

Factorization in the Color Glass Condensate (CGC).

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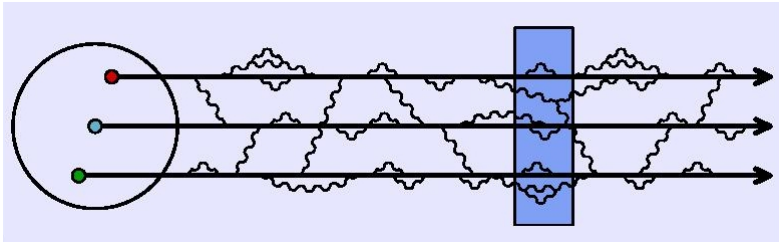


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 - The CGC.
 - Power counting.
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 - Inclusive quantities.
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 - A further step : the leading log approximation.
 - Technical difficulties.
 - Choice of initial surface.
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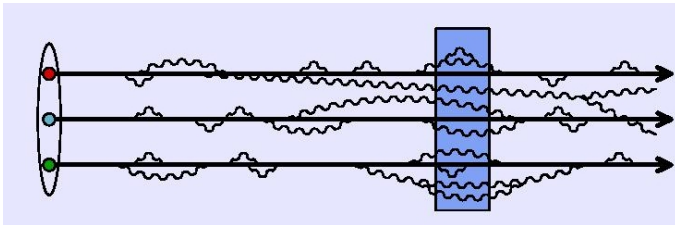
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Let us consider a proton at low energy : plenty of virtual fluctuations whose lifetime are $\Delta t \lesssim \frac{1}{M_p}$.



What happens if the nucleus travels faster ? Lifetime of fluctuations increase by Lorentz time dilatation.
If it travels fast enough, these fluctuations live longer than the collision time between two nuclei.



Let P be the momentum of the proton which is assumed to be large and xP the momentum carried by a gluon.

The probability of emission of this gluon behaves like :

$$\alpha_s \ln \left(\frac{1}{x} \right).$$

Small x gluons are the dominant ones.

This is experimentally observed.

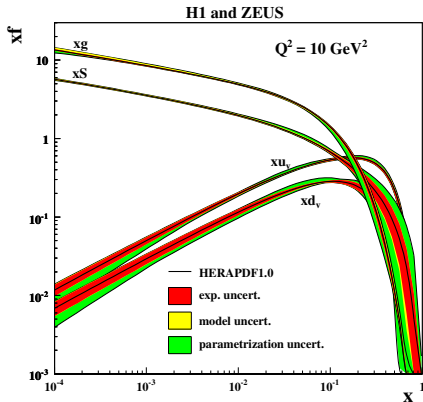
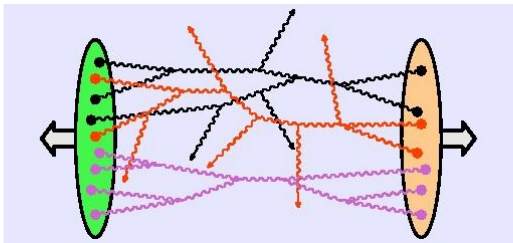


Figure: Gluon distribution at small x .

We consider the scattering of two fast nuclei, many gluons interact and are emitted.



Usual perturbation theory becomes impractical.

- Fast partons, with $p^\pm > \Lambda^\pm$, can be considered as static during the scattering process. At leading order, one can consider these partons to be only gluons. They are treated as classical static sources on the light cone :

$$\mathcal{J}^{\mu a} = \delta^{+\mu} \rho_1^a(\mathbf{x}_\perp) \delta(x^-) + \delta^{-\mu} \rho_2^a(\mathbf{x}_\perp) \delta(x^+).$$

- The slower ones must be considered as usual quantum fields.

This is the so called Color Glass Condensate (CGC). It is an effective theory in the sense of the Wilson renormalization group approach.

- The classical sources \mathcal{J} (or equivalently ρ) are random sources.
- These random sources have a probability distribution $W_\Lambda[\rho]$ that depends on the cutoff.
- Under a change of Λ , $W_\Lambda[\rho]$ evolves as :

$$\frac{\partial}{\partial \ln \Lambda} W_\Lambda[\rho] = \mathcal{H} \left[\rho, \frac{\delta}{\delta \rho} \right] W_\Lambda[\rho].$$

\mathcal{H} is known as the JIMWLK hamiltonian and W_Λ obeys the JIMWLK equation.

For very fast nuclei, gluons saturate.

In this regime, the classical sources ρ are of order $\frac{1}{g}$.

