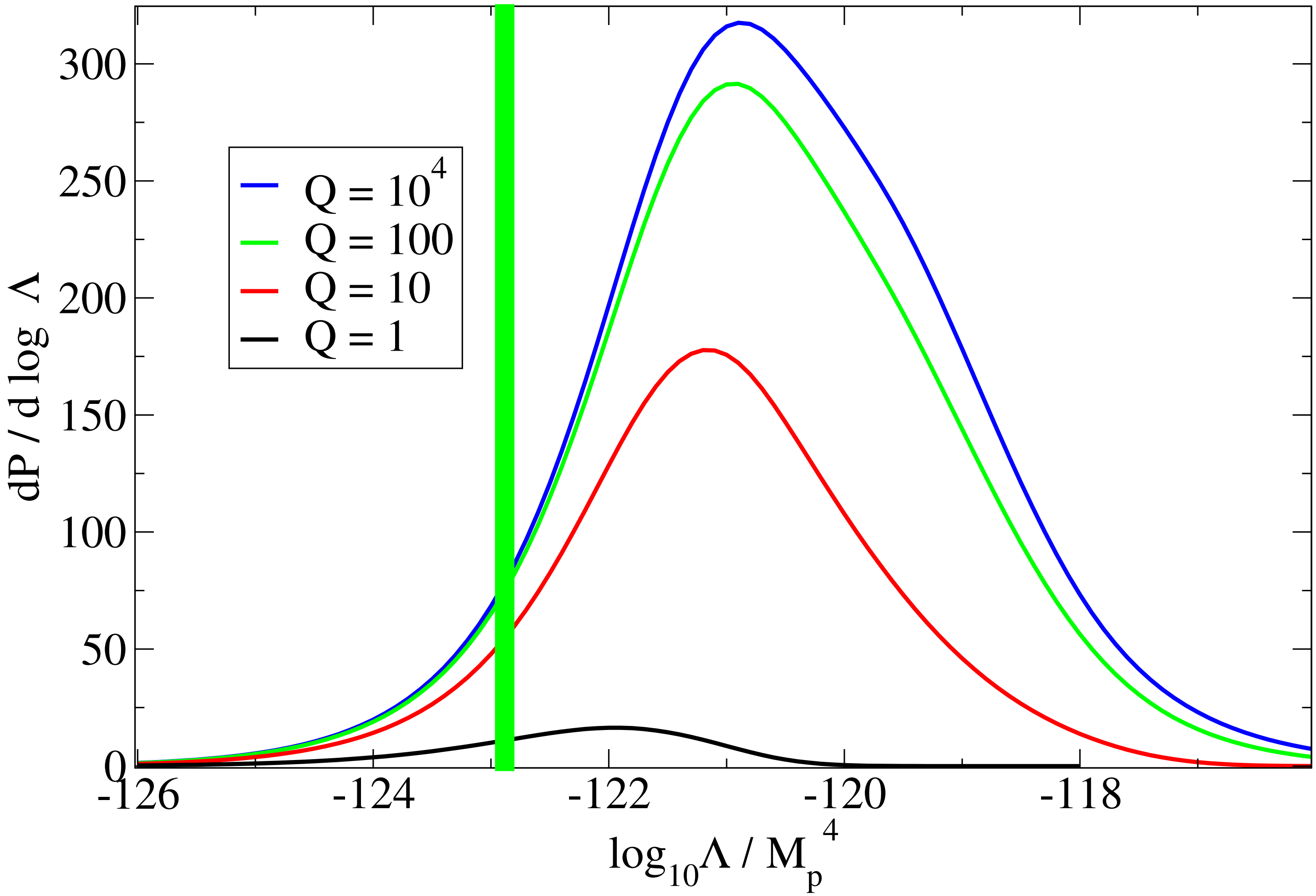
Comparison of SFR’s

The three different star formation rates considered:



Dependence on Density Contrast

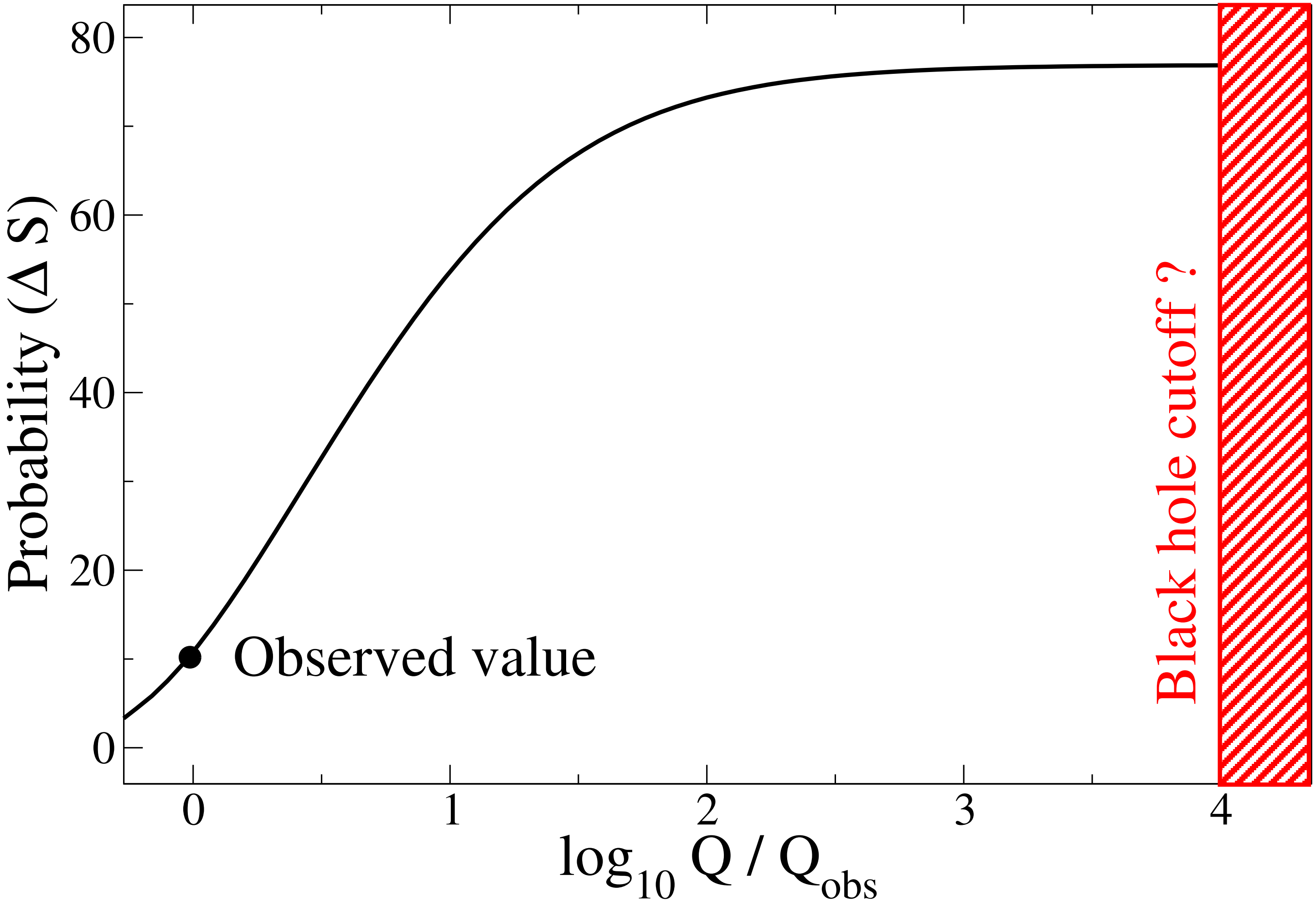
We can use the H-S SFR to see effect of varying Q (renormalized to $Q = 1$ in our universe):



Most likely value of Λ is fairly insensitive to value of Q—unlike anthropic argument where $\Lambda \sim Q^3$.

