Gluon saturation at higher orders and improvement of kinematics

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Heavy Ion Collisions in the LHC Era

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Saturation phenomenology in dense-dilute processes

Color Glass Condensate and related formalisms with gluon saturation effects: useful to study dense-dilute processes, like DIS observables and forward particle production.

Phenomenological state of the art:

Successful description of inclusive (and diffractive) DIS
Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
Kuokkanen, Rummukainen, Weigert (2011)

and of single inclusive forward particle production in pA or pp collisions
Albacete, Marquet (2010)

using the dipole-target amplitude from numerical simulations of the BK equation (at Leading Logs + running coupling) together with the LO impact factors.

Recent theoretical progresses towards NLO/NLL

- NLL corrections to the BK equation
  Balitsky, Chirilli (2008)
- NLO corrections to DIS structure functions
  Balitsky, Chirilli (2011)
  G.B. (2012)
- NLO corrections to forward single inclusive particle production in pA or pp
  Chirilli, Xiao, Yuan (2012)
- Demonstration that the same BK equation at NLL applies for both DIS and particle production (non-trivial crossing of Wilson lines)
  Mueller, Munier (2012)

Need for collinear resummations

Main obstacle to gluon saturation phenomenology at NLO/NLL: Pathologically large corrections appears in the NLL BK equation (like in the NLL BFKL equation), making solutions of that equation unstable, and indicating a breakdown of the formalism.

$\Rightarrow$ Need to resum these large contributions in order to obtain a stable and reliable the NLL BK equation.

Similar resummations have been performed in the BFKL case,
Salam (1998)
Ciafaloni, Colferai, Salam, (Staśto) (1999-2007)
Altarelli, Ball, Forte (1999-2008)
but non-trivial to adapt to BK.

Main topic of this talk: one step in that direction
G.B., *in preparation*

Outline

- DIS impact factors at LO and NLO

  $\rightarrow$ mostly based on:

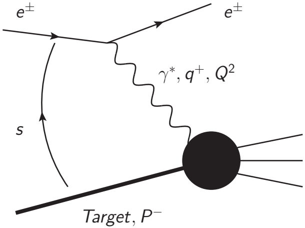
  G.B., Phys.Rev. D85 (2012) 034039, arXiv:1112.4501 [hep-ph].

- Factorization of high-energy Leading Logs in DIS

- Improvement of kinematics in the BK equation

  G.B., *in preparation*

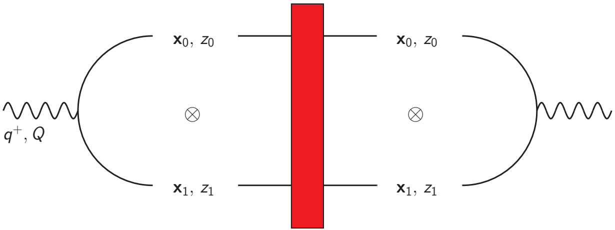
Deep inelastic Scattering (DIS) cross-section



$$\frac{\mathrm{d}^2\,\sigma^{DIS}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{\alpha_{em}}{\pi\,x\,Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)\,\sigma_T^{\gamma}(x_{Bj},Q^2)+(1-y)\;\sigma_L^{\gamma}(x_{Bj},Q^2)\right\}$$

$$x_{Bj} = \frac{Q^2}{2(q\cdot P)} \simeq \frac{Q^2}{2q^+\,P^-} \quad \text{and} \quad y = \frac{Q^2}{x_{Bj}\,s}$$

Dipole factorization of DIS at LO order

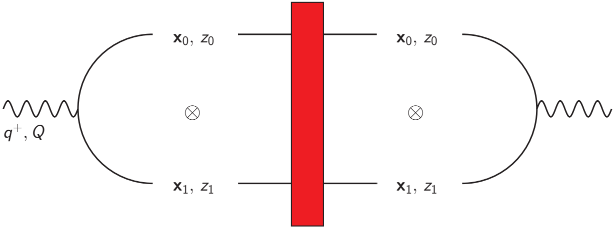


$$\sigma^{\gamma}_{T,L}(x_{Bj},Q^2) \quad = \quad 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1\ \ \mathcal{I}^{LO}_{T,L}(x_{01},1-z_1,z_1)\ \left[1-\langle \mathcal{S}_{01}\rangle\right]$$