Thus, the order of a connected diagram is

$$g^{(\text{number of external legs})+2*(\text{number of loops})-2}.$$

This does not depend on the number of sources.

Only the number of particles in the initial and final state and the number of loops matter.

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We are interested in the average number of gluons of any polarization and any color produced in a momentum space element d^3k :

$$\frac{dN_g}{d^3k}.$$

Inclusive = which does not constrain the final number of gluons.

Exclusive = (for instance) the probability to get exactly n gluons in the final state.

$\frac{dN_g}{d^3k}$ can be expressed in terms of Green's functions by the reduction formula :

$$\begin{aligned}\frac{dN_g}{d^3k} &= \frac{1}{(2\pi)^3 2\omega_{\mathbf{k}}} \sum_{a,\lambda} \sum_{\text{states}} |\langle \mathbf{k}, a, \lambda; \alpha_{out} | 0_{in} \rangle|^2 \\ &= \frac{1}{(2\pi)^3 2\omega_{\mathbf{k}}} \int d^4x d^4y e^{ik \cdot (x-y)} \epsilon_{\lambda}^{\mu*}(\mathbf{k}) \epsilon_{\lambda}^{\nu}(\mathbf{k}) \\ &\quad \square_x \square_y \left[\mathcal{D}_{+-}^{aa}{}_{\mu\nu}(x, y) + A_{+\mu}^a(x) A_{-\nu}^a(y) \right].\end{aligned}$$

$\mathcal{D}_{+-\mu\nu}$ is the exact gluon propagator, $\mathcal{D}_{+-\mu\nu} \sim g^{2 \times \text{number of loops}}$.

$A_{\pm\mu}$ are the dressed gluon's 1-point functions,

$A_{\pm\mu} \sim g^{2 \times \text{number of loops} - 1}$.

The leading order (LO) is given by :

$$\left. \frac{dN_g}{d^3k} \right|_{LO} = \frac{1}{(2\pi)^3 2\omega_{\mathbf{k}}} \int d^4x d^4y e^{ik \cdot (x-y)} \epsilon_{\lambda}^{\mu*}(\mathbf{k}) \epsilon_{\lambda}^{\nu}(\mathbf{k})$$

$$\square_x \square_y [A_{+\mu}^a(x) A_{-\nu}^a(y)] \Big|_{\text{tree}}.$$

One can show that $A_{\pm\mu}(x)|_{\text{tree}}$ is the classical field configuration, vanishing at time $x^0 \rightarrow -\infty$.

And the next to leading order (NLO) :

$$\left. \frac{dN_g}{d^3k} \right|_{NLO} = \frac{1}{(2\pi)^3 2\omega_{\mathbf{k}}} \int d^4x d^4y e^{ik \cdot (x-y)} \epsilon_{\lambda}^{\mu*}(\mathbf{k}) \epsilon_{\lambda}^{\nu}(\mathbf{k})$$

$$\square_x \square_y \left[\mathcal{D}_{+-}^{aa}{}_{\mu\nu}(x, y) \Big|_{\text{tree}} + \right.$$

$$\left. + A_{+\mu}^a(x) \Big|_{1\text{loop}} A_{-\nu}^a(y) \Big|_{\text{tree}} + A_{+\mu}^a(x) \Big|_{\text{tree}} A_{-\nu}^a(y) \Big|_{1\text{loop}} \right] .$$

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The aim is to write down a simple relation between $\left. \frac{dN_g}{d^3k} \right|_{LO}$ and $\left. \frac{dN_g}{d^3k} \right|_{NLO}$.

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$\left. \frac{dN_g}{d^3k} \right|_{LO}$ is a functional of the classical field $A_{\pm\nu}(x)|_{\text{tree}}$ which is a solution of the Yang-Mills equations.

By use of Green's formulae, one can express formally the field $A_{\pm\nu}(x)|_{\text{tree}}$ at any x in terms of the initial conditions.

A remarkable property is that $\left. \frac{dN_g}{d^3k} \right|_{NLO}$ is obtained by perturbing the initial field in $\left. \frac{dN_g}{d^3k} \right|_{LO}$.

Such a relation reads :

$$\left. \frac{dN_g}{d^3k} \right|_{NLO} = \mathcal{O} \left[\frac{\delta}{\delta A_{\text{init.}}} \right] \left. \frac{dN_g}{d^3k} \right|_{LO}.$$

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- $\mathcal{O}\left[\frac{\delta}{\delta A_{\text{init}}}\right]$ contains first and second derivatives.
- the coefficients contain at worst $\ln \Lambda$ divergences that can be computed analytically. These pieces dominate the high energy behaviour \rightarrow leading log approximation.

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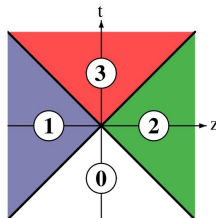
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Region (0) contains no sources. The classical field is zero everywhere.

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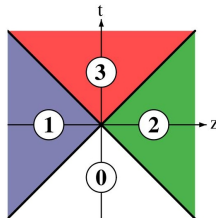
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Regions (1) and (2) are causally disconnected. Thus in region (1) (resp. (2)), the field is given by the Yang-Mills equations in presence of the only right (resp. left) moving sources.

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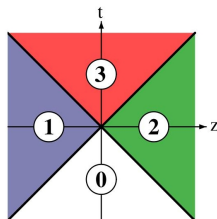
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The field in region (3) is very complicated : numerical solutions only.

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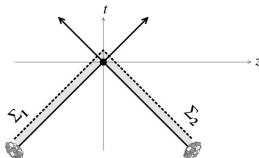
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Since interesting things happen "above" the sources, it is quite natural to take the initial surface at a time just after the sources.

Fluctuations of the fields on this surface = small variations of the sources.

Working out the structure of the operator \mathcal{O} and picking up the leading logs, we can show that it is a sum of JIMWLK hamiltonians (one for each nucleus) :

$$\begin{aligned} \left. \frac{dN_g}{d^3k} \right|_{NLO} &= \left[\ln \left(\frac{\Lambda^+}{\Lambda'^+} \right) \mathcal{H} \left[\rho_1, \frac{\delta}{\delta \rho_1} \right] + \right. \\ &\quad \left. + \ln \left(\frac{\Lambda^-}{\Lambda'^-} \right) \mathcal{H} \left[\rho_2, \frac{\delta}{\delta \rho_2} \right] \right] \left. \frac{dN_g}{d^3k} \right|_{LO}. \end{aligned}$$

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Are these equations also true for the average number of quarks produced in a momentum space element d^3p :

$$\frac{dN_q}{d^3p}?$$

$\frac{dN_q}{d^3p}$ is not only a functional of the gluon field A but also of the quark field ψ .

Since there are no quark sources, quark 1-point functions are 0 and the reduction formula reads :

$$\frac{dN_q}{d^3p} = \frac{-1}{(2\pi)^3 2E_{\mathbf{p}}} \int d^4x d^4y e^{ip \cdot (x-y)} \bar{u}^s(\mathbf{p})(i\overrightarrow{\not{\partial}}_x - m)S_{+-}^{ii}(x,y)(i\overleftarrow{\not{\partial}}_y + m)u^s(\mathbf{p})$$

S_{+-} is the exact quark propagator (depending implicitly on the gluon's field).

We need to go one order higher in perturbation theory :

- the leading order is given by the computation of the dressed propagator at tree level.
- the next to leading order requires the propagator with a 1-loop correction.

By dropping vacuum $q\bar{q}$ fluctuations, we obtain an LO to NLO relation analogous to the one for gluons :

$$\left. \frac{dN_q}{d^3p} \right|_{NLO} = \tilde{\mathcal{O}} \left[\frac{\delta}{\delta A_{\text{init.}}}, \frac{\delta}{\delta \psi_{\text{init.}}} \right] \left. \frac{dN_q}{d^3p} \right|_{LO}.$$

The work in progress is to identify the leading log part of the operator $\tilde{\mathcal{O}}$ with the JIMWLK Hamiltonian :

$$\begin{aligned} \left. \frac{dN_q}{d^3p} \right|_{NLO} = & \left[\ln \left(\frac{\Lambda^+}{\Lambda'^+} \right) \mathcal{H}_1 \left[\rho_1, \frac{\delta}{\delta\rho_1} \right] + \right. \\ & \left. + \ln \left(\frac{\Lambda^-}{\Lambda'^-} \right) \mathcal{H} \left[\rho_2, \frac{\delta}{\delta\rho_2} \right] \right] \left. \frac{dN_q}{d^3p} \right|_{LO}. \end{aligned}$$

Likely true but non trivial...

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- For inclusive observables, mere formal relations exist between LO and NLO.
- These relations can be used to compute the leading logs at NLO (proven for gluons, in progress for quarks).
- Open questions :
 - extension to more general inclusive observables (multi-point correlations).
 - what about exclusive observables ?