Nonlocal effective theories

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Outlines

1 Degrees of freedom of QCD

2 Nonlocal effective theories
- Quasiparticles
- Bound states
- QCD near the critical temperature
- Black body radiation of a strongly interacting system

3 Conclusion

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Energy regimes of QCD

QCD! $\Rightarrow$ interacting theory of quark and gluon fields

Is it a theory of quarks and gluons?

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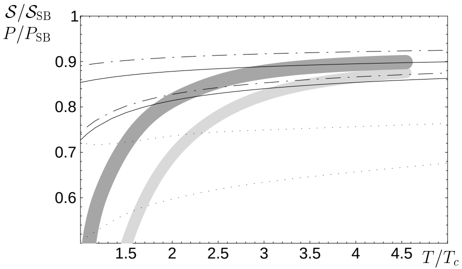
- at high energy: quark, gluons & perturbation theory



(J. P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. Lett. **83**, 2906 (1999) [arXiv:hep-ph/9906340])

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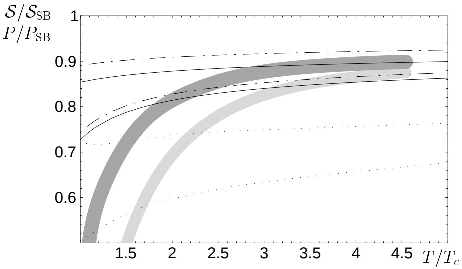

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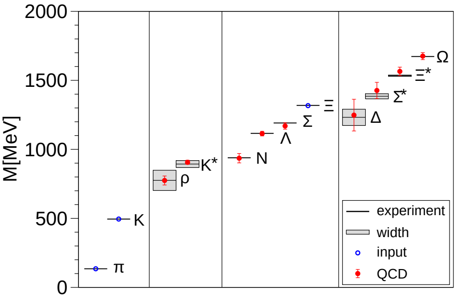

(S. Durr *et al.*, Science **322**, 1224 (2008) [arXiv:0906.3599 [hep-lat]])

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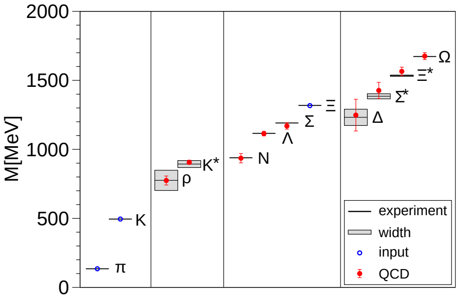
$\Downarrow$

description (physical picture) depends on the energy range



(S. Durr *et al.*, Science **322**, 1224 (2008) [arXiv:0906.3599 [hep-lat]])

Elementary excitations at high energy

- At asymptotically large energy $\alpha_s(E) \to 0$ (asymptotic freedom) $\Rightarrow$ we expect to recover quarks and gluons as stable elementary excitations
- At large energy/high temperature
  - quark- and gluon-like excitations (cf. quark-gluon plasma)
  - quasiparticles with mass *and* lifetime
  - no sharp mass shell $\Rightarrow$ several energy levels, spectrum

Low energy/temperature

- At low energy/temperature
  - strongly interacting regime from the point of view of QCD
  - effective description: bound states (hadrons)
  - effective models: "weakly" interacting hadrons (e.g. sigma models, NJL model, Polyakov loop models etc.)
    Hadron resonance gas – no interaction
  - realistic excitations: quasiparticles with finite (sometimes large) width
  - not fundamental degrees of freedom – independent?
- Near T_c
  - near perfect liquid $\Rightarrow$ strongly interacting from the point of view of both QCD and hadronic description
  - not necessarily non-perturbative: we should find the relevant degrees of freedom
  - only representable by the complete spectral function

Nonlocal effective models

- lifetime of the quasiparticles $\Rightarrow$ time dependent Hamiltonian?
  unitarity? E-conservation?
- consistent with considering the complete spectral function (causality, unitarity, symmetries, E-conservation etc.)
- applicable also when small-width quasiparticle description fails (eg. off-shell transport, liquids)

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The free model

- at large energy: QCD weakly interacting
- elementary excitations: free quarks and gluons
  $\Rightarrow$ energy and momentum eigenstates with $E_{\mathbf{k}}^2 = \mathbf{k}^2 + m^2$ dispersion relation