Probability distribution for Q

Keeping $\Lambda = \Lambda_{\mathrm{obs}}$,



Observed value is ~ 8 times less likely than at high Q

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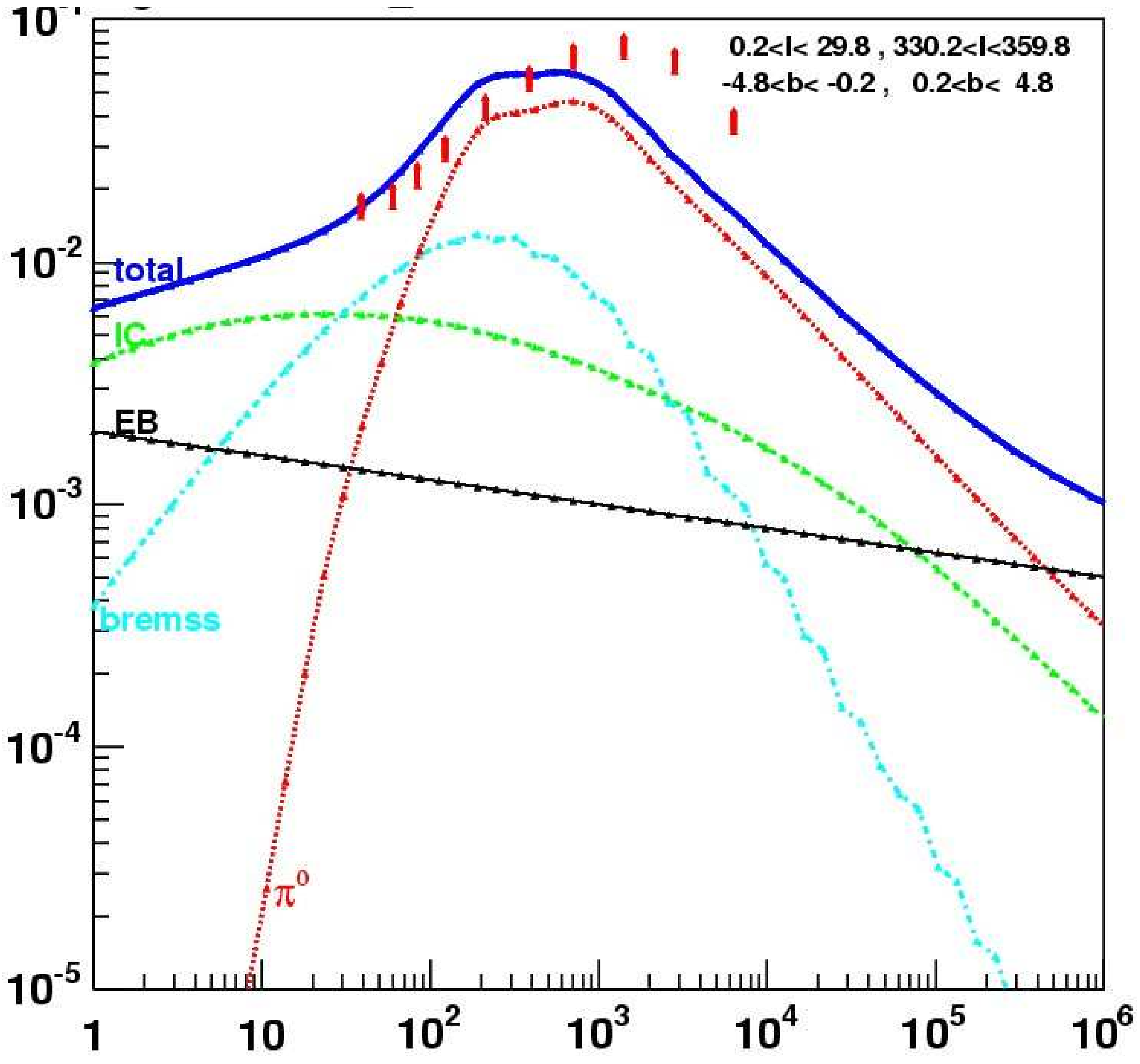
Dark matter can also be a source of entropy if it decays

Hints of decaying dark matter

- EGRET observes excess γ rays with $E \sim 2 - 10$ GeV.

