QCD critical point, charge fluctuations, and final state interactions

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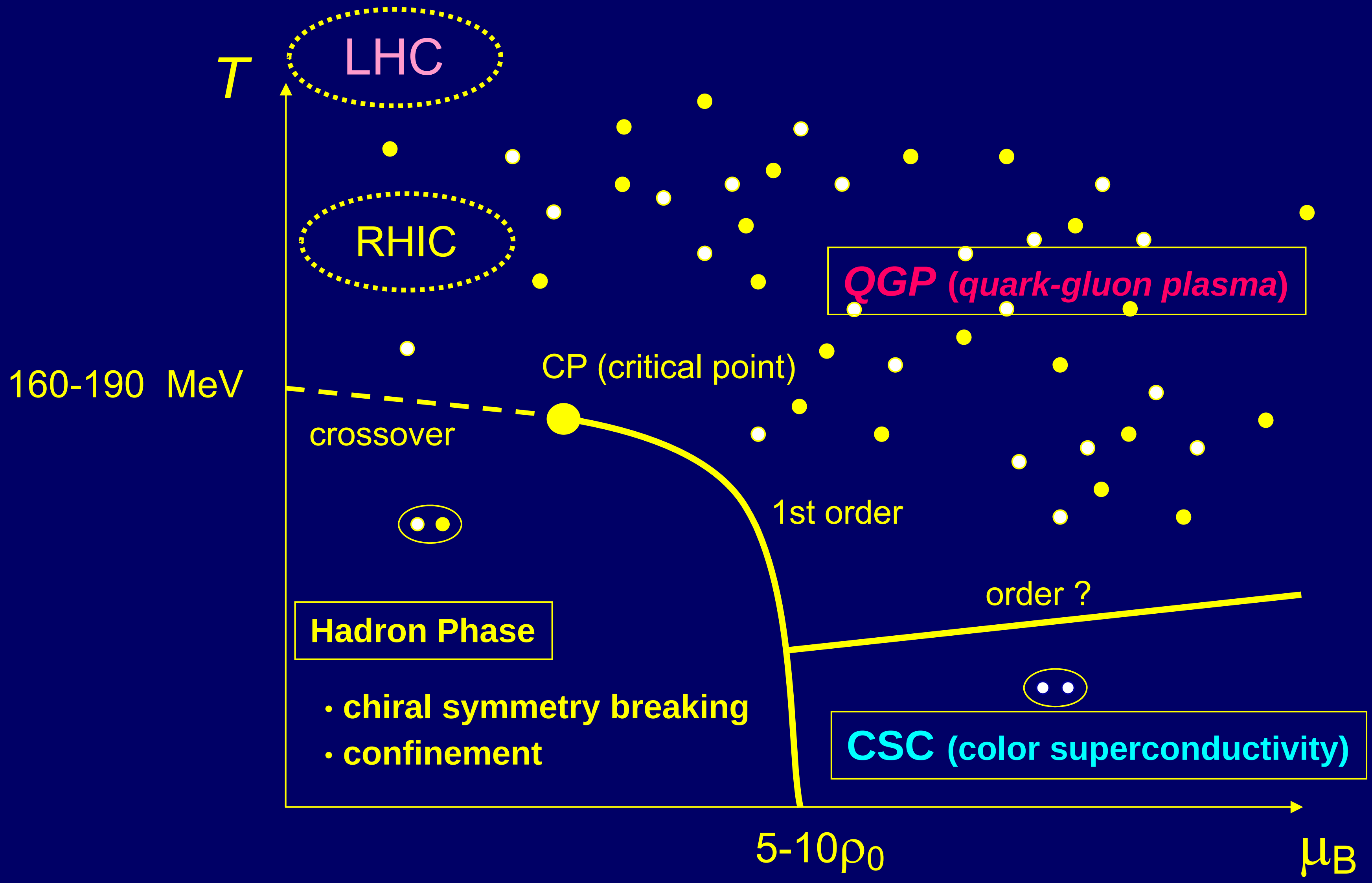
S. Ejiri, M. Kitazawa, and M.A., PRL 103 (2009) 262301
M. Kitazawa and M.A., PRC 85 (2012) 021901R
M. Kitazawa and M.A., (2012) to be published

July 2012

@ Heavy Ion Collisions in the LHC Era

QCD Phase Diagram



M. Asakawa (Osaka University)

How CP in QCD started

Nuclear Physics **A504** (1989) 668–684
North-Holland, Amsterdam

CHIRAL RESTORATION AT FINITE DENSITY AND TEMPERATURE

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Received 2 May 1988
(Revised 24 April 1989)

M. Asakawa, K. Yazaki / Chiral restoration

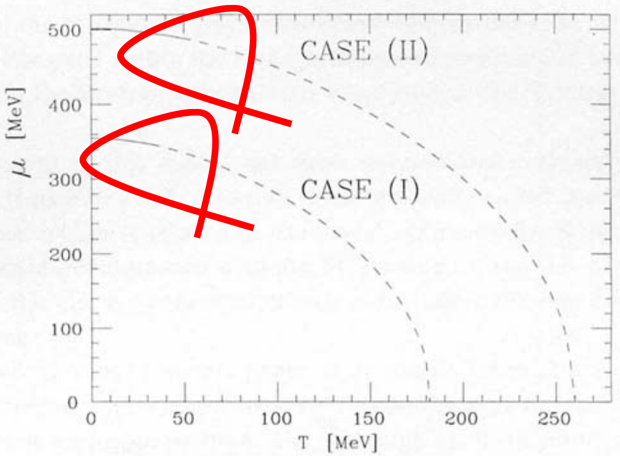
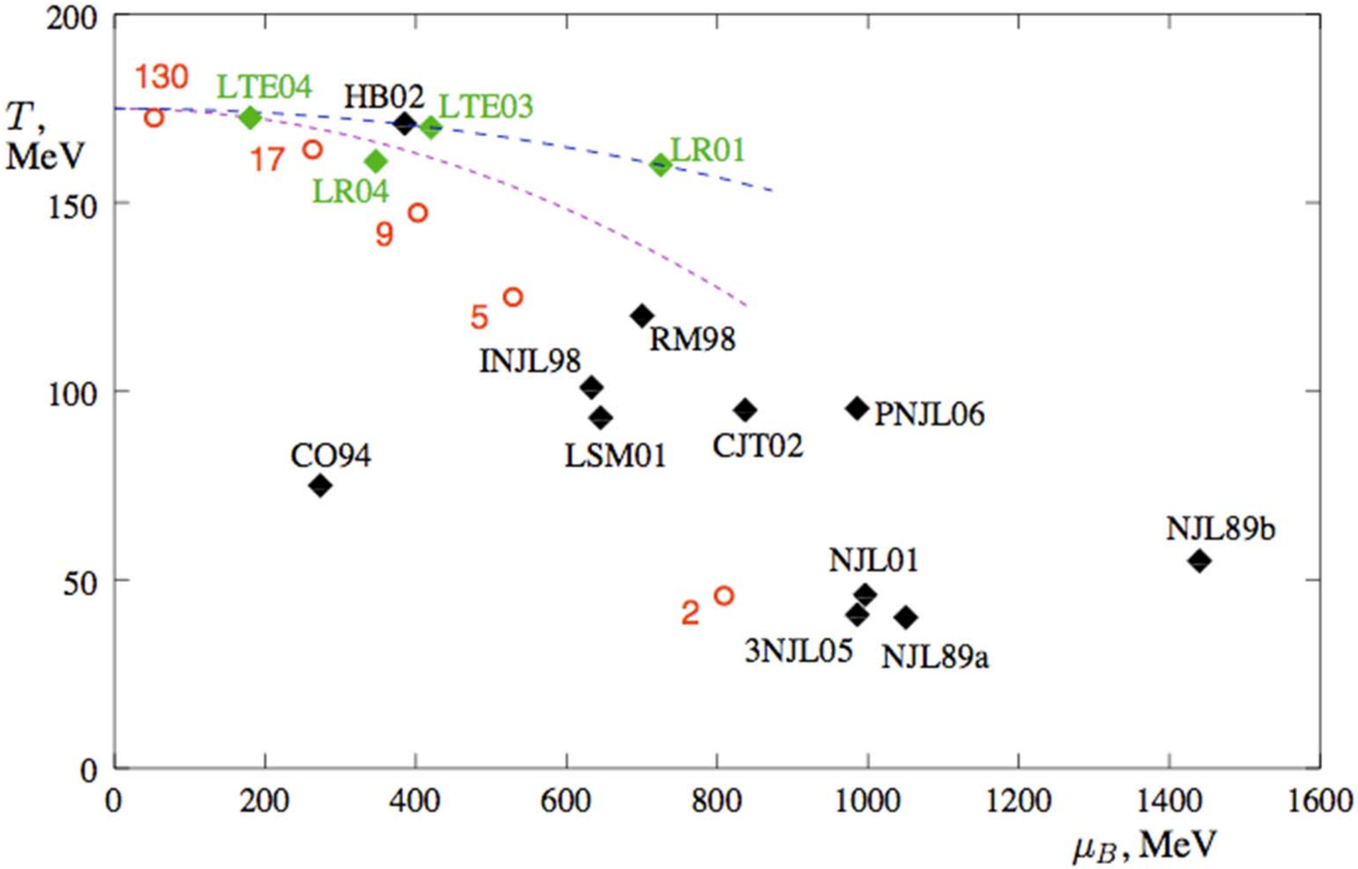


Fig. 8. The phase transition line in the cases (I) and (II).