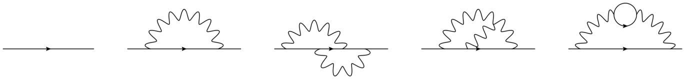
Basic model

Non-interacting, free particles (infinite lifetime)

Perturbation theory

Weak interaction: expand expectation values with respect of the coupling constant $\Rightarrow$ perturbation theory (PT)

- direct PT: Feynman-diagrams

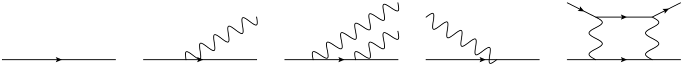


- divergences (UV and IR) $\Rightarrow$ renormalization, resummation (self-energy, RG, OPE, thermal masses, dimensional reduction, screened PT, HTL, 2PI, etc.)

Spectrum

Result of PT: states with the same quantum numbers mix

e.g. one-particle states mix with multi-particle states



multi-particle states have no "mass-shell"

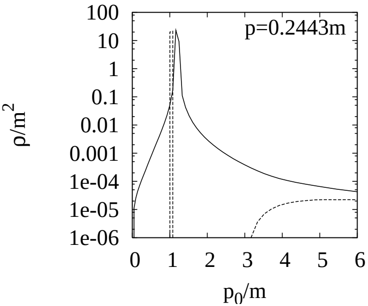
$\Rightarrow$ Characterization: spectrum

- energy levels at fixed momentum (and other Q-numbers)
- to measure it: take an operator A

$$\varrho_A(x) = \langle 0 | [A(x), A(0)] | 0 \rangle \Rightarrow \varrho_A(\omega > 0, \mathbf{k}) = \sum_n \alpha_{n,\mathbf{k}} \delta(\omega - E_{n,\mathbf{k}})$$

spectral function is proportional to the spectrum $\left(\alpha_{n,\mathbf{k}} = 2\pi |\langle 0 | A | n, \mathbf{k} \rangle|^2\right)$.

Typical spectrum



Φ^4 model, 2-loop renormalized 2PI resummation

$(T = 0, m, \lambda = 10)$

(AJ, PRD76 (2007) 125004 [hep-ph/0612268])

- $T = 0$: mass-shell shifts, multiparticle thresholds
- $T > 0$: mass-shell shifts and acquires width, $\varrho > 0$ everywhere

Quasiparticles

Good approximation if we consider only the peak of the spectral function

$\Rightarrow$ quasiparticle: (modified) mass and finite lifetime

Lifetime? decaying particle $\Rightarrow$ violates E-conservation, unitarity

What can we do instead?

2PI approximation

(2PI approximation: 2-particle irreducible)

Idea of 2PI

use the "exact" excitation spectrum for the quasiparticles

Consistent resummed PT: all energy levels are taken into account!
technically for scalar field theory we start from the form:

$$\mathcal{L} = \frac{1}{2}\Phi G^{-1}\Phi + \mathcal{L}_{int}$$

$\Rightarrow$ G comes from self-consistent propagator equation (2PI)

(J. M. Cornwall,R. Jackiw and E. Toumbolis, Phys. Rev. D10, 2428 (1974).)
(J. Berges and J. Cox, Phys. Lett. B 517 (2001) 369)

$$G^{-1}(p) = G_0^{-1}(p) - \Sigma[G](p)$$

and in the self-energy calculation we use the G propagator.

- renormalizability ✓

  (H. van Hees, J. Knoll, PRD66 (2002) 025028)
  (A. Jakovac, Zs. Szep PRD71 (2005) 105001 [hep-ph/0405226])
  (A. Patkos, Zs. Szep, Nucl.Phys. A811 (2008) 329, [arXiv:0806.2554])

- unitarity: no missing state ✓
- global and local symmetries (Ward-identities) ✗

  (U. Reinosa, J. Serreau, Ann.Phys. 325 (2010) 969, [arXiv:0906.2881])

- deep IR physics ✗
  Bloch-Nordsieck model, cf. P. Mati's talk

  (P. Mati, BME, diploma work, paper in preparation)

Lesson

Non-local inverse propagator consistently treatable within resummed PT framework

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QCD at low energy/temperature

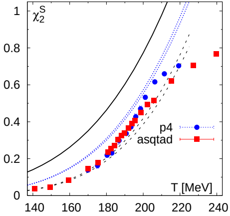
- Strongly interacting, nonperturbative from the point of view of the quark-gluon picture
- observation: "weakly" interacting bound states (hadrons)

Basic model

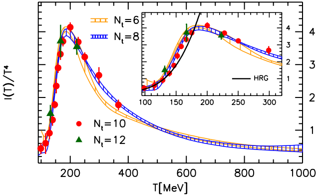
non-interacting hadrons

Basic model

HRG (hadron resonance gas) – masses from the experiments



(P. Huovinen and P. Petreczky, Nucl. Phys. A **837**)
(26 (2010) [arXiv:0912.2541 [hep-ph]].)