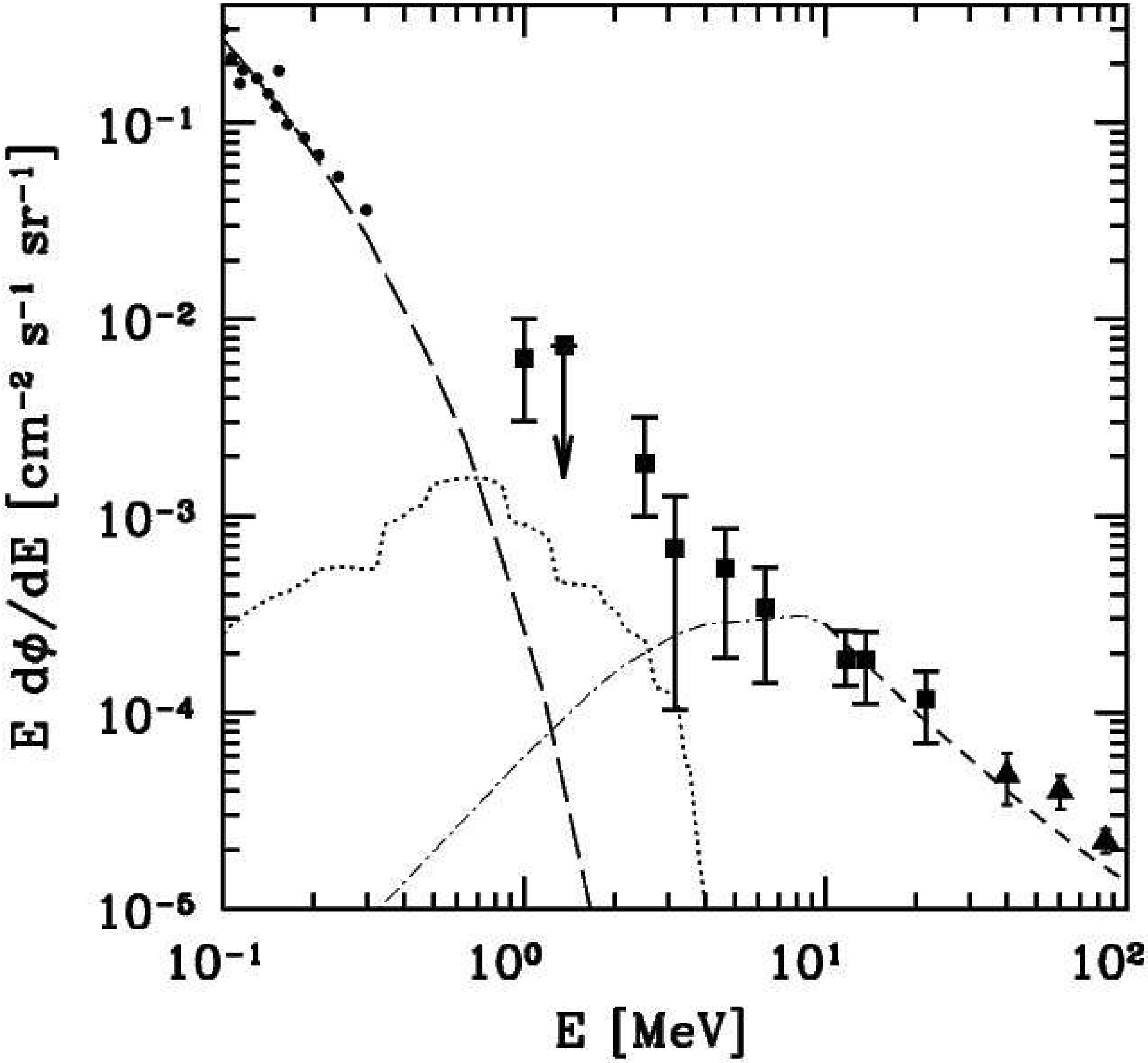


From de Boer *et al.*, astro-ph/0408272.

Buchmuller, Covi, Hamaguchi, Ibarra, Yanagida, hep-ph/0702184, interpret as due to gravitino DM decays

Hints of decaying dark matter

- COMPTEL observes excess γ rays with $E \sim 1 - 5$ MeV.



