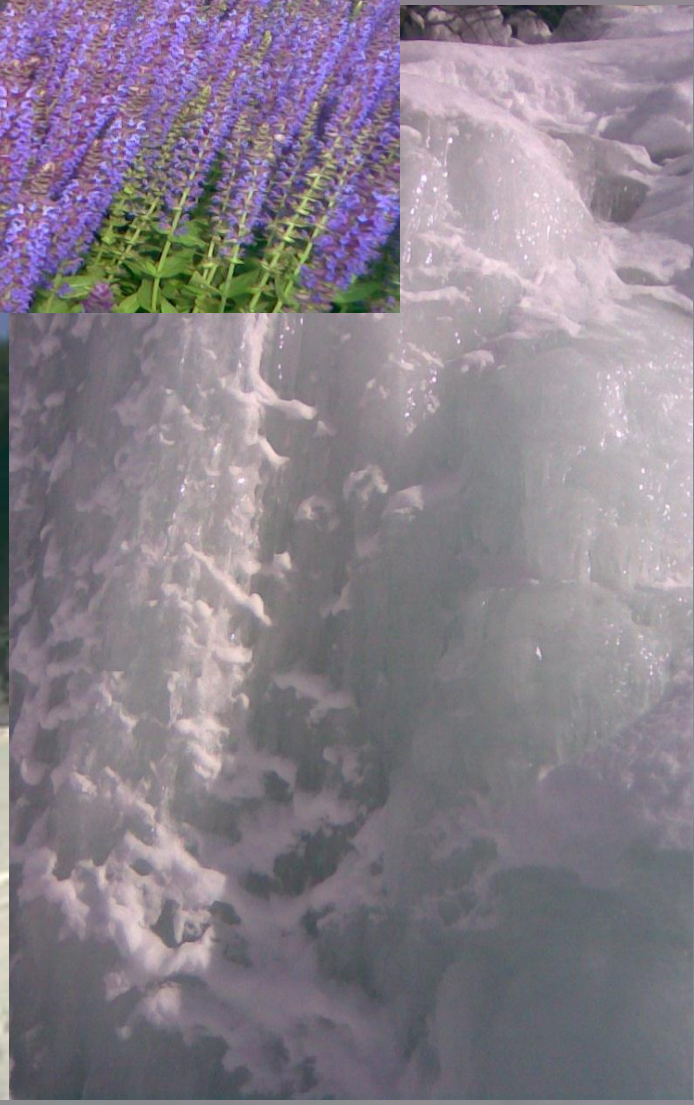
30°
JCDecaux



Fundamental Theories of Physics 1014

Tamás Sándor Biró

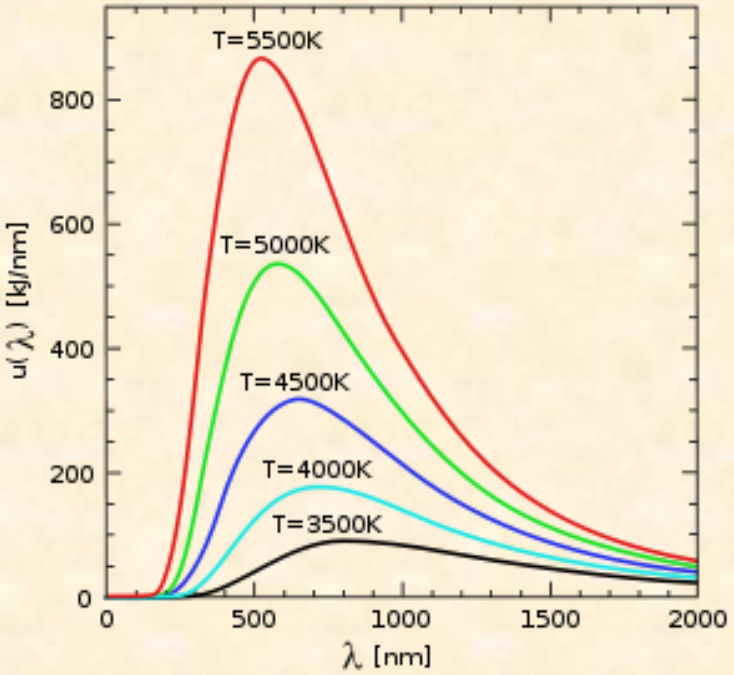
Is There a Temperature?

Conceptual Challenges at High Energy, Acceleration and Complexity

Springer

Measuring the temperature

- Thermometer (direct contact)
  - dilatation
  - air pressure
  - mechanical or electric stress
- Chemistry (mixture)
  - color
  - mass ratios
  - hadronic composition
- Spectra (telemetrics)
  - astronomy (photons)
  - pT spectra of light and heavy particles
  - multiplicity fluctuations



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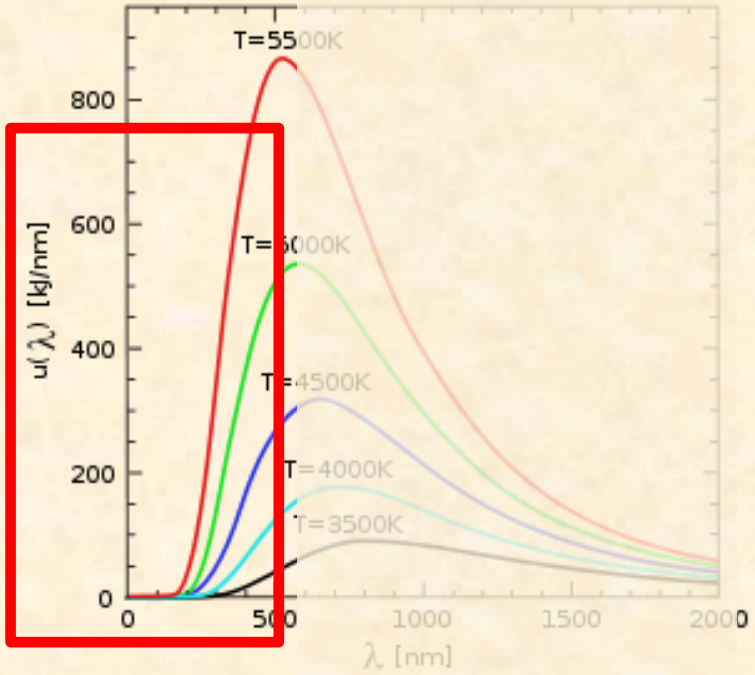
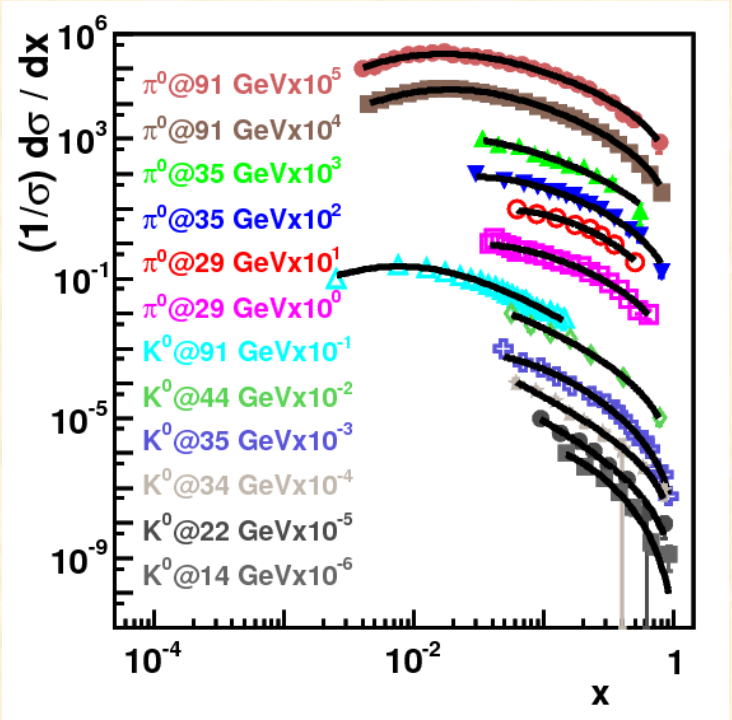


Interpreting the temperature

- Thermodynamics (universality of equilibrium)
  - zeroth theorem (Biro+Van PRE 2011)
  - derivative of entropy
  - Boyle-Mariotte type equation of state
- Kinetic theory (equipartition)
  - energy / degree of freedom
  - fluctuation - dissipation
  - other average values
- Spectral statistics (abstract)
  - logarithmic slope parameter
  - observed energy scale
  - dispersion / power -law effects



arXiV: 1111.4817 Unruh gamma radiation at RHIC ?

Why do thermal models work?

Are there alternatives?

T.S.Biró[1], M.Gyulassy[2], Z. Schram[3]

[1] MTA KFKI RMKI [2] Columbia University [3]University of Debrecen

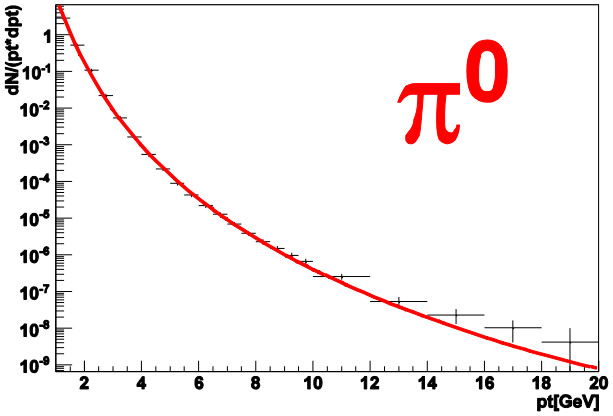
Constant acceleration = temperature ?
Soft bremsstrahlung interpolates between high k_T exponential and low k_T 1 / square

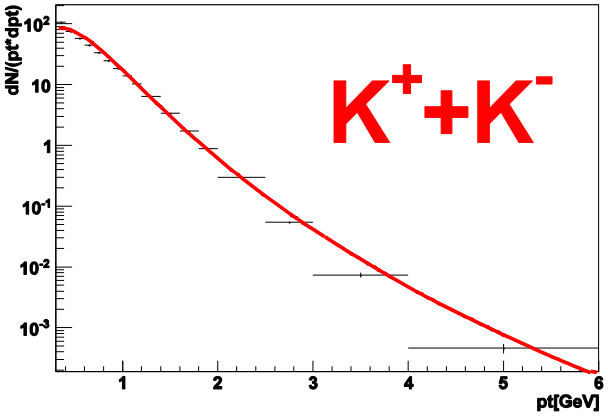
Zimányi School 2011, Dec. 1. 2011, KFKI RMKI Budapest, HU.EU

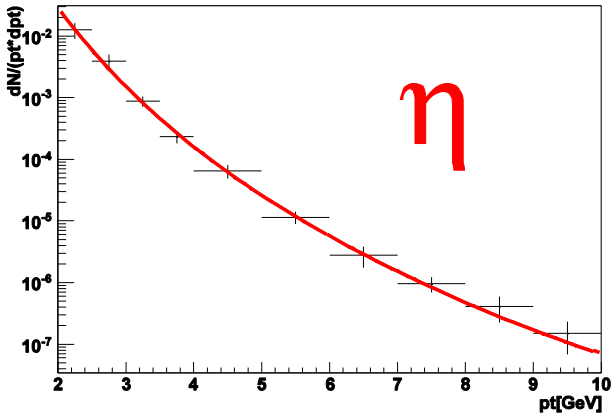
Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