Where is CP, if any?



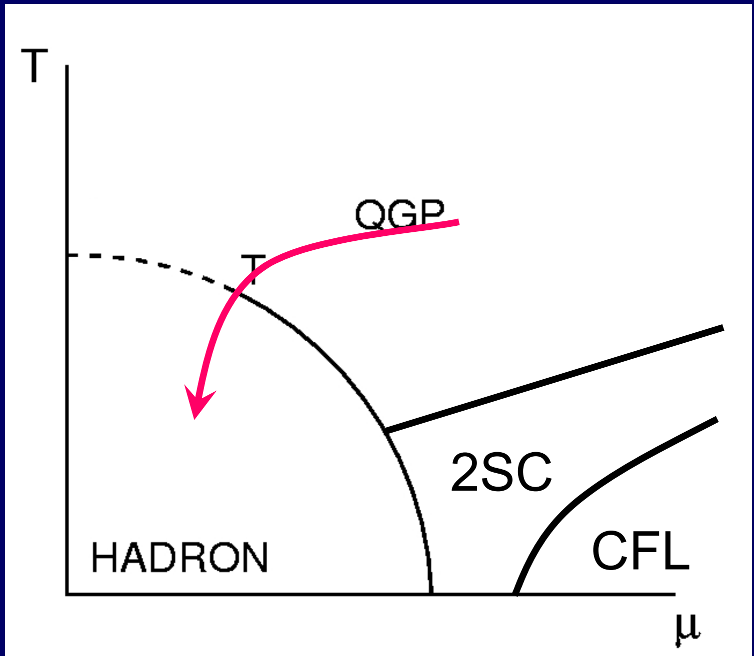
Stephanov, hep-lat/0701002

CP = 2nd order phase transition, but...

CP =
2nd Order Phase Transition Point

→ Divergence of Fluctuation
Correlation Length
Specific Heat ?

If expansion is adiabatic



even if the system goes right through the critical point…

There is no conservation law that slows down the change of those quantities !

→ Subject to Final State Interactions

Fluctuations: Higher Moments

- Recently higher moments have attracted quite a lot of attention
  - (Roughly) Two Reasons
    - Larger critical exponents around CP (=Aξ^z) Stephanov (2008)
    - Sign change across the phase transition (crossover) line Asakawa, Ejiri, Kitazawa (2009)

⇨ Next Slide

Odd Power Fluctuation Moments

- Fluctuations of Conserved Charges: not subject to final state interactions
  - Usually even power fluctuations such as $\left\langle (\delta Q)^2 \right\rangle$ have been considered
  - Usual Fluctuations such as $\left\langle (\delta Q)^2 \right\rangle$: positive definite

→ Absolute values carry information of states (D-measure)

Asakawa, Heinz, Müller, Jeon, Koch

On the other hand,

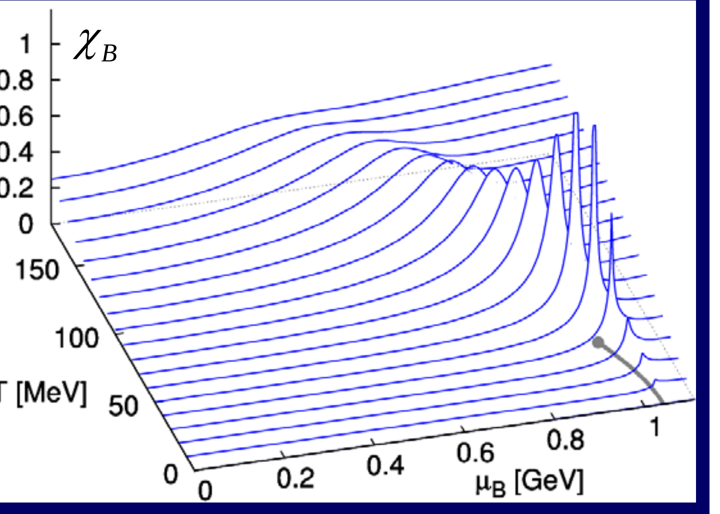
✓ Odd power fluctuations : NOT positive definite

- In general, do not vanish (exception, $\left\langle \delta A \right\rangle$)
- *Sign also carries information of states*

Physical Meaning of 3rd Fluc. Moment

χ_B : Baryon number susceptibility

in general, has a peak along phase transition



M. Kitazawa, S. Ejiri, and M.A., 2009

⇨ $\dfrac{\partial \chi_B}{\partial \mu_B}$ changes the sign at QCD phase boundary !

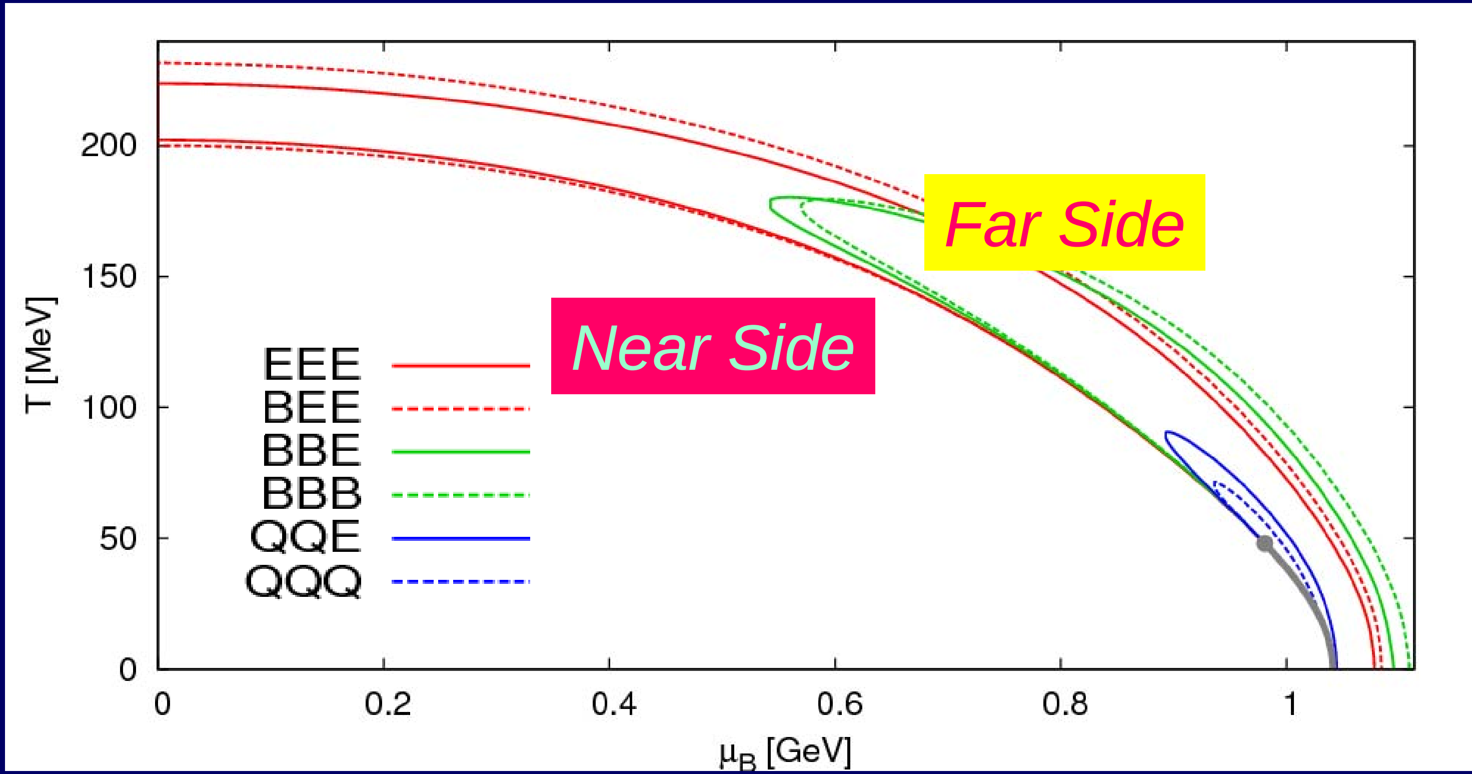
- In the language of fluctuation moments:

$$\chi_B = -\frac{1}{V}\frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\left\langle (\delta N_B)^2 \right\rangle}{VT}$$

$$\frac{\partial \chi_B}{\partial \mu_B} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\left\langle (\delta N_B)^3 \right\rangle}{VT^2} \equiv m_3(\mathrm{BBB})$$

more information than usual fluctuation

Comparison of Various Moments



2-flavor NJL with standard parameters

G=5.5GeV^{-2}
m_q=5.5MeV
Λ=631MeV

- Different moments have different regions with negative moments

⇒ By comparing the signs of various moments, possible to pin down the origin of moments

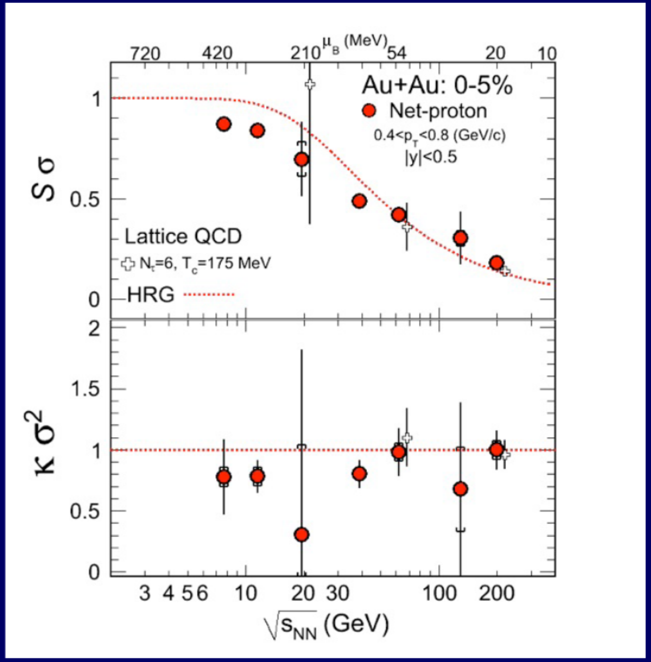
- Negative m_3(EEE) region extends to T-axis (in this particular model)

Recent Progress: Proton Number Cumulants

- Proton Number Fluctuation has been attracting a lot of interest because it can be observed experimentally
- Proton Number Fluctuation diverges at CP

Hatta and Stephanov, 2003

- Comparisons of experimental results and lattice predictions have been made (e.g. Gupta et al., Science 2011)



$$\chi_B^{(n)}\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)=\frac{1}{T^n}\frac{\partial^n}{\partial\left(\mu_B/T\right)^n}P\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)\Bigg|_{T/T_c}$$

$$S\sigma=\frac{T\chi_B^{(3)}}{\chi_B^{(2)}}$$

$$\kappa\sigma^2=\frac{T^2\chi_B^{(4)}}{\chi_B^{(2)}}$$

STAR, QM2011

Experiment: Net Proton
Theory: Net Baryon

Is this harmless?

Protons and Baryons

The question here is how these are related to each other:

$$\left\langle \left(\delta N_p \right)^n \right\rangle_c \Leftrightarrow \left\langle \left(\delta N_{\mathrm{B}} \right)^n \right\rangle_c$$

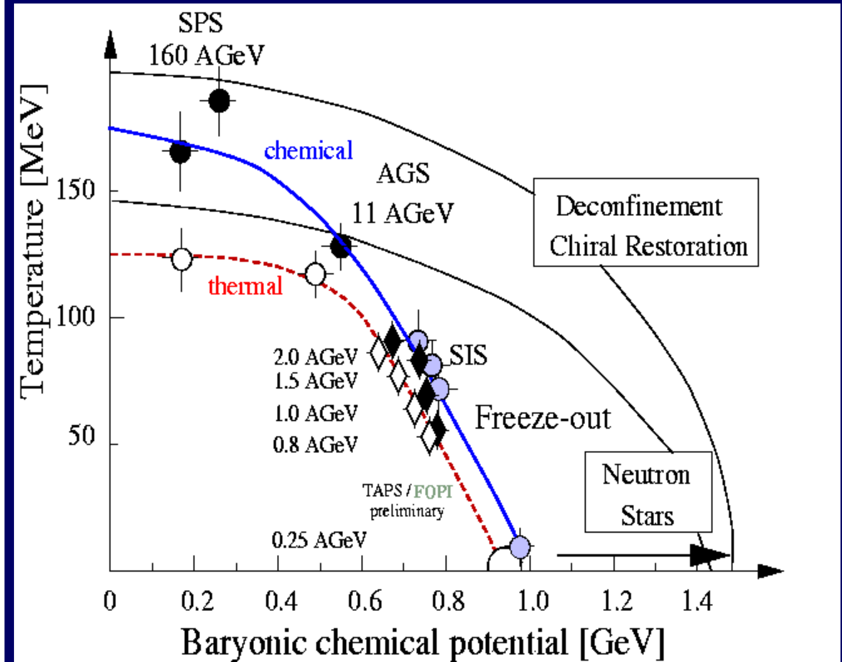
In free nucleon gas in equilibrium,

$$\left\langle \left(\delta N_{\mathrm{B}} \right)^n \right\rangle_c = 2 \left\langle \left(\delta N_p \right)^n \right\rangle_c$$

Otherwise, in general,

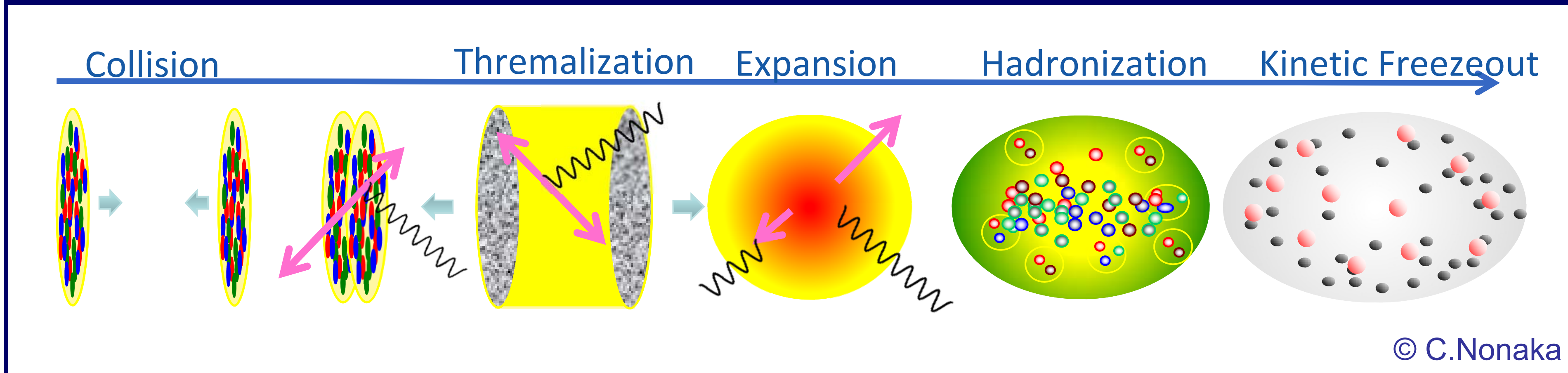
$$\left\langle \left(\delta N_{\mathrm{B}} \right)^n \right\rangle_c \neq 2 \left\langle \left(\delta N_p \right)^n \right\rangle_c$$

Freezeouts



- Net proton number may be considered as an alternative of net baryon number
- Chemical freezeout is close to the crossover, and (anti)proton number is expected to be fixed early (?)