From Cembranos, Feng, Strigari, astro-ph/0704.1658

They consider KK and SUSY DM with small Δm, $\Delta m = M_{DM^*} - M_{DM}$

Hints of decaying dark matter

- INTEGRAL/SPI observes excess 511 keV γ's from galactic center

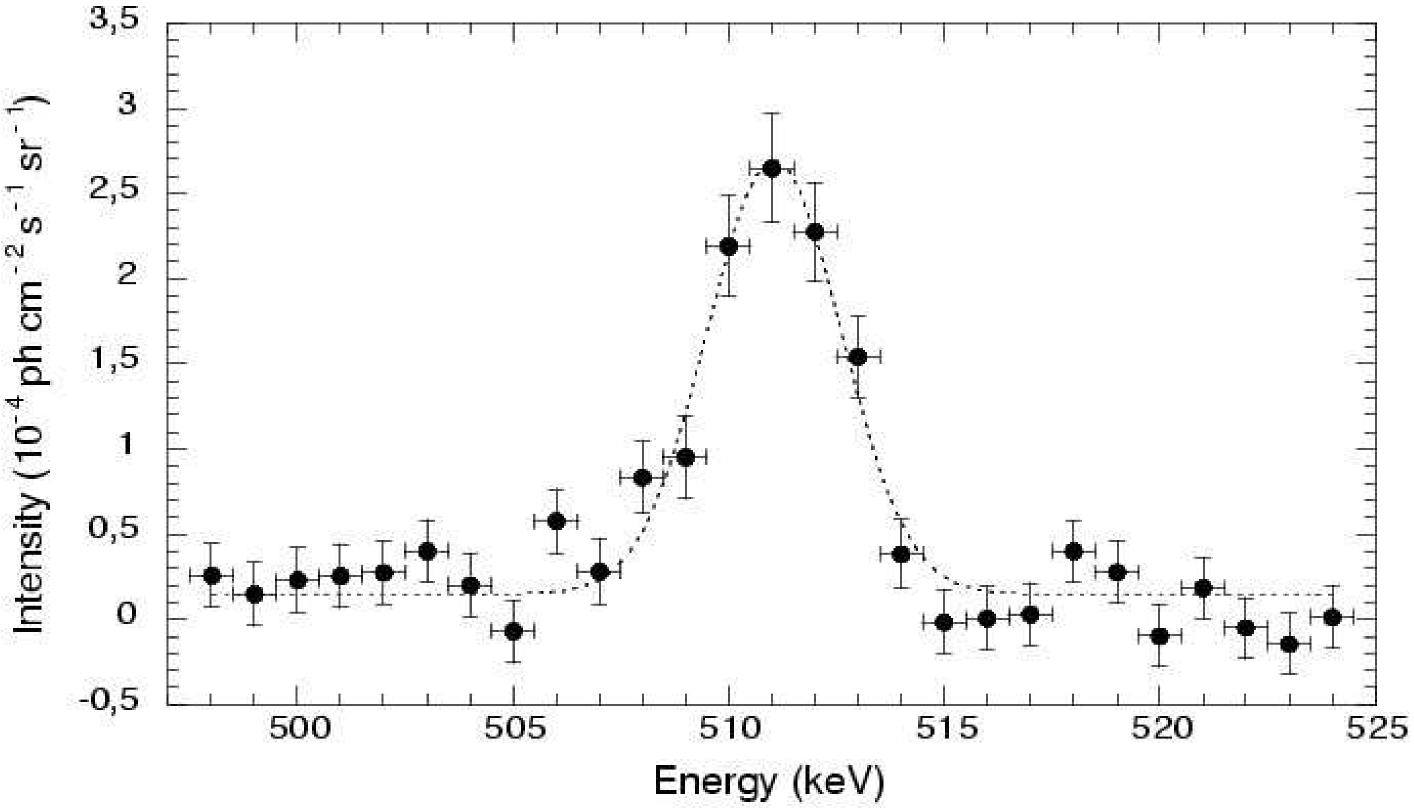


Fig. 3. 511 keV flux spectrum obtained using a gaussian centred on the GC with a FWHM of 10°.