Nikolaev, Zakharov (1991)

$\mathcal{I}^{LO}_{T,L} \propto |\text{virtual photon light-front wave-function}|^2$

Dipole factorization of DIS at LO order

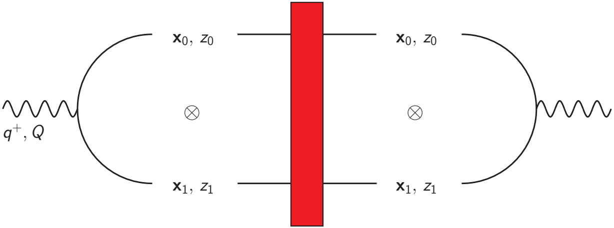


$$\sigma^{\gamma}_{T,L}(x_{Bj},Q^2) \quad = \quad 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1\;\; \mathcal{I}^{LO}_{T,L}(x_{01},1-z_1,z_1)\;\; \Big[1-\langle \mathcal{S}_{01}\rangle\Big]$$

Nikolaev, Zakharov (1991)

$$\mathcal{I}^{LO}_L(x_{01},z_0,z_1) \quad = \quad 4Q^2 z_0^2 z_1^2\, \mathrm{K}_0^2\!\Big(Q\sqrt{z_0 z_1 x_{01}^2}\Big)$$

Dipole factorization of DIS at LO order



$$\sigma^{\gamma}_{T,L}(x_{Bj},Q^2) \quad = \quad 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1\,\int_0^1 \mathrm{d}z_1\;\mathcal{I}^{LO}_{T,L}(x_{01},1-z_1,z_1)\;\Big[1-\langle\mathcal{S}_{01}\rangle\Big]$$

Nikolaev, Zakharov (1991)

$$\mathcal{I}^{LO}_{T}(x_{01},z_0,z_1) \quad = \quad \Big[z_0^2+z_1^2\Big]\,z_0 z_1\,Q^2\mathrm{K}_1^2\Big(Q\sqrt{z_0 z_1 x_{01}^2}\Big)$$

Dipole-target amplitude

In the CGC formalism, the dipole-target elastic S-matrix $\mathcal{S}_{01}$ is related by

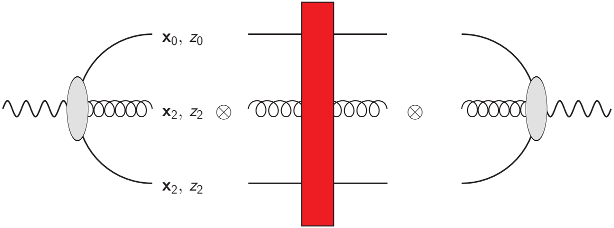
$$\mathcal{S}_{01} \quad = \quad \frac{1}{N_c}\,\mathrm{tr}\left(U(\mathbf{x}_0)\;U^\dagger(\mathbf{x}_1)\right)$$

to the fondamental Wilson line in the semiclassical gluon field $\mathcal{A}_a^-$ of the target:

$$U(\mathbf{x}) \quad = \quad \mathcal{P}\,\exp\left[ig\int \mathrm{d}x^+\;T^a\,\mathcal{A}_a^-(x^+,\mathbf{x},0)\right]$$

$\langle\cdots\rangle$: statistical average over the target gluon field $\mathcal{A}_a^-$ in the Color Glass Condensate formalism.