(Sz. Borsanyi, G. Endrodi, Z. Fodor, A.J., S. D. Katz)
(S. Krieg, C. Ratti, K.K. Szabo, JHEP 1011 (2010) 077)

basic model works quite well for thermodynamics!

- Is it allowed to consider short lifetime resonances as independent degrees of freedom ($\tau \sim 10^{-24}$ s lifetime)?
- How independent are these degrees of freedom?

Resonances from scattering theory

contribution of resonant states to the partition function

scattering theory, elastic scattering: $\delta_\ell(\varepsilon)$ phase shifts
$\Rightarrow$ modify the free energy
(Uhlenbeck-Beth formula)

$$\sum_i e^{-\beta E_i} \quad \Rightarrow \quad \delta Z \sim \int_0^\infty \frac{d\omega}{\pi} \frac{\partial \delta}{\delta \omega} e^{-\beta\omega}$$

at bound states δ has a π-t jump $\Rightarrow$ $\delta Z \sim e^{-\beta E}$

(Landau, Lifsitz V.; R.F Dashen, R. Rajaraman, PRD10 (1974), 694.)

- thermodynamical degrees of freedom (quasiparticles)
- independence?
  - simplest approach: consider them independent
  - complex amplitudes to the S-matrix, unitarity gives relations between them $\Rightarrow$ not independent

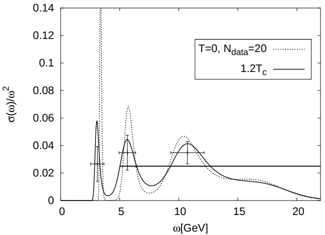
    (M. Svec, PRD64 (2001) 096003 [hep-ph/0009275])

Resonances in field theory

we want consider bound states as quasiparticles
can be consistent only with taking into account the complete spectrum (cf. 2PI)!

Take spectrum as an input!

- from PT using interacting hadron models
- from experiments
- from MC simulation



(AJ., P. Petreczky, K. Petrov, A. Velytsky, PRD75 (2007) 014506)

Nonlocal Lagrangian

Nonlocal scalar quadratic model: similar to 2PI

(AJ. arXiv:1102.5629)

$$\mathcal{L} = \frac{1}{2}\Phi(x)\mathcal{K}(i\partial)\Phi(x)$$

- relation of $\mathcal{K}$ kernel and ϱ spectral function:

  $$G_R(k_0,\mathbf{k}) = \mathcal{K}^{-1}(k_0+i\varepsilon,\mathbf{k}), \qquad \varrho = -2\,\mathrm{Im}\,G_R$$

  $$G_R(k_0,\mathbf{k}) = \int\frac{d\omega}{2\pi}\frac{\varrho(\omega,\mathbf{k})}{k_0-\omega+i\varepsilon}, \qquad \mathcal{K} = \mathrm{Re}\,G_R^{-1}$$

  spectrum completely determines phyisics!
- can be proven: unitarity, causality, E-conservation, Lorentz-invariance (like in 2PI)

Energy density

time translation symmetry $\Rightarrow$ energy density (Noether-thm)

$$\varepsilon = T_{00} = \int \frac{d^4 p}{(2\pi)^4} \Theta(p_0) \left(p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K} \right) n(p_0) \varrho(p)$$

- pressure, entropy, etc come from standard thermodynamics
- nonlinear functional of ϱ! (because $\varrho \Rightarrow \mathcal{K}$)

Number of degrees of freedom

Number of the bound states?

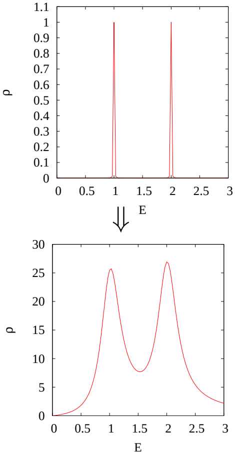
- not evident in case of a general spectrum!
- consistency (for independent particles, or for one Breit-Wigner form)
- consistent with usual physical picture (Williams-Weizsacker)

$$N_{dof} = \int_0^\infty \frac{dp_0}{2\pi}\,\frac{1}{p_0}\left(p_0\frac{\partial\mathcal{K}}{\partial p_0} - \mathcal{K}\right)\,\varrho(p).$$

$\Rightarrow$ number of bound states (degreed of freedom) is a dynamical quantity!

On the independence of the bound states

Change the width and compute the number of degrees of freedom!



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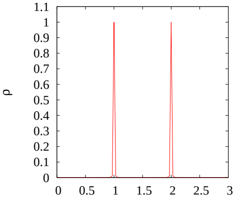

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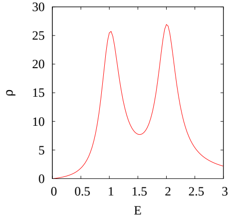
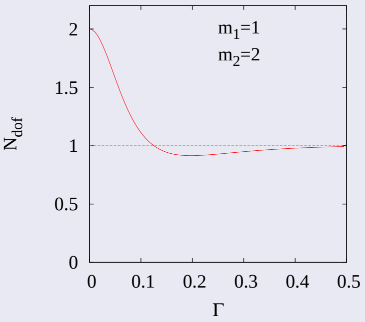


number of DoF



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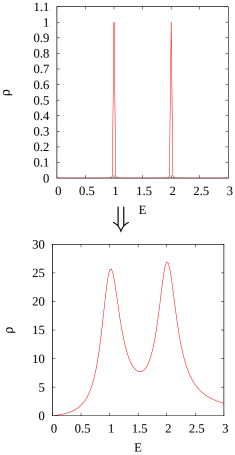

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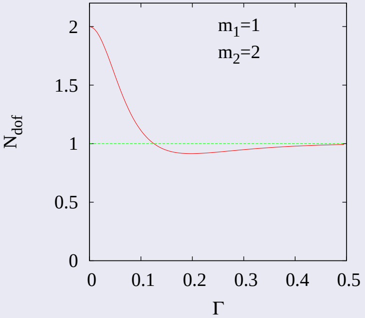
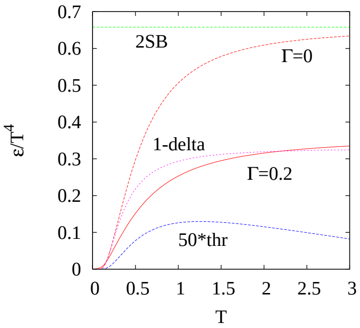


independence: separation is larger then width

Independece of bound states from thermodynamics



$m_1 = 1,\ m_2 = 2$

- $\Gamma = 0$: 2 Dirac-delta ($\Gamma_1 = \Gamma_2 = 0$)
- $\Gamma = 0.2$: finite width peaks
  $\Gamma_1 = \Gamma_2 = 0.2$: if we had only one particle!
  $\Rightarrow$ reduction of the number of degrees of freedom is observable in thermodynamics, too

Lowest curve: multiparticle threshold $\varrho(p) = \sqrt{1 - \frac{m^2}{p^2}}$

- negligible contribution to thermodynamics!
- overlapping Breit-Wigners $\Rightarrow$ destructive interference

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experimental evidence: liquid-like matter ("almost perfect liquid")

- more precisely: $\frac{\eta}{s} \sim \eta\ell^3 \sim \frac{1}{4\pi}$ small (on ℓ internal scale)

  $\Rightarrow$ very far from an ideal gas
- kinetic theory: $\frac{\eta}{s} \sim E\tau$ small

  $\Rightarrow$ very short lifetime excitations are needed (!?)
  (cf. jet suppression)

$\Rightarrow$ nonperturbative regime both from hadronic and quark-gluon side

how to treat it?

I. method: exactly solvable model

$\mathcal{N} = 4$ SYM theory with large N_c and $\lambda = g^2 N_c$

- CFT $\Rightarrow$ AdS/CFT duality $\Rightarrow$ 5D AdS gravitation $\Rightarrow$ computable
- indeed liquid: $\eta/s = 1/4\pi$ if $\lambda \to \infty$

  (P. Kovtun, D.T. Son, A.O. Starinets JHEP 0310, (2003) 064.)

  (A. Buchel, R.C. Myers, M.F. Paulos, A. Sinha, Phys.Lett.B669:364-370,2008.)

BUT: $\mathcal{N} = 4$ SYM $\neq$ QCD (symmetries, particle content)

- similar when we apply Φ^3 model instead QCD
- Hope: some universality is in the background, and so the details are not important

Universality?

How specific is QCD?

Generic fluid:

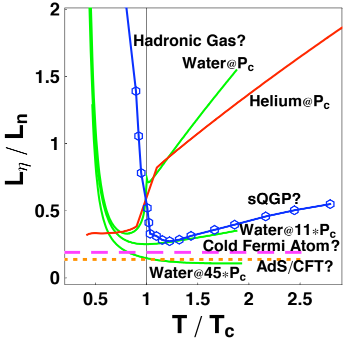
- fluidity measure $\frac{\eta}{s} \to \frac{L_\eta}{L_n}$
- smallest to supercritical fluids

Lesson:

- QCD not extraordinary
- behaviour near T_C

$\Downarrow$

supercritical fluids?



(J. Liao, V. Koch, Phys. Rev. C81, 014902 (2010))

II. method: nonlocal model

can we build a quadratic model which describes liquid?

$$\mathcal{L} = \frac{1}{2}\Phi(x)\mathcal{K}(i\partial)\Phi(x) \qquad \mathcal{K} \Leftrightarrow \varrho$$

- in the spectrum must be no sharp peaks $\Rightarrow$ they would lead to large free mean path, gas-like behaviour
- excitations are not particle-like "non-particles", "non-shell particles", "unparticles"

  (N.P. Landsman, Annals Phys. 186 (1988) 141)

  (H. Georgi, Phys. Rev. Lett. **98**, 221601 (2007). [hep-ph/0703260].)

Viscosity for broad spectral functions

One can calculate η/s for a generic spectral function

(AJ., PRD81 (2010) 045020 [arXiv:0911.3248])

generic structure:

$$\frac{\eta}{s} \sim \frac{\int f_1 \varrho^2}{\int f_2 \varrho + \ln \int f_3 \varrho} \xrightarrow{\textit{rescaling}} \frac{\langle \varrho^2 \rangle}{\langle \varrho \rangle, \ln \langle \varrho \rangle}.$$

sum rule: $\int \varrho = 1$

- large peak in ϱ $\Rightarrow$ even larger peak in ϱ^2 $\Rightarrow$ η/s large
- shallow ϱ $\Rightarrow$ ϱ^2 even shallower $\Rightarrow$ η/s small

robust result: broad spectral function describes liquid!

is it the universality in the background. . . ?

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Particle yields

In the plasma: distribution function $e^{-\beta E}$ $\Rightarrow$ what is the outgoing particle current?

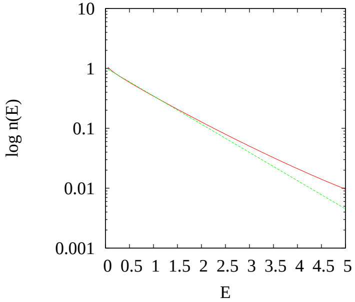
- quasiparticles in the plasma $\neq$ vacuum particles
- dressing ("hadronization"): assume some conserved quantity: energy and momentum! (works also with other assumptions)
- observed energy spectrum:

$$\omega_p n_{obs}(\omega_p) = \int_0^\infty \frac{dp_0}{2\pi} \left(p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K} \right) \varrho(p_0, p) n(p_0)$$

- if ϱ peaked near ω_p $\Rightarrow$ at small energies the peak region dominates
- at large energies peak suppressed by $n(p_0)$ exponentially $\Rightarrow$ small p_0 regime dominates $\Rightarrow$ off-shell effects

Példa

in case of a Breit-Wigner spectrum ($\Gamma = 0.1E$)



Prediction

- exponential behaviour at small energies
- power-law at large energies

(details depend on the form of the spectral function at small energies)

. . . work in progress. . .

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Conclusions

- description of quasiparticles is consistent only with taking into account the complete spectrum
  - gives nonlocal theory
  - unitary, causal, E-conserving
  - number of exciations is dynamical question
    $\Rightarrow$ independence of excitations, change in the number of excitations is possible to describe
- applications
  - quasiparticles in PT: 2PI method
  - description of bound states
  - description of liquids, transport coefficients
  - black body radiation, off-shell effects: lower-law at large energies