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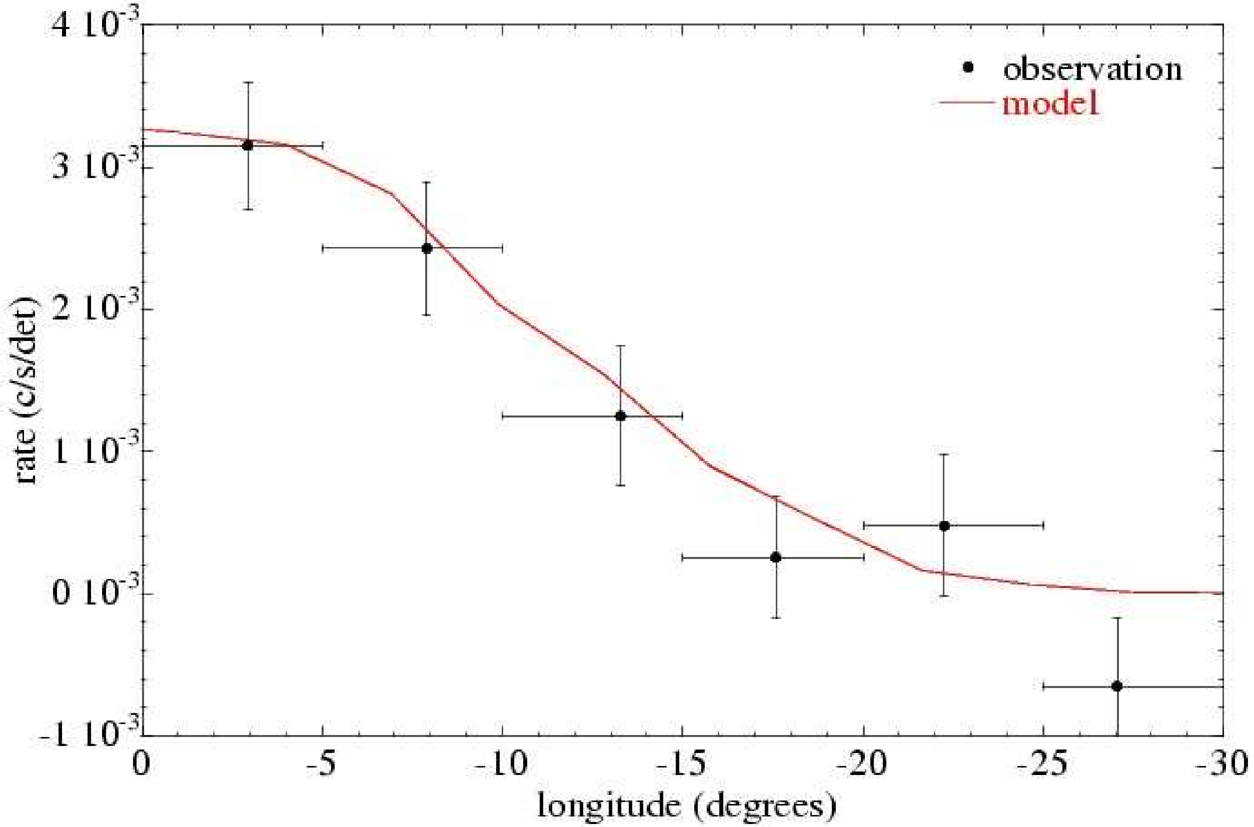


Fig. 2. Rate induced by galactic 511 keV photons as a function of longitude. The response to a gaussian source (FWHM = 10°) is also shown for comparison.