NLO corrections to DIS at high energy



+ virtual corrections.

$$\sigma^{\gamma}_{T,L} = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2\int \mathrm{d}^2\mathbf{x}_0\int \mathrm{d}^2\mathbf{x}_1\int_0^1 \mathrm{d}z_1\,\Bigg\{\mathcal{I}^{LO}_{T,L}(x_{01},1{-}z_1,z_1)\,\Big[1-\langle \mathcal{S}_{01}\rangle\Big]$$

$$+\bar{\alpha}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\int_0^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\,\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,1{-}z_1{-}z_2,z_1,z_2)\,\langle \mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle\Bigg\}$$

Longitudinal NLO impact factor

$$\mathcal{I}_L^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2)=4Q^2\,\mathrm{K}_0^2(QX_3)\left\{z_1^2(1-z_1)^2\,\frac{\mathcal{P}\left(\frac{z_2}{1-z_1}\right)}{x_{20}^2}\right.$$

$$\left.+z_0^2(1-z_0)^2\,\frac{\mathcal{P}\left(\frac{z_2}{1-z_0}\right)}{x_{21}^2}-2z_1(1-z_1)z_0(1-z_0)\left[1-\frac{z_2}{2(1-z_1)}-\frac{z_2}{2(1-z_0)}\right]\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\;x_{21}^2}\right)\right\}$$

with the notation

$$X_3^2 \quad = \quad z_1\,z_0\,x_{10}^2+z_2\,z_0\,x_{20}^2+z_2\,z_1\,x_{21}^2$$

and the DGLAP quark to gluon splitting function

$$\mathcal{P}(z)=\frac{1}{2}\left[1+(1-z)^2\right]$$

Transverse NLO impact factor

$$
\begin{aligned}
\mathcal{I}_T^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2) = \left[\frac{QX_3\,\mathrm{K}_1\big(QX_3\big)}{X_3^2}\right]^2 \Bigg\{ & z_1^2(1-z_1)^2\left[z_1^2+(1-z_1)^2\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)^2 \frac{\mathcal{P}\left(\frac{z_2}{1-z_1}\right)}{x_{20}^2} \\
& + z_0^2(1-z_0)^2\left[z_0^2+(1-z_0)^2\right]\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)^2 \frac{\mathcal{P}\left(\frac{z_2}{1-z_0}\right)}{x_{21}^2} \\
& + 2z_1(1-z_1)z_0(1-z_0)\left[z_1(1-z_0)+z_0(1-z_1)\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\cdot\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right) \\
& \quad \times \left[1-\frac{z_2}{2(1-z_1)}-\frac{z_2}{2(1-z_0)}\right]\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\;x_{21}^2}\right) \\
& + \frac{z_0\,z_1\,z_2^2\,(z_0-z_1)^2}{(1-z_1)(1-z_0)}\;\frac{\left(\mathbf{x}_{20}\wedge\mathbf{x}_{21}\right)^2}{x_{20}^2\;x_{21}^2} + z_0\,z_1^2\,z_2\left[\frac{z_0\,z_1}{(1-z_1)}+\frac{(1-z_1)^2}{(1-z_0)}\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}\right) \\
& + z_0^2\,z_1\,z_2\left[\frac{z_0\,z_1}{(1-z_0)}+\frac{(1-z_0)^2}{(1-z_1)}\right]\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)\left(\frac{\mathbf{x}_{21}}{x_{21}^2}\right) + \frac{z_0^2\,z_1^2\,z_2^2}{2}\left[\frac{1}{(1-z_1)^2}+\frac{1}{(1-z_0)^2}\right]\Bigg\}
\end{aligned}
$$

Formation time interpretation of the prefactors

The DIS impact factors contain a prefactor dependent on the variable

$$X_2^2 = z_1\, z_0\, x_{10}^2 \quad (\text{with } z_0 + z_1 = 1)$$

for the LO ones and

$$X_3^2 = z_1\, z_0\, x_{10}^2 + z_2\, z_0\, x_{20}^2 + z_2\, z_1\, x_{21}^2 \quad (\text{with } z_0 + z_1 + z_2 = 1)$$

for the NLO ones.

