A relativistic hydro model compared to LHC data

Sándor Lökös

Eötvös University

Outline

1. The basic equations
2. Few relativistic solutions
3. The investigated solution
4. Comparing to data

Basic equations

- Continuity equation: $\partial_\mu (n u^\mu) = 0$
- The energy-momentum tensor: $T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$
- Euler-equation: $(\epsilon + p) u^\nu \partial_\nu u^\mu = (g^{\mu\rho} - u^\mu u^\rho) \partial_\rho p$
- Energy conservation: $(\epsilon + p) \partial_\nu u^\nu + u^\nu \partial_\nu \epsilon = 0$
- Equations of state: $p = nT \quad \epsilon = \kappa(T) p$

Few relativistic solution

- Landau–Khalatnikov-solution (LK)
  (S. Belen'kji and L. Landau, Il Nuovo Cimento (1955-1965) 3, 15 10.1007/BF02745507 (1956).)
  1. 1+1 dimesion
  2. Implicit
  3. Longitudinal
  4. Can be applied to $p^+ - p^+$ collision

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- Hwa–Bjørken-solution (HB)
  (R. C. Hwa, Phys. Rev. D 10, 2260 (1974). ; J. D. Bjorken, Phys. Rev. D 27, 140 (1983).)
  1. 1+1 dimension
  2. Accelerationless
  3. Explicit
  4. Underestimating the initial energy density

Few relativistic solution

- Nagy–Csörgő–Csanád-solution (NCC)
  (T. Csorgo, M. I. Nagy, and M. Csanad, Phys. Lett. B663, 306 (2008) [arXiv:nucl-th/0605070].)
  1. Written in Rindler-coordinates
  2. Acceleration
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  4. Contain the HB-solution

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- Bialas-solution
  (Phys.Rev.C76:054901,2007)
  1. Written in light-cone coordinate
  2. 1+1 dimension
  3. Connects the HB-solution and LK-solution

The investigated solution

(Csörgő, Csernai, Hama, Kodama Heavy.Ion.Phys. A21:73-84, 2004)

This solution has self-similar, ellipsoidal symmetry:
$s = \frac{x^2}{X^2(t)} + \frac{y^2}{Y^2(t)} + \frac{z^2}{Z^2(t)}$ is the scaling variable

- The velocity field is Hubble-type field:
  - $u^\mu = \gamma \left(1, \frac{\dot{X}}{X}x, \frac{\dot{Y}}{Y}y, \frac{\dot{Z}}{Z}z\right) = \frac{x^\mu}{\tau}$
  - $\dot{X}, \dot{Y}, \dot{Z} = const. \rightarrow$ accerelationless
- The thermodynamical functions:
  - $n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$
  - $T = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\nu(s)}$

$\nu(s)$ is an arbitrary function (e.g. $\nu(s) = e^{-bs/2}$)

The source function, the momentum distribution

- Relativistic Maxwell–Boltzmann-distribution $\rightarrow$ Maxwell–Jüttner-distribution:

$$N_1(p) = \int_{\mathbb{R}^4} S(x,p) d^4x = \int \mathcal{N} n \exp\left[\frac{p_\mu u^\mu}{T}\right] H(\tau) d\tau p_\mu d^3\Sigma_\mu(x),$$

- the freeze-out hyper surface can be written: $d^3\Sigma(x) = \frac{u^\mu d^3x}{u^0}$ if $H(\tau) = \delta(\tau - \tau_0)$
- $p_z = 0$ and average ϕ
- the transverse momentum distribution:

$$N_1(p_t) = \frac{1}{2\pi} \int_0^{2\pi} N_1(p) d\phi$$

The elliptic flow – azimuthal asymmetry

$$N_1(p) = N_1(p_t)\left[1 + 2\cdot\sum_{n=1}^{\infty} v_n \cos(n\phi)\right]$$

$$v_2(p_t) = \frac{\int_0^{2\pi} d\phi N_1(p_t,\phi)\cos(2\phi)}{\int_0^{2\pi} N_1(p_t,\phi)}$$