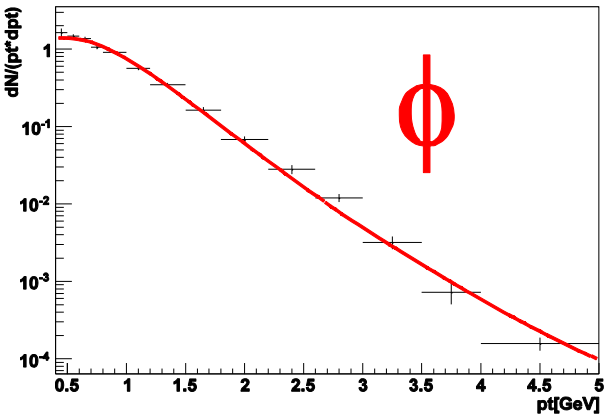
with Károly Ürmössy

RHIC data





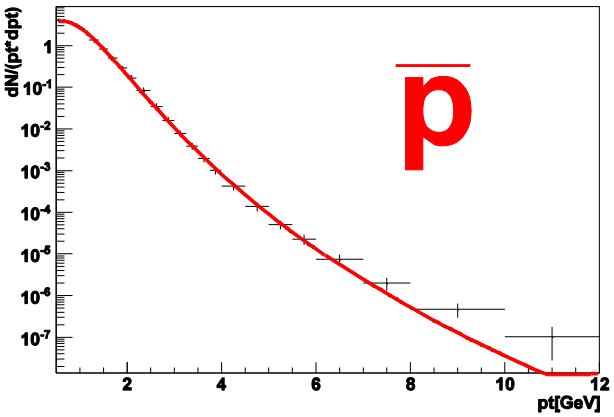


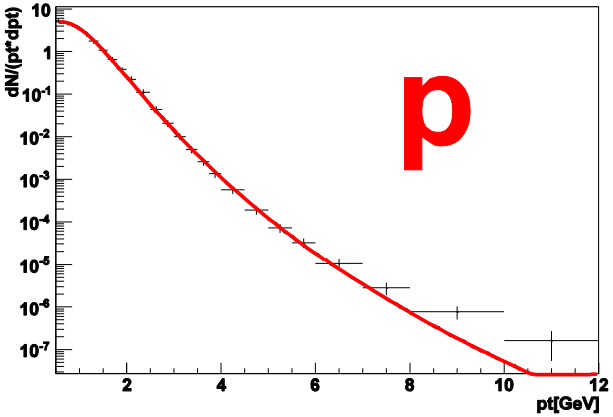


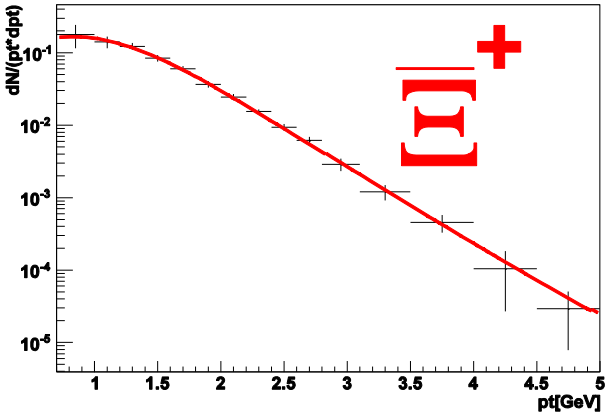
Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

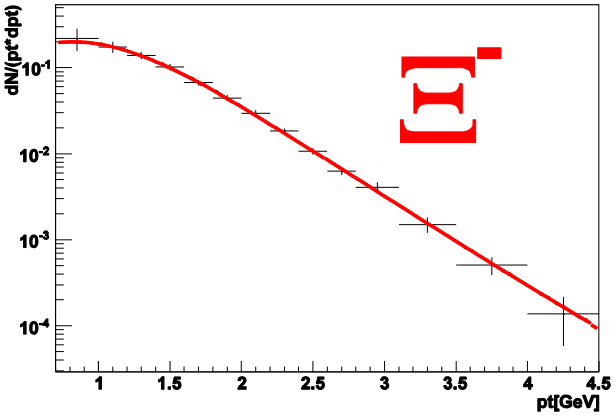
with Károly Ürmössy

RHIC data



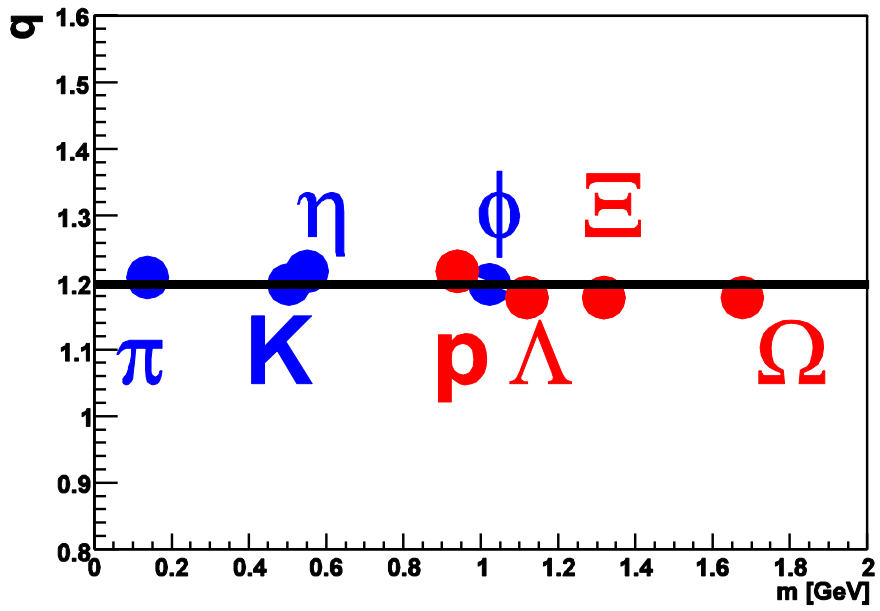


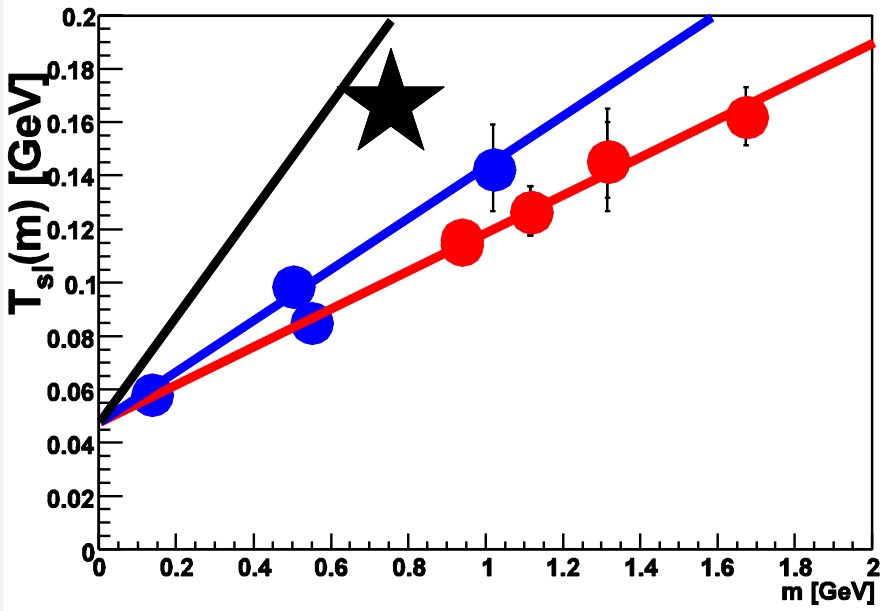




Blast wave fits and quark coalescence

with Károly Ürmössy





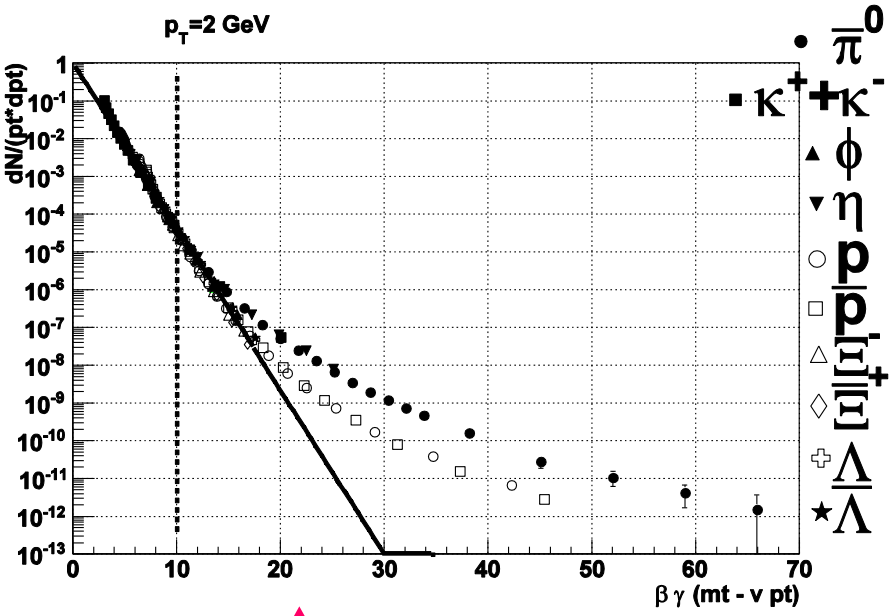
$$q = 1 + \frac{2}{\pi^2} \approx 1.202642$$

arXiV: 1011.3442

SQM 2008, Beijing

All particle types follow power-law

with Károly Ürmössy



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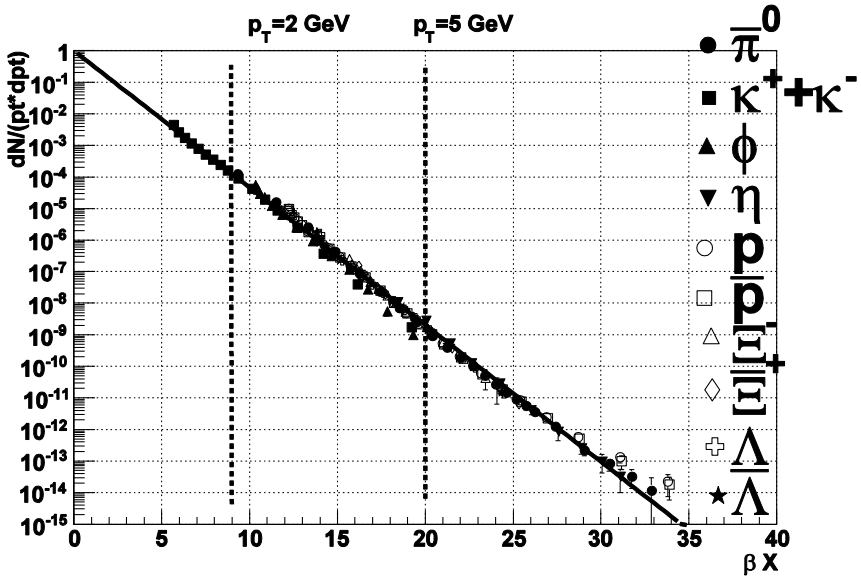
BETTER!



L(E)

Power-law tailed spectra

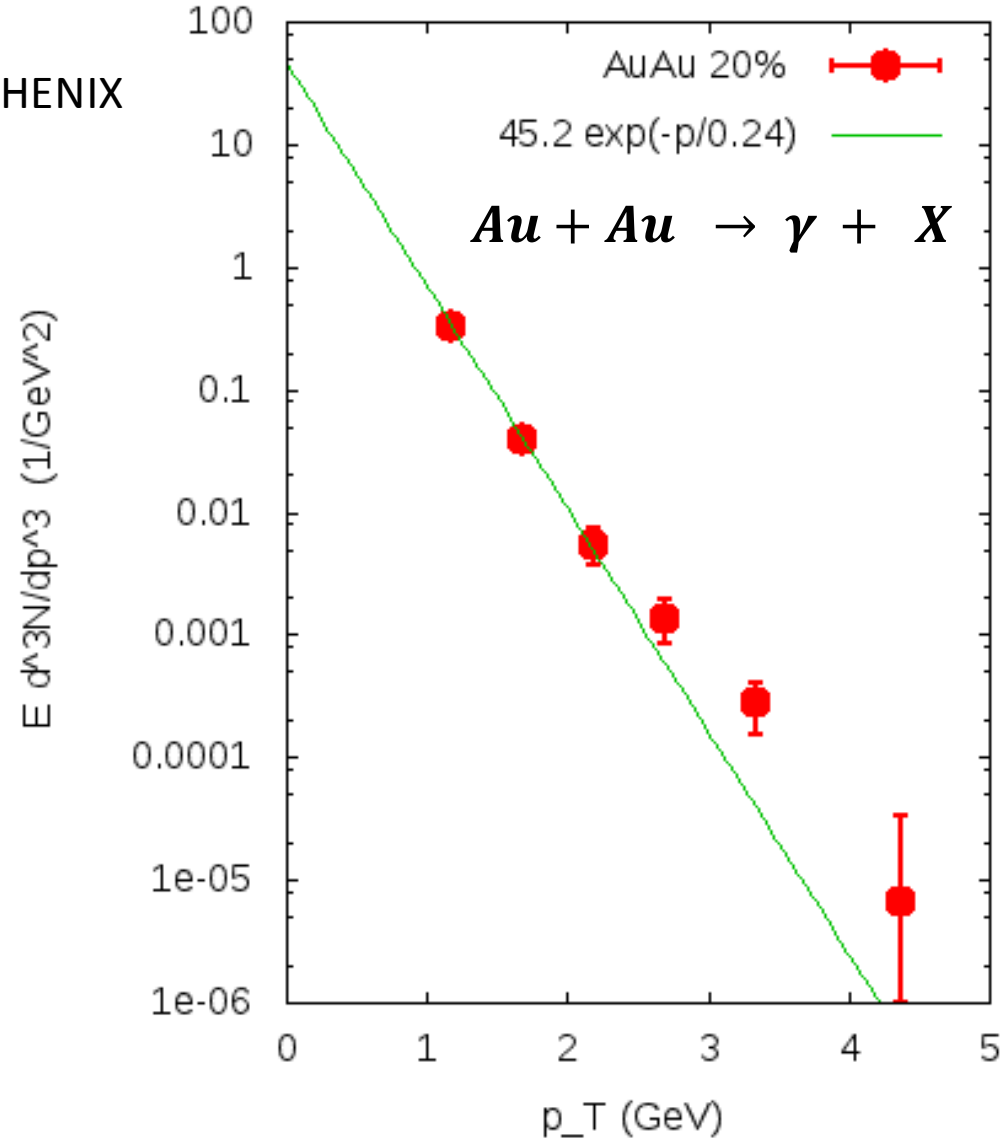
- particles and heavy ions: (SPS) RHIC, LHC
- fluctuations in financial returns
- natural catastrophes (earthquakes, etc.)
- fractal phase space filling
- preferential connection in networks
- turbulence, critical behavior
- near Bose condensates
- citation of scientific papers….

Why do statistics work ?

- **Independent** observation over 100M events
- **Universal** laws for large numbers
- **Steady noise** in environment: ‚reservoir'
- **Phase space** dominance
- **By chance** the dynamics mimics thermal behavior

Experimental motivation

RHIC: PHENIX



Theoretical motivation

- **Deceleration due to stopping**
- **Schwinger formula + Newton + Unruh = Boltzmann**

$$E_p \frac{dN}{d^3p} \propto e^{-2\pi m_T^2 / qE}, \quad qE = m_T a, \quad T = \frac{a}{2\pi}$$

$$E_p \frac{dN}{d^3p} \propto e^{-m_T / T}$$

Satz, Kharzeev, …

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Unruh gamma radiation at RHIC?

Arxiv: 1111.4817

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Zsolt Schram
Dept. of Theoretical Physics, University of Debrecen, Hungary

(Dated: October 23, 2011)

Varying the proposition that acceleration itself would simulate a thermal environment, we investigate the semiclassical coherent photon radiation as a possible telemetric thermometer of accelerated charges. Based on the classical Jackson formula we obtain the equivalent photon intensity spectrum stemming from a constantly accelerated charge and demonstrate its resemblances to a thermal distribution for high transverse momenta. The transverse slope temperature *differs* from the famous Unruh formula: it is larger by a factor of π. We compare the resulting direct photon spectrum with experimental data for AuAu collisions at RHIC and speculate about further, analytically solvable acceleration histories.

PACS numbers: 24.10.Pa, 25.75.Ag, 25.20.Lj
Keywords: Thermal models, Unruh temperature, bremsstrahlung, photon spectra

THERMAL LOOKING CLASSICAL RADIATION Satz and Kharzeev [19–22] A calculation demonstrating

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Why Photons (gammas) ?

- **Zero mass**: flow – Doppler, easy kinematics
- **Color neutral**: escapes strong interaction
- **Couples to charge**: Z / A sensitive
- **Classical** field theory also predicts spectra

Soft bremsstrahlung

- Jackson formula for the amplitude:

$$\vec{A} = K \int e^{i\phi} \frac{d}{dt}\left(\frac{\vec{n}\times\left(\vec{n}\times\vec{\beta}\right)}{1-\vec{n}\cdot\vec{\beta}}\right)\mathrm{dt}$$

$\vec{u}$

With $K^2 = \frac{e^2}{8\pi c^2}$, $\vec{\beta} = \frac{\vec{v}}{c} = \frac{1}{c}\frac{d\vec{r}}{dt}$

and the retarded phase $\phi = \omega\left(t - \frac{\vec{n}\cdot\vec{r}}{c}\right) = k \cdot x$

Soft bremsstrahlung

- Covariant notation:

$$k = (\omega, \omega\vec{n}) = k_\perp(\cosh\eta, \sinh\eta, \cos\psi, \sin\psi\ )$$

$$u = (\gamma, \gamma\vec{v}) = (\cosh\xi\ , \sinh\xi\ , 0{,}0)$$

$$\aleph = \int e^{i\varphi}\ \frac{d}{d\tau}\left(\frac{\epsilon\cdot u}{k\cdot u}\right)\ d\tau$$

IR div, coherent effects

Feynman graphs

The Unruh effect cannot be calculated by any finite number of Feynman graphs!

Kinematics, source trajectory

- Rapidity: $\beta = \frac{v}{c} = \tanh(\xi + \xi_0)$

$$\xi = \frac{g}{c}\,\tau$$

Trajectory:

$$t = t_0 + \frac{c}{g}\left(\sinh(\xi + \xi_0) - \sinh\xi_0\right)$$

$$z = z_0 + \frac{c^2}{g}\left(\cosh(\xi + \xi_0) - \cosh\xi_0\right)$$

Let us denote $\xi + \xi_0$ by ξ in the followings!

Kinematics, photon rapidity

- Angle and rapidity:

$$\cos\theta = \tanh\eta$$

$$\sin\theta = \frac{1}{\cosh\eta}$$

$$\cot\theta = \sinh\eta$$

$$\boldsymbol{\eta = \ln\cot\frac{\theta}{2}}$$

Kinematics, photon rapidity

- Doppler factor:

$$\mathrm{k}\cdot\mathrm{u} = \omega\gamma(1-\beta\cos\theta) = \omega\frac{\cosh(\xi-\eta)}{\cosh\eta} = \frac{d\phi}{d\tau}$$

Phase:

$$\phi = \frac{\omega c}{g}\frac{\sinh(\xi-\eta)}{\cosh\eta} = \ell k_{\perp}\sinh(\xi-\eta)$$

Magnitude of projected velocity:

$$u = \frac{\sinh\xi}{\cosh(\xi-\eta)}, \qquad \frac{du}{d\xi} = \frac{\cosh\eta}{\cosh^2(\xi-\eta)}$$

Intensity, photon number

Amplitude as an integral over rapidities on the trajectory:

$$\vec{A} = K\vec{e} \int_{\xi_1}^{\xi_2} e^{i\ell k_\perp \sinh(\xi-\eta)} \frac{\cosh\eta}{\cosh^2(\xi-\eta)} d\xi$$

Here $\ell = \frac{c^2}{g}$ is a characteristic length.

Intensity, photon number

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from –c to +c):

$$\vec{A} = 2K\vec{e}\,\ell k_\perp \cosh\alpha\;\; K_1(\ell k_\perp)$$

With K1 Bessel function!

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi}\;\ell^2\;K_1^2(\ell k_\perp)$$

Flat in rapidity !

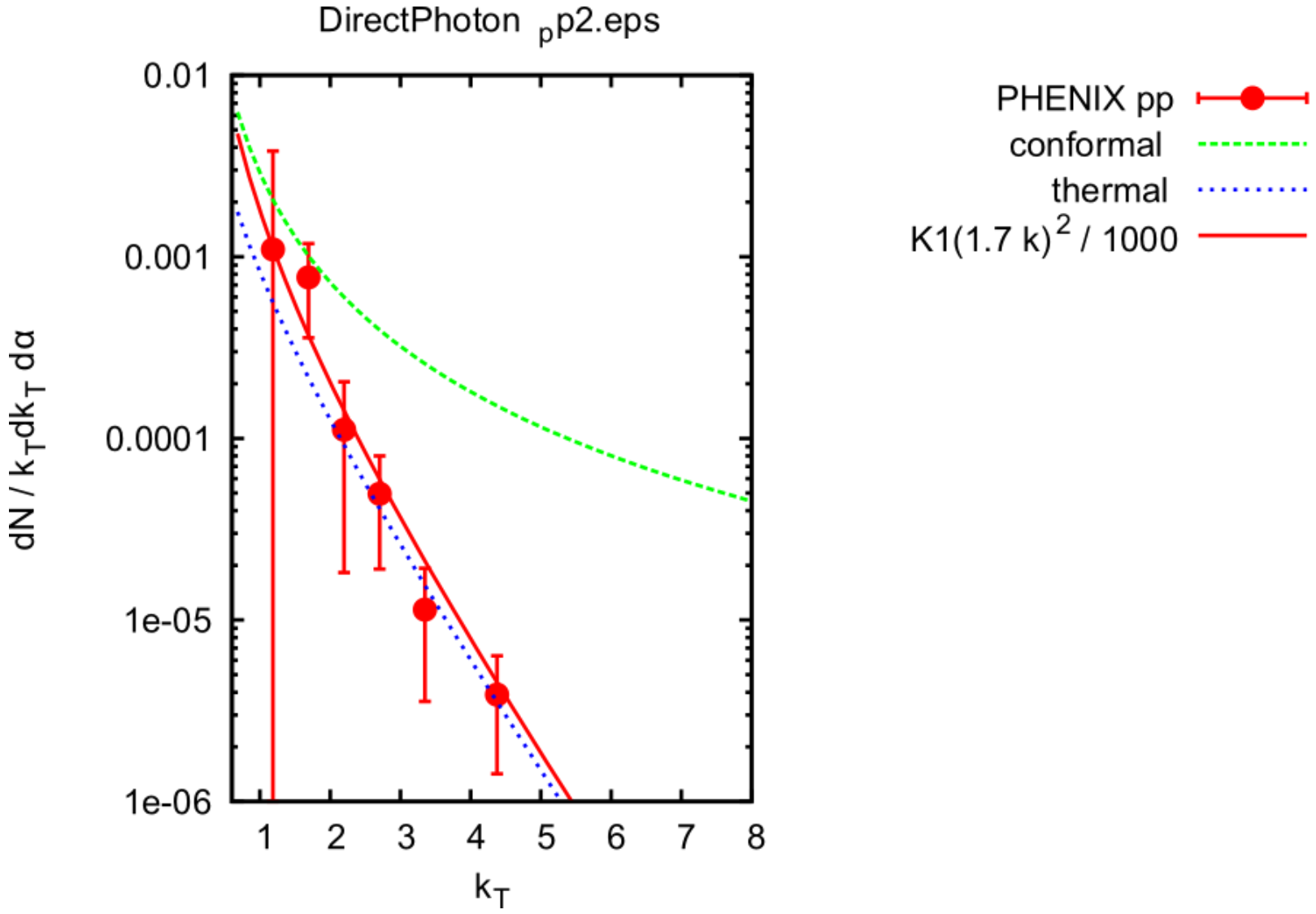
Photon spectrum, limits

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from –c to +c):

$$\frac{dN}{k_\perp dk_\perp d\eta d\psi} = \frac{4\alpha_{EM}}{\pi}\ \frac{1}{k_\perp^2} \qquad \text{for } \ell k_\perp \to 0$$

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = 2\alpha_{EM}\ \frac{\ell}{k_\perp}\ e^{-2\ell k_\perp} \quad \text{for } \ell k_\perp \to \infty$$

Photon spectrum from pp backgroound, PHENIX experiment



Apparent temperature

- High - $k_\perp$ infinite proper time acceleration:

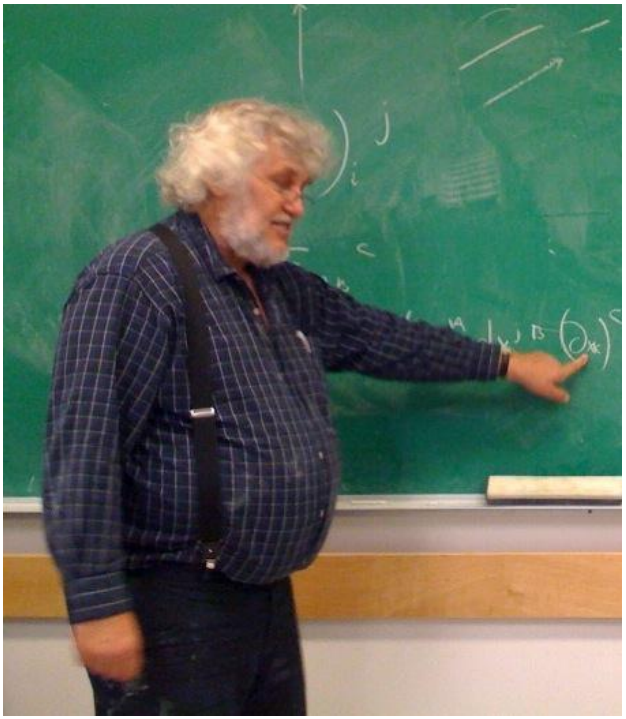
$$k_B\, T = \frac{\hbar c}{2\ell} = \frac{\hbar g}{2c} = \pi\, k_B\, T^{Unruh}$$

Connection to Unruh:

$$\frac{du}{d\tau} \rightarrow e^{-i\nu\tau}$$

proper time Fourier analysis of a monochromatic wave

Unruh temperature



Unruh

- **Entirely classical effect**
- **Special Relativity suffices**

$$I(\nu) \propto \left| \int e^{i\left[\int \omega \sqrt{\frac{1-V(\tau)/c}{1+V(\tau)/c}} d\tau - \nu\tau\right]} d\tau \right|^2$$

$$I(\nu) \propto \left| \int_0^\infty e^{ic\omega z/g} z^{-i\nu c/g - 1} dz \right|^2 \propto \frac{1}{e^{2\pi c\nu/g} - 1}$$

Constant ‚g' acceleration in a comoving system: $dv/d\tau = -g(1-v^2)$

Unruh temperature

Planck-interpretation:

$$\frac{2\pi\,\mathrm{c}}{\mathrm{g}}\nu = \frac{\hbar\nu}{k_B T}$$

The temperature in Planck units:

$$T = \frac{g}{2\pi}$$

The temperature more commonly:

$$k_B T = \frac{\hbar}{c}\frac{g}{2\pi} = M_P g \cdot \frac{L_P}{2\pi}$$

Unruh temperature

Small for Newtonian gravity

$$g = \frac{GM}{R^2}$$

$$k_B T = \frac{Mc^2}{2\pi} \cdot \frac{L_P^2}{R^2}$$

On Earth' surface ist is 10^(-19) eV, while at room temperature about 10^(-3) eV.

Unruh temperature

Not small in heavy ion collisions

$$g = \frac{c^2}{2L} = \frac{mc^3}{\hbar}$$

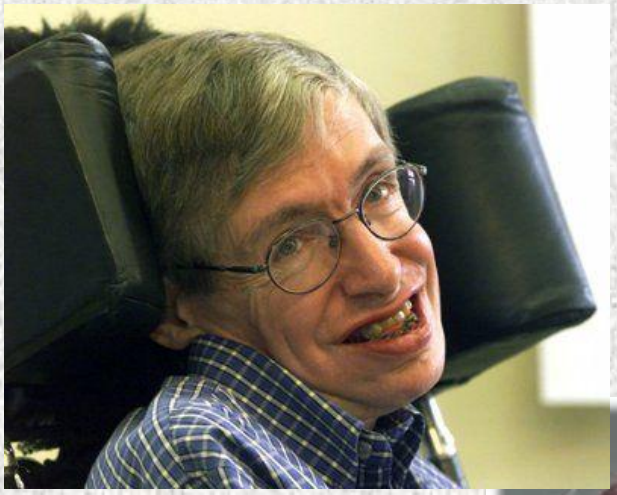
$$k_B T = \frac{mc^2}{2\pi}$$

Braking from +c to -c in a Compton wavelength:

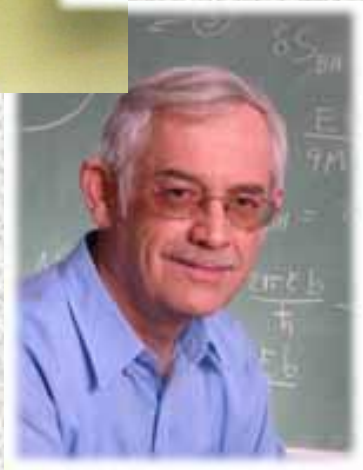
kT ~ 150 MeV if $mc^2 \sim 940$ MeV (proton)

Bekenstein-Hawking entropy

- **Unruh temperature at the event horizon**
- **Clausius: 1 / T is an integrating factor**



Hawking



Bekenstein

$$R = \frac{2GM}{c^2} \qquad g = \frac{GM}{R^2} = \frac{c^2}{2R}$$

$$k_B T = \frac{\hbar g}{2\pi c}$$

$$\frac{S}{k_B} = \int \frac{d(Mc^2)}{k_B T} = \frac{c^3}{\hbar G}\pi R^2$$

$$\frac{S}{k_B} = \frac{1}{4}\frac{A}{L_P^2}$$

Connection to Unruh

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{\alpha_{EM}}{2\pi k_\perp^2 \cosh^2 \eta} \left| \int_{-\infty}^{+\infty} e^{i\phi(\tau)} \frac{du}{d\tau} d\tau \right|^2$$

Fourier component for the retarded phase:

$$f_k = \int_{-\infty}^{+\infty} e^{i\phi(\tau)}\, e^{iv\tau} d\tau = \frac{\ell}{c} \int_{-\infty}^{+\infty} e^{i\ell k_\perp \sinh \xi}\, e^{ik\xi} d\xi$$

Connection to Unruh

Fourier component for the projected acceleration:

$$a_k = \int_{-\infty}^{+\infty} \frac{du}{d\tau} e^{i\nu\tau}\, d\tau = \cosh\eta \int_{-\infty}^{+\infty} \frac{1}{\cosh^2 \xi}\, e^{ik\xi} d\xi$$

Photon spectrum in the incoherent approximation:

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{\alpha_{EM}}{2\pi k_\perp^2 \cosh^2\eta} \int_{-\infty}^{+\infty} |f_k|^2 |a_k|^2 \frac{c}{\ell} \frac{dk}{2\pi}$$

Connection to Unruh

Fourier component for the retarded phase at constant acceleration:

$$f_k = \frac{\ell}{c}\int_{-\infty}^{+\infty} e^{i\ell k_\perp \sinh\xi}\, e^{ik\xi}\, d\xi = \frac{2\ell}{c}\, K_{ik}(\ell k_\perp)\, e^{-\pi k/2}$$

KMS relation and Planck distribution:

$$f_{-k} = e^{k\pi}\, f_k^*, \qquad |f_{-k}|^2 = e^{2\pi k}\, |f_k|^2$$

$$\mathrm{n}(-\nu) = e^{2\pi\ell\nu/c}\, n(\nu) = 1 + n(\nu)$$

$$n(\nu) = \frac{1}{e^{2\pi\ell\nu/c} - 1}$$

Connection to Unruh

KMS relation and Planck distribution:

$$2\pi k = \frac{2\pi c}{g}\,\nu = \frac{\hbar}{k_B T_U}\,\nu\,;$$

$$T_U = \frac{\hbar}{2\pi k_B c}\,g$$

Connection to Unruh

Note:

$$a_k = \cosh\eta \, \frac{k\pi}{\sinh k\pi/2}$$

It is peaked around k = 0, but relatively wide! (an unparticle…)

Bessel-K compendium

1. $$\int_{-\infty}^{+\infty} \frac{e^{ibw}}{(1+w^2)^{3/2}}\, dw = 2|b|\, K_1(|b|)$$

2. $$\int_1^{\infty} e^{-bx}\,(x^2-1)^{1/2}\, dx = \frac{1}{|b|}\, K_1(|b|)$$

3. $$\iint_0^{\infty} \sqrt{x_1 x_2}\, e^{-x_1} e^{-x_2}\, \delta(x_1 - x_2 - b) = \frac{|b|}{4}\, K_1(|b|)$$

3 → 1 : expand Dirac delta as Fourier integral,

3 → 2 : rewrite integral over difference and the sum bx = x1 + x2

Transverse flow interpretation

Mathematica knows: (I derived it using Feynman variables)

$$\int_0^{\pi} \frac{d\theta}{\sin\theta}\; K_2\left(\frac{z}{\sin\theta}\right) = K_1^2\left(\frac{z}{2}\right)$$

$$\frac{1}{\sin\theta} = \cosh\eta$$

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}\,\hbar c}{\pi(2\pi k_B T_U)^2} \int_{-\infty}^{+\infty} K_2\left(\frac{\hbar c\, k_\perp}{\pi k_B T_U}\cosh(\zeta-\eta)\right) d\zeta$$

$$\boldsymbol{\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi\, g} \int_{-\infty}^{+\infty} K_2\left(\frac{k\cdot u}{\pi T_U}\right) d\tau}$$

Alike Jüttner distributions integrated over the flow rapidity…

Finite time (rapidity) effects

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4K^2}{\hbar\, {k_\perp}^2}\left|\int_{w_1}^{w_2} \frac{e^{i\ell k_\perp w}}{(1+w^2)^{3/2}}\right|^2$$

with $w = \sinh(\xi - \eta)$

Short-time deceleration → Non-uniform rapidity distribution; → Landau hydrodynamics

Long-time deceleration → uniform rapidity distribution; → Bjorken hydrodynamics

Short time constant acceleration

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi}\,\frac{1}{k_\perp^2}\,\frac{(w_2-w_1)^2}{(1+w_0^2)^3}$$

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi}\,\frac{4}{\omega^2}\,\frac{1}{\cosh^2\eta}$$

Non-uniform rapidity distribution;

→ Landau hydrodynamics

The reverse problem

- **Photon spectrum → amplitude**
- **Amplitude → Fourier component**
- **Phase → velocity**
- **Velocity → rapidity**
- **Rapidity → acceleration**
- **Acceleration → Path in t -- x**

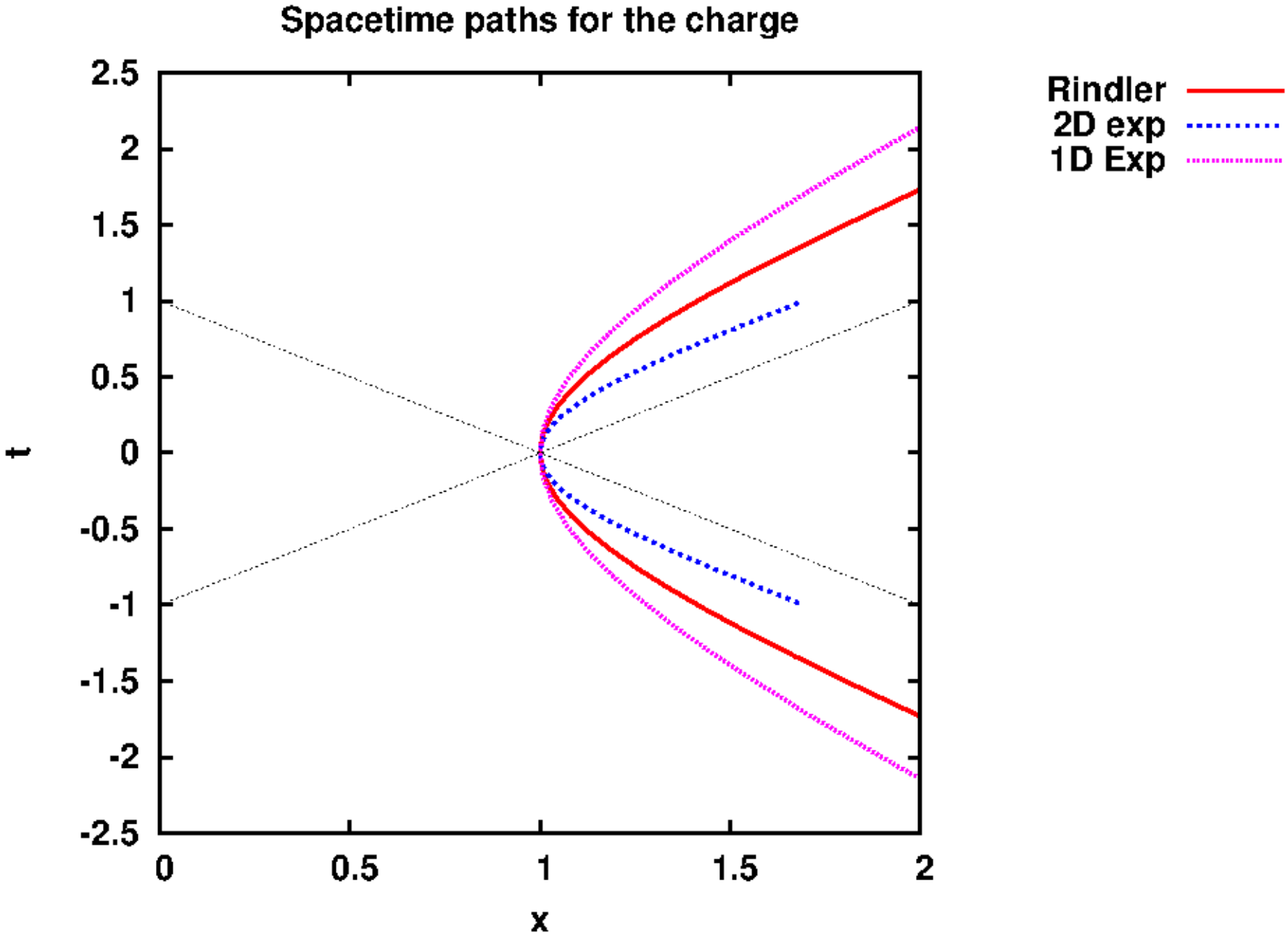
Analytic results

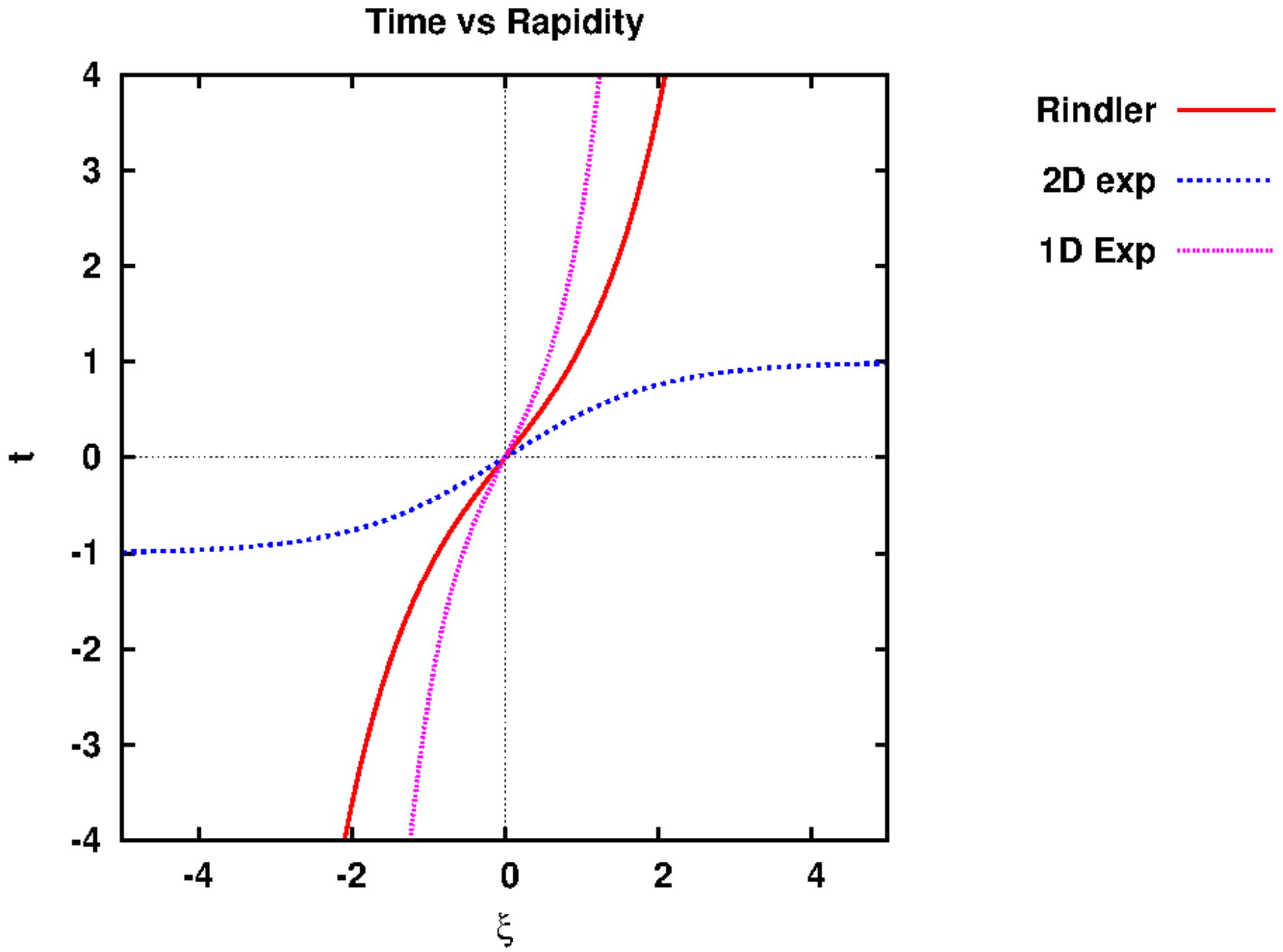
x(t), v(t), g(t), τ(t), $\mathcal{A}$, dN/kdkdη limit

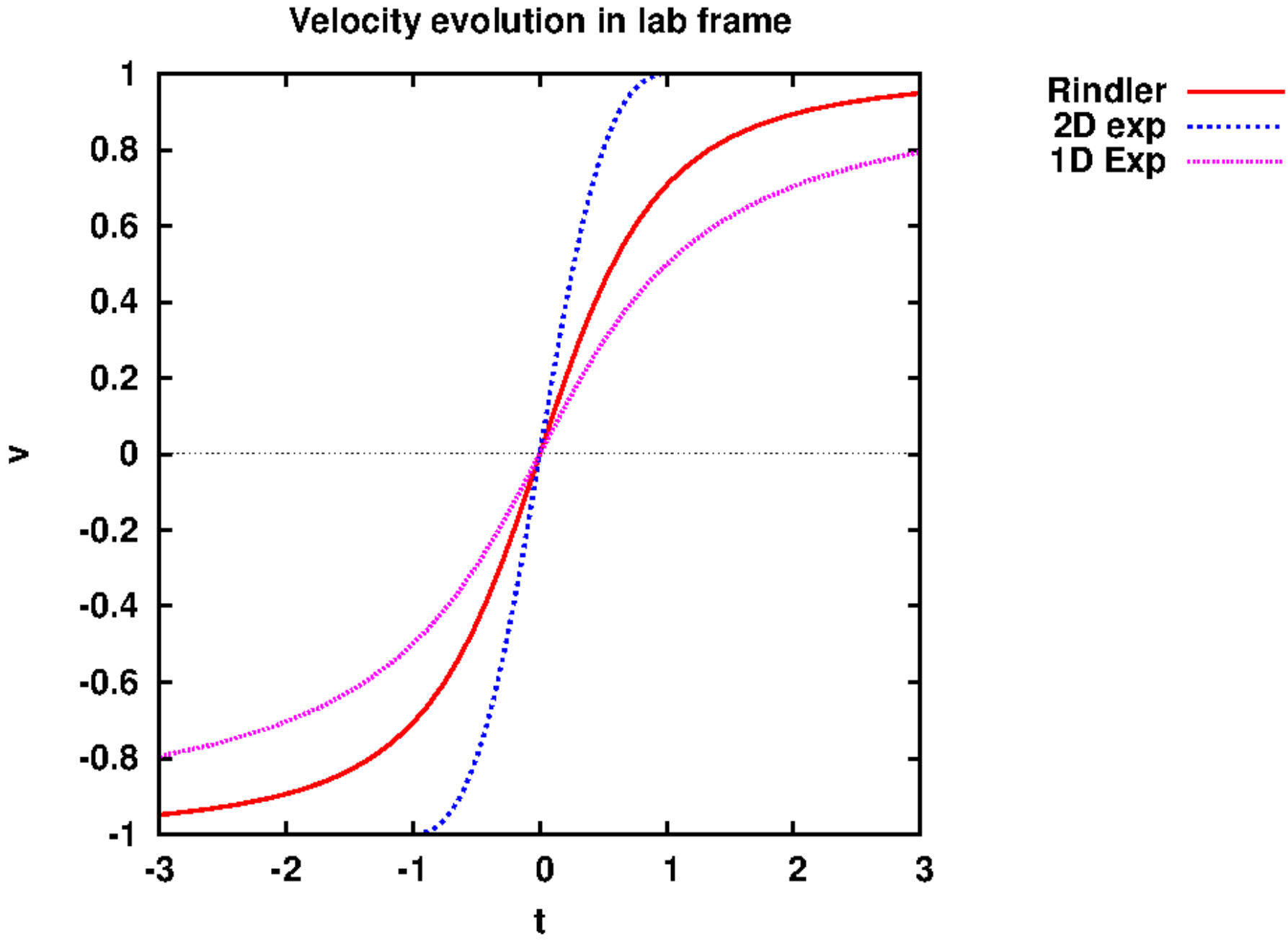
$\sqrt{1+t^2}$, $\frac{t}{\sqrt{1+t^2}}$, 1, $\mathrm{Arc\,sh}\,t$, $bK_1(b)$, $\frac{\ell}{k}e^{-2\ell k}$

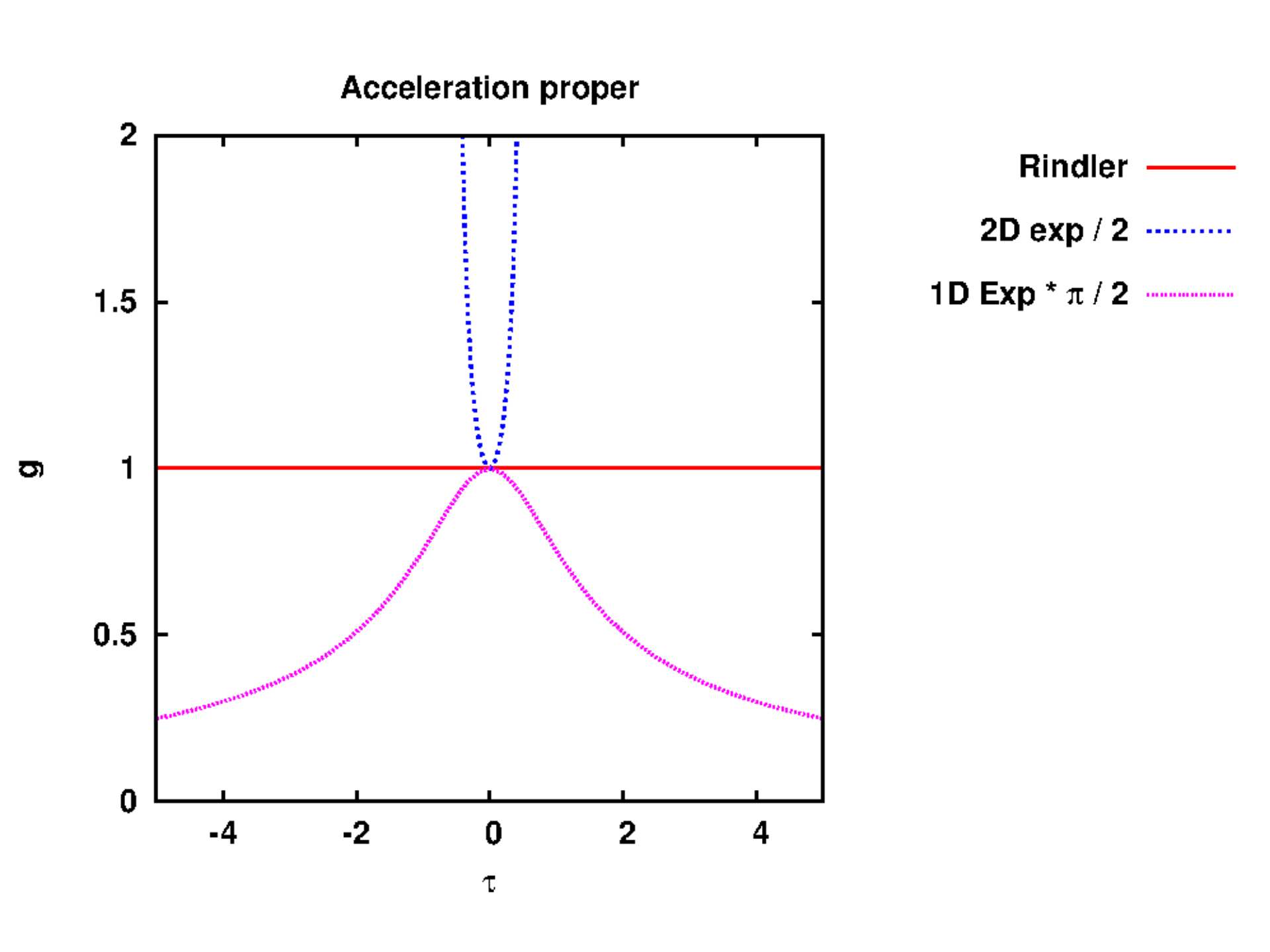
$1+\ln(1+t^2)$, $\frac{2t}{1+t^2}$, $\frac{2(1+t^2)}{(1-t^2)^2}$, $2\,\mathrm{atn}\,t-t, be^{-b}, \ell^2 e^{-2\ell k}$

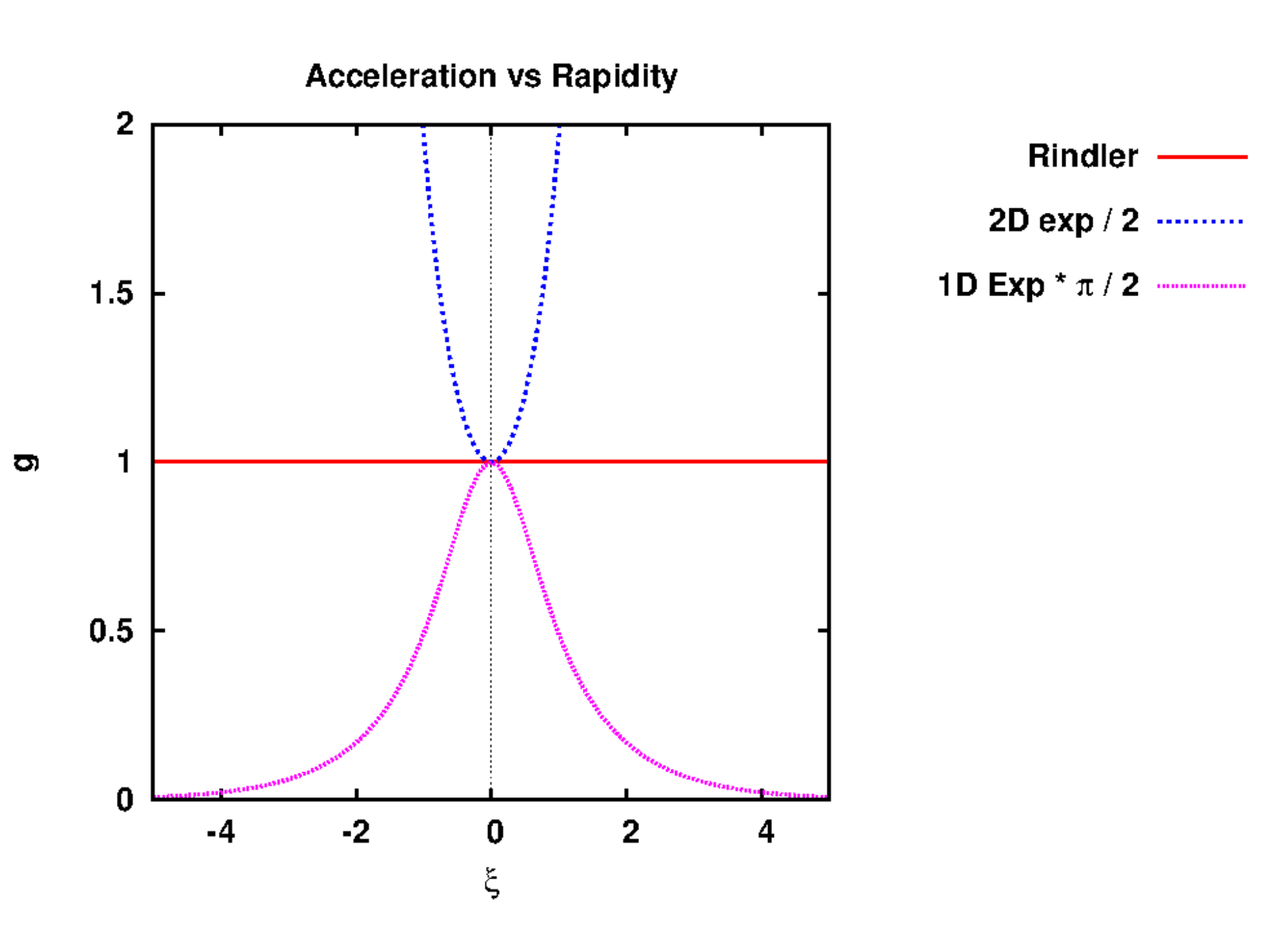
$1+\frac{2t}{\pi}\mathrm{atn}\left(\frac{t}{\pi}\right)-\ln\left(1+\frac{t^2}{\pi^2}\right), \frac{2}{\pi}\mathrm{atn}\left(\frac{t}{\pi}\right), \frac{2\gamma^3}{\pi^2+t^2}, \frac{\pi^2}{2}\int\frac{\sqrt{1-v^2}\,dv}{\cos^2\left(\frac{\pi v}{2}\right)}, e^{-b}, \frac{1}{k^2}e^{-2\ell k}$

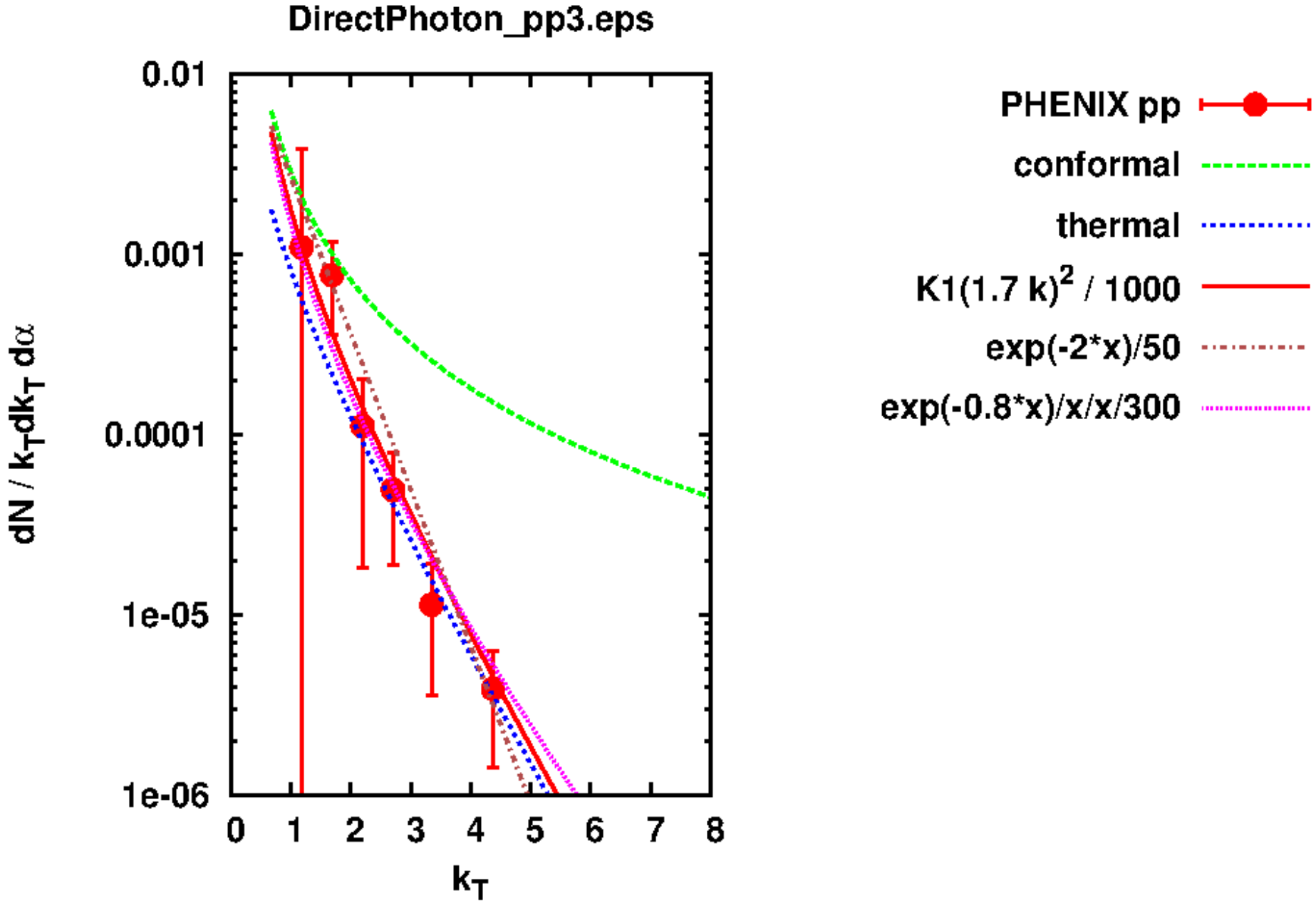

Spacetime paths for the charge
Rindler
2D exp
1D Exp
t
x
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2

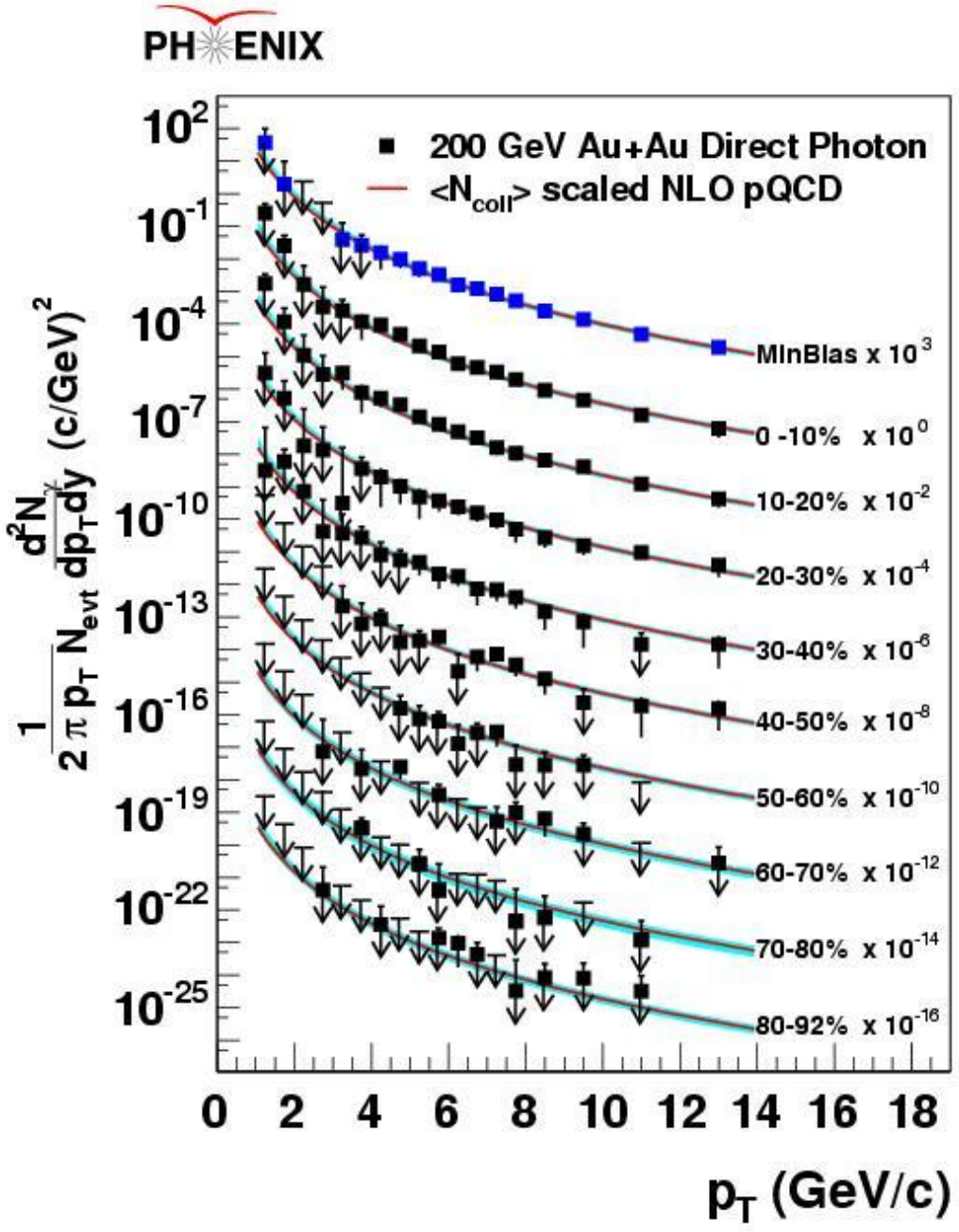

Time vs Rapidity
t
ξ
4
3
2
1
0
-1
-2
-3
-4
-4
-2
0
2
4
Rindler
2D exp
1D Exp


Velocity evolution in lab frame
v
t
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-3
-2
-1
0
1
2
3
Rindler
2D exp
1D Exp


Acceleration proper
2
1.5
1
0.5
0
g
-4
-2
0
2
4
τ
Rindler
2D exp / 2
1D Exp * π / 2


Acceleration vs Rapidity
2
1.5
1
0.5
0
g
-4
-2
0
2
4
ξ
Rindler
2D exp / 2
1D Exp * π / 2


DirectPhoton_pp3.eps
0.01
0.001
0.0001
1e-05
1e-06
0
1
2
3
4
5
6
7
8
k_T
$dN / k_T dk_T d\alpha$
PHENIX pp
conformal
thermal
K1(1.7 k)^2 / 1000
exp(-2*x)/50
exp(-0.8*x)/x/x/300



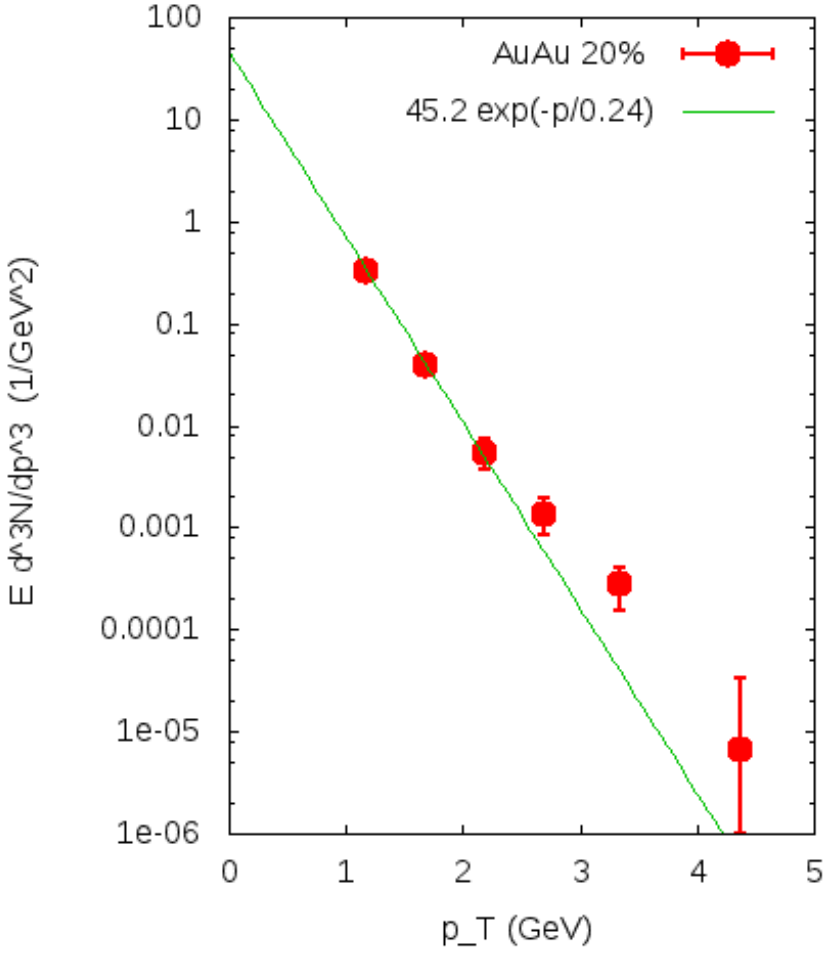
Large pT:

NLO QCD

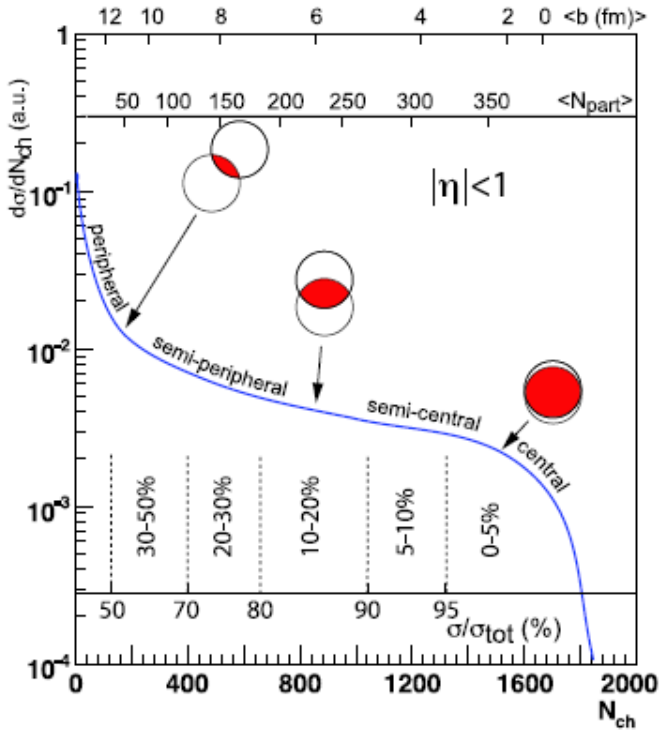
Smaller pT:

plus direct gammas

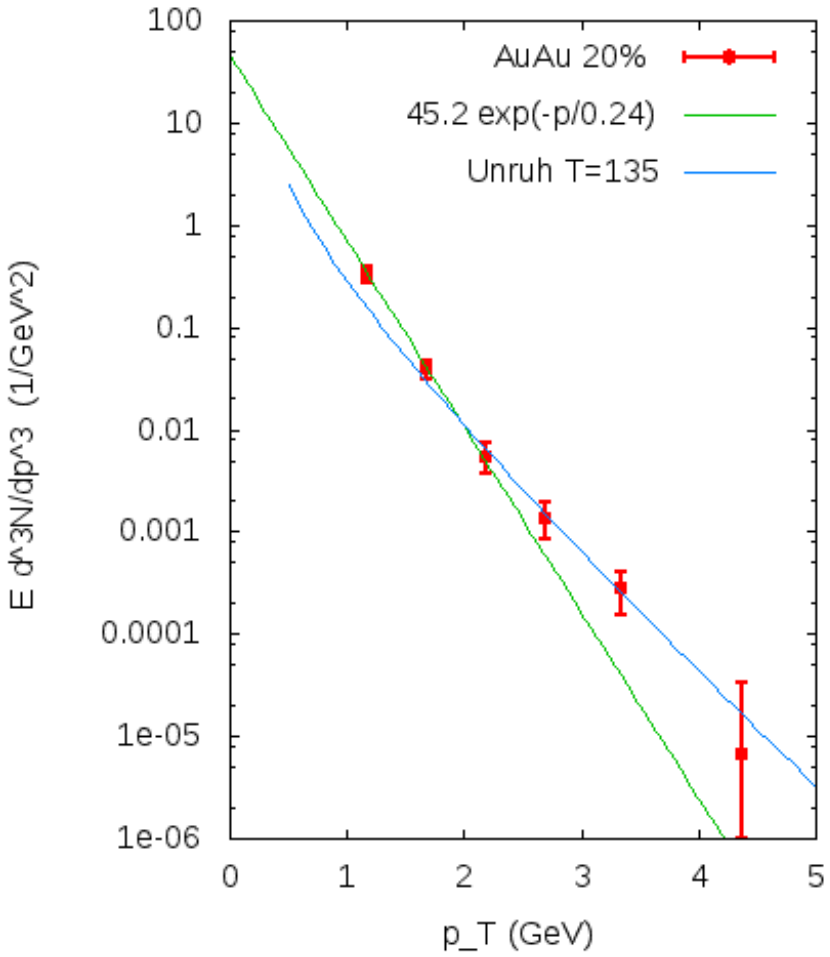
$$Au + Au \rightarrow \gamma + X$$



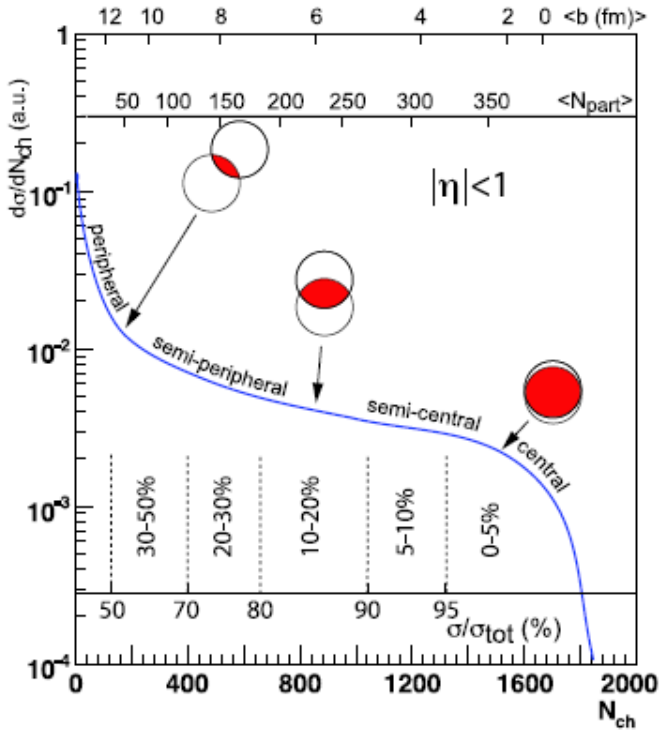
Glauber model

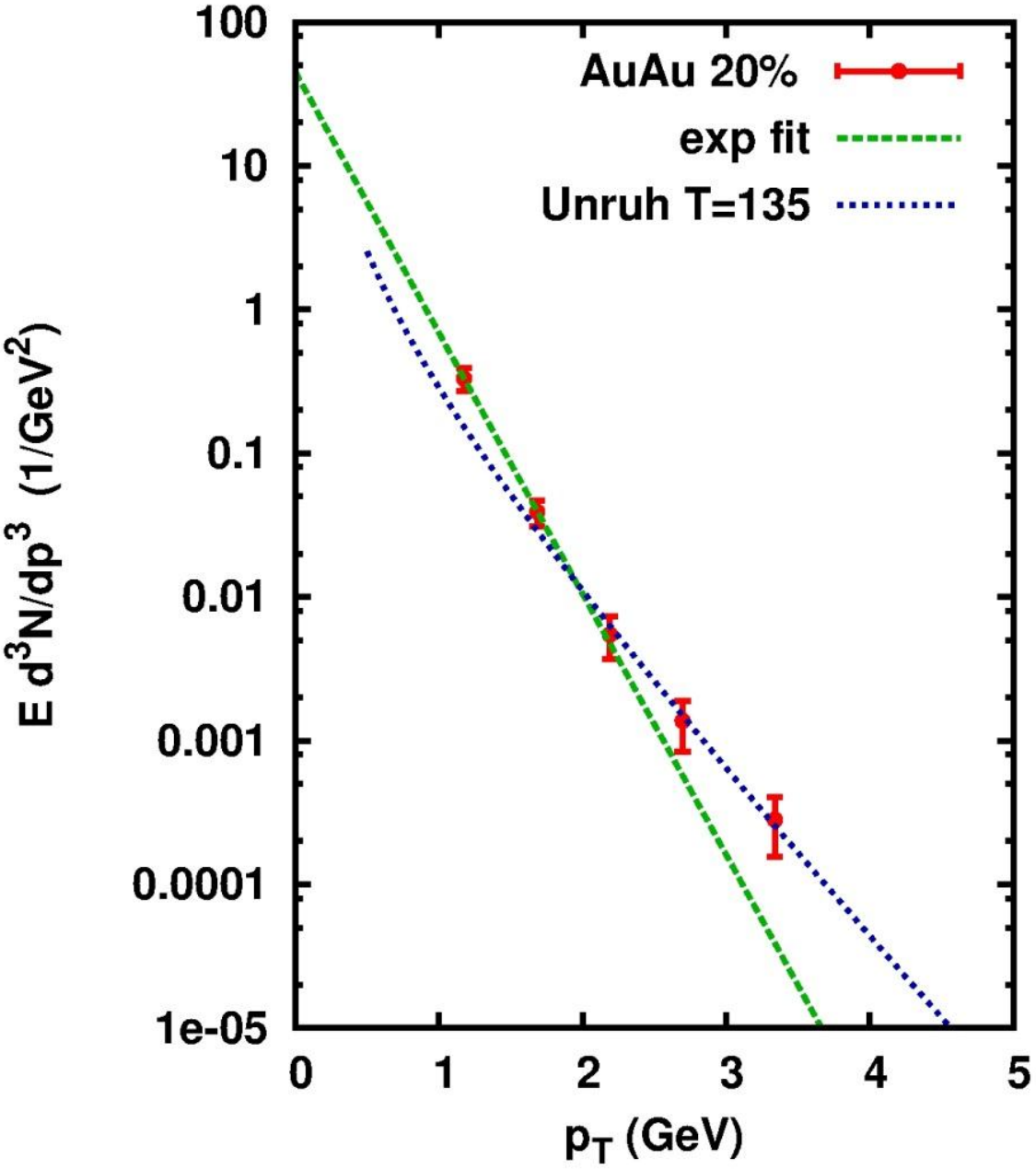


$$Au + Au \rightarrow \gamma + X$$



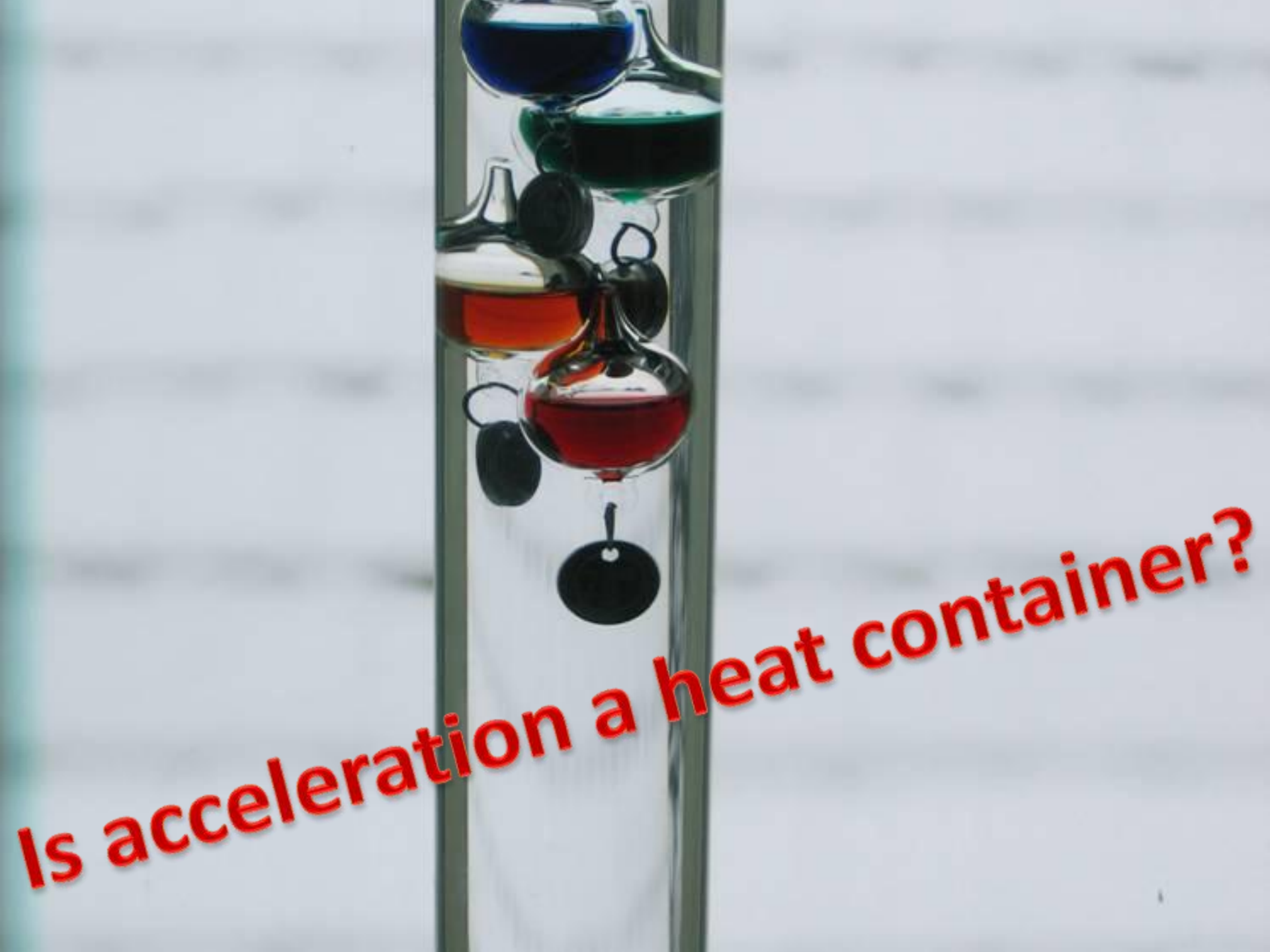
Glauber model




AuAu 20%
exp fit
Unruh T=135
E d^{3}N/dp^3 (1/GeV^2)
p$_T$ (GeV)
100
10
1
0.1
0.01
0.001
0.0001
1e-05
0
1
2
3
4
5

Summary

- Semiclassical radiation from constant accelerating point charge occurs **rapidity-flat** and **thermal**
- The thermal tail develops at high enough k_perp
- At low k_perp the **conformal** NLO result emerges
- Finite time/rapidity acceleration leads to **peaked** rapidity distribution, alike **Landau** - hydro
- Exponential fits to surplus over NLO pQCD results reveal a ’’**pi-times Unruh-**’’ temperature


Is acceleration a heat container?