Hints of decaying dark matter

- INTEGRAL/SPI observes excess 511 keV γ's from galactic center

$\sim$ MeV (or weak scale*) DM decaying into e^+e^- has been suggested,

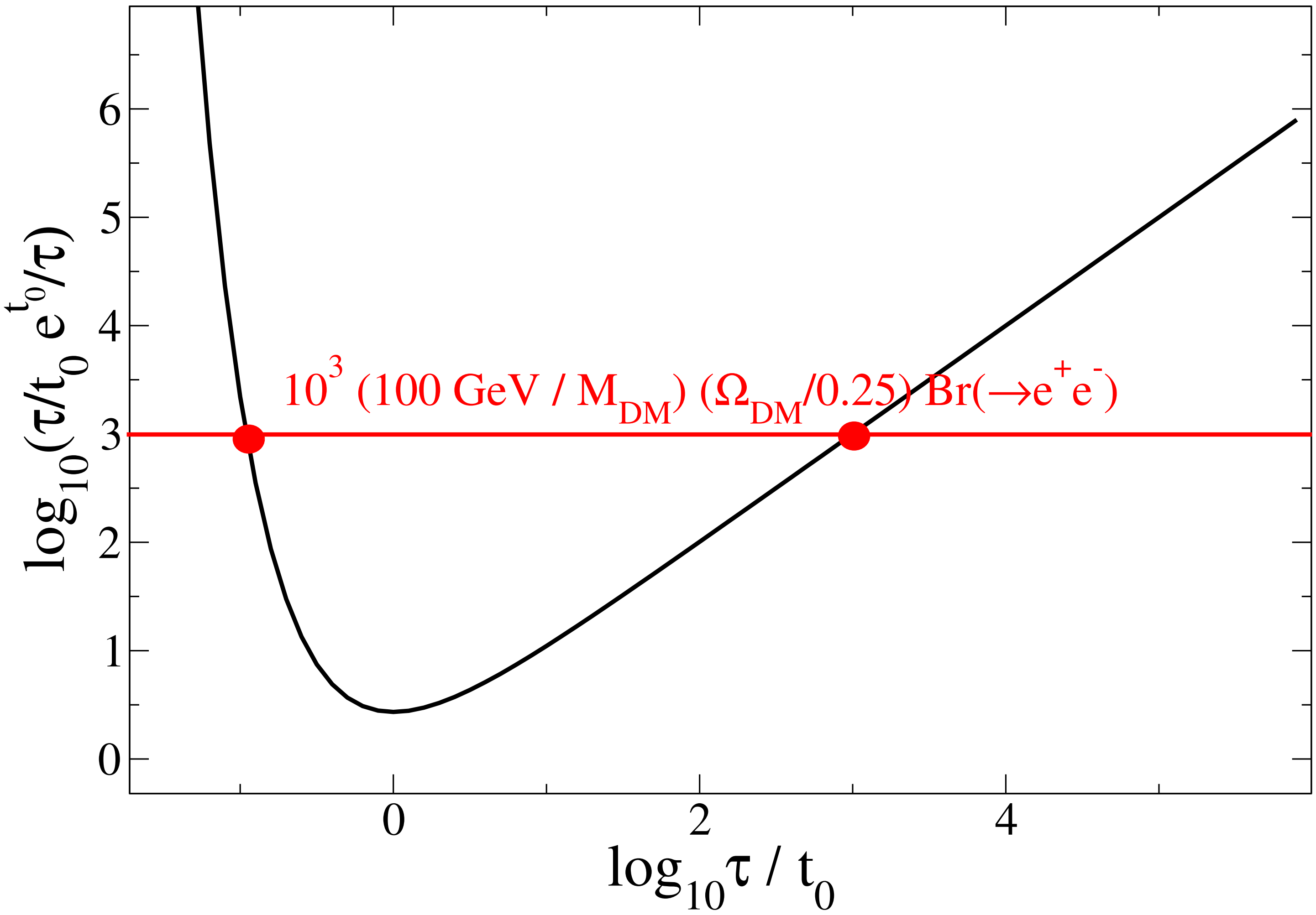
Picciotto, Pospelov	hep-ph/0402178:	sterile neutrinos
Hooper, Wang	hep-ph/0402220:	axinos
Kawasaki, Yanagida	hep-ph/0505167:	moduli
Kasuya, Takahashi	astro-ph/0508391:	Q-balls
Kasuya, Kawasaki	astro-ph/0602296:	moduli
*Pospelov, Ritz	hep-ph/0703128:	heavy excited WIMPs
Conlon, Quevedo	hep-ph/0705.3460:	string theory volume modulus

But why should DM be decaying only today? ⇒

Lifetime needed for SPI anomaly

To match observed 511 keV line, need

$$\frac{\tau}{t_0} e^{t_0/\tau} \cong 10^3 \left(\frac{100\ \mathrm{GeV}}{M_{DM}} \right) \left(\frac{\Omega_{DM}}{0.25} \right) \mathrm{Br}(\mathrm{DM} \to e^+ e^-)$$



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But why should DM be decaying only today?
New coincidence problem