➢ But Not all particle numbers and fluctuations are fixed at chemical freezeout



M. Asakawa (Osaka University)

Exception

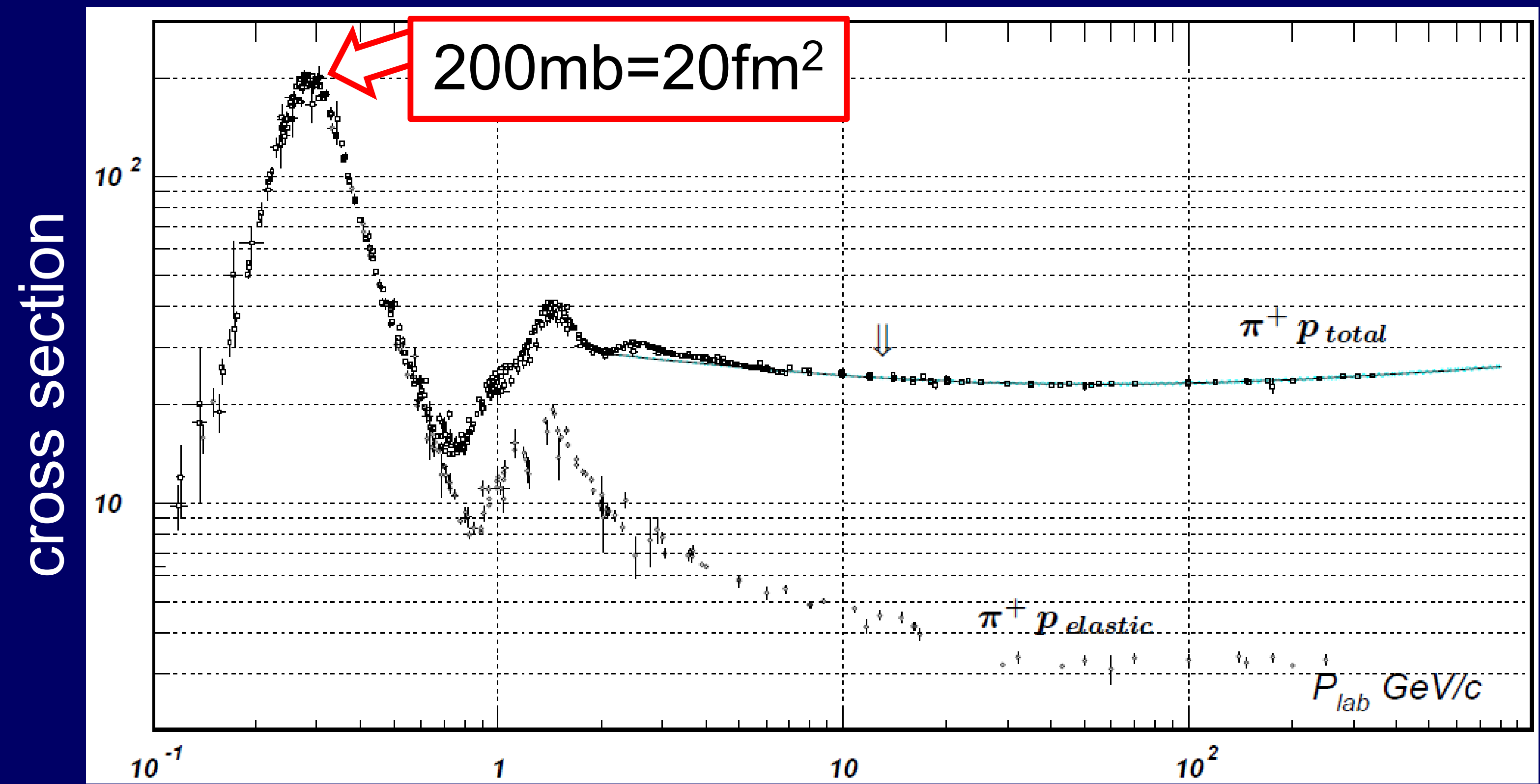
If there are low mass resonances, this exception happens

In our case at hand, Δ resonances

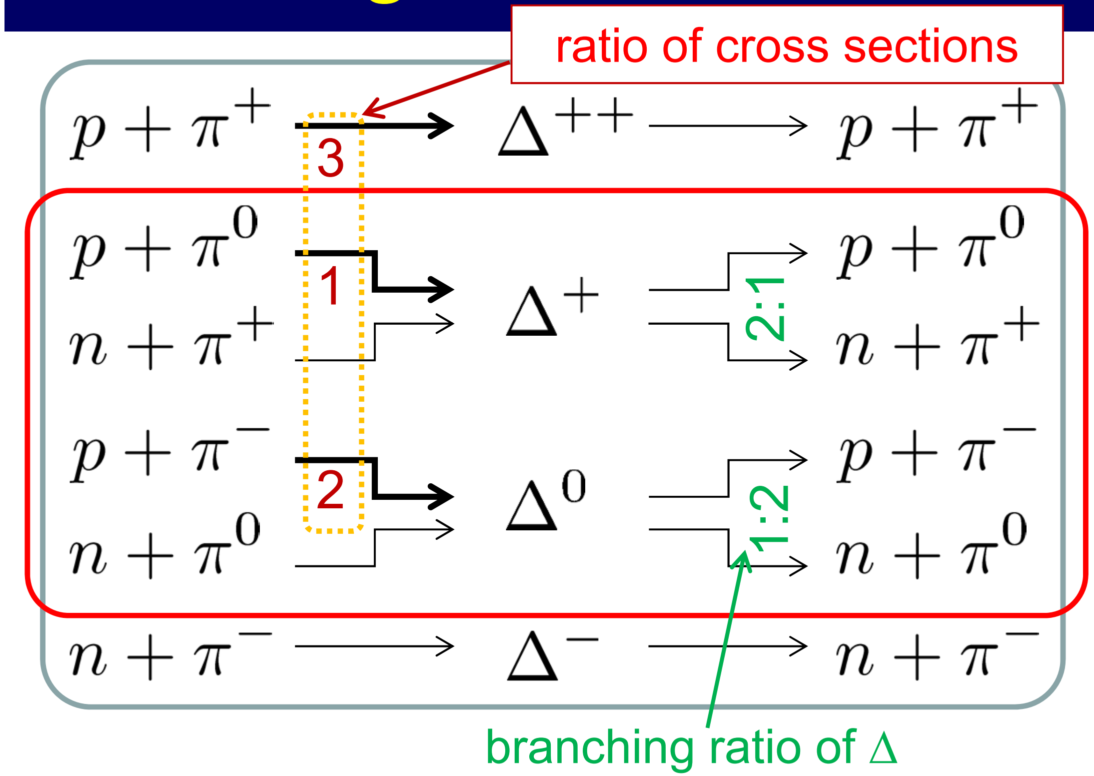
$$p, n + \pi \rightarrow \Delta(1232) \rightarrow p, n + \pi$$

$$I = 3/2$$

$$\Gamma \simeq 1.8 \text{ [fm]}$$



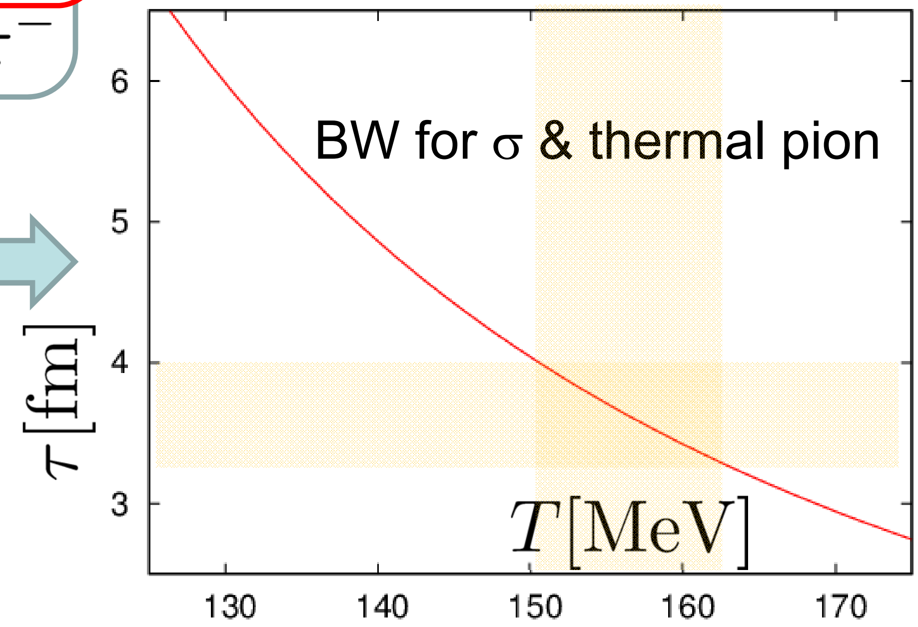
How long is the mean free time?