$2q^+X_2^2$ and $2q^+X_3^2$ are the formation time of the $q\bar{q}$ and $q\bar{q}g$ Fock states in the photon wave-function.

The $\mathrm{K}_{0,1}^2(QX_n)$ prefactors then suppress exponentially the Fock states whose formation time is larger than the virtual photon lifetime $2q^+/Q^2$.

High-energy factorization

Rapidity divergence of $\langle \mathcal{S}_{01} \rangle$ and soft divergence of the z_2 integration: regularized by properly formulating a high-energy factorization scheme, avoiding double counting of gluons.

Convenient (but not unique) choice of factorization scheme:

- only gluons with $k^+ < k_f^+$ included into the shockwave field $\mathcal{A}$ of the target
- other gluons, with $k^+ > k_f^+$: kept into the NLO impact factor

see *e.g.* Balitsky, Chirilli (2007)

Available range for the evolution of the target

The presence of the target sets a physical lower bound on the k^+ of the gluons and on the factorization scale k_f^+:

$$k_f^+ \geq k_{min}^+ = \frac{Q_0^2}{2x_0 P^-} = \frac{x_{Bj}\, Q_0^2}{x_0\, Q^2}\, q^+$$

Available range for the evolution of the target

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$\Rightarrow$ Range for LL evolution from the target to the factorization scale:

$$Y_f^+ = \log\left(\frac{k_f^+}{k_{\mathrm{min}}^+}\right) = \log\left(\frac{x_0\, Q^2\, k_f^+}{x_{Bj}\, Q_0^2\, q^+}\right)$$

$\rightarrow$ Not a rapidity range, and not $\log(x_0/x_{Bj})$ either, beyond LL accuracy.

LL BK evolution for $\langle \mathcal{S}_{01} \rangle$

In the impact factor: interpret $\langle \mathcal{S}_{01} \rangle$ as the non-perturbative un-evolved $\langle \mathcal{S}_{01} \rangle_0$, and accordingly put a lower cut-off on the z_2 integration at $k^+_{\min}/q^+$.

Then, make use of the integrated version

$$1 - \langle \mathcal{S}_{01} \rangle_0 = 1 - \langle \mathcal{S}_{01} \rangle_{Y_f^+} - \bar{\alpha} \int_0^{Y_f^+} \mathrm{d}Y_2^+ \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \; \langle \mathcal{S}_{01} - \mathcal{S}_{02}\mathcal{S}_{21} \rangle_{Y_2^+}$$

of the LL BK equation

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \; \langle \mathcal{S}_{02}\mathcal{S}_{21} - \mathcal{S}_{01} \rangle_{Y^+}$$

Final result for inclusive DIS at NLO/LL accuracy

$$\sigma^{\gamma}_{T,L}=2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2\int \mathrm{d}^2\mathbf{x}_0\int \mathrm{d}^2\mathbf{x}_1\int_0^1 \mathrm{d}z_1\,\Bigg\{\mathcal{I}^{LO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,1-z_1,z_1)$$

$$\times\Bigg[1-\left\langle\mathcal{S}_{01}\right\rangle_{Y_f^+}-\bar{\alpha}\int_0^{Y_f^+}\mathrm{d}Y_2^+\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\left\langle\mathcal{S}_{01}-\mathcal{S}_{02}\mathcal{S}_{21}\right\rangle_{Y_2^+}\Bigg]$$

$$+\bar{\alpha}\int_{k^+_{min}/q^+}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2)\,\left\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\right\rangle_{Y_2^+}\Bigg\}$$

The LL contributions $(\bar{\alpha}\,Y_f^+)^n$ essentially cancel between the last two terms, and remain only in $\left\langle\mathcal{S}_{01}\right\rangle_{Y_f^+}$.

