# Linear sigma model with vector mesons

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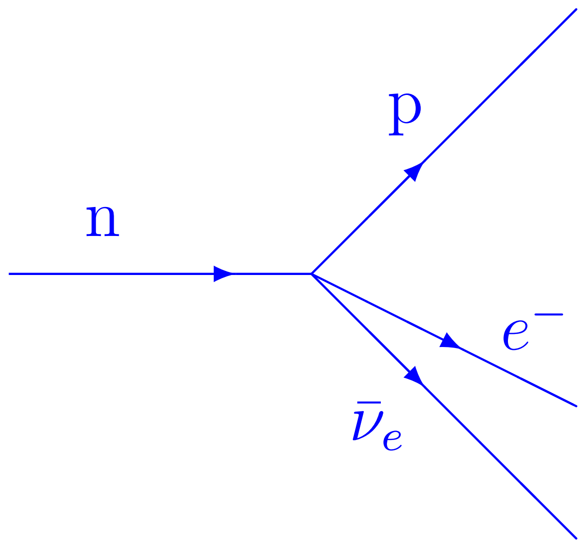
- Symmetries of the strong interaction
- Effective models of strong interaction
- Linear Sigma model and it symmetries
- Masses and decay widths
- Summary

# Chiral Symmetry in the weak interaction




The classical theory of $\beta$-decay:

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} J^{\dagger}_{\mu} J^{\mu} + \text{h.c.} \qquad J^{\mu} = J^{\mu}_h + j^{\mu}_l$$

$$j^{\mu} = \sum_l \bar{\nu}_l \gamma^{\mu}(1 - \gamma_5) l$$

$\beta$-decay of muons and neutrons can be described by:

$$\mathcal{L}_\mu = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\mu\bar{e}\gamma^\lambda(1-\gamma_5)\nu_e$$

$$\mathcal{L}_n = -\frac{G_F}{\sqrt{2}}\bar{p}\gamma^\lambda(g_V - g_A\gamma_5)n\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e\left[g_V V^\lambda - g_A A^\lambda\right]$$

where the vector and axial vector currents are:

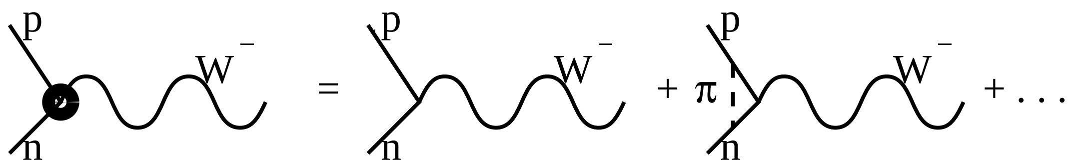
$$V^\lambda = \bar{p}\gamma^\lambda n = \bar{N}\tau_+\gamma^\lambda N \qquad A^\lambda = \bar{p}\gamma^\lambda\gamma_5 n = \bar{N}\tau_+\gamma^\lambda\gamma_5 N$$

Experimentally

$$g_V = 0.974\,; \qquad g_A/g_V = 1.25$$

# CVC and PCAC

$g_V = 1$, but for neutron $\beta$-decay only the strangeness conserving component of the hadronic current should be included $\cos\theta_c V$, where $\theta_c$ is the Cabibo angle.




Why $g_V = 1$? Coupling constant renormalization may be different for weakly and strongly interactin particles.

CVC: conserved vector (isospin or even SU(3)) current

$$\partial_\mu V^\mu = 0$$

PCAC: since $g_A$ is close to 1, axial vector current is almost conserved.

$$\partial^\mu A_\mu^k = f_\pi m_\pi^2 \pi^k$$

Symmetry of the strong interaction is: $U(1)_V x SU(3)_V x SU(3)_A$. The symmetry is slightly broken.

# QCD Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm QCD} &\equiv -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}_f \, (i\gamma^\mu D_\mu - m_f) \, q_f \\
&= -\frac{1}{4} (\partial^\mu G^\nu_a - \partial^\nu G^\mu_a)(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) \\
&\quad + \sum_f \bar{q}^\alpha_f \, (i\gamma^\mu \partial_\mu - m_f) \, q^\alpha_f \\
&\quad + g_s \, G^\mu_a \sum_f \bar{q}^\alpha_f \gamma_\mu \left( \frac{\lambda^a}{2} \right)_{\alpha\beta} q^\beta_f \\
&\quad - \frac{g_s}{2} f^{abc} (\partial^\mu G^\nu_a - \partial^\nu G^\mu_a) \, G^b_\mu G^c_\nu \\
&\quad - \frac{g_s^2}{4} f^{abc} f_{ade} \, G^\mu_b G^\nu_c G^d_\mu G^e_\nu
\end{aligned}
$$

All terms except the mass term are symmetric under the global $U(3)_V$x$U(3)_A$ group. $U(1)_A$ is broken by topological charges.

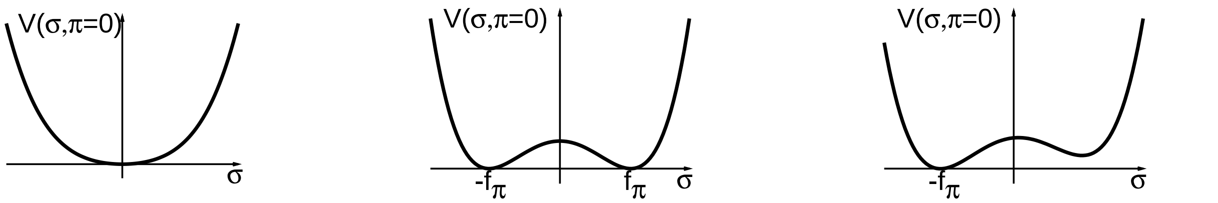
## Effective models

It is very difficult to solve QCD (lattice or perturbative QCD), so effective models are used incorporating the symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa)
- Chiral perturbation theory
- Linear sigma model

# Chiral Symmetry

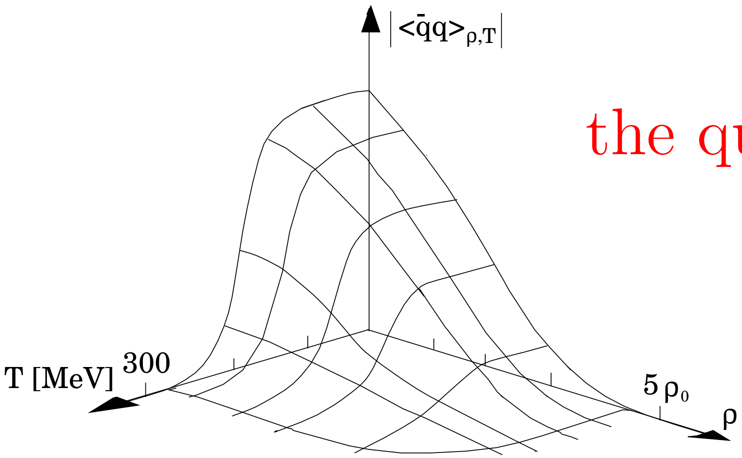
- for massless fermions: helicity
  spin has the same or opposite direction to the momentum, right-handed or left-handed particles
- $m_u \approx 4MeV$, $m_d \approx 7MeV$, $m_s \approx 150MeV$
  $m_q << m_p$, QCD is approximatelly chiral symmetric
- The vacuum symmetric $\Rightarrow$ parity dublets
  not symmetric $\Rightarrow$ SSB $\Rightarrow < \bar{q}q >$ condensates in the vacuum




symmetry spontaneous explicit breaking

# Quark condensate

order parameter of the chiral phase transition: $<\bar{q}q>$

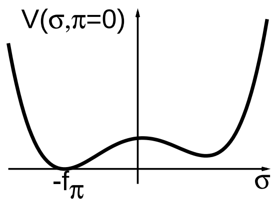



the quark condensate as a function of T and $\rho$

$$\frac{<\bar{q}q>_{\rho_O}}{<\bar{q}q>_{vac}} \approx 0.7$$

Proposed signals: disoriented chiral condensate (DCC) and change of hadron masses

- DCC: Large fluctuations in pion charges
  coherent decay of the false vacuum

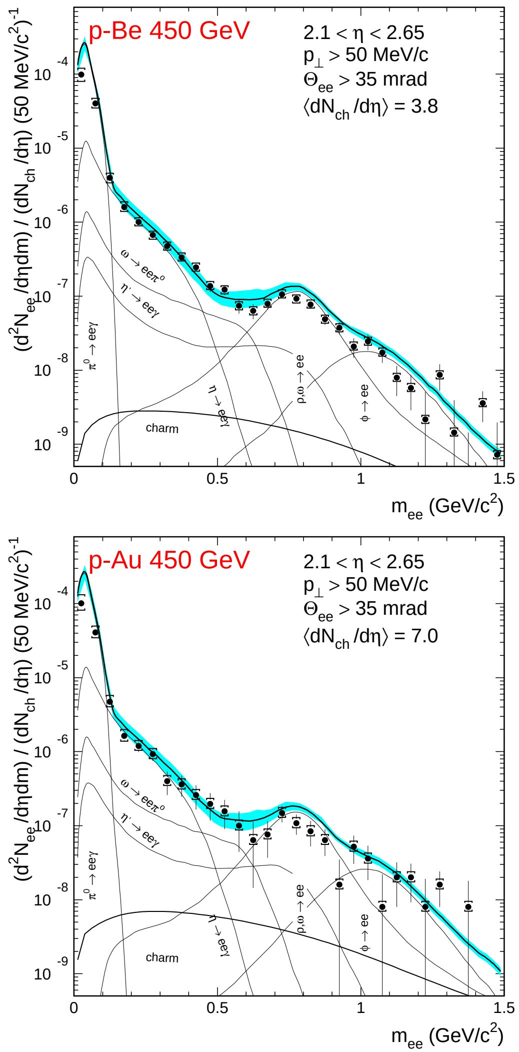


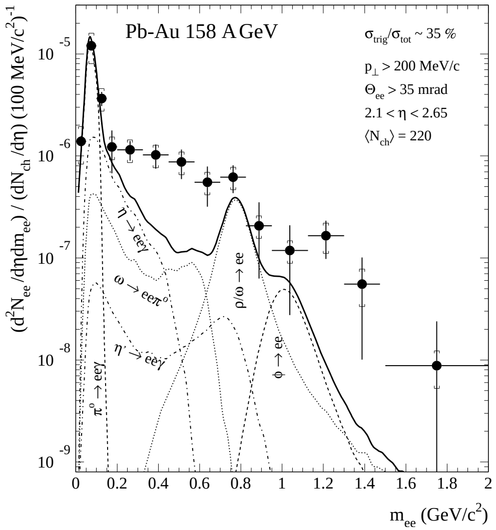

- Hadron masses: light vector meson masses shifted down
  $\Rightarrow$ dilepton production

## Chiral symmetry and vector mesons

- vector mesons can be observed by their dileptonic decay
- Chiral symmetry requires: in restored phase the spectral functions of parity partners ($\rho$ and $A_1$) are the same.
- Two simplified scenarios for chiral restoration
  masses become equal
  spectral functions mix
- What do we know about the masses
  $A_1$: nothing, but in the vacuum
  $\rho$: QCD sum rules (mass shifts)
      hadronic models
- Study spectral functions

# CERES data

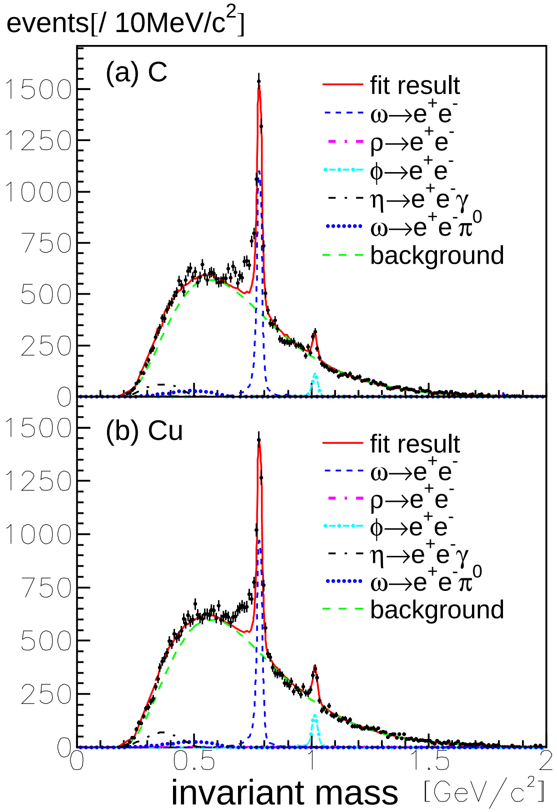



G. Agakichiev *et al.*
Eur. Phys. J. C4 (1998) 231

G. Agakichiev *et al.*
Phys. Lett. B422 (1998) 405

# p + A 12 GeV at KEK




Naruki et al., Phys.Rev.Lett. 96 (2006) 092301

excess below the $\omega$-mass, cannot be described by free coctail

$m_v = m_{v_o} * (1 - 0.09\rho/\rho_o), \qquad v : \rho, \omega$

# $\rho$-A1 mixing

- $\Pi_\rho = (1-\xi)\Pi^o_\rho + \xi\Pi^o_{A1}$ $\qquad$ $\Pi_{A1} = (1-\xi)\Pi^o_{A1} + \xi\Pi^o_\rho$
- $\xi$ from QCD Sum Rules
- the sum rules for the $\rho$ and $A_1$ is very similar:

$$\int_0^\infty ds\ s^3\ (\Pi_V(s) - \Pi_A(s)) = -\frac{1}{2}\pi\alpha_s\langle\mathcal{O}_4\rangle,$$
$$\langle\mathcal{O}_4\rangle = \langle(\bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d)^2\rangle - \langle(\bar{u}\gamma_\mu\lambda^a u - \bar{d}\gamma_\mu\lambda^a d)^2\rangle,$$

- use our ansatz inside the integral

$$\begin{aligned}\Pi_V - \Pi_A &= [(1-\xi)\Pi^0_V + \xi\Pi^0_A] - [(1-\xi)\Pi^0_A + \xi\Pi^0_V] \\ &= (1-2\xi)(\Pi^0_V - \Pi^0_A).\end{aligned}$$

$$\xi = \frac{1}{2}\left[1 - \frac{\langle\mathcal{O}_4\rangle}{\langle\mathcal{O}_4\rangle_0}\right]$$

# Evaluation of $\xi$

Let us try to evaluate $\xi$ with a simple factorization assumption

$$\begin{aligned}\langle(\bar{u}\gamma_\mu\lambda^a u)^2\rangle &\simeq -\langle(\bar{u}\gamma_\mu\gamma_5\lambda^a u)^2\rangle \\ &\simeq -\frac{16}{9}\langle(\bar{u}u)^2\rangle \simeq -\frac{16}{9}\langle(\bar{u}u)\rangle^2,\end{aligned}$$

with

$$\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_0} \simeq 1 - \frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2}\rho.$$

Then the ratio $\langle\mathcal{O}_4\rangle/\langle\mathcal{O}_4\rangle_0$ is estimated as

$$\frac{\langle\mathcal{O}_4\rangle}{\langle\mathcal{O}_4\rangle_0} \simeq \frac{\langle(\bar{q}q)^2\rangle}{\langle(\bar{q}q)^2\rangle_0} \simeq \left(\frac{\langle\bar{q}q\rangle_N}{\langle\bar{q}q\rangle_0}\right)^2 = 1 - 2\rho\frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} + \mathcal{O}(\rho^2).$$

Thus we arrive at

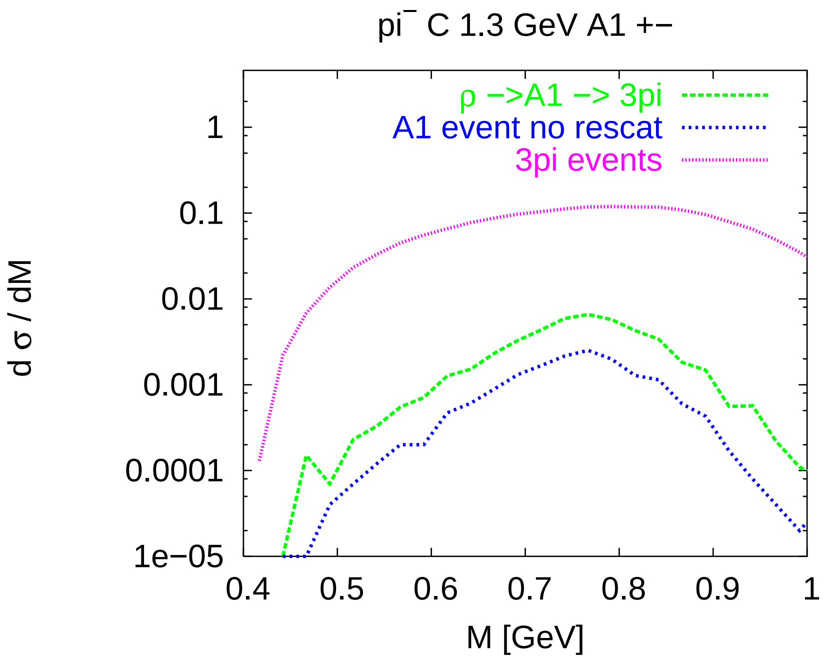
$$\xi \simeq \frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2}\rho \approx \frac{1}{2}0.3\frac{\rho}{\rho_o}.$$

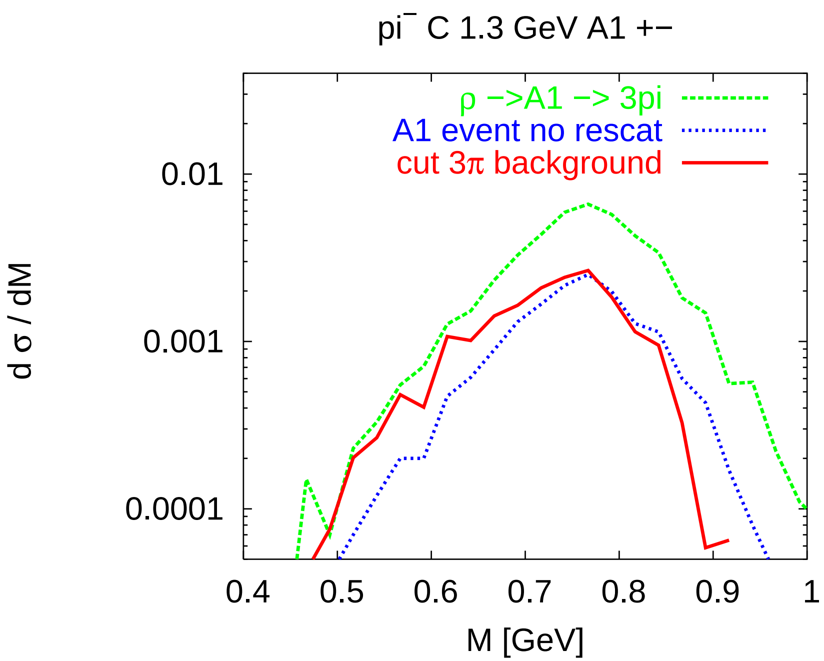
# $\rho$-A1 mixing

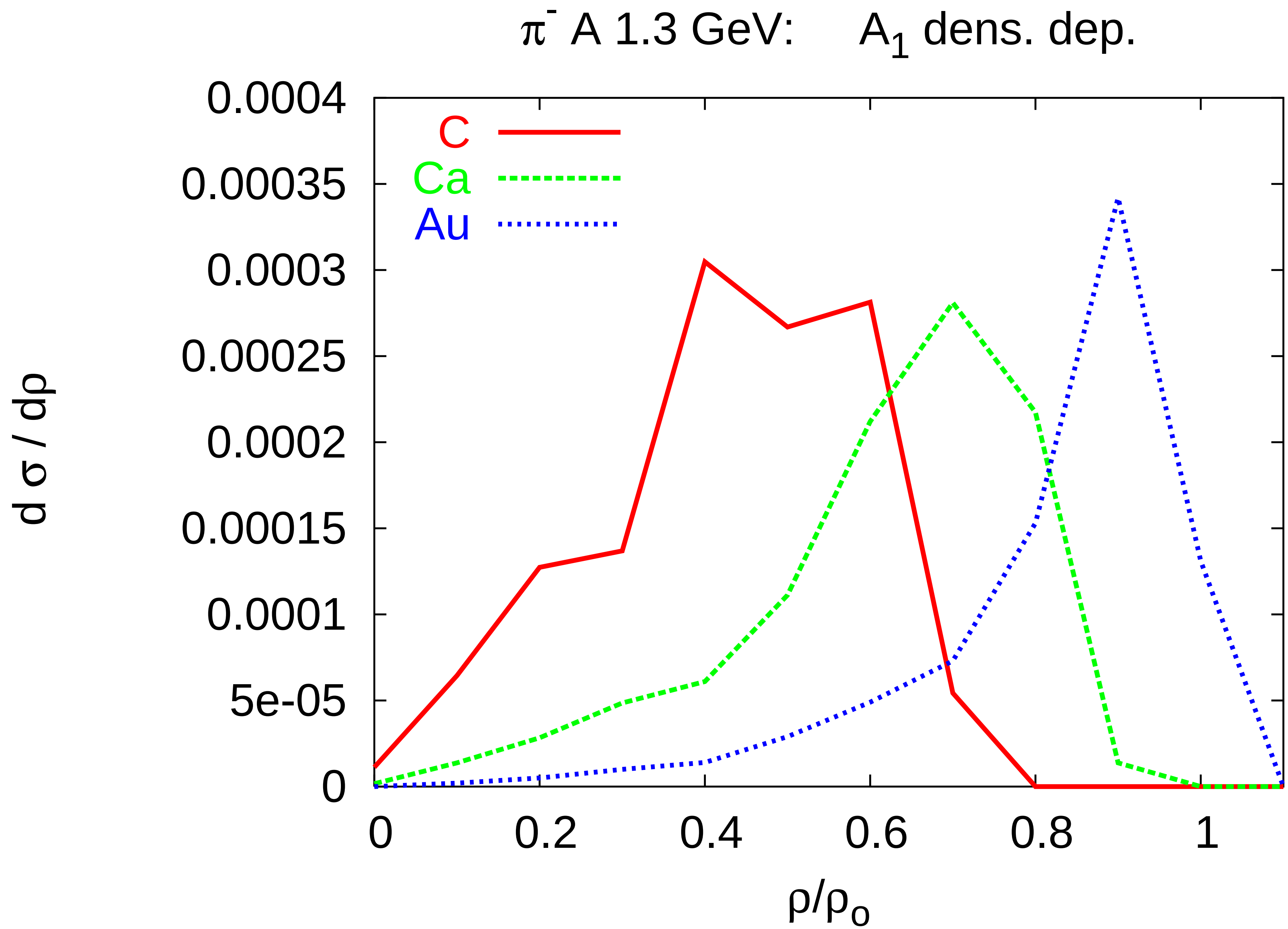
- $\overline{\Pi_\rho} = (1-x)\Pi^v_\rho + x\Pi^v_{A1}$ where $x = \frac{1}{2}0.3 * \frac{density}{normal\,density}$
  $\overline{\Pi_{A1}} = (1-x)\Pi^v_{A1} + x\Pi^v_\rho$ 0.3 comes from chiral symmetry
- Signal: $3\pi$ decay of $A_1$ with the mass of $\rho$-meson
- $\pi$-nucleus collisions
- 1 million central event

| system | energy | A1 | 3 pions | A1 no rescatt. |
|---|---|---|---|---|
| $\pi^- Au$ | 1.3 GeV | 6400 | 56 000 | 100 |
| $\pi^- C$ | 1.3 GeV | 1200 | 43 000 | 450 |
| $\pi^- Ca$ | 1.3 GeV | 3200 | 54 000 | 400 |
| $\pi^- Ca$ | 1.1 GeV | 3100 | 46 000 | 350 |
| $\pi^- Ca$ | 1.5 GeV | 2400 | 55 000 | 360 |

- Light system is prefered


pi$^-$ C 1.3 GeV A1 +−
ρ −>A1 −> 3pi
A1 event no rescat
3pi events
d σ / dM
M [GeV]
1
0.1
0.01
0.001
0.0001
1e−05
0.4
0.5
0.6
0.7
0.8
0.9
1


pi$^-$ C 1.3 GeV A1 +−
ρ −>A1 −> 3pi
A1 event no rescat
cut 3π background
d σ / dM
M [GeV]
0.01
0.001
0.0001
0.4
0.5
0.6
0.7
0.8
0.9
1



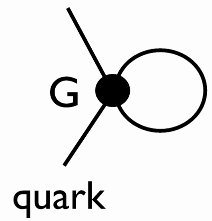
π⁻ A 1.3 GeV: A1 dens. dep.
d σ / dρ
ρ/ρo
C
Ca
Au
0.0004
0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
0
0.2
0.4
0.6
0.8
1

# Nambu–Jona-Lasinio model

"Integrating out" the gluons → four-fermion interaction

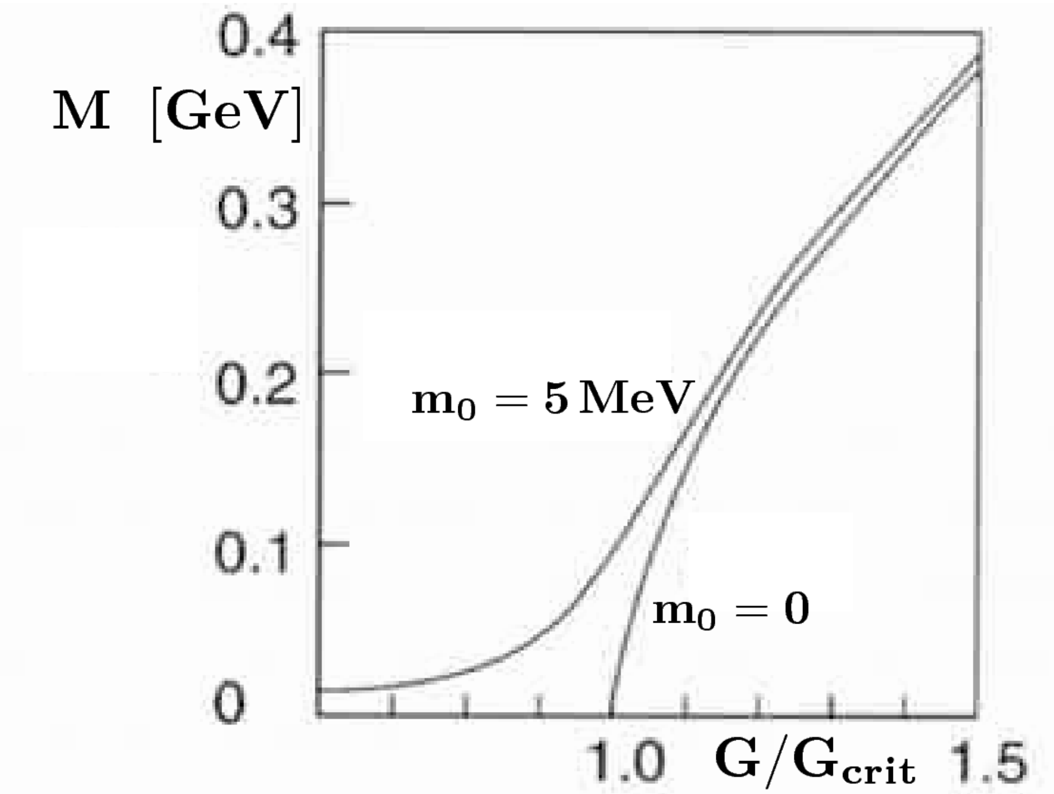
$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m_0)\psi + \frac{G}{2}\left[(\bar{\psi}\lambda_a\psi)^2 + \sum_{a=0,8}(\bar{\psi}i\lambda_a\gamma_5\psi)^2\right] + g_D\,\det[\bar{\psi}_i(1-\gamma_5)\psi_j]$$

- global symmetries of QCD but no confinement
- constituent quark mass: $M = m_0 - G\langle\bar{\psi}\psi\rangle$




$$\langle\bar{\psi}\psi\rangle = -Tr \lim_{x\to 0^+}\langle\mathcal{T}\psi(0)\bar{\psi}(x)\rangle = -2iN_fN_c\int\frac{d^4p}{(2\pi)^4}\frac{M\,\theta(\Lambda^2-\vec{p}^{\,2})}{p^2-M^2+i\varepsilon}\,.$$

- regularization with a cut off $\Lambda$, selfconsistent equation for M




- Masses of mesons are the poles in the $\bar{q}q$ scattering amplitudes
- coupling and cut off is fitted to the pion mass and pion decay constant
- in medium: use in-medium propagator of the quarks
- assumption: cut off and coupling do not change in matter

# Mesons in the Nambu–Jona-Lasinio model

An interesting observation: introduce the bosonic matrix

$$\Phi_{ij} = \bar{\psi}_J (1 - \gamma_5) \psi_i$$

The NJL Lagrangian can be rewritten in terms of that matrix, except the kinetic energy term:

$$\mathcal{L} - \mathcal{L}_{kin} = G\,\mathrm{Tr}(\Phi^\dagger \Phi) - 1/2\,\mathrm{Tr}(m_0(\Phi + \Phi^\dagger)) + g_D(\det \Phi + \mathrm{h.c.})$$

It is just a simplified version of the linear sigma model without its kinetic term.

# Meson nonets

$$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix},$$

$$L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0 \\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu,$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0 \\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu,$$

# Linear sigma model

$$
\begin{aligned}
\mathcal{L} \;=\;& \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& -\frac{1}{4}\,\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)] \\
& + c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
+\;& \frac{h_1}{2}\,\mathrm{Tr}(\Phi^\dagger\Phi)\,\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\,\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\,\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\
& + g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) \\
& + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)] + g_5\,\mathrm{Tr}\,(L_\mu L^\mu)\;\mathrm{Tr}\,(R_\nu R^\nu) \\
& + g_6[\mathrm{Tr}(L_\mu L^\mu)\;\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\;\mathrm{Tr}(R_\nu R^\nu)]
\end{aligned}
$$

$$D^\mu\Phi \equiv \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA^\mu[T_3, \Phi]$$

$$L^{\mu\nu} \equiv \partial^\mu L^\nu - ieA^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu[T_3, L^\mu]\} \quad \text{and} \quad L \leftrightarrow R$$

# Symmetry properties of the model

Global $U(3)_L$x$U(3)_R$ transformation:

$$\begin{aligned} \Phi &\rightarrow U_L \Phi U_R^\dagger \\ L^\mu &\rightarrow U_L L^\mu U_L^\dagger \\ R^\mu &\rightarrow U_R R^\mu U_R^\dagger \end{aligned}$$

Consequences (using the unitarity of U's):

$$\begin{aligned} D^\mu \Phi &\rightarrow U_L D^\mu \Phi U_R^\dagger \\ L^{\mu\nu} &\rightarrow U_L L^{\mu\nu} U_L^\dagger \\ R^{\mu\nu} &\rightarrow U_R R^{\mu\nu} U_R^\dagger \end{aligned}$$

$$(\mathrm{Tr}(\Phi^\dagger\Phi))' = \mathrm{Tr}(U_R\Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger) = \mathrm{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \mathrm{Tr}(\Phi^\dagger\Phi U_R^\dagger U_R) = \mathrm{Tr}(\Phi^\dagger\Phi)$$

All terms are invariant except the determinant and the explicit symmetry breaking term.

## Determinant term

$$U_L = e^{-i\omega_L^a T^a} \qquad U_R = e^{-i\omega_R^a T^a}$$

$$\omega_V^a = 0.5(\omega_L^a + \omega_R^a) \qquad \omega_A^a = 0.5(\omega_L^a - \omega_R^a)$$

By SU(3)$_L$xSU(3)$_R$ transformation (if $\omega_L^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$)

$$(\det \Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det \Phi \det U_R^\dagger = \det \Phi$$

Similarly $\det \Phi^\dagger$ is also invariant.

If $\omega_V^0 \neq 0$ and all the other $\omega$'s are 0 ($[T^a, T^0] = 0$)

$$(\det \Phi)' = \det(e^{-i\omega_V^0 T^0} \Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0} \Phi) = \det \Phi$$

On the other hand, if $\omega_A^0 \neq 0$ and all the other $\omega$'s are 0

$$(\det \Phi)' = \det(e^{-i\omega_A^0 T^0} \Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0} \Phi) = e^{-i2\omega_A^0 \det \Phi \, \mathrm{Tr}\, T^0}$$

So the determinant term is invariant under U(3)$_V$xSU(3)$_A$ transformation and breaks explicitely the U(1)$_A$ symmetry.

## $\mathrm{Tr}[\hat{\epsilon}(\Phi+\Phi^\dagger)]$

$$\hat{\epsilon} = \sum_{i=0}^{8} \epsilon_i T_i = \begin{pmatrix} \frac{\epsilon_N}{2} & 0 & 0 \\ 0 & \frac{\epsilon_N}{2} & 0 \\ 0 & 0 & \frac{\epsilon_S}{\sqrt{2}} \end{pmatrix} \qquad \text{only} \quad \epsilon^0, \epsilon^8 \neq 0$$

- axial transformation: if at least $\epsilon^0 \neq 0$ $\mathrm{U}(3)_A$ is broken:

$$(\mathrm{Tr}[\hat{\epsilon}(\Phi)])' = \mathrm{Tr}(e^{-i2\omega_A^a T^a} \hat{\epsilon}\Phi)$$

- vector transformation

$$(\mathrm{Tr}[\hat{\epsilon}(\Phi)])' = \mathrm{Tr}(e^{-i\omega_V^a T^a} \hat{\epsilon} e^{i\omega_V^a T^a} \Phi)$$

Since $[\hat{\epsilon}, T^0] = 0$, $\mathrm{U}(1)_V$ symmetry is preserved.
If all $\epsilon^a = 0$ except $\epsilon^0$, $\mathrm{U}(3)_V$ is preserved.
If $\epsilon^8$ also non zero, then since $[T^K, T^8] = 0$ if $k = 1, 2, 3$, $\mathrm{U}(1)_V \mathrm{x} \mathrm{SU}(2)_V$ survives (isospin symmetry)
(If $\epsilon^3 \neq 0$ too, then the isospin symmetry is broken, only $\mathrm{U}(1)_V$.)

## Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + V_{N/S} \qquad V_{N/S} \equiv < \sigma_{N/S} >$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: -g_1 V_N a^\mu_{1N} \partial_\mu \pi_N, \\
\pi - a^\mu_1 &: -g_1 V_N (a_1^{\mu +} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.}, \\
\pi_S - a^\mu_{1S} &: -\sqrt{2} g_1 V_S a^\mu_{1S} \partial_\mu \pi_S, \\
K_S - K^\star_\mu &: \frac{i g_1}{2} (\sqrt{2} V_S - V_N)(\bar{K}^{\star 0}_\mu \partial^\mu K^0_S + K^{\star -}_\mu \partial^\mu K^+_S) + \text{h.c.}, \\
K - K^\mu_1 &: -\frac{g_1}{2} (V_N + \sqrt{2} V_S)(K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu +} \partial_\mu K^-) + \text{h.c.}.
\end{aligned} \tag{4}$$

# Mixing

$$
\begin{aligned}
a^{\mu}_{1N/S} &\longrightarrow a^{\mu}_{1N/S} + w_{a_{1N/S}}\partial^{\mu}\pi_{N/S}, \\
a_1^{\mu\pm,0} &\longrightarrow a_1^{\mu\pm,0} + w_{a_1}\partial^{\mu}\pi^{\pm,0}, \\
K_1^{\mu\pm,0} &\longrightarrow K_1^{\mu\pm,0} + w_{K_1}\partial^{\mu}K^{\pm,0}, \\
\bar{K}_1^{\mu 0} &\longrightarrow \bar{K}_1^{\mu 0} + w_{K_1}\partial^{\mu}\bar{K}^{0}, \\
K^{\star\mu +} &\longrightarrow K^{\star\mu +} + w_{K^\star}\partial^{\mu}K_S^{+}, \\
K^{\star\mu -} &\longrightarrow K^{\star\mu -} + w^{\star}_{K^\star}\partial^{\mu}K_S^{-}, \\
K^{\star\mu 0} &\longrightarrow K^{\star\mu 0} + w_{K^\star}\partial^{\mu}K_S^{0}, \\
\bar{K}^{\star\mu 0} &\longrightarrow \bar{K}^{\star\mu 0} + w^{\star}_{K^\star}\partial^{\mu}\bar{K}_S^{0}.
\end{aligned}
$$

$w$'s are appropriate, calculable constants.

Since the coefficients of the kinetic energy term is sometimes not one, one has to renormalize the fields.

Mixing inside of the nonets can be resolved by orthogonal transformations.

## Results

The model parameters are obtained by a fit with the minuit program:

$V_N, V_S, m_0^2, \lambda_1, \lambda_2, m_1^2, \delta_S, \epsilon_N, \epsilon_S, c, g_1, h_1, h_2, h_3$

Since the identification of the scalars are not straightforward, we calculate their masses as predictions. There is a conjecture: the low lying scalars are tetraquark states, the true two-quark states are above 1 GeV. Our calculations seems to support that.

From the interaction terms we can predict the following decay widths as well: $\Gamma_{a1 \to \pi\gamma}, \Gamma_{\rho}, \Gamma_{\phi}, \Gamma_{K^*}, \Gamma_{Ks}, \Gamma_{f_0}, \Gamma_{a_0}$

# Mass fits

| | Calculations(GeV) | Mass | error |
|---|---|---|---|
| $f_\pi$ | 0.0901 | 0.0930 | 0.0010 |
| $f_K$ | 0.1073 | 0.1100 | 0.0010 |
| $m_\pi$ | 0.1389 | 0.1390 | 0.0010 |
| $m_\eta$ | 0.5321 | 0.5480 | 0.0010 |
| $m_{\eta'}$ | 0.9753 | 0.9580 | 0.0050 |
| $m_K$ | 0.5089 | 0.4940 | 0.0010 |
| $m_\rho$ | 0.7481 | 0.7750 | 0.0200 |
| $m_\phi$ | 1.0166 | 1.0200 | 0.1000 |
| $m_{K*}$ | 0.8935 | 0.8930 | 0.0100 |
| $m_{a_1}$ | 1.2019 | 1.2300 | 0.2000 |
| $m_{f_1 L}$ | 1.2019 | 1.2800 | 0.2000 |
| $m_{f_1 H}$ | 1.4426 | 1.4260 | 0.0500 |
| $m_{K_1}$ | 1.3271 | 1.2720 | 0.0300 |
| $m_{a_0}$ | 1.4526 | 0.9850 | 0.0300 |
| $m_{Ks}$ | 1.5497 | 1.4250 | 0.0500 |

## Outlook

- Calculations at nonzero temperature and densities. Phase diagram.
- Including baryons: nucleon octett and delta decouplett. Calculations of dilepton production.
- More precise vacuum phenomenology: tetraquarks, glueballs.

# Summary

- The in-medium modification of the $\rho$ is contraversal
- A possible signal of chiral restoration in $A_1 - \rho$ mixing
- Needs very large statistics
- several theoretical uncertainties