The freeze-out expansion anisotropy can be chosen to compared with data:

$$\epsilon = \frac{\dot{X}^2 - \dot{Y}^2}{\dot{X}^2 + \dot{Y}^2}$$

and the transverse expansion rate:

$$\frac{1}{u_t^2} = \left(\frac{1}{\dot{X}^2} + \frac{1}{\dot{Y}^2}\right)$$

The two-particle correlation

The two-particle correlation is the Fourier-transforming of source function

$$C_2 = \frac{N_2(p_1, p_2)}{N_1(p_1)N_2(p_2)} = 1 + \left| \frac{\tilde{S}(q,K)}{\tilde{S}(q=0,K)} \right|^2$$

where: $q = p_1 - p_2$, $K = 0.5 \cdot (p_1 + p_2)$
If the source function is Gaussian $\rightarrow$

$$C_2 = 1 + \exp\left(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2\right)$$

To compare the R_x, R_y, R_z HBT radii with data the Bretsch-Pratt frame:

- if $H(\tau) = \delta(\tau - \tau_0)$
- $R_{out}^2 = \frac{R_x^2 + R_y^2}{2}$ $\quad R_{long}^2 = R_z^2$ $\quad R_{side}^2 = R_{out}^2$

Comparison with RHIC/LHC data

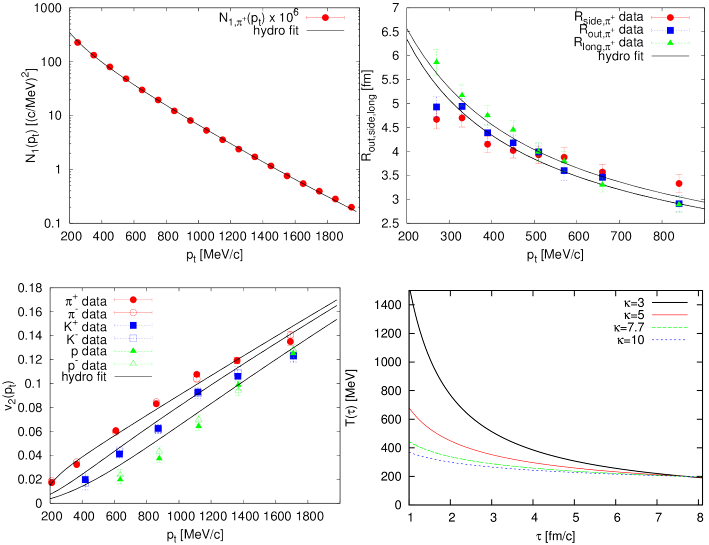
RHIC: (Csanád, M. and Vargyas, M. Eur.Phys.J. A44:473-478,2010)

Parameters	RHIC values	LHC values
T_0 [MeV]	199 $\pm$ 3	270 $\pm$ 3
ϵ	0.80 $\pm$ 0.02	0.95 $\pm$ 0.04
u_t^2/b	-0.84 $\pm$ 0.08	-1.44 $\pm$ 0.22
τ_0 [fm/c]	7.7 $\pm$ 0,1	8.10 $\pm$ 0.22
$\dot{Z}_0^2/b$	-1.6 $\pm$ 0.3	fixed
NDF	41	46
χ^2	24	67
Probablity		2.3%

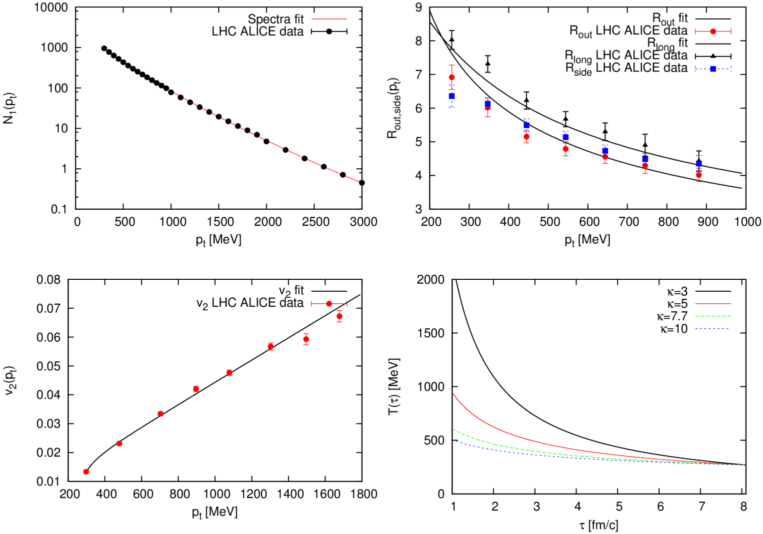
Parameters	RHIC values	LHC values
$T_0[MeV]$	204 $\pm$ 7	272 $\pm$ 10.0
ϵ	0.34 $\pm$ 0.01	0.23 $\pm$ 0.13
u_t^2/b	-0.34 $\pm$ 0.01	-0.300 $\pm$ 0.023
NDF	34	5
χ^2	66	14
Probablity		1.6 %

Comparison with RHIC data

(Csanád, M. and Vargyas, M. Eur.Phys.J. A44:473-478,2010)



Comparison with LHC data



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- The freeze-out proper times: $\tau_{RHIC} = 7.7$, $\tau_{LHC} = 8.1$
- It's seems, the model can be applied to data of LHC too.