Incorrect subtraction of Leading Logs

Low z_2 contribution to σ_L^γ at NLO (for $z_2 \ll z_1,\ 1-z_1$):

$$\sim \bar{\alpha} \frac{\mathrm{d}z_2}{z_2} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \mathrm{K}_0^2(QX_3)\, \langle \mathcal{S}_{01} - \mathcal{S}_{02}\,\mathcal{S}_{21} \rangle_{Y_2^+}$$

Low z_2 term used to subtract LL from σ_L^γ at NLO:

$$\sim \bar{\alpha} \frac{\mathrm{d}z_2}{z_2}\, \mathrm{K}_0^2\left(Q\sqrt{z_1(1-z_1)x_{01}^2}\right) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \langle \mathcal{S}_{01} - \mathcal{S}_{02}\,\mathcal{S}_{21} \rangle_{Y_2^+}$$

At low z_2: $X_3^2 \simeq z_1(1-z_1)x_{01}^2$ in most of the available range for $\mathbf{x}_2$
But mismatch in the regime $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2 x_{12}^2$, where $X_3^2 \simeq z_2 x_{02}^2 \simeq z_2 x_{12}^2$.

Incorrect subtraction of Leading Logs

In the regime $z_2 \ll z_1$, $1-z_1$ and $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2 x_{12}^2$:

- $\mathrm{K}_0(QX_3)$ is exponentially smaller than $\mathrm{K}_0\left(Q\sqrt{z_1(1-z_1)x_{01}^2}\right)$
- and no contribution to leading logs is present in σ_L^γ at NLO.

$\Rightarrow$ More leading logs subtracted with the BK equation than present in σ_L^γ (and σ_T^γ).

Incorrect treatment in a kinematical regime parametrically narrow, but quantitatively important:

- spoils the DGLAP DLL limit.
- LL subtraction with the standard BK equation spoils the suppression of Fock states with too large formation time

Link with the problems of NLL BK and BFKL

BK and BFKL usually derived in strict multi-Regge kinematics:

- strong ordering in k^+ (or k^-, or rapidity)
- all **k**'s (or dipole sizes) of the same order

and kinematical approximations are performed accordingly.
For example, in light-front perturbation theory each energy denominator is approximated by the contribution of the last emitted gluon.
$\rightarrow$ momentum space analog of the approximation $X_3^2 \simeq z_1(1-z_1)x_{01}^2$ in the NLO impact factor.

Problem: unrestricted integration over $\mathbf{k}$ or $\mathbf{x}_2$ in BFKL and BK
$\Rightarrow$ Second assumption not consistent!

This is the origin of the largest NLL, NNLL and so on corrections in the BFKL and BK equations.

Link with the problems of NLL BK and BFKL

Strict ordering of the gluons *both* in k^+ and k^- simultaneously:

- self-consistent and sufficient condition for the kinematical approximation of energy denominators
- sufficient condition to have ordering in the *formation time* of the gluonic fluctuations of the projectile

High-energy factorization automatically implies ordering wrt. its factorization variable k^+, but not wrt. k^-.

In mixed space, k^- ordering $\Leftrightarrow$ $z_1(1-z_1)x_{01}^2 \gg z_2 x_{02}^2$ and $z_1(1-z_1)x_{01}^2 \gg z_2 x_{12}^2$

Need to impose by hand the missing k^- ordering in the equation via a kinematical constraint.
Ciafaloni (1988)
Kwieciński, Martin, Sutton (1996)
Andersson, Gustafson, Kharraziha, Samuelsson (1996)

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}z_2}{z_2}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y_2^+}$$

Kinematical constraint: forbid gluon emission in the regime $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2\, x_{12}^2$

$\Rightarrow$ Multiply the real contribution by $\theta\left(z_1(1-z_1)\,x_{01}^2 - z_2\;\min(x_{02}^2, x_{21}^2)\right)$

Same general idea as in the previous study in mixed space: Motyka, Staśto (2009)

However: inappropriate treatment of virtual corrections there.