$$p+\pi \to \Delta^{+,0}$$
$$\to p:n$$
$$=5:4$$

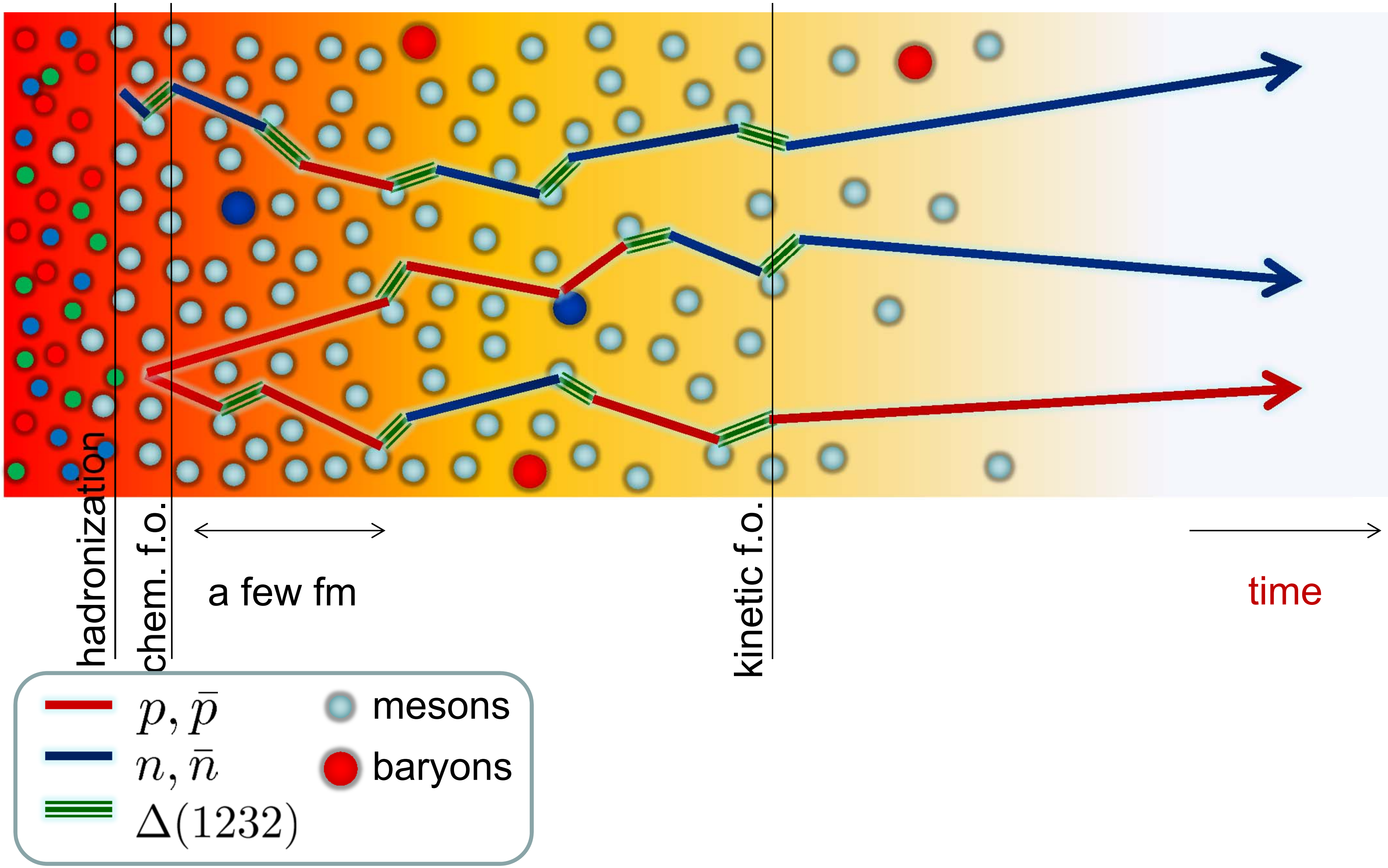
Meantime to create Δ^{+} or Δ^{0}

$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

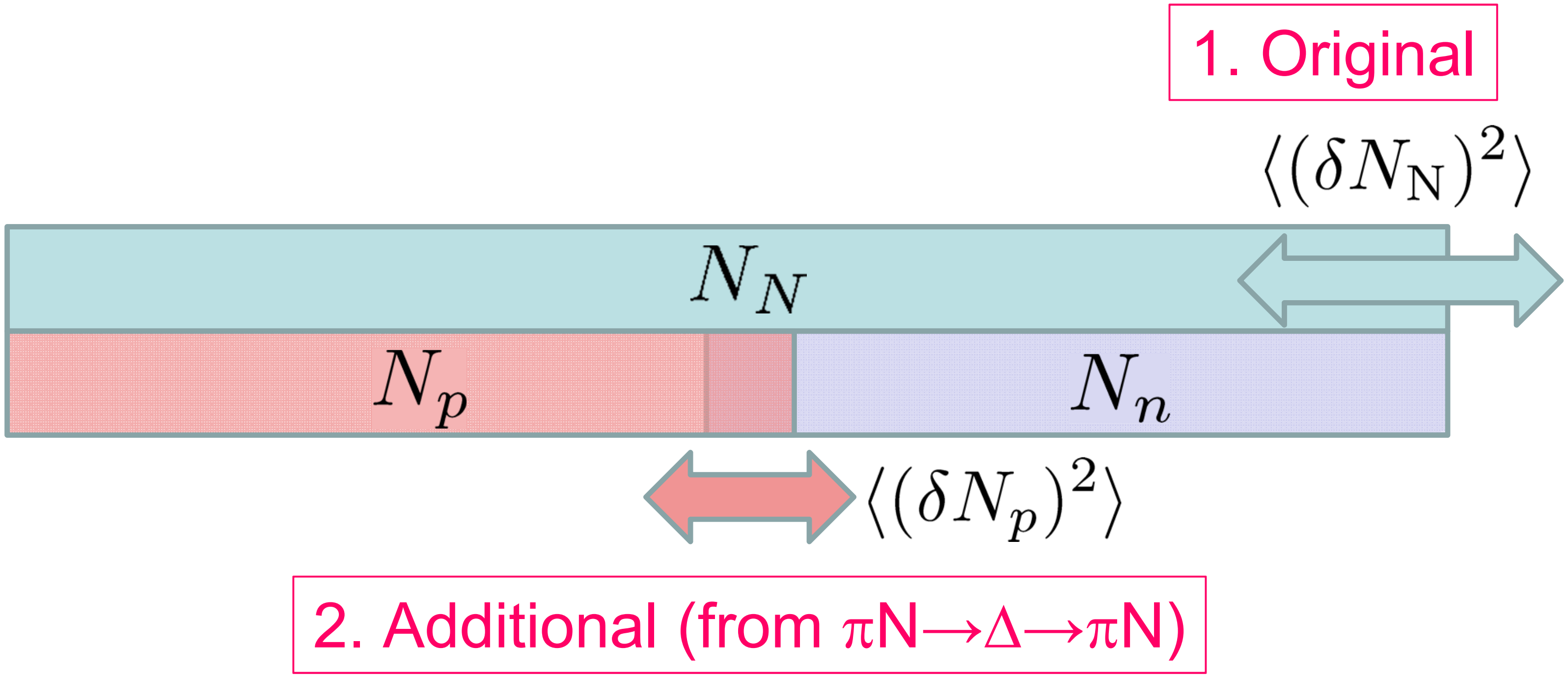


$\tau \leq$ a few fm

Nucleon Isospin Randomization in Pion Gas

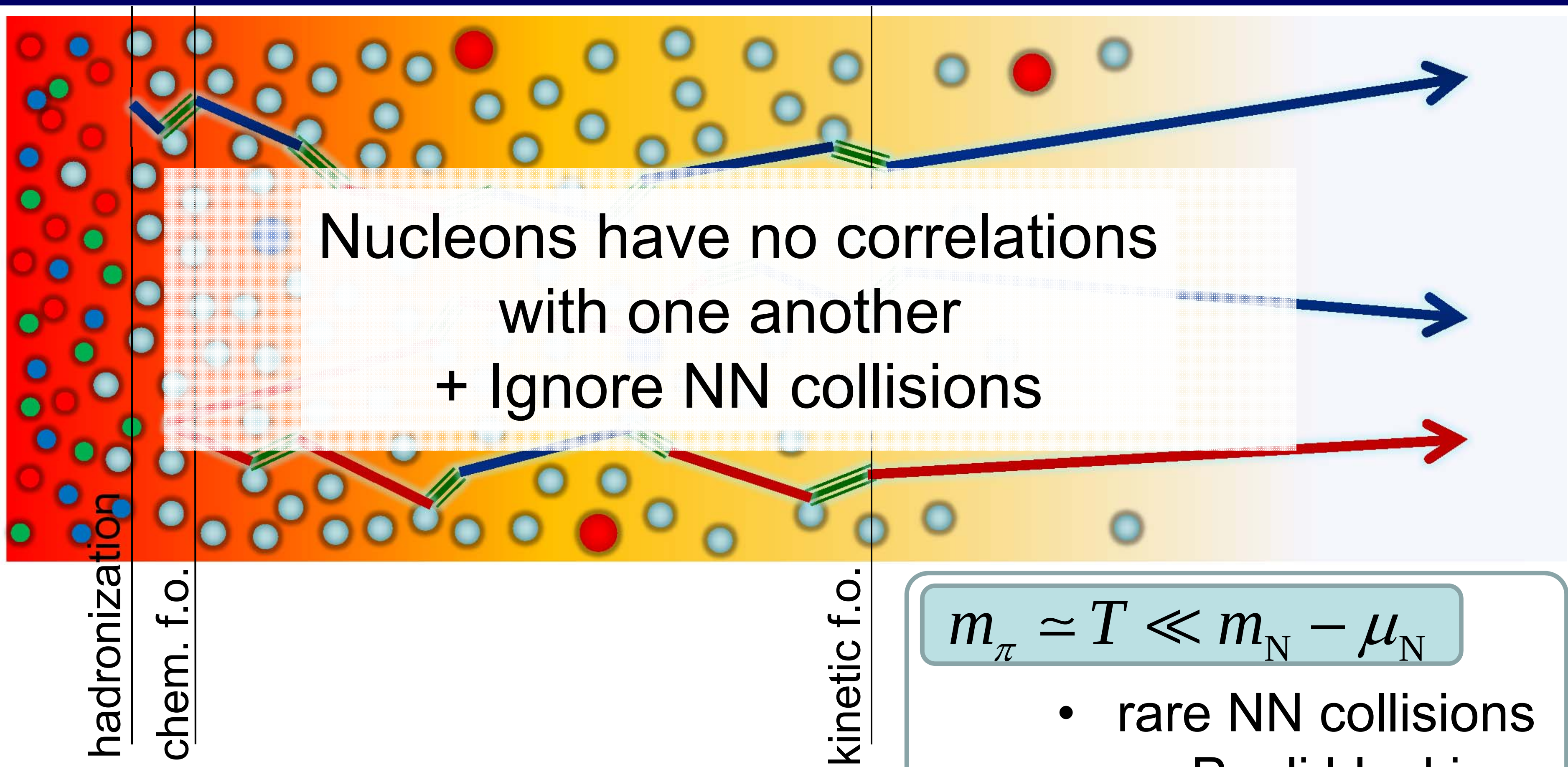


Production of Additional Fluctuation



- In, general, fluctuations of N_N and N_p are different
- Additional N_p fluctuations are created by (thermal) pions

Dilute Nucleon Approximation



Nucleons have no correlations
with one another
+ Ignore NN collisions

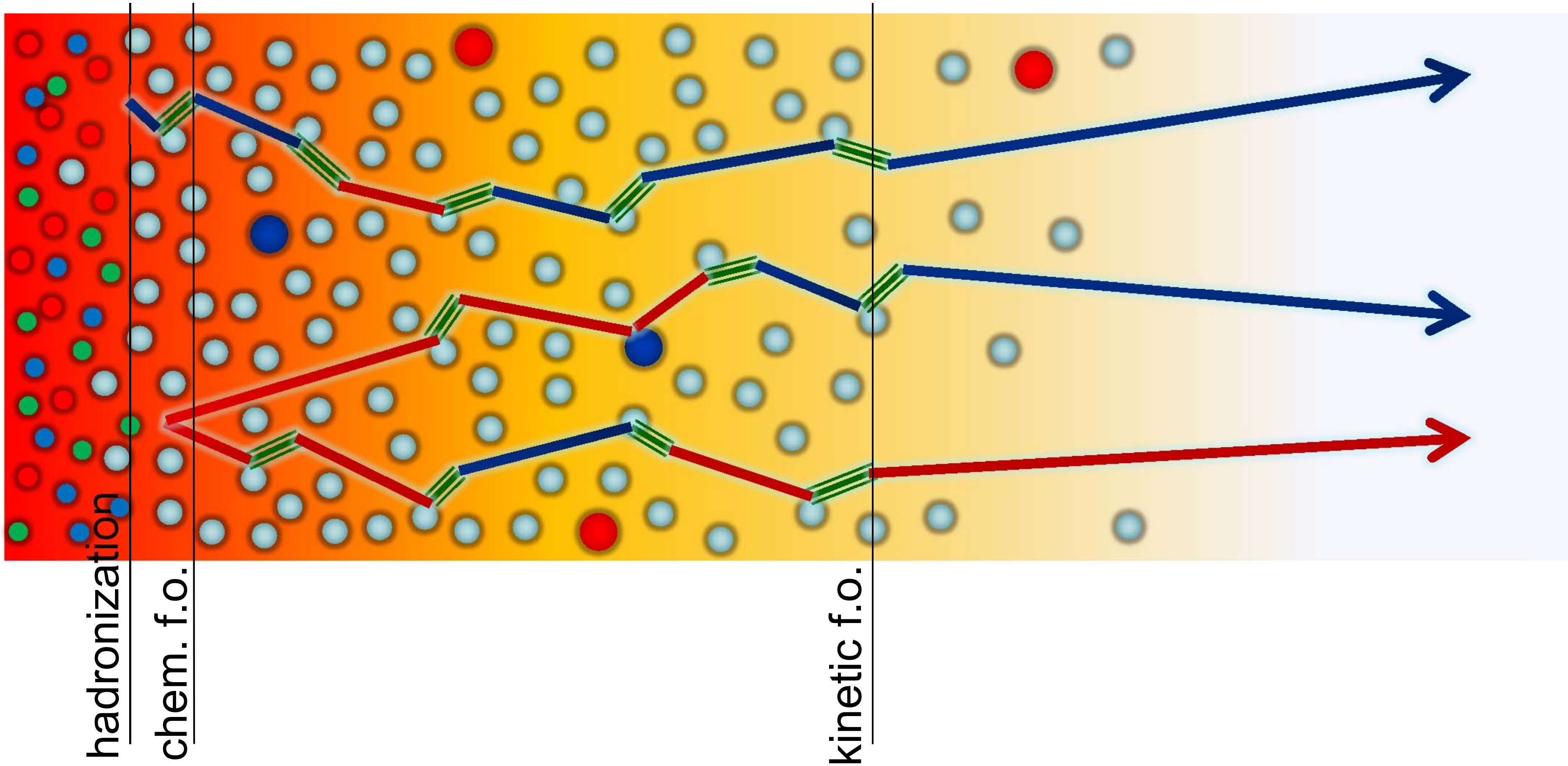
$$m_\pi \simeq T \ll m_N - \mu_N$$

- rare NN collisions
- no Pauli blocking

$n_N \ll n_\pi$

- many pions

Probability Distribution



$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) \Rightarrow P_f(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$

Probability Distribution

$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = P(N_{\mathrm{N}}^{(\mathrm{net})}, N_{\mathrm{N}}^{(\mathrm{tot})}, N_p, N_{\bar{p}})$$

$$N_{\mathrm{N}}^{(\mathrm{net})} = N_p + N_n - N_{\bar{p}} - N_{\bar{n}}$$

$$N_{\mathrm{N}}^{(\mathrm{tot})} = N_p + N_n + N_{\bar{p}} + N_{\bar{n}}$$

In the dilute nucleon approximation, $N_{\mathrm{N}}^{(\mathrm{net})}$ and $N_{\mathrm{N}}^{(\mathrm{tot})}$ are conserved, i.e. $N_p + N_n$ and $N_{\bar{p}} + N_{\bar{n}}$ are conserved separately