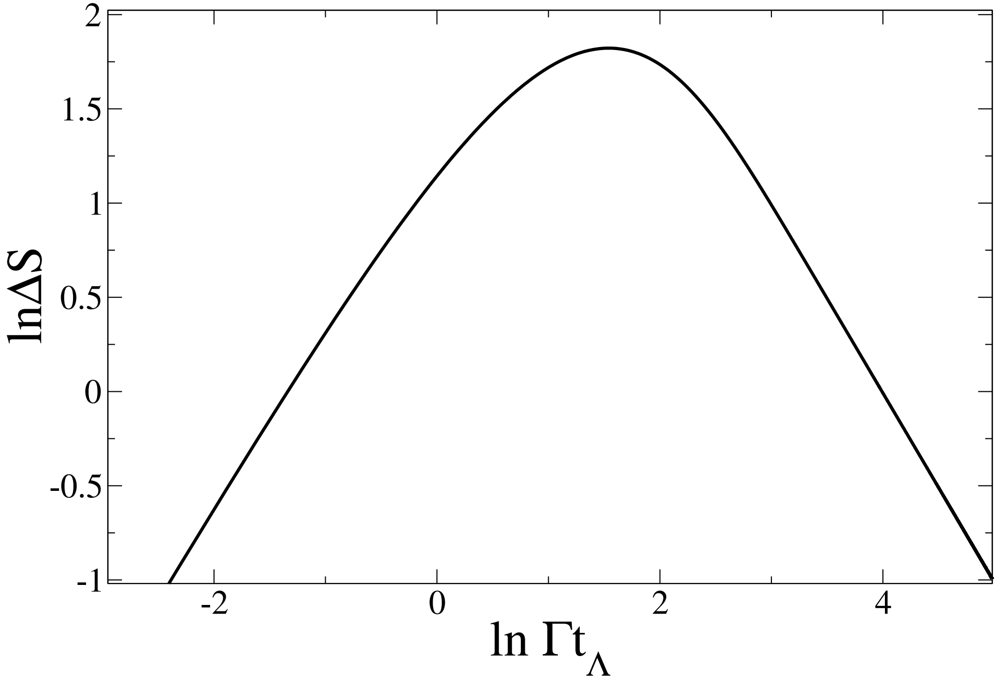
Entropy from Decaying DM

Decaying DM could be major new source of entropy production in causal diamond. Simpler to compute than star contribution:

$$\frac{dS}{dV_c dt} = g_s \, \Gamma \, n \, e^{-\Gamma t}$$

Leads to entropy change $\Delta S_d = t_\Lambda f(\Gamma t_\Lambda)$, where $t_\Lambda \equiv \sqrt{3/\Lambda}$

f is maximized when $1/\Gamma \cong t_\Lambda/4.7$. Since $t_\Lambda = 16.7$ Gyr, could explain why DM is decaying just today



Decaying DM versus stars

How does ΔS_d compare to ΔS from stars? From decays,

$$\frac{dS_d}{dV_c dt} \cong 10^{66} g_s \left(\frac{\Gamma}{\mathrm{Gyr}^{-1}}\right) \left(\frac{\mathrm{eV}}{m}\right) \left(\frac{\rho_{DM}}{(10^{-3}\mathrm{eV})^4}\right) \mathrm{Mpc}^{-3}\,\mathrm{y}^{-1}$$

where $m =$ DM mass, $g_s =$ number of decay products.

From stars, peak value is

$$\frac{dS_*}{dV_c dt} = 10^{63}\,\mathrm{Mpc}^{-3}\,\mathrm{y}^{-1}$$

$$\Longrightarrow \quad \Delta S_d > \Delta S_* \quad \text{if} \quad \frac{m}{g_s} \leq \text{ keV.}$$

Need $m \sim$ MeV, $g_s \sim 10^3$ to explain excess positrons. Such large g_s can come from synchrotron radiation by decay e^-'s.

If decay is GUT-suppressed, then $\Gamma^{-1} \sim \frac{M_{\mathrm{GUT}}^2}{m^3} \sim 10$ Gyr

Conclusions

- Causal entropic principle overcomes many deficiencies of anthropic principle
- Predicts Λ better than anthropic approach
- No problem from eternal inflation for defining measure on landscape
- No arbitrary assumptions about nature of observers
- Robust against variation of density contrast
- Can be applied to universes in the landscape which look quite different from ours
- Explanation for decaying dark matter?

“cosmo-ph” archive

the cosmologically-oriented papers in astro-ph:

http://www.physics.mcgill.ca/∼jcline/astroph/