Thank you for your attention!

Skipped frame

- Jet quenching – missing high-monument particle (K. Adcox et al. Phys.Rev.Lett. 88.022301 (2002)).

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- Scaling property (Phys. Rev. Lett. 98, 162301 (2007))
- Almost a perfect fluid (Phys. Rev. Lett. 98, 172301 (2007))
- High temperature (Phys. Rev. Lett. 104, 132301 (2010))

The source function

Relativistic Maxwell–Boltzmann-distribution $\rightarrow$ Maxwell–Jüttner-distribution:

$$S(x,p)d^4x = \mathcal{N} n e^{-\frac{p_\mu u^\mu}{T}} H(\tau) d\tau p_\mu d^3\Sigma_\mu(x), \qquad (1)$$

where the Cooper–Frye-faktor ($\tau =$*áll.* and $H(\tau) = \delta(\tau - \tau_0)$):
$d^3\Sigma_\mu(x) = \frac{u^\mu d^3x}{u^0}$.

if a second order Gaussian approximation is applied:

$$N_1(p) = \int_{\mathbb{R}^4} S(x,p) = \overline{N} \cdot \overline{E} \cdot \overline{V} \cdot e^{\frac{p^2}{2ET_0} - \frac{p_x^2}{2ET_x} - \frac{p_y^2}{2ET_y} - \frac{p_z^2}{2ET_z}} \qquad (2)$$

Momentum distribution

$$T_x = T_0 + \frac{ET_0\dot{X}_0^2}{b(T_0 - E)} \tag{3}$$

After some calculations:

$$N_1(p_t) = \overline{NV}\left(E - \frac{p_t^2(T_{\mathrm{eff}} - T_0)}{ET_{\mathrm{eff}}}\right) e^{\left[\frac{p_t^2}{2ET_{\mathrm{eff}}} + \frac{p_t^2}{ET_0} - \frac{E}{T_0}\right]} \tag{4}$$

where $1/T_{\mathrm{eff}} = 0.5(1/T_x + 1/T_y)$ and

$$w = \frac{p_t^2}{4m_t}\left(\frac{1}{T_y} - \frac{1}{T_x}\right) \tag{5}$$

have been introduce.

Elliptic flow

$$v_2 = \frac{\int_0^{2\pi} d\phi N_1(p_t,\phi)\cos(2\phi)}{\int_0^{2\pi} d\phi N_1(p_t,\phi)}. \tag{6}$$

with $I_1(w) \approx 2wI_0(w)$, $I_2(w) \approx 0$ approximations:

$$v_2(p_t) = \frac{I_1(w)}{I_0(w)} \left(1 + \frac{2T_0}{E - \frac{p_t^2(T_{\mathrm{eff}} - T_0)}{ET_{\mathrm{eff}}}}\right). \tag{7}$$

Correlation function (HBT)

Symmetric two-particle wavefunction:

$$\Psi_{1,2} = 1/\sqrt{2}\left(e^{ik_1 r_1} e^{ik_2 r_2} + e^{ik_1 r_2} e^{ik_2 r_1}\right) \tag{8}$$

Two-particle momentum distribution:
$N_2(p_1, p_2) = \int S(r_1, p_1) S(r_2, p_2) |\Psi_{1,2}|^2 d^4 r_1 d^4 r_2$

where $|\Psi_{1,2}|^2 = 1 + (e^{-i(k_1 - k_2) r_1} e^{-i(k_1 - k_2) r_2})$

Put in the integrals to formula $\rightarrow$ Fourier-integrals, where
$q = r_1 - r_2$ és $K = 0.5(p_1 + p_2) \rightarrow C_2 = 1 + \left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|$

If the source function is Gaussian, the correlation function:

$$C_2(q, K) = 1 + \exp\left(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2\right) \tag{9}$$