Calculating virtual corrections from unitarity

Assume the kinematical constraint to preserve the probabilistic interpretation of the parton cascade.
Evolution of $\langle S_{01}\rangle$ over a finite range $Y_f^+ = \log(k_f^+/k_{\min}^+)$:

$$\begin{aligned}\langle S_{01}\rangle_{Y_f^+} &= \langle S_{01}\rangle_0 \; D_{01}(Y_f^+) + \bar{\alpha}\int_0^{Y_f^+} \mathrm{d}Y_2^+ \; D_{01}(Y_f^+ - Y_2^+) \\ &\times \int \frac{\mathrm{d}^2\mathbf{x_2}}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\theta\left(Y_f^+ - Y_2^+ - \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right) \\ &\times \left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+}\end{aligned}$$

with the probability $D_{01}(Y^+)$ of no splitting for the dipole 01 in the range Y^+.

Calculating virtual corrections from unitarity

In the vacuum (absence of target), $S_{01} = S_{02} = S_{21} = 1$.
$\rightarrow$ equation determining $D_{01}(Y^+)$.
Solution:

$$D_{01}(Y^+) = \exp\left[-\bar{\alpha}\,\frac{2C_F}{N_c}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left(Y^+ - \Delta_{012}\right)\;\theta\left(Y^+ - \Delta_{012}\right)\right]$$

with the notation

$$\Delta_{012} = \max\left\{0,\, \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right\}$$

Typical behavior:

$$\begin{aligned} \Delta_{012} &= 0 \quad \text{for} \quad x_{02}^2 \ll x_{01}^2 \quad \text{or} \quad x_{21}^2 \ll x_{01}^2 \\ \Delta_{012} &\sim \log\left(\frac{x_{02}^2}{x_{01}^2}\right) \sim \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \quad \text{for} \quad x_{01}^2 \ll x_{02}^2 \sim x_{21}^2 \end{aligned}$$

Kinematically constrained BK equation (kcBK)

Rewriting the new evolution equation as a differential equation and discarding irrelevant terms explicitly of order NLL:

$$\partial_{Y_f^+} \langle \mathcal{S}_{01} \rangle_{Y_f^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y_f^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y_f^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y_f^+} \right\}$$

G.B., *in preparation*

Only gluon emission at large transverse distance is modified, and regime of very large transverse distances completely removed.

This should slow down significantly the BK evolution!

However, the range for evolution $Y_f^+ = \log\left(\frac{x_0\, Q^2\, k_f^+}{x_{Bj}\, Q_0^2\, q^+}\right)$ can be larger than the usually assumed $\log\left(\frac{x_0}{x_{Bj}}\right)$.

Kinematically constrained BK equation (kcBK)

$$\partial_{Y_f^+} \left\langle \mathcal{S}_{01} \right\rangle_{Y_f^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta\!\left(Y_f^+ - \Delta_{012}\right) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y_f^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \left\langle \mathcal{S}_{01} \right\rangle_{Y_f^+} \right\}$$

That modification of the LL BK equation resums precisely the largest and most pathological corrections appearing in the known NLL BK equation.
$\Rightarrow$ Necessary step towards a stable and reliable version of the NLL BK equation.

When regularizing the NLO DIS impact factors and removing the LL contribution using the kcBK equation:
fully correct subtraction the LL contributions, with no mismatch in the collinear regime, by contrast to the standard LL BK case.

Conclusions

1. Explicit expression for NLO DIS impact factors provides one of the ways to understand kinematical issues in the BK equation
2. The same modification (kcBK) of the LL BK equation allows to
   - resum the largest and unphysical corrections to BK of order NLL and beyond
   - restore the correct DGLAP DLL limit in the BK equation and impact factors
   - restore ordering in the formation time of fluctuations along the evolution

Outlook

- Kinematical improvement: can be trivially combined with running coupling effects.

  $\Rightarrow$ Kinematically constrained running coupling BK: new standard for future phenomenological studies.

- Missing step towards a fully stable NLL BK equation:

  understanding and resummation of the finite x part of the DGLAP evolution in mixed space, both in the collinear and anti-collinear limits.

- kinematical constraint for the JIMWLK equation?
  for the evolution equation of quadrupoles?