When $N_p + N_n \equiv N_{\mathrm{N}}$ and $N_{\bar{p}} + N_{\bar{n}} \equiv N_{\bar{\mathrm{N}}}$ are fixed and hadron phase is long enough compared to the mean free time of (anti)nucleons, the final state (anti)proton distribution is given by the binomial distribution

$$B(N_p; N_{\mathrm{N}})\left(B(N_{\bar{p}}; N_{\bar{\mathrm{N}}})\right)$$

Probability Distribution

- As a result, the final state distribution is factorized as follows:

$$P_f(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = F(N_{\mathrm{N}}^{(\mathrm{net})}, N_{\mathrm{N}}^{(\mathrm{tot})}) B(N_{\mathrm{N}}; N_p) B(N_{\bar{\mathrm{N}}}; N_{\bar{p}})$$

$$F(N_{\mathrm{N}}^{(\mathrm{net})}, N_{\mathrm{N}}^{(\mathrm{tot})}) = \sum_{N_p, N_{\bar{p}}} P(N_{\mathrm{N}}^{(\mathrm{net})}, N_{\mathrm{N}}^{(\mathrm{tot})}, N_p, N_{\bar{p}})$$

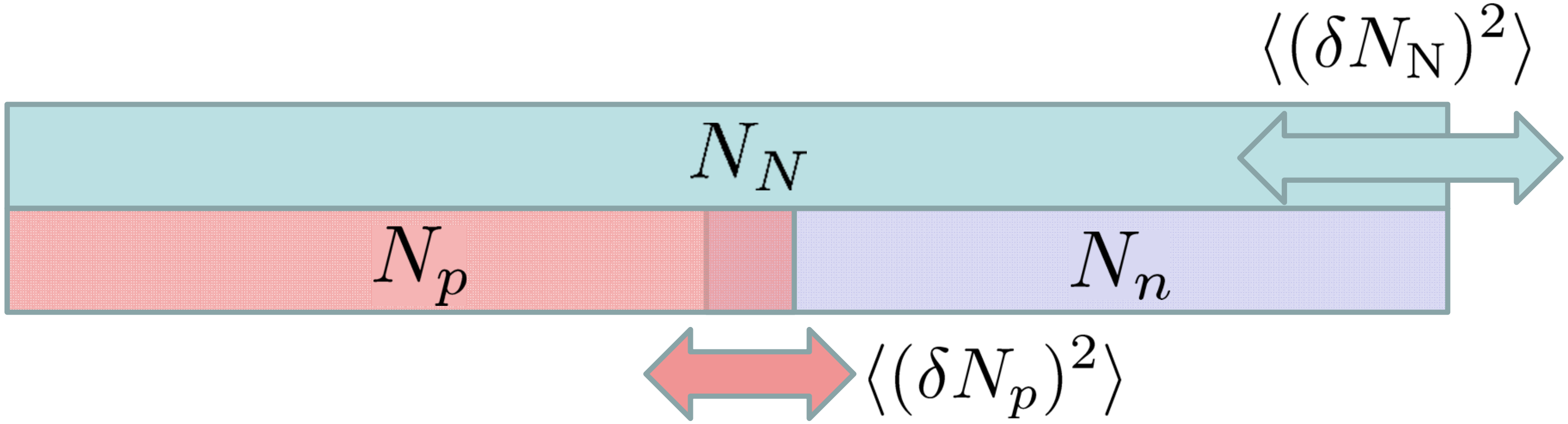
$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = P(N_{\mathrm{N}}^{(\mathrm{net})}, N_{\mathrm{N}}^{(\mathrm{tot})}, N_p, N_{\bar{p}})$$
$$= P'(N_{\mathrm{N}}, N_{\bar{\mathrm{N}}}, N_p, N_{\bar{p}})$$

The two variable function "F" includes the initial information (correlation)

This form of P_f enables to relate proton moments and nucleon moments

Proton and Nucleon Moments

1. Original



2. Additional (from πN→Δ→πN)

$$\left\langle\left(\delta N_p^{(\mathrm{net})}\right)^2\right\rangle = \frac{1}{4}\left\langle\left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2\right\rangle + \frac{1}{4}\left\langle N_{\mathrm{N}}^{(\mathrm{tot})}\right\rangle$$

$$\left\langle\left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2\right\rangle = 4\left\langle\left(\delta N_p^{(\mathrm{net})}\right)^2\right\rangle - 2\left\langle N_p^{(\mathrm{tot})}\right\rangle$$

for isospin symmetric matter

- For free nucleon gas

$$\left\langle\left(\delta N_p^{(\mathrm{net})}\right)^2\right\rangle = \frac{1}{2}\left\langle\left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2\right\rangle$$

Proton and Nucleon Moments

Similarly,

$$N_{\mathrm{N}} \longrightarrow N_p$$

$$\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^3 \right\rangle = \frac{1}{8}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^3 \right\rangle + \frac{3}{8}\left\langle \delta N_{\mathrm{N}}^{(\mathrm{net})} \delta N_{\mathrm{N}}^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^4 \right\rangle_c = \frac{1}{16}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^4 \right\rangle_c + \frac{3}{8}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2 \delta N_{\mathrm{N}}^{(\mathrm{tot})} \right\rangle + \frac{3}{16}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2 \right\rangle - \frac{1}{8}\left\langle N_{\mathrm{N}}^{(\mathrm{tot})} \right\rangle$$

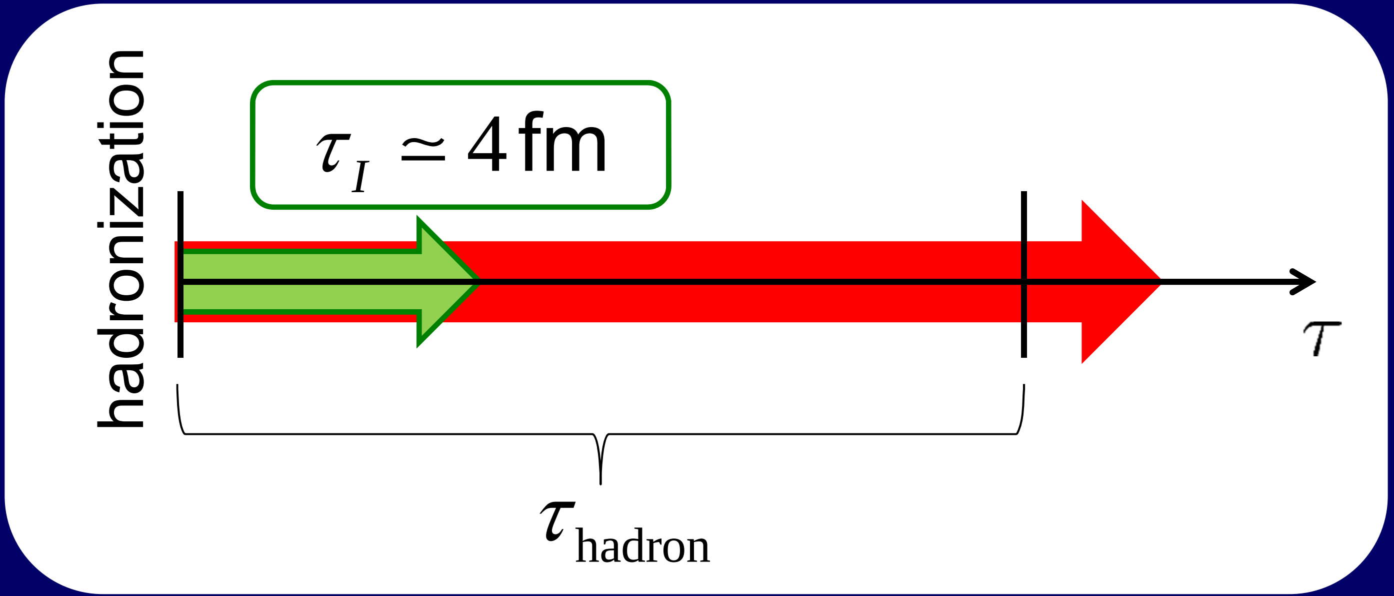
$$N_p \longrightarrow N_{\mathrm{N}}$$

$$\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^3 \right\rangle = 8\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^3 \right\rangle - 12\left\langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \right\rangle + 6\left\langle N_p^{(\mathrm{net})} \right\rangle$$

$$\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^4 \right\rangle_c = 16\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^4 \right\rangle_c - 48\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^2 \delta N_p^{(\mathrm{tot})} \right\rangle + 48\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^2 \right\rangle + 12\left\langle \left(\delta N_p^{(\mathrm{tot})}\right)^2 \right\rangle - 26\left\langle N_p^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \delta N^4 \right\rangle_c = \left\langle \left(\delta N\right)^4 \right\rangle - 3\left\langle \left(\delta N\right)^2 \right\rangle^2$$

Time Scales



τ_I : time scale to realize isospin randomization

τ_{hadron} : time scale of hadron phase duration

τ_{hadron} ⇐ result of state-of-art hydro + cascade calculation

Result of Hydro+Cascade Calculation

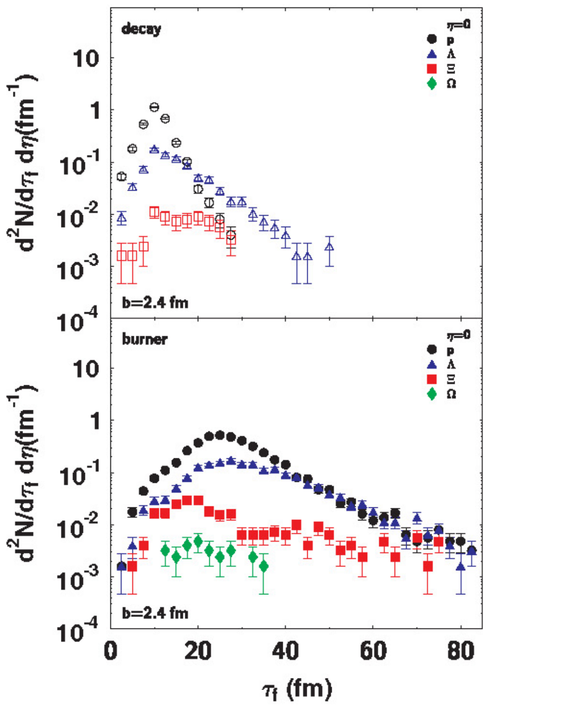


FIG. 22. (Color online) Freeze-out time distribution of baryons for hydro+decay (open symbols, above) and hydro+UrQMD (solid symbols, below) at midrapidity.

providing us with an estimate on the lifetime of the hadronic phase around 10–20 fm/*c*. Note that this estimate is subject to the same systematic uncertainties discussed previously in the context of the overall lifetime of the system.

Freezeout time distribution

without after-burner

with after-burner

$\tau_{\rm hadron}$: 10 ~ 20 fm

Nonaka and Bass, PRC 2007

$\tau_I \ll \tau_{\rm hadron}$ isospin: randomized

N_N or N_B: Strange Baryon Contribution

Strange baryon contribution is mainly from Λ and Σ

Branching Ratios:

Λ $m_\Lambda \simeq 1116[\mathrm{MeV}]$

Σ $m_\Sigma \simeq 1190[\mathrm{MeV}]$

Λ+Σ go to p and n with approximately equal probabilities, 1:1

(Anti)protons from these decays can be incorporated into the binomial distribution, then, $N_N \to N_B$

Decay modes:

$\Lambda \to p + \pi^-$ 64%

$\Lambda \to n + \pi^0$ 36%

$\Sigma^+ \to p + \pi^0$ 52%

$\Sigma^+ \to n + \pi^+$ 48%

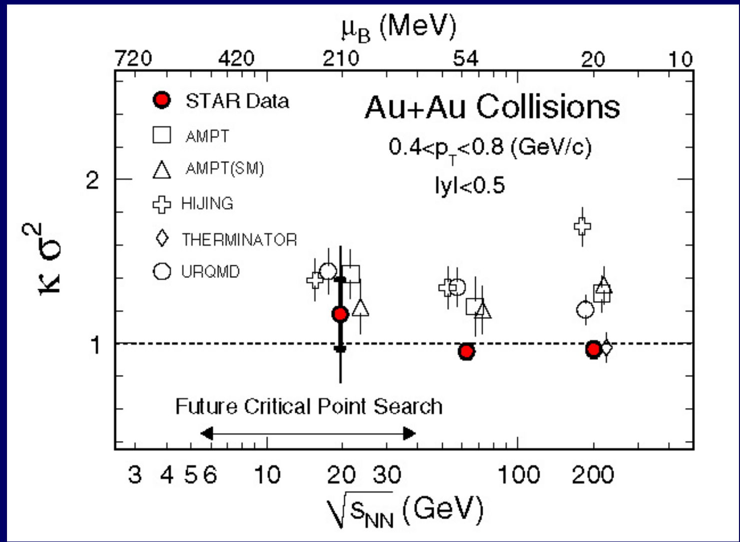
$\Sigma^0 \to \Lambda \to p + \pi^-$ 64%

$\Sigma^0 \to \Lambda \to n + \pi^0$ 36%

$\Sigma^- \to n + \pi^-$

Theoretical & Experimental Questions

Isn't the effect of Δ included in theoretical calculations?



$$\chi_B^{(n)}\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)=\frac{1}{T^n}\frac{\partial^n}{\partial(\mu_B/T)^n}P\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)\Bigg|_{T/T_c}$$

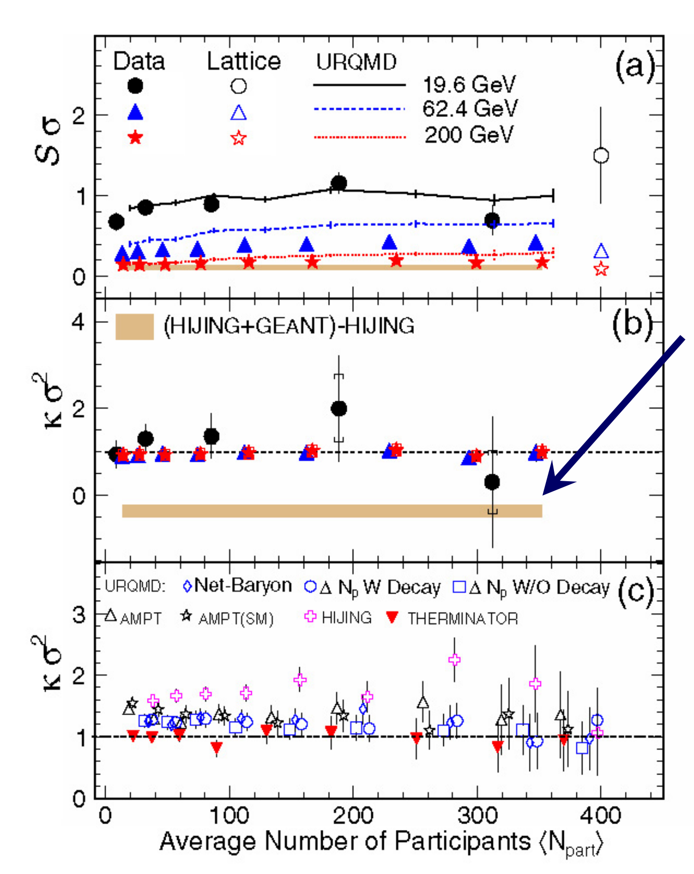
$$\kappa\sigma^2=\frac{T^2\chi_B^{(4)}}{\chi_B^{(2)}}$$

STAR, PRL(2010)

	Final State Interaction
AMPT, URQMD	YES
HRG, THERMINATOR, HIJING	NO

But none of them includes effects of the critical point

Theoretical & Experimental Questions



STAR PRL(2010)

(HIJING+GEANT) - HIJING < 0

detector effeiciency ⇒ Poissonian

$\kappa\sigma^2 \rightarrow 1$

← Poissonian case (no correlation)

Summary

- **Conserved Charges and Higher Moments:**
  - Third Fluctuation Moments of Conserved Charges take negative values in regions on the FAR SIDE of Phase Transition (more information!)

- **Proton Number Cumulants and Baryon Number Ones:**
  - Proton Number Cumulants are not frozen at chemical freezeout
  - Mean Free Time of Protons in hadron phase is very short owing to Δ formation and isospin is randomized
  - Final p, $\bar{p}$, n, $\bar{n}$ distributions are factorized (NOT an assumption!)
  - This makes it possible to relate the initial baryon number cumulants and final proton number cumulants, and vice versa
  - Extension to isospin nonsymmetric case is straightforward, and it turned out its effect is very weak (Kitazawa and M.